ECON 221

# Assignment 1

Due: September 6, 2012

1. Generalize the law of iterated expectations. Assuming that $X$ and $Y$ are scalars having joint distribution with density $f_{x,y}$, show that

$$\mathbb{E}_X[\mathbb{E}[g(Y)|X]] = \mathbb{E}[g(Y)]$$

2. {S.W. 2.7} In a given population of two-earner male/female couples, male earnings, X, have a mean of \$40,000 per year and a standard deviation of \$12,000. Female earnings, Y, have a mean of \$45,000 per year and a standard deviation of \$18,000. The correlation between male and female earnings for a couple is 0.80[1]. Let Z denote the combined earnings for a randomly selected couple.

   (a) What is the mean of Z?

   (b) What is the covariance between male and female earnings?

   (c) What is the standard deviation of Z?

   (d) Convert the above answers from U.S. dollars (\$) to euros (€) (assume exchange rate of \$1 = €0.81).

3. {Wackerly 5.114} For the daily output of an industrial operation, let $Y_1$ denote the amount of sales and $Y_2$, the costs, in thousands of dollars. Assume that the density functions for $Y_1$ and $Y_2$ are given by

$$f_1(y_1) = \begin{cases} (1/6)y_1^3 e^{-y_1}, & y_1 > 0 \\ 0, & y_1 \leq 0 \end{cases}$$

   and

$$f_2(y_2) = \begin{cases} (1/2)e^{-y_2/2}, & y_2 > 0 \\ 0, & y_2 \leq 0 \end{cases}$$

   Thus, the daily profit is given by $\Pi = Y_1 - Y_2$

[1] The correlation between X and Y, $\rho_{XY}$, is given by $\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$

(a) Find $\mathbb{E}(\Pi)$

(b) Assuming that $Y_1$ and $Y_2$ are independent, find $Var(\Pi)$

(c) Would you expect the daily profit to drop below zero very often? Why?

4. {S.W. 2.6} Use the following table giving the joint pmf between unemployment status and college graduation among those either employed or looking for work (unemployed) in the working age U.S. population for 2008 to answer the following questions:

Table 1: Joint Distribution of Employment Status and College Graduation in the U.S. Population Aged 25 and Greater, 2008

| | Unemployed (Y=0) | Employed (Y=1) | Total |
|---|---|---|---|
| Non-college grads (X=0) | 0.037 | 0.622 | 0.659 |
| College grads (X=1) | 0.009 | 0.332 | 0.341 |
| Total | 0.046 | 0.954 | 1.000 |

(a) Find the conditional expectation function $\mathbb{E}[Y|X]$

(b) Find the best linear predictor $\mathbb{E}^*[Y|X]$

(c) Compare $\mathbb{E}[\cdot|\cdot]$ and $\mathbb{E}^*[\cdot|\cdot]$

5. {S.W.} On ICON, you will find a data file **cps92_08.dta** that contains an extended version of the data set used in Table 3.1 for 2008. It contains data for full-time, full-year workers, age 25-34, with a high school diploma or B.A./B.S. as their highest degree. A detailed description is given in **CPS08_Description**, also available on the Stock and Watson Web site. In a brief memo (no longer than one page, single spaced), provide an analysis of the relationship between a worker's age and earnings. In your analysis, address the following points:

- How much do earnings increase as workers age by 1 year? Is this statistically significant? That is, can you reject the null hypothesis $H_0 : \beta_1 = 0$ versus a two-sided alternative at, for example, the 10%, 5%, or 1% significance level?
- Construct a 95% confidence interval for that relationship.
- What are the predicted earnings of the average-aged worker in the sample?
- Does age account for a large fraction of the variance in earnings across individuals?