ECO 518: Homework 6

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Note:

Question 2

Part i

Let's try using the non-parametric bootstrap. For each $b = 1, ..., B$ for total bootstrap draws B,

1. Sample n draws of (X_i, Z_i^*) with replacement. Call the result the bootstrap sample.
2. Use maximum likelihood to estimate $\widehat{\gamma}_b$ and $\widehat{\sigma}_b^2$ from the bootstrap sample.
3. Regress retirement age Y_i (corresponding to the bootstrap sample) on a constant and $\widehat{Z}_i^*$. Store regression coefficients $\widehat{\alpha}_b^*$ and $\widehat{\beta}_b$.

Then, the standard deviation of $\widehat{\beta}_b$ yields the bootstrap standard error.

Part ii

After reading chapter 8.2 of Hayashi (and taking the hint from the question itself), we did not manually take the first order conditions of the MLE censored normal regression problem. We leave this task as an exercise to the RA for his development as a budding econometrician.

The moment conditions can be written as follows

$$
\begin{aligned}
& g(Z^*, X, Y, \gamma, \sigma^2, \beta, \alpha) \\
&= \begin{pmatrix} \frac{\partial l_n}{\partial \gamma} \\ \frac{\partial l_N}{\partial \sigma^2} \\ Z_i(Y_i - \alpha - \beta Z_i) \\ (Y_i - \alpha - \beta Z_i) \end{pmatrix} \\
&= \begin{pmatrix} \frac{\partial l_n}{\partial \gamma} \\ \frac{\partial l_N}{\partial \sigma^2} \\ (Z_i \mathbb{1}[Z_i < 10^6] + \gamma' X_i \mathbb{1}[Z_i = 10^6])(Y_i - \alpha - \beta(Z_i \mathbb{1}[Z_i < 10^6] + \gamma' X_i \mathbb{1}[Z_i = 10^6])) \\ (Y_i - \alpha - \beta(Z_i \mathbb{1}[Z_i < 10^6] + \gamma' X_i \mathbb{1}[Z_i = 10^6])) \end{pmatrix}
\end{aligned}
$$

Consider the first line. The first two entries correspond to the FOCs of the censored normal linear regression problem. The last two entries correspond to the moment conditions of the linear regression model in the second state. In the following lines, we explicitly include imputed earnings.

Then, with four equations and four unknowns, the system is just-identified, so there is no need to worry about weighting matrices. Assuming all the regularity conditions are satisfied, let $G = \nabla g$, the first derivative matrix of the moment conditions. Let $\Omega = gg'$. Then, using results of asymptotic normality of the GMM estimator, the asymptotic variance is

$$(G'WG)^{-1}G'W\Omega WG(G'WG)^{-1}$$

from Matt Masten's 2018 ECON 707D lecture notes. The asymptotic variance simplifies to

$$V = (G'\Omega^{-1}G)^{-1}$$

under the optimal weighting matrix. Checking for consistency, true parameter is not on the boundary of the parameter space, g is once continuously differentiable, the second moment exists, local dominance of the derivative, and G full rank, we know

$$\sqrt{n}(\widehat{\theta} - \theta_0) \xrightarrow{d} \mathcal{N}(0, V)$$

where V is defined above and $\theta = (\gamma, \sigma^2, \beta, \alpha)$.