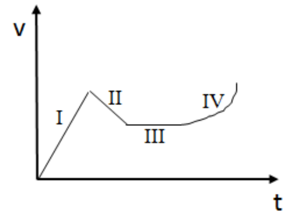
Directions: Show the steps required to arrive at the answer (if applicable). For all problems, $g = 10.0\ m/s^2$. Work out the problems on separate page.

3.1 – Forces and Laws of Motion

1. If only one force acts on an object, can it be in equilibrium? Explain

No, as the net force on the object must be zero, which can only happen with no force or counteracting forces.

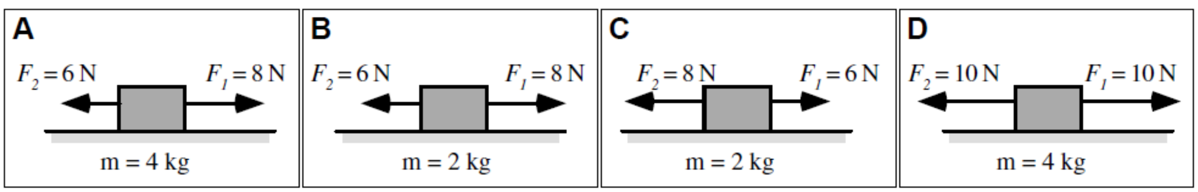
2. Consider the velocity vs. time graph shown. List the intervals that fit each description.



a) A constant, non-zero force is applied to the object.
b) A non-zero force acts in the direction the object is moving.
c) The net force on the object is zero.
d) A variable force is applied to the object.

a) I, II **b) I, IV** **c) III** **d) IV**

3. Two forces act on a block that is initially at rest on a frictionless surface.



Rank the speed of the block after 3 seconds. Explain your reasoning.

B=C>A>D. This can be found using F=ma.

4. Cart A has a mass of 20 kg and rolls downhill at a constant speed of 10 m/s. Cart B has a mass of 10 kg and rolls downhill at a constant speed of 30 m/s. Compare the net force on each cart. Justify your answer.

Both forces are zero because there is constant speed in both cases.

5. A man ties a rope to a box and pulls the box across the floor with the rope. Identify the action-reaction pairs.

6. Shohei Ohtani throws a ball straight up into the air at a speed of 40.0 m/s. In the process, he moves his hand through a distance of 1.50 m. If the ball has a mass of 0.150 kg, find the force he exerts on the ball to give it this upward speed.

$V_i = 0$ m/s
$V_f = 40$ m/s
$\Delta x = 1.5$
$a = ?$

$V_f^2 = V_i^2 + 2a\Delta x$

$a = \frac{V_f^2}{2\Delta x} = \frac{40^2}{2(1.5)} = 533\ m/s^2$

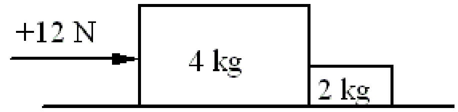
$\Rightarrow F = ma = (.15\ kg)(533\ m/s^2)$

$\boxed{F = 80\ N}$

7. A 4-kg block and a 2-kg block can move on the horizontal frictionless surface. The blocks are accelerated by a +12-N force that pushes the larger block against the smaller one.

a) Determine the force that the 2-kg block exerts on the 4-kg block.

b) Determine the force that the 4 kg block exerts on the 2 kg block.



The acceleration of the system is 2 m/s^2. The force on the forward block, the 2 kg one, is 4 N (2 kg x 2 m/s^2). The force on the 4 kg block is 4 N to the left. The force on the 2 kg block is 4 N to the right.

8. Students collect the following data from an experiment where they exerted the same force, F, to identical sized boxes with different masses and recorded the acceleration.

Trial	1	2	3	4	5	6	7
Mass	2 kg	4 kg	5 kg	7 kg	12 kg	15 kg	18 kg
Acceleration	1.5 m/s^2	0.75 m/s^2	0.60 m/s^2	0.40 m/s^2	0.30 m/s^2	0.20 m/s^2	0.15 m/s^2

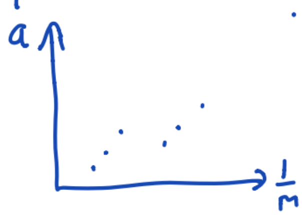
a) Suppose the data is graphed with one of the variables on the x-axis and the other on the y-axis. Which variable should be chosen as the y-variable in order for the graph to be a straight line.

b) Using your relationship from a), what would the slope of the graph represent?

c) Calculate the value of F, the constant force.

$F = ma \Rightarrow a = \frac{1}{m}F$ $y = kx$
$a = F\frac{1}{m}$

a)

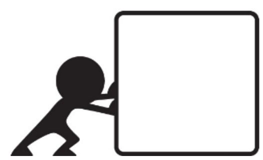


b) Force would be the slope.

c) slope = $\underline{3\ N}$

3.2 – F=ma

1. Some dude pushes a block across a horizontal surface at constant speed. The push (F_{push}), gravity (F_g), friction (F_f), and the normal force (F_N) act on the block.



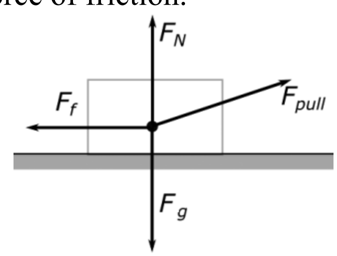
a) Draw a free body diagram of the block.

b) Starting from Newton's second law, show that the force of gravity equals the normal force in this situation.

