# Homework 1 Answers

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## Question 1

See separate IJulia/Jupyter notebook file.

## Question 2

### Question 2 (a)

*What are the decision variables in the above problem? Describe them (textually) and then assign them to shorthand variables of your choice (write the variables). Are these decision variables indexed across any sets? If so, describe how you define the sets.*

#### Textual description

Part Tiger Electronics Co (PTE) has three factories, and each factory is specialised in the production of a single good type. For each day over the next three days, PTE must decide how much each factory should produce.

PTE can also choose how many silicon wafers to use each day (although this decision/state variable is actually completely defined through the production choices of the other three variable, I include it as a decision variable for ease of notation).

Therefore, PTE's decision variables are;

- How many phones to produce on each of the three days (how much Factory 3 should produce).
- How many circuit boards to produce on each of the three days (how much Factory 1 should produce).
- How many processors to produce on each of the three days (how much Factory 2 should produce).
- How many of its reserves of silicon wafers to use from its inventory on each of the three days.

#### Shorthand notation: variables indexed across sets

Writing this as shorthand variables, lets define $X_{f,g,t}$ as the number of units produced at Factory $f$ of good type $g$ on day $t$ that are directly sold. Note, we have: $f \in \{1, 2, 3\}$, $g \in \{board, processor, phone\}$, and $t \in \{1, 2, 3\}$. Since each factory can only produce one type of good, we can remove the $f$ index, and refer to our decision variables as simply $X_{g,t}$.

Also, we have $Inv_t$, which represents the amount of silicon chips used on day $t \in \{1, 2, 3\}$ from the inventory.

Then, the total set of decision variables is:

$$\{ X_{board,t}, X_{processor,t}, X_{phone,t}, Inv_t : \forall t \in \{1, 2, 3\} \}$$

This gives us a total of 12 decision variables.

## Question 2 (b)

*Write down a set of linear constraints that define the feasible region for this problem, over the domain of the decision variables. Feel free to make use of set and sum notations to write the constraints compactly.*

**Production requirement constraint 1**: PTE must produce at least 30 circuit boards and 6 smart phones over the next three days.

$$\sum_{t=1}^{3} X_{board,t} \geq 30 \tag{1}$$

**Production requirement constraint 2**: PTE must produce at least 6 smart phones over the next three days.

$$\sum_{t=1}^{3} X_{phone,t} \geq 6 \tag{2}$$

**Technology constraint set 1**: It takes 2 silicon wafers to produce a circuit board, and 1 silicon wafer to produce a processor.

$$2X_{board,t} + X_{processor,t} = Inv_t \quad \forall t \in \{1,2,3\} \tag{3}$$

**Technology constraint set 2**: Factory 1 can produce at most 50 circuit boards per day. Factory 2 can produce 10 processor units per day. Factory 3 can produce 2 smart phones per day.

$$X_{board,t} \leq 50 \quad \forall t \in \{1,2,3\} \tag{4,5,6}$$

$$X_{processor,t} \leq 10 \quad \forall t \in \{1,2,3\} \tag{7,8,9}$$

$$X_{phone,t} \leq 2 \quad \forall t \in \{1,2,3\} \tag{10,11,12}$$

**Technology constraint set 3**: Factory 3 must consume 5 circuit boards and 1 processor to produce a smart phone.

$$X_{board,t} - 5X_{phone,t} \geq 0 \quad \forall t \in \{1,2,3\} \tag{13,14,15}$$

$$X_{processor,t} - X_{phone,t} \geq 0 \quad \forall t \in \{1,2,3\} \tag{16,17,18}$$

**Inventory constraint**: Tiger Electronics has 150 silicon wafers in inventory for the next three days (assume it cannot get another shipment until day 4).

$$\sum_{t=1}^{3} Inv_t \leq 150 \tag{19}$$

**Non-negativity constraints**: Tiger Electronics cannot produce a negative quantity (e.g. the quantities for all products can take any continuous non-negative values).

$$\forall Y \in \{\, X_{board,t}, X_{processor,t}, X_{phone,t}, Inv_t : \forall t \in \{1,2,3\}\,\}\,, Y \geq 0 \tag{20:31}$$

## Question 2 (c)

*What is the objective function of this problem? Write it algebraically as a linear function of the decision variables described above (please simplify terms as much as possible, although this will not impact your grade for this part).*

The objective function for this problem is the function that represents the maximisation the company profits. Profits are equal to total revenues generated from the sale of all goods, minus total costs of production.

Let $D$ represent the set of decision variables identified in part (a).

Then, revenues from production will be equal to the sum of sales of boards, phones, and processors:

$$Revenues = \sum_{t=1}^{3} 6(X_{board,t} - 5X_{phone,t}) + 15(X_{processor,t} - X_{phone,t}) + 200X_{phone,t} \tag{1}$$

Total costs can be represented as:

$$Costs = \sum_{t=1}^{3} 5X_{board,t} + 10X_{processor,t} + 50X_{phone,t} \tag{2}$$

Then we can write the objective function as follows:

$$\begin{aligned} \pi^* &= \max_{d_i \in D} \quad Revenues - Costs \\ &= \max_{d_i \in D} \sum_{t=1}^{3} \left(X_{board,t} + 5X_{processor,t} + 105X_{phone,t}\right) \end{aligned} \tag{3}$$

## Extension - Solve the model!

Solving the model gives the following optimal values of our decision variables:

| Day | Circuit Board Production | Processor Production | Phone Production | Inventory used as input |
|---|---|---|---|---|
| 1 | 20.0 | 10.0 | 2.0 | 50.0 |
| 2 | 20.0 | 10.0 | 2.0 | 50.0 |
| 3 | 20.0 | 10.0 | 2.0 | 50.0 |
| Total | 60.0 | 30.0 | 6.0 | 150.0 |

In total, PTE makes $840 profit over the three days.

One thing to note - there are an infinite number of other optimal solutions, that give exactly the same profit. All of these solutions produce the same amount of processors and phones on each day. However, the timing on when they make the circuit boards can be shifted. For example, PTE could make the same profit by making 40 circuit boards on the first day, and the ten on the two subsequent days. All of the solutions will produce the same total number of circuit boards over the three days.