ECON 221

Assignment 3

Due: September 27, 2012

1. {S.W. 6.11} Consider the regression model

$$Y_i = \beta_1 X_{1i} + \beta_2 X_{2i} + u_i$$

   for $i = 1, ..., n$. (Notice that there is no constant term in the regression.)

   (a) Specify the least squares function that is minimized by OLS.

   (b) Compute the partial derivatives of the objective function with respect to b_1 and b_2.

   (c) Suppose $\sum_{i=1}^{n} X_{1i}X_{2i} = 0$. Show that $\hat{\beta}_1 = \sum_{i=1}^{n} X_{1i}Y_i / \sum_{i=1}^{n} X_i^2$.

   (d) Suppose $\sum_{i=1}^{n} X_{1i}X_{2i} \neq 0$. Derive an expression for $\hat{\beta}_1$ as a function of the data (Y_i, X_{1i}, X_{2i}), $i = 1, ..., n$.

   (e) Suppose that the model includes an intercept: $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i$. Show that the least squares estimators satisfy $\hat{\beta}_0 = \overline{Y} - \hat{\beta}_1 \overline{X}_1 - \hat{\beta}_2 \overline{X}_2$.

   (f) As in (e), suppose that the model contains an intercept. Also suppose that $\sum_{i=1}^{n}(X_{1i} - \overline{X}_1)(X_{2i} - \overline{X}_2) = 0$. Show that $\hat{\beta}_1 = \sum_{i=1}^{n}(X_{1i} - \overline{X}_1)(Y_i - \overline{Y}) / \sum_{i=1}^{n}(X_{1i} - \overline{X}_1)^2$. How does this compare to the OLS estimator of β_1 from the regression that omits X_2?

2. {S.W. 7.10} Consider the following formulations of the "Homoskedasticity-Only" F-statistic:

$$F = \frac{(SSR_{restricted} - SSR_{unrestricted})/q}{SSR_{unrestricted}/(n - k_{unrestricted} - 1)} \tag{1}$$

$$F = \frac{(R^2_{unrestricted} - R^2_{restricted})/q}{(1 - R^2_{restricted})/(n - k_{unrestricted} - 1)} \tag{2}$$

   Show that the two formulas are equivalent and explain.

3. {Wackerly 5.15, 5.33} The management at a fast-food outlet is interested in the joint behavior of the random variables Y_1, defined as the total time between a customer's arrival at the store and departure from the service window, and Y_2, the time a customer waits in line before reaching the service window. Because Y_1 includes the time a customer waits in line, we must have $Y_1 \geq Y_2$. The relative frequency distribution of observed values of Y_1 and Y_2 can be modeled by the probability density function

$$f(y_1, y_2) = \begin{cases} e^{-\theta y_1}, & 0 \leq y_2 \leq y_1 < \infty \\ 0, & \text{elsewhere} \end{cases} \tag{3}$$

For part (a), assume $\theta = 1$.

(a) Find the conditional expectation function $\mathbb{E}[Y_1|Y_2]$ and the best linear predictor $\mathbb{E}^*[Y_1|Y_2]$. Compare.

(b) Now assume θ is unknown. Using the data set `line.csv` on ICON, use MLE to estimate the model assuming that the joint distribution is defined by (3). Is there evidence to suggest that $\theta = 1$? Now, estimate the BLP using OLS. Does the data seem to be generated from (3)?

4. {S.W. E8.4} On ICON, you will find a data file **Growth.dta** that contains data on average growth rates from 1960 through 1995 for 65 countries along with variables that are potentially related to growth. A detailed description is given in **Growth_Description**, also available on the Web site. In this exercise, you will be investigating the relationship between growth and trade. Excluding the data for Malta (consider why this is appropriate), run the following five regressions: *Growth* on (1) *TradeShare* and *YearsSchool*; (2) *TradeShare* and ln(*YearsSchool*); (3) *TradeShare*, ln(*YearsSchool*), *Rev_Coups*, *Assassinations* and ln(*RGDP60*); (4) *TradeShare*, ln(*YearsSchool*), *Rev_Coups*, *Assassinations*, ln(*RGDP60*), and $TradeShare \times ln(YearsSchool)$; and (5) $TradeShare$, $TradeShare^2$, $TradeShare^3$, $\ln(YearsSchool)$, *Rev_Coups*, *Assassinations*, and $\ln(RGDP60)$.

(a) Construct a scatterplot of $Growth$ on $YearsSchool$. Does the relationship look linear or nonlinear? Explain. Use the plot to explain why regression (2) fits better than regression (1).

(b) In 1960, a country contemplated an education policy that would increase average years of schooling from 4 years to 6 years. Use regression (1) to predict the increase in $Growth$. Use regression (2) to predict the increase in $Growth$.

(c) Test whether the coefficients on *Assassinations* and *Rev_Coups* are equal to zero (individually and jointly) using regression (3). Compare results to the same regression with "Robust" standard errors (ie. White covariance matrix).

(d) Using regression (4), is there evidence that the effect of $TradeShare$ on $Growth$ depends on the level of education in the country?

(e) Using regression (5), is there evidence of a nonlinear relationship between $TradeShare$ and $Growth$?

(f) In 1960, a country contemplated a trade policy that would increase the average value of $TradeShare$ from 0.5 to 1. Use regression (3) to predict the increase in $Growth$. Use regression (5) to predict the increase in $Growth$.