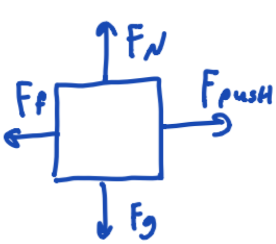
c) Starting from Newton's second law, show the force of the push equals the force of friction.

d) The dude gets tired of pushing the block and instead beings to pull up with force F_{pull} as shown. The block still travels at constant speed. Some guy claims: *"The velocity of the block is constant, so the net force exerted on the block must be zero. Thus, the normal force F_N equals the weight F_g, and the force of friction F_f equals the applied force F_{pull}."*

What, if anything, is wrong with this statement?



a)



b) $\Sigma F_y = 0 = F_N - F_g$
$\Rightarrow F_N = F_g$

c) $\Sigma F_x = 0 = F_{push} - F_f$
$\Rightarrow F_{push} = F_f$

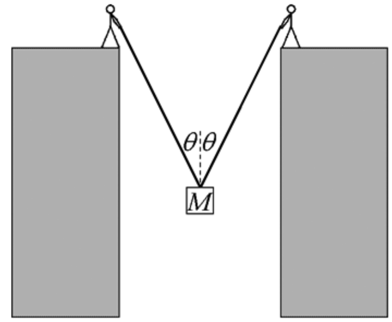
d) The normal force is less than the force of gravity, since the force lifts up on the block.

2. Two people standing on equal-height buildings are lifting a box of mass M between the buildings using two ropes. The people keep the two ropes the same length between their hands and the box so that both ropes make an angle θ with the vertical. Each person pulls the box with the same tension force, F_T.

a) Draw a free-body diagram of the forces acting on the box.

b) Derive an expression for the magnitude of the tension F_T in the two ropes in terms of M, θ, and fundamental constants.

c) The two people lift the box higher and higher and notice they have to exert more and more force. Why is this?



a)

b) $\sum F_y = 0 = T\cos\theta + T\cos\theta - Mg$

$\Rightarrow 2T\cos\theta = Mg$

$\Rightarrow T = \dfrac{Mg}{2\cos\theta}$

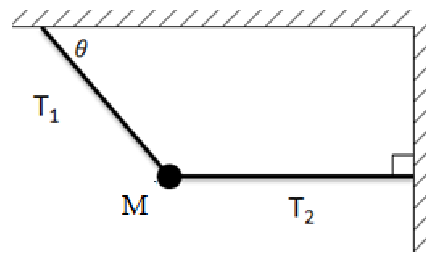
c) Less of the force is acting vertically, and more of it is acting horizontally.

3. A mass M is suspended from two massless strings as shown.

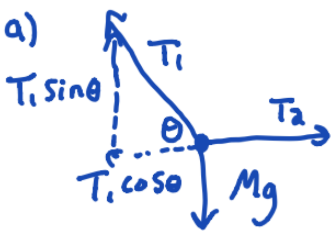
a) Draw a free-body diagram of the forces on the mass M.

b) What is the tension in string T_1?

c) What is the tension in string T_2?



a)



b) $\sum F_y = 0 = T_1 \sin\theta - Mg$

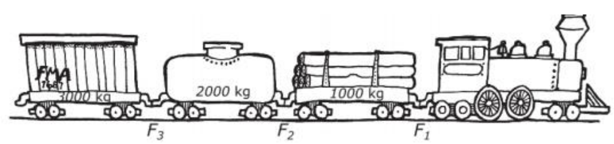
$\Rightarrow T_1 = \dfrac{Mg}{\sin\theta}$

c) $\sum F_x = 0 = T_2 - T_1\cos\theta$

$\Rightarrow T_2 = \dfrac{Mg}{\sin\theta}\cos\theta$

$\Rightarrow T_2 = Mg\cot\theta$

4. A train engine pulls a train with three cars. Each car has the mass shown. Suppose that the cars are connected by metal bars with the tensions indicated in the diagram. The engine accelerates at a rate of 2 m/s^2. Assume that the cars travel on bearings with negligible friction.



a) Use the equations you wrote above to find each of the three tensions: F_1, F_2, and F_3.

b) Without referencing any math or any numbers, explain why F_1 is the greatest tension and F_3 is the smallest tension, even though F_3 is connected to the greatest mass.

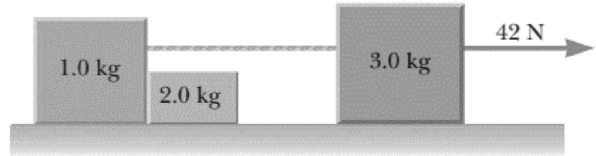
a) $F_1 = (6000\ kg)(2\ m/s^2) = 12{,}000\ N$

$F_2 = (5000\ kg)(2\ m/s^2) = 10{,}000\ N$

$F_3 = (3000\ kg)(2\ m/s^2) = 6{,}000\ N$

b) F1 has to pull the most mass since it pulls everything behind it, while F3 only pulls the last mass.

