ECON 221

Assignment 2

Due: September 13, 2012

1. {S.W. 2.24} Suppose Y_i is distributed i.i.d. $N(0, \sigma^2)$ for $i = 1, 2, ..., n$.
   (a) Show that $\mathbb{E}[Y_i^2/\sigma^2] = 1$
   (b) Show that $W = (1/\sigma^2)\sum_{i=1}^{n} Y_i^2$ is distributed χ_n^2.
   (c) Show that $\mathbb{E}[W] = n$. [*Hint*: Use your answer to (a).]
   (d) Show that $V = Y_1 / \sqrt{\frac{\sum_{i=2}^{n} Y_i^2}{n-1}}$ is distributed t_{n-1}.

2. {Goldberger 8.3} Consider these alternative populations for a random variable X:
   - Bernoulli with parameter $p = 0.5$.
   - Normal with parameters $\mu = 0.5$, $\sigma^2 = 0.25$.
   - Exponential with parameter $\lambda = 2$.

   Let A be the event $\{0.3 < X \leq 0.7\}$. For each population, find $\mathbb{E}[X]$, $V(X)$, and $\Pr(A)$.

3. {S.W. 2.15} Suppose Y_i, $i = 1, 2, ..., n$, are i.i.d. random variables, each distributed $N(10, 4)$.
   (a) Compute $\Pr(9.6 \leq \overline{Y} \leq 10.4)$ when (i) $n = 20$, (ii) $n = 100$, and (iii) $n = 1,000$.
   (b) Suppose c is a positive number. Show that $\Pr(10 - c \leq \overline{Y} \leq 10 + c)$ becomes close to 1.0 as n grows large.
   (c) Use your answer in (b) to argue that $\overline{Y}$ converges in probability to 10.

4. {Goldberger 9.3} Let $\overline{X}$ denote the sample mean in random sampling, sample size n, from a population in which the random variable $X \sim$ exponential(λ). For convenience, let $\theta = \mathbb{E}[X] = 1/\lambda$. So $\mathbb{E}[\overline{X}] = \theta$, $V(\overline{X}) = \theta^2/n$, plim $\overline{X} = \theta$, and the limiting distribution of $\sqrt{n}(\overline{X} - \theta)$ is $N(0, \theta^2)$. Consider the sample statistic $U = 1/\overline{X}$.
   (a) Use a Slutsky theorem to show that plim $U = \lambda$.

(b) Use the Delta method to find the limiting distribution of $\sqrt{n}(U - \lambda)$.

(c) Use your result to approximate $\Pr(U \leq 5/2)$ in random sampling, sample size 16, from an exponential population with $\lambda = 2$.

(d) Find the exact $\Pr(U \leq 5/2)$.

5. {Ruud, 6.1 & 8.1} Carry out the following Monte Carlo (i.e. simulation) experiment[1]:

- Generate 100 draws of a pseudorandom variable $x_i \sim \text{Unif}[0, 10]$.
- Generate pseudorandom variable y_i from a normal distribution with conditional mean $\mathbb{E}[y_i|x_i] = 10 - x_i$, $i = 1, 2, ...100$ and conditional variance 25 (or conditional standard deviation 5).
- Compute the OLS fitted coefficients of the regression $\mathbb{E}[Y|X] = \beta_0 + \beta_1 X$

(a) Repeat the second two steps of the above procedure 1000 times (i.e. holding x constant) and compute the sample means of the coefficient estimates each time. How do the sample means compare with the population coefficients? Show a histogram or density estimate of each coefficient to see the distribution of $\hat{\beta}_0$ and $\hat{\beta}_1$.

(b) Also save s^2 for each fit and check whether it appears to be an unbiased estimate of the conditional variance of $y_i|x_i$.

[1] If you choose to do this in STATA, a useful primer can be found here: http://www.learneconometrics.com/pdf/MCstata/MCstata.pdf