Fast-Slow Dynamics

Jonna, Kieran, Tom

Fast- Slow systems are systems of differential equations that can be viewed on two different time scales, which are separated by a parameter, ϵ. The transition between those two systems is done by a transformation of the time variable $t = \frac{\tau}{\epsilon}$. In this project two and three dimensional systems are considered. Fast System:

$$x' = \frac{dx}{dt} = f(x, y, \lambda, \epsilon), \quad y' = \frac{dy}{dt} = \epsilon g(x, y, \lambda, \epsilon),$$
$$\epsilon \dot{x} = \epsilon \frac{dx}{d\tau} = f(x, y, \lambda, \epsilon), \quad \dot{y} = \frac{dy}{d\tau} = g(x, y, \lambda, \epsilon),$$

Motivation and Setup

Fast- Slow systems are systems of differential equations that can be viewed on two different time scales, which are separated by a parameter, ϵ. The transition between those two systems is done by a transformation of the time variable $t = \frac{\tau}{\epsilon}$. In this project two and three dimensional systems are considered.

$$\begin{cases} x' &= \frac{dx}{dt} = f(x, y, \lambda, \epsilon), \\ y' &= \frac{dy}{dt} = \epsilon g(x, y, \lambda, \epsilon) \end{cases}$$
$$\begin{cases} \epsilon \dot{x} &= \epsilon \frac{dx}{d\tau} = f(x, y, \lambda, \epsilon), \\ \dot{y} &= \frac{dy}{d\tau} = g(x, y, \lambda, \epsilon), \end{cases}$$

Van der Pol System

Fast System:

$$\begin{cases} x' = y - \frac{x^3}{3} + x \\ y' = -\epsilon x, \end{cases}$$

Slow System:

$$\begin{cases} \epsilon \dot{x} = y - \frac{x^3}{3} + x \\ \dot{y} = -x, \end{cases}$$

Canard Cycle

Layer Problem:

$$\begin{cases} x' = y - \frac{x^3}{3} + x, \\ y' = 0, \end{cases}$$

Reduced Problem:

$$\begin{cases} 0 = y - \frac{x^3}{3} + x := f, \\ \dot{y} = -x. \end{cases}$$

Phase Portrait: Van der Pol System

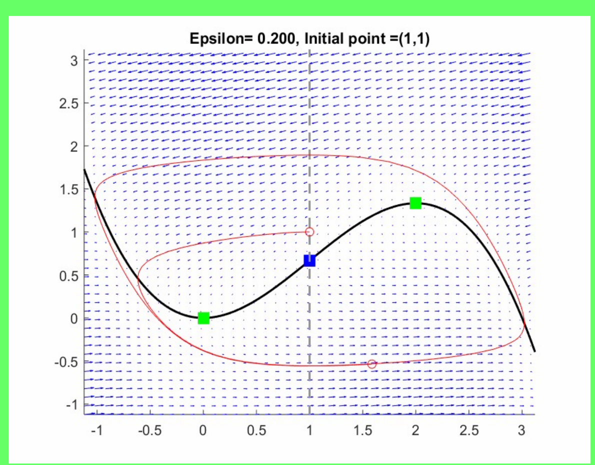


Fig. 1: h!

Phase Portrait: Canard Cycle

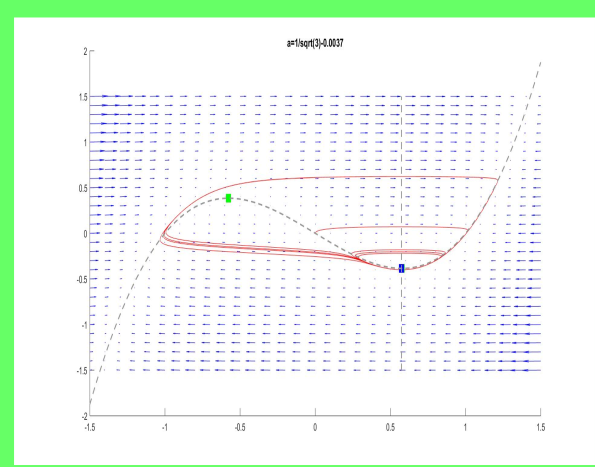


Fig. 2: h!