5. Assume the three blocks shown move on a frictionless surface and a 42 N force acts as shown on the 3.0 kg block.



a) Determine the acceleration given this system.

b) Determine the tension in the cord connecting the 3.0 kg and the 1.0-kg blocks

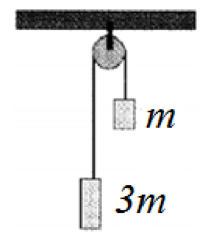
c) Calculate the force exerted by the 1.0 kg block on the 2.0 kg block.

a) $a = \frac{42\ N}{6\ kg} = 7\ m/s^2$

b) $T = (3\ kg)(7\ m/s^2) = 21\ N$

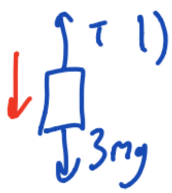
c) $F = (2\ kg)(7\ m/s^2) = 14\ N$

6. Consider the pulley system shown above.

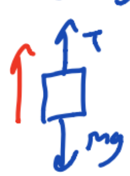


a) What is the tension in the string connecting the masses?

b) What is the magnitude of the acceleration of the masses?



1) $\Sigma F = 3ma = 3mg - T$



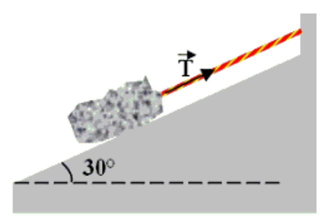
2) $\Sigma F = ma = T - mg$

Adding equations: $4\cancel{m}a = 2\cancel{m}g$

$\Rightarrow \boxed{a = g/2}$

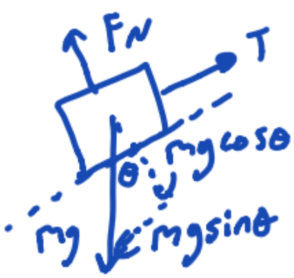
Sub a into 2) $m(g/2) = T - mg \Rightarrow \boxed{T = \frac{3}{2}mg}$

7. A rope holds a 10-kg rock at rest on a *frictionless* inclined plane as shown.



a) Determine the tension in the rope.

b) Suppose the rope snaps. Determine the acceleration of the block as it slides down the include.



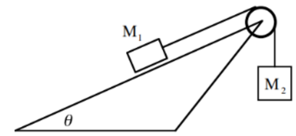
a) $\Sigma F_{\parallel} = 0 = T - mg\sin\theta$

$\Rightarrow T = mg\sin\theta$

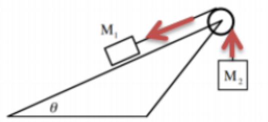
b) $\Sigma F_{\parallel} = \cancel{m}a = \cancel{m}g\sin\theta$

$\Rightarrow a = g\sin\theta$

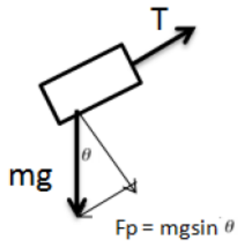
8. Suppose M_1 = 15 kg, M_2= 9.0 kg and θ = 40 °. Ignore the effects of friction. Assume mass M_1 moves down the incline and mass M_2 is lifted. What is the acceleration of the masses?



Take the direction of motion as down the incline (from problem statement)



FBD of M_1:



Do F= ma along incline (down incline is +)

$\sum F = ma = Fp - T$

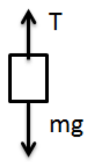
$ma = mgsin\theta - T$

$15a = 15(9.8)sin40 - T$ (plug in knowns)

15a = 94.5 - T (solve for T)

$T = 94.5 - 15a$

FBD of M_2:



$\sum F = ma = T - mg$

$9a = T - 9 * 9.8$ (plug in knowns)

$9a = T - 88.2$ (plug in T from other mass)

$9a = 94.5 - 15a - 88.2$

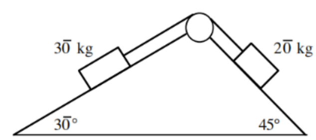
$24a = 6.3$

a = .26 m/s²

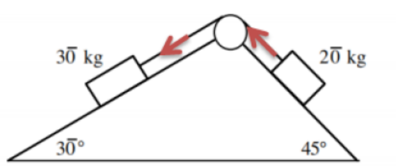
To find T, plug a=.26 into T = 94.5 - 15a:

$T = 94.5N - 15(.26\frac{m}{s^2})$=91 N

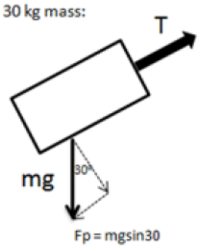
9. The 30 kg block slides down the left incline in the following frictionless system. Find the magnitude of the acceleration of the blocks.



Left mass is heavier so take direction of motion as follows:



30 kg mass:



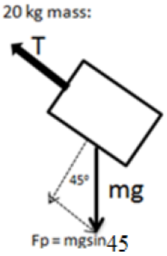
Do F = ma parallel to incline: (Fp is force of weight that acts parallel to incline)

$\sum F = ma = Fp - T$

$ma = mgsin30 - T => 30a = 30\ kg\ \left(9.8\frac{m}{s^2}\right)\sin(30) - T$

$30a = 147 - T => T = 147 - 30a$

20 kg mass:



Do F = ma parallel to incline: (Fp is force of weight that acts parallel to incline)

$\sum F = ma = T - Fp$ (direction of motion is down the other incline)

