6E:221
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Problem Set on Matrix Algebra

1. Consider the following Matrix and Vectors

$$A = \begin{bmatrix} -1 & 8 & 7 \\ 0 & -2 & 4 \end{bmatrix}, \qquad b = \begin{bmatrix} 9 \\ 6 \\ 0 \end{bmatrix}, \qquad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Calculate:

a. Ab
b. AIb
c. $x'IA$
d. $x'A$
e. Does the insertion of I in b change the result in a?
f. Does the deletion of I in d change the result in c?

2. Let A, B, C, D, E, F, G and H be Square Matrices, where E and F are nonsingular.

Expand the matrix product

$$X = \left(\left[AB + (CD)' \right] \left[(EF)^{-1} + GH \right] \right)'$$

3. Consider the following system of equations

$$2x_1 + 4x_2 - x_3 = 15$$

$$x_1 - 3x_2 + 2x_3 = -5$$

$$6x_1 + 5x_2 + x_3 = 28$$

Solve by

a. Matrix Inversion
b. Cramer's rule

4. Let $x = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 1 \\ 1 & 3 \end{pmatrix}$

a. Compute $A = \left[- x(x'x)^{-1}x' \right]$

b. Show that A is idempotent and determine its rank.

5. Let $a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$, $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$, $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$

Define $L = a'x$ and $Q = x'Ax$, where L is called a linear form in x and Q is called a quadratic form in x. Follow the definitions of matrix derivatives given in Greene's textbooks of econometrics

Find the following derivatives:

a. $\dfrac{\partial L}{\partial x}$

b. Assume A is a symmetric matrix. Find

$$\frac{\partial Q}{\partial x} \quad \text{and} \quad \frac{\partial Q}{\partial A}$$

c. Let $y = Ax$. Find

$$\frac{\partial y}{\partial x'} \quad \text{and} \quad \frac{\partial y}{\partial x}$$

6. Let $e'e = y'y - 2b'x'y + b'x'xb$, where e and y are $n \times 1$ vectors, b is a $p \times 1$ vector, and x is a $n \times p$ matrix. Find $\dfrac{\partial e'e}{\partial b}$

7. Let $R = a'x - x'Ax$

where $$a = \begin{pmatrix} 5 \\ 4 \\ 2 \end{pmatrix}, \qquad A = \begin{pmatrix} 6 & 1 & 4 \\ 1 & 4 & -1 \\ 4 & -1 & 5 \end{pmatrix}$$

a. Maximize R with respect to x.

b. Verify that the second order condition is satisfied.