Blow-up

Fast- Slow systems are systems of differential equations that can be viewed on two different time scales, which are separated by a parameter. These systems are generally of the form

$$\begin{cases} x' &= \frac{dx}{dt} = f(x, y, \lambda, \epsilon), \\ y' &= \frac{dy}{dt} = \epsilon g(x, y, \lambda, \epsilon), \end{cases}$$

which is known as the fast system. Using a scaling for the time, $t = \frac{\tau}{\epsilon}$, we find that this can be rewritten as

$$\begin{cases} \epsilon \dot{x} &= \epsilon \frac{dx}{d\tau} = f(x, y, \lambda, \epsilon), \\ \dot{y} &= \frac{dy}{d\tau} = g(x, y, \lambda, \epsilon), \end{cases}$$

which is called the slow system.

Phase Portrait: Van der Pol System

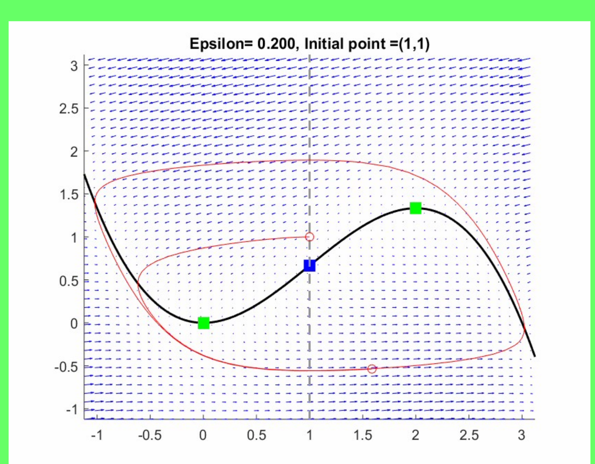


Fig. 3: h!

Phase Portrait: Singular Limit

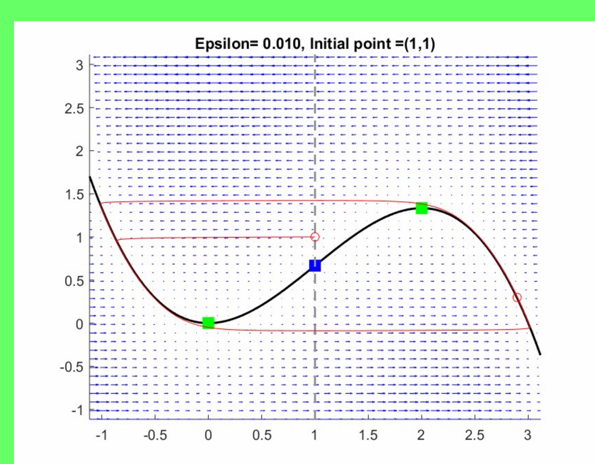
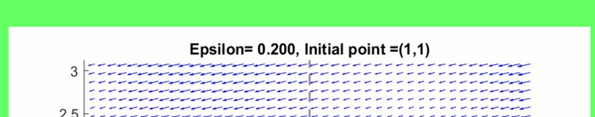


Fig. 4: h!

Mixed-Mode Oscillations In the planar case, canards only occure within $O(\epsilon)$ of the fold point. To get more readily observable canards, another variable is introduced.

$$\dot{x} = y - x^2 - x^3, \qquad \dot{y} = \epsilon(z - x), \qquad \dot{z} = \epsilon(-\nu - ax - by - cz)$$

Phase Portrait: Van der Pol System



Phase Portrait: Singular Limit