$ma = T - mgsin45 => 20a = T - 20kg\left(\frac{9.8m}{s^2}\right)sin45)$

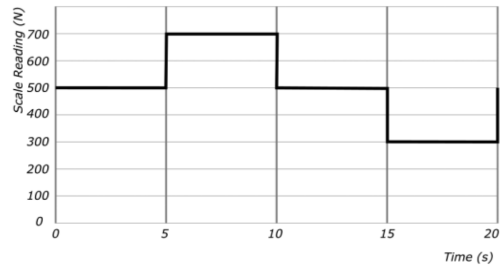
$20a = T - 138.5$ (Plug in T from the other mass)

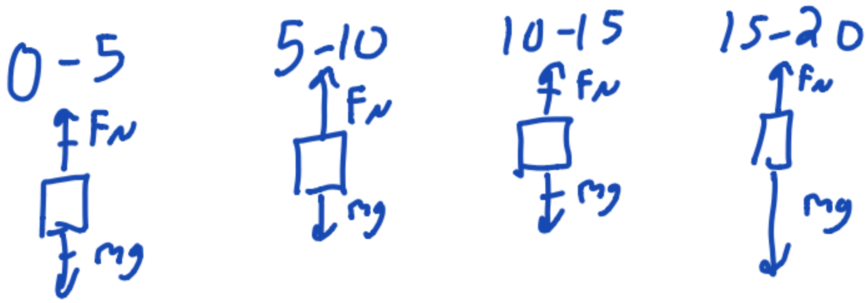
$20a = 147 - 30a - 138.5 => 50a = 8.5 => a = \mathbf{.17\ m/s^2}$

10. A student whose normal weight is 500 N stands on a scale in an elevator and records the scale reading as a function of time. The data are shown in the graph below.

a) Draw a free-body diagram of the forces acting on the student for each 5 s segment.

b) Describe the motion of the elevator for each segment.





b) 0-5: At rest   5-10: Going up, speeding up   10-15 s: constant speed   15-20: going up, slowing down

11. Some guy weights 750 N. He brings a scale on an elevator. At one point, the scale reads 680 N.

a) Draw free body diagrams of the man when i) when not on an elevator and ii) when the scale reads 680 N on the elevator. Have the relative lengths of the vectors represent relative magnitudes

b) Calculate the acceleration of the elevator.

c) Describe a possible state of motion of the elevator at this point.

a) i)

ii)

b) $\Sigma F = ma = F_N - mg$

$\Rightarrow a = \dfrac{F_N - mg}{m} = \dfrac{680-750}{75} = -.9\ m/s^2$

c) The elevator could be going up and slowing down.

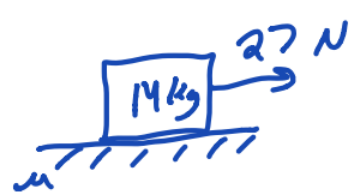
3.3 – Friction

1. The equation below results from the applications of Newton's Laws to an object:

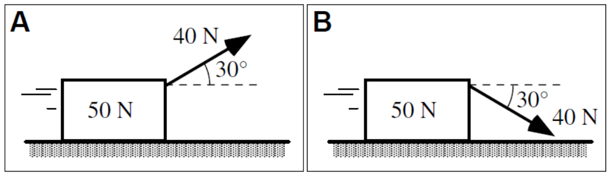
$$27\text{ N} - (\mu)(14\text{ kg})(9.8\text{ m/s}^2) = 0$$

Draw a physical situation that would result in this equation, and explain how your drawing is consistent with the equation.

A 14 kg mass is dragged by a 27 N force across a rough horizontal surface.

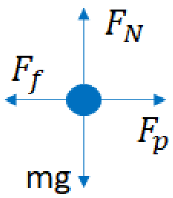


2. A 50 N box has an applied force on it of 40 N that makes an angle of 30° with the horizontal. The box is moving to the right at a constant speed in both cases.



Will the frictional force exerted on the box by the rough surface be *greater* in Case A, *greater* in Case B, or *the same* in both cases? Explain your reasoning.
Case B will have a greater frictional force since the normal force is greater.

3. A 60 N force is need to slide a 440 N table at constant speed across a horizontal floor. What is the coefficient of sliding friction?

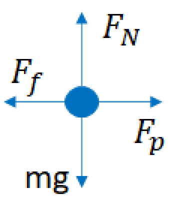


$$\sum F_x = F_p - F_f = 0 \ \ (constant\ speed)$$

$$F_p = F_f \rightarrow F_p = \mu F_N$$

$$\rightarrow \mu = \frac{F_p}{F_N} = \frac{60\ N}{440\ N} = 0.14$$

4. A horizontal force of 275 N is applied to a 15.0 kg box. The coefficient of sliding friction between the box and horizontal surface is 0.32. At what rate will the box accelerate?

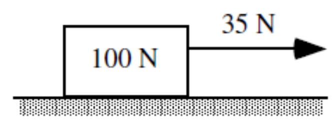


$$\sum F_x = ma = F_p - F_f$$

$$ma = F_p - \mu F_N \rightarrow ma = F_p - \mu mg$$

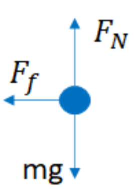
$$\rightarrow a = \frac{F_p}{m} - \mu g = \frac{275\ N}{15\ kg} - (0.32)\left(10\frac{m}{s^2}\right) = 15\frac{m}{s^2}$$

5. A 100 N box is initially at rest on a rough, horizontal surface. The coefficient of static friction is 0.6, and the coefficient of kinetic friction is 0.4. A constant 35 N horizontal force to the right is applied to the box. Calculate the frictional force on the block.



