Chapter Five

The Theory of Cold Electron Bolometers

5.1 Introduction

As with all areas of study within physics the fabrication and testing of Cold Electron Bolometers brings together elements from several fields including. These include concepts from quantum mechanics such as electron tunnelling, cryogenics and low temperature physics, electronics, as well as solid state physics. The study and testing of a Cold Electron Bolometer requires a strong understanding of these areas as well as the general grounding in the field of instrumentation and its associated vocabulary. These are covered in the following sections and chapters.

5.2 Tunnelling Barriers

As will be explained in the following sections the Cold Electron Bolometer direct removes hot electrons from the detector's absorber. This thermally selective removal of charges is made possible through the use of a tunnelling barrier. These tunnelling barrier allows the electron systems on either side to be separated (i.e., the energy levels in the two do not have to be aligned).

Several types of tunnelling barriers exist however only those involving a superconductor shall be considered here since this is a requirement of the thermal selection required for a Cold Electron Bolometer. The four main types of contact used in Cold Electron Bolometers are:

Normal metal-Insulator-Superconductor (NIS) The simplest (at least conceptually). The two sides (the normal metal and the superconductor) are separated by an insulating layer (typically an oxide layer).

Superconductor-Insulator-Superconductor (SIS) This is essentially the same as the arrangement described above expect the normal metal is replaced by the same material as is used on the other side of the barrier.

Superconductor-Insulator-(different) Superconductor (SIS′) A further progression of the systems already described. Here the materials on either side of the insulator are both superconductors but have different energy gaps (they are different superconductors).

Semiconductor-Superconductor (SmS) This structure replaces the insulator with a Schottky barrier which forms naturally between the semiconductor and a metal (or superconductor). Typically (and for all the work describe din this thesis) a highly doped semiconductor, which can be thought of as being metallic (there is no discernible band gap), is used.

5.2.1 Formation of Insulating Layers

From the above list it can be seen that only two types of insulating barriers are typically used in the fabrication of Cold Electron Bolometers. These are: oxide layers and Schottky barriers. While both of these can be thought of as performing the same function their formation is very different. An oxide layer requires an additional stage during the device fabrication process, where oxygen is introduced to the evacuated deposition chamber. A Schottky barrier on the other hand will form naturally between a semiconductor and a metal (or superconductor), this means no additional fabrication stages are required.

An oxide insulating layer forms ionic bonds between the atoms of the metal and the oxygen atoms. This causes the outer electrons in the metal, which previously were *free* and available for current flow, to no longer be able to flow as current and thus the resistance of the material is greatly increased.

The formation of an oxide layer (illustrated form the growth of aluminium oxide in Figure 5.1) is conceptually simple. Since aluminium is commonly used as one side of a tunnelling contact aluminium oxide often forms the insulating

