ECON 221

Assignment 5

Due: November 2, 2012

1. {Goldberger 19.2, 20.1} The Classical Normal Regression model applies to $E(\mathbf{y}) = \mathbf{X}\beta$, with $\sigma^2 = 2$ and

$$\mathbf{X}'\mathbf{X} = \begin{pmatrix} 5 & 1 \\ 1 & 2 \end{pmatrix}$$

Your sample has $b_1 = 3$, $b_2 = 2$ with $n = 32$.

   (a) Construct a 95% confidence interval for $\theta = \beta_1 + \beta_2$.

   (b) Construct a 90% confidence region for the pair (β_1, β_2).

   (c) Test at the 5% significance level the joint null hypothesis that $\beta_1 = 3 = \beta_2$.

   (d) State the alternative hypothesis against which you are testing.

2. {Goldberger 20.2} The Classical Normal Regression model applies to $E(\mathbf{y}) = \mathbf{X}\beta$, with $\sigma^2 = 11$ and

$$\mathbf{X}'\mathbf{X} = \begin{pmatrix} 4 & 1 \\ 1 & 3 \end{pmatrix}$$

The null hypothesis $\beta_2 = 1$ will be tested at the 10% significance level, against the alternative that $\beta_2 \neq 1$. What is the probability of rejecting that null hypothesis, if the true value of β_2 is 3?

3. {Greene Ex 2.3} Use Matlab - without using the statistical toolbox - to do the following exercise (be sure to include your code).

On ICON, you can find greenDataFormat.csv, data on the U.S. gasoline market for the years 1953-2004. You will use these data to obtain, among other things, estimates of the income, own price, and cross-price elasticities of demand in this market. Assuming the Classical Normal Regression model applies, consider the following model of per capita gasoline consumption:

$$\ln(G/pop) = \beta_1 + \beta_2 \ln Income/pop + \beta_3 \ln price_G + \beta_4 \ln P_{newcars} + \beta_5 \ln P_{usedcars} + \varepsilon.$$

This model will provide estimates of the income and price elasticities of demand for gasoline and an estimate of the elasticity of demand with respect to the prices of

new an used cars. What should we expect for the sign of β_4? Cars and gasoline are complementary goods, so if the prices of new cars rise, ceteris paribus, gasoline consumption should fall. Or should it? If the prices of new cars rise, then consumers will buy fewer of them; they will keep their used cars longer and buy fewer new cars. If older cars use more gasoline that newer ones, then the rise in the prices of new cars would lead to higher gasoline consumption than otherwise, not lower.

Consider these questions and test your predictions using the data. In doing so, give coefficient estimates, both robust and non-robust standard errors (ie. White covariance matrix), some measure of their significance using the robust errors (eg. test statistic, p-value, or 95% CI), and the coefficient of determination, R^2.[1] Also include a joint-significance test on the price variables for the null hypothesis $H_0 : \beta_4 = \beta_5 = 0$.

[1]Note that R^2 has the matrix formulation $1 - \frac{e'e}{y'M^0y}$, where $e = y - X(X'X)^{-1}X'y$ and $M^0 = I - \frac{1}{n}ii'$, where i is an $n \times 1$ vector of ones and is the matrix used to transform data to deviations from their mean.