The maximum static friction is $F_f = \mu F_N = (.60)(100\ N) = 60N$. The pulling force is not sufficient to pull the block, so the force of friction is 35 N (to overcome the applied force).

6. A block's initial velocity is 13 m/s. The block slides 36 m on a horizontal surface before coming to rest. What is the coefficient of sliding friction between the block and surface?



1) Find a using kinematics

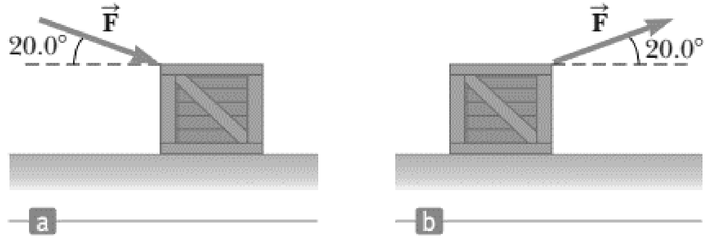
$$v_i = 13\frac{m}{s}, v_f = 0, \Delta x = 36\ m, a = ?$$

$$\cancel{v_f^2} = v_i^2 + 2a\Delta x \rightarrow a = \frac{-v_i^2}{2\Delta x} = \frac{-\left(13\frac{m}{s}\right)^2}{2(36\ m)} = -2.3\ m/s^2$$

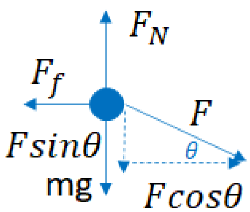
2) Find mu using F=ma

$$ma = -\mu F_N \rightarrow ma = -\mu mg \rightarrow \mu = -\frac{a}{g} = -\frac{-2.3\frac{m}{s^2}}{10\frac{m}{s^2}} = 0.23$$

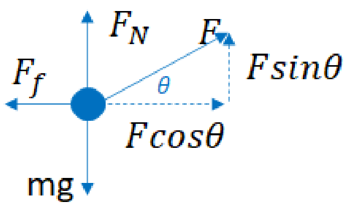
7. A 100 kg crate is being pushed across a level floor at a constant speed by a force = 300 N at an angle of 20.0° below the horizontal, as shown figure a.



a) What is the coefficient of kinetic friction between the crate and the floor?
b) If the 3.00 N force is instead pulling the block at an angle of 20.0° above the horizontal, as shown in figure b, what will be the acceleration of the crate? Assume that the coefficient of friction is the same as that found in a.

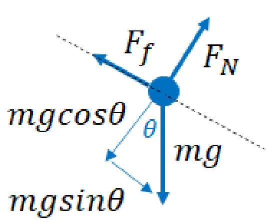


$$a)\ \Sigma F_y = 0 = F_N - mg - Fsin\theta \rightarrow F_N = mg + Fsin\theta$$
$$\Sigma F_x = 0 = Fcos\theta - F_f \rightarrow Fcos\theta - \mu F_N = 0$$
$$Fcos\theta = \mu(mg + Fsin\theta) \rightarrow \mu = \frac{Fcos\theta}{mg + Fsin\theta}$$
$$\mu = \frac{300cos20}{(100)(10) + 100sin20} = 0.27$$



$$b)\ \Sigma F_y = 0 = F_N - mg + Fsin\theta \rightarrow F_N = mg - Fsin\theta$$
$$\Sigma F_x = ma = Fcos\theta - F_f \rightarrow ma = Fcos\theta - \mu F_N$$
$$ma = Fcos\theta - \mu(mg - Fsin\theta)$$
$$\rightarrow a = \frac{Fcos\theta - \mu mg + \mu Fsin\theta}{m}$$
$$\rightarrow a = \frac{300cos20 - .27(100)(10) + .27(300sin20)}{100} = 0.4\ m/s^2$$

8. A wooden block is placed on an inclined plane and given a push to start it in motion. The coefficient of sliding friction is .30. What angle will allow the block to slide down the incline at constant speed?



Some of the forces parallel to the incline is zero b/c constant speed

$\Sigma F_{||} = 0 = mgsin\theta - \mu mgcos\theta$

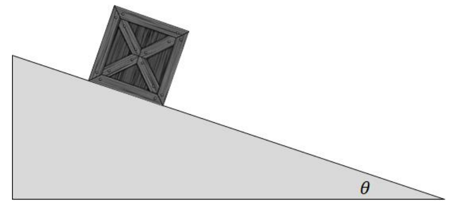
$\rightarrow mgsin\theta = \mu mgcos\theta \rightarrow sin\theta = \mu cos\theta \rightarrow tan\theta = \mu$

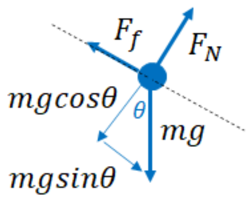
$\theta = \tan^{-1} 0.3 = 17°$

9. A crate of mass M = 8.0 kg is placed on a ramp that is inclined at $\theta = 30°$ with the horizontal as shown. The coefficient of static friction between the block and incline is $\mu_s = .60$ and the coefficient of kinetic friction is $\mu_k = .30$.

