

Assignment 6

Due: November 8, 2012

1. {Goldberger 25.1} Suppose that x and y are bivariate-normally distributed with $E(y|x) = \alpha + \beta x$, $V(y|x) = \sigma^2$, and $V(x) = \sigma_x^2$. In random sampling, sample size n from this population, let b be the sample slope and let s_x^2 be the sample variance of x . Let

$$z = \sqrt{n}(b - \beta)/(\sigma/s_x)$$

$$w = ns_x^2/\sigma_x^2$$

$$u = \sqrt{(n-1)}(b - \beta)/(\sigma/\sigma_x)$$

- (a) Show that $z \sim N(0, 1)$, that $w \sim \chi^2(n-1)$, and that z and w are independent.
 - (b) Show that $u \sim t(n-1)$.
 - (c) Explain how the result in (b) completely specifies the marginal distribution of the sample slope in terms of parameters and sample size.
2. {S.W. 11.10} Suppose that a random variable Y has the following probability distribution: $\Pr(Y = 1) = p$, $\Pr(Y = 2) = q$, and $\Pr(y = 3) = 1 - p - q$. A random sample of size n is drawn from this distribution and the random variables are denoted Y_1, \dots, Y_n .
 - (a) Derive the likelihood function for the parameters p and q .
 - (b) Derive formulas for the MLE of p and q .
 - (c) Derive the variance for the score variable.
3. Consider again model (1) from Assignment 3. Once again using the **Growth** data set, but this time including the data for Malta, compare the coefficient estimates when using OLS and using LAD for the variable *TradeShare*. Now exclude Malta and once again compare the OLS and LAD estimators. Including Malta, which estimator might be more appropriate? What about excluding Malta? Should Malta be excluded? Explain.

4. {Greene Ex 23.1} On ICON, you will find FLP.txt, data on female labor force participation. Estimate the following model

$$\Pr[LFP = 1] = F(\text{constant}, \text{age}, \text{age}^2, \text{family income}, \text{education}, \text{kids})$$

for $F(\mathbf{x}, \beta) = \mathbf{x}'\beta$ (linear probability model), $F(\mathbf{x}, \beta) = \Phi(\mathbf{x}'\beta)$ (Probit), and $F(\mathbf{x}, \beta) = \frac{e^{\mathbf{x}'\beta}}{1+e^{\mathbf{x}'\beta}}$ (Logit).

- (a) Compare the marginal effects from the three models at averages
- (b) Find the expected probability of labor force participation for a woman with average values of each variable (rounded up to the nearest integer, with age^2 the squared value of the average of age) using the LPM and Logit models. What does this say about their comparative effectiveness?
- (c) Using the Probit model,
 - i. Interpret the marginal effects (and their statistical and economic significance)
 - ii. Does age seem to have a nonlinear effect on labor force participation? At what age is a female most likely to participate in the labor force?