a) The block sits at rest on the incline. Calculate the force of friction.

b) The block is given a nudge downwards and starts going down the incline. Calculate its acceleration.





a) Some of the forces parallel to the incline is zero b/c not moving

$\Sigma F_{||} = 0 = mgsin\theta - F_F$

$\rightarrow F_f = mgsin\theta = (8\ kg)\left(10\frac{m}{s^2}\right) sin30 = 40\ N$

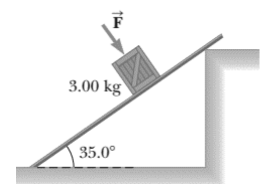
$*\mu_s mgcos\theta$ is the max value of the force of static friction, the value here is just equal to the downward force (which is gravity down the incline)

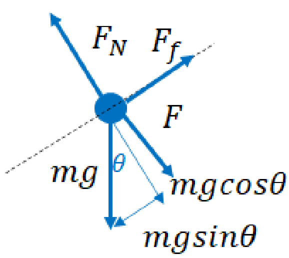
b) $\Sigma F_{||} = ma = mgsin\theta - F_F$

$\rightarrow ma = mgsin\theta - \mu_k mgcos\theta \rightarrow a = gsin\theta - \mu_k gcos\theta$

$\rightarrow a = \left(10\frac{m}{s^2}\right) sin30 - (0.3)\left(10\frac{m}{s^2}\right) cos30 = 2.4\ m/s^2$

10. The coefficient of static friction between the 3.00-kg crate and the 35.0° incline shown is 0.300. What minimum force F must be applied to the crate perpendicular to the incline to prevent the crate from sliding down the incline?





In the questions so far with friction on an incline, the normal force was $mgcos\theta$. The normal force increases from that in this question due to the extra downwards force, so the first step is to determine an expression for the normal force in terms of the downwards force F. In this case, the Normal force is $mgcos\theta$ + F since it has to negate both those forces.

1) $\Sigma F_{\perp} = 0 = F_N - mgcos\theta - F \rightarrow F_N = mgcos\theta + F$

2) $\Sigma F_{||} = 0 = mgsin\theta - F_F \rightarrow mgsin\theta = F_f$

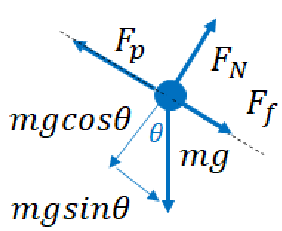
$mgsin\theta = \mu F_N \rightarrow mgsin\theta = \mu(mgcos\theta + F)$

$mgsin\theta = \mu mgcos\theta + \mu F \rightarrow \mu F = mgsin\theta - \mu mgcos\theta$

$$\rightarrow F = \frac{mgsin\theta - \mu mgcos\theta}{\mu}$$

$$\rightarrow F = \frac{(3\ kg)\left(10\frac{m}{s^2}\right) sin35 - (0.35)(3\ kg)\left(10\frac{m}{s^2}\right) cos35}{0.35} = 24.5\ N$$

11. A 15 kg block is pulled up an incline plane by a force F acting parallel to the incline. The incline makes an angle of 32° with the horizontal. The coefficient of friction between the block and the incline is 0.35. The block accelerates up the incline at a rate of 4.0 m/s^2. What is the magnitude of the force F?



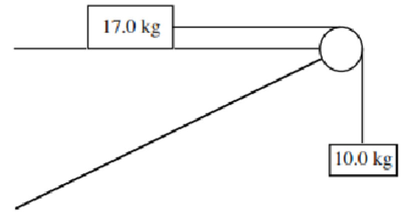
$\Sigma F_{||} = ma = F_p - mgsin\theta - \mu mgcos\theta$

$F_p = ma + mgsin\theta + \mu mgcos\theta$

$F_p = (15\ kg)\left(4\frac{m}{s^2}\right) + (15\ kg)\left(10\frac{m}{s^2}\right) sin32 + 0.35(15\ kg)\left(10\frac{m}{s^2}\right) cos32$

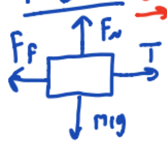
$F_p = 184\ N$

12. The coefficient of sliding friction between the block and the horizontal surface is 0.48. The initial velocity of the 17.0 kg block is 3.00 m/s to the right.

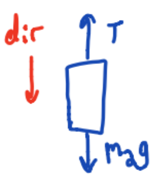


a) What is the acceleration of the 10.0 kg block?
b) Suppose the 17 kg is never given an initial velocity. What is the minimum coefficient of static friction to prevent motion?

FBDs dir



a) $\Sigma F = m_1 a = T - \mu m_1 g$



$\Sigma F = m_2 a = m_2 g - T$

$m_1 a + m_2 a = m_2 g - \mu m_1 g$

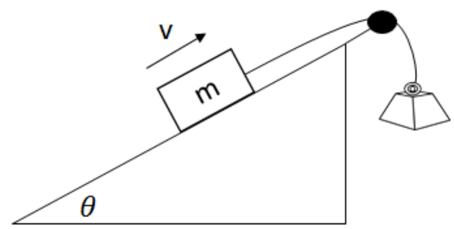
$\Rightarrow a = \frac{m_2 g - \mu m_1 g}{m_1 + m_2}$

$\Rightarrow a = \frac{(10)(10) - (.48)(17)(10)}{17 + 10} = \underline{.68\ \frac{m}{s^2}}$

b) $a = 0 \Rightarrow 0 = m_2 g - \mu m_1 g$

$\Rightarrow \mu = \frac{m_2 \cancel{g}}{m_1 \cancel{g}} = \frac{10}{17} = \underline{.59}$

13. A block of mass m is being pulled up a ramp by a hanging weight as shown on the right. The blocks are attached by a light string that passes through a frictionless pulley. Both the block and hanging weight move at constant speed. The incline makes an angle of θ and the coefficient of kinetic friction between the block and incline is μ.



a) Draw free body diagrams of block the block and the hanging weight. Label each force.
b) Derive an expression for the mass of the hanging weight.

FBDs

constant v

$\Sigma F = \dot{0} = T - mg\sin\theta - \mu mg\cos\theta$

$\Sigma F = 0 = Mg - T$

+

$0 = Mg - mg\sin\theta - \mu mg\cos\theta$

$\Rightarrow \boxed{M = m\sin\theta + \mu m\cos\theta}$

3.4 – Circular Motion

1. A skateboarder is skating over a circular bump. He is at the top of the bump and is moving rightward. Is the normal force exerted on the skateboarder by the bump *greater than*, *less than*, or *equal to* the weight of the skateboarder? Explain your reasoning.

F_N, mg

$$\Sigma F = \frac{mv^2}{r} = mg - F_N \Rightarrow F_N = mg - \frac{mv^2}{r} \quad \therefore \text{less}$$

The skateboarder is in circular motion, so the net force towards the center must equal mv^2/r. The normal force is direction away from the center while gravity is the only force acting to keep the skateboarder in a circle, so the normal force must be small in this case or the rider will not maintain circular motion.

2. An object of mass *m* moves in a circular path with a constant speed *v*. The centripetal force on the object is *F*. If the objects speed were halved and the mass was tripled, what would happen to the centripetal force?

$$\frac{F}{F'} = \frac{\frac{mv^2}{r}}{\frac{3m(\frac{v}{2})^2}{r}} \Rightarrow \frac{F}{F'} = \frac{1}{\frac{3}{4}} \Rightarrow \boxed{F' = \frac{3}{4}F}$$

3. A 3.5 kg mass is swung horizontally in a circle of radius 2.9 m. The centripetal force supplied by the rope is 17 N. What is the period of the mass's motion?

$$\frac{mv^2}{r} = F_c \Rightarrow v = \sqrt{\frac{F_c r}{m}} = \sqrt{\frac{(17N)(2.9m)}{3.5\,kg}} = 3.8\,m/s$$

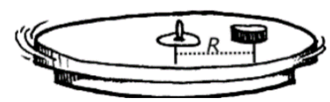
$$\Rightarrow T = \frac{2\pi r}{v} = \frac{2\pi(2.9)}{3.8} = \underline{4.9\,s}$$

4. An object travels around a circular path 2.0 times in 8.0 seconds. What is the frequency and period of the motion?

Period $= \frac{8s}{2\,rev} = \underline{4\,s}$ per revolution

freq $= \frac{1}{\text{period}} = \underline{\frac{1}{4}\,Hz}$

5. Consider a coin of mass m=0.1 kg placed on a rotating surface a distance R=1.0 m from the axis of rotation. The surface rotates with a period T. There are some locations on the surface where the coin can be placed and the force of static friction will not allow the coin to slip. At other locations, the coin will slip because static friction is not strong enough to prevent the coin from slipping. The coefficient of static friction between the coin and the surface is μ=0.2.



a) Calculate the maximum period of rotation of the turntable so that the coin does not slip.

b) Suppose the turn table is now spun faster, and the coin is to remain on the table. Should it be moved closer to the center of the table or further away? Justify your answer conceptually.

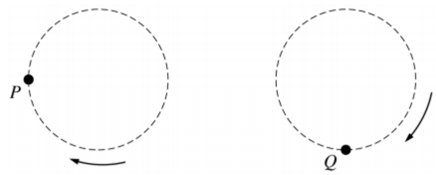
F_f a) $\frac{mv^2}{r} = F_f \Rightarrow \frac{\cancel{m}v^2}{r} = \mu \cancel{m} g$

$$\Rightarrow v = \sqrt{r\mu g} = \sqrt{(1.0m)(.2)(10\,m/s^2)}$$

$$\Rightarrow \underline{v = 4.5\,m}$$

b) Centripetal force required is mv^2/r, so increasing r would decrease the amount of force required for the object to stay in circular motion.

6. A ball of mass M is attached to a string of length R and negligible mass. The ball moves clockwise in a vertical circle, as shown. When the ball is at point P, the string is horizontal. Point Q is at the bottom of the circle and point Z is at the top.



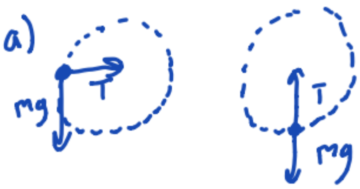
a) Draw and label all forces on the ball at points P and Q.

b) Derive an expression for v_{min}, the minimum speed the ball can have at point Z without leaving the circular path.

c) The maximum tension the string can have without breaking is T. Derive an expression for v_{max}, the maximum speed the ball can have at point Q without breaking the string. Answer in terms of M, R, T, and fundamental constants.

d) Suppose that the string breaks when the ball is at point P. Describe the ball's velocity and acceleration after the string breaks.

a)



b)

$$\Sigma F = \frac{mv^2}{r} = \cancel{T}^{\,0} + mg$$

$$\Rightarrow v = \sqrt{gr}$$

c) $\Sigma F = \frac{mv^2}{r} = T - mg \Rightarrow mv^2 = Tr - mg$

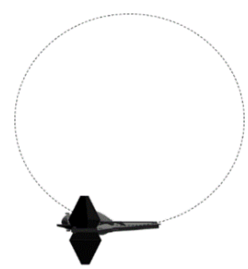
$$\Rightarrow v = \sqrt{\frac{Tr}{m} - g}$$

$$\Rightarrow V_{max} = \sqrt{\frac{T_{max}\, r}{m} - g}$$

d)

tangent, then falls b/c gravity

7. This is where the fun begins. Anakin Skywalker pilots a Jedi starfighter in Earth's atmosphere (with Earth gravity acting) as shown and completes in a vertical circle. At the bottom of the circle, Anakin feels a normal force of 990 N acting on him from his pilot chair and is traveling at 220 m/s at this point. Anakin's mass = 70 N



a) Calculate the radius of the circular loop completed by the airplane.

b) Take a seat, young Skywalker. Without doing any calculations, will the normal force of the seat on the pilot at the top of the loop be greater than, less than, or equal to 990 N? Justify your answer qualitatively.

a)

$$\Sigma F = \frac{mv^2}{r} = F_N - mg$$

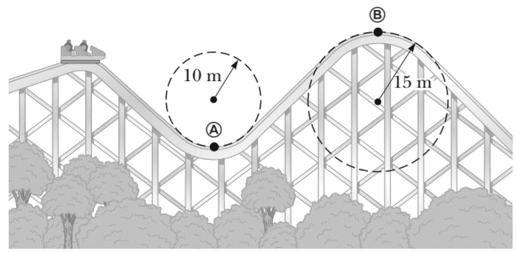
$$\Rightarrow r(F_N - mg) = mv^2$$

$$\Rightarrow r = \frac{mv^2}{F_N - mg} = \frac{70(220)^2}{990 - 70(10)}$$

$$\Rightarrow r = \underline{11{,}682 \text{ m}}$$

b) The normal force would be less. The net force on the pilot must be equal to mv^2/r to keep the pilot in circular motion. At the top of the loop, gravity also points towards the center of the circle, so the normal force doesn't need to be as large at the top. On the bottom, the normal force must supply mv^2/r and cancel out mg, so it's much larger at the bottom.

8. A roller-coaster vehicle has a mass of 500 kg when fully loaded with passengers.



a) If the vehicle has a speed of 20.0 m/s at point A, what is the force of the track on the vehicle?

b) What is the maximum speed the vehicle can have at point B for gravity to hold it on the track?

a) F_N, mg $\quad \Sigma F = \frac{mv^2}{r} = F_N - mg$

$\Rightarrow F_N = \frac{mv^2}{r} + mg = \frac{(500)(20)^2}{10} + (500)(10)$

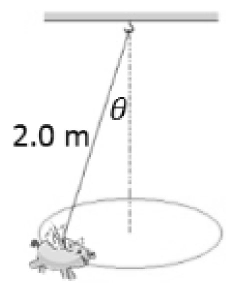
$\Rightarrow \underline{F_N = 25{,}000\ N}$

b) F_N, mg $\quad$ If $F_N < 0$, cart loses contact

$\frac{mv^2}{r} = mg - \cancel{F_N} \Rightarrow v = \sqrt{gr}$

$\Rightarrow v = \sqrt{(10)(15)} = \underline{12.2\ m/s}$

9. A pig of mass 1.0 kg is swung at the end of 2.0 m long string in a conical pendulum as shown. The string makes an angle of $\theta = 30°$ with the vertical.



a) Explain why θ cannot be 90? (horizontal string)

b) Find the speed of the pig's circular motion.

c) Will θ increase or decrease as the pig's speed increases? Justify your answer.

a) If the angle were 90, all the tension force would be horizontal, with none in the x-direction. Tension must overcome gravity in the y-direction, so it must have a vertical component.

b) In the y-direction, tension must cancel out gravity to keep the pig from falling down/going up. In the x-direction, the tension force must equal mv^2/r to keep the pig in circular motion. The radius of the circle here is found to be 2sin30 using right-triangle trig.

b) T, $T\cos\theta$, $T\sin\theta$, mg $\quad \Sigma F_y = 0 = T\cos\theta - mg$

$\Rightarrow T = \frac{mg}{\cos\theta}$

$\Sigma F_x = \frac{mv^2}{r} = T\sin\theta$

$\Rightarrow \frac{\cancel{m}v^2}{r} = \frac{\cancel{m}g}{\cos\theta}\sin\theta$

$r = 2\sin 30$

$\Rightarrow \frac{v^2}{r} = g\tan\theta$

$\Rightarrow v = \sqrt{gr\tan\theta}$

$\Rightarrow v = \sqrt{(10)\,2\sin 30\,\tan 30} = 2.4$

c) If theta increases, more of the tension force is in the x-direction. Part b shows that in the x-direction, $\frac{mv^2}{r} = T\sin\theta$, so if theta increases, there's more tension in the x-direction, leaving the velocity to increase.