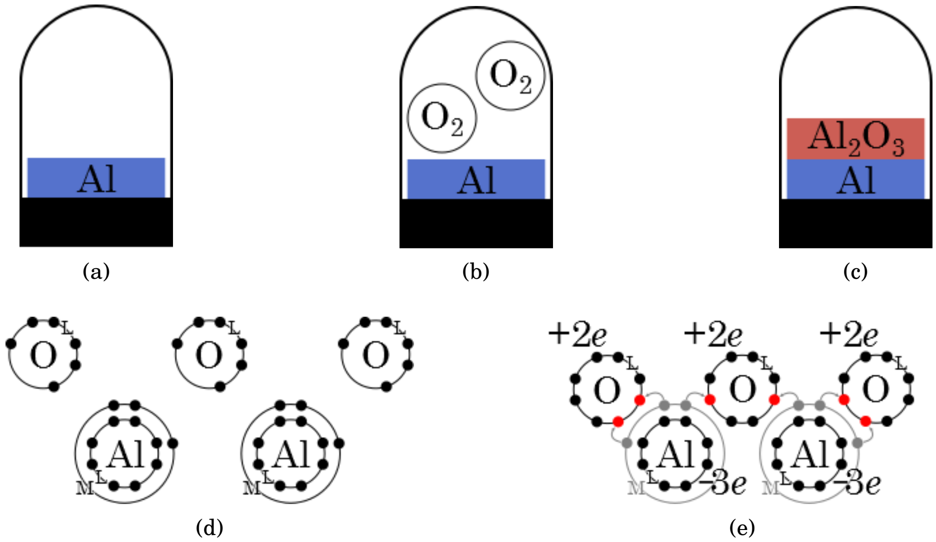

Figure 5.1: Growth (a)–(c) and ionic bond formation (d) and (e) of an aluminium oxide (Al_2O_3) layer. (a) Aluminium has been deposited (usually via evaporation) in a vacuum. (b) Oxygen is introduced into the evaporation chamber. (c) The oxygen atoms form ionic bond with the aluminium, this causes the growth of an aluminium oxide layer of the surfaces of the deposited aluminium. (d) Oxygen and aluminium atoms prior to bonding; oxygen contains six electrons in the L electron shell ($1s^2$ $2s^2$ $2p^4$), aluminium contains a full L electron shell and has three electrons in its M shell ($1s^2$ $2s^2$ $2p^6$ $3s^2$ $3p^1$). (e) The electrons from the M shells in the aluminium atoms (shown in grey for clarity) move to the L shell of the oxygen atoms (shown as the red electrons), this is the formation of ionic bonds and results in both the oxygen and aluminium atoms having their L shells filled (for tidiness the full K shell is not shown).

oxide, because of this Figure 5.1 and the following explain the growth of an aluminium oxide layer (there are several other oxide layers used). After the metal has been deposited by evaporation or other means (Figure 5.1a) oxygen is introduced into the deposition chamber (Figure 5.1b). The outermost electrons from the aluminium (those in the third shell, the M shell) move to the vacant states in the outer shell of the oxygen atoms (oxygen has two vacant electron states in its second shell, the L shell) forming ionic bonds between the aluminium and the oxygen, this is shown in Figures 5.1d and 5.1e. This results in a layer of aluminium oxide

(Al_2O_3) forming on top of the deposited aluminium (Figure 5.1c).

While conceptually simple, in order to produce an even, high quality layer of a desired thickness great care needs to be taken as to both the quantity of gas introduced an the temperature of the during the introduction of the oxygen gas (Cabrera and Mott, 1949; Jaeger et al., 1991). The addition of an oxide layer also necessitates an additional step in the fabrication along with the required equipment to add and monitor the flow of gas into the deposition chamber.

As opposed to an oxide layer a Schottky barrier will form naturally between a metal and a semiconductor. the barrier is formed as the electrons in the two materials move to cause the Fermi-energy in the two materials to be aligned. The concept of a naturally forming potential barrier between a semiconductor and a metal was first suggested by Schottky (1939) whose original explanation is illustrated in Figure 5.2

Schottky's explanation was that, after the semiconductor has been brought into contact with the normal metal, the electrons in the conduction band of the semiconductor (the most energetic) are able to move to the lower (energetically favourable) states above the Fermi-level in the metal (illustrated in Figure 5.2b) leaving behind positive;y charged *donor* ions. This causes the Fermi-level within the semiconductor to decrease since there are fewer electrons in the conduction band. This movement of electrons continues until there is an equilibrium established between the electron systems in the two materials (i.e. when the Fermi-levels are aligned). Away from the interface between the metal and the semiconductor the valence and conduction bands move relative to the Fermi-level however at the interface the bands move differently since it is these electrons which have moved. This causes the phenomenon of *band-bending* and the formation of a depletion region (from E_f to $E_f + \Phi_B$) in the semiconductor at the interface.

Schottky barriers are, in theory, simple to fabricate. All that is required is for the two materials to be deposited sequentially. An important caveat is that one must ensure that the first material deposited (often the semiconductor) is free of impurities or unwanted surface films (such as oxide layers). This is simple if the entire fabrication process can be performed in a single system under continuous vacuum. If however the device needs to be removed from the protection of the evacuated deposition system, to be patterned for example, then it is important to ensure that the surfaces is thoroughly cleaned prior to the deposition of the second material (a good description of surface preparation requirements is given by

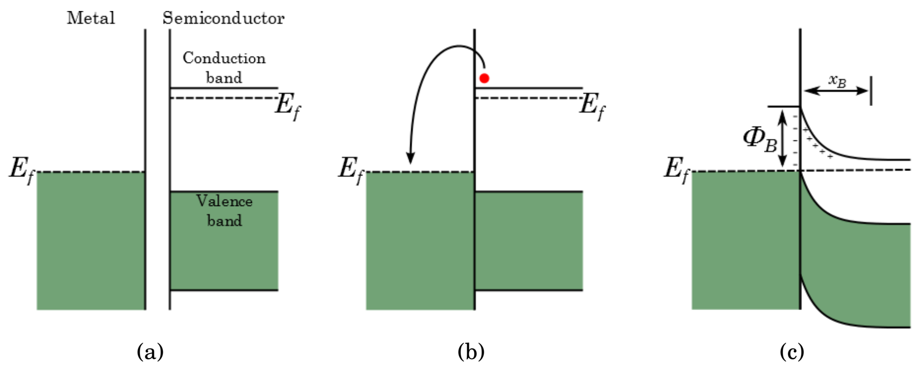

Figure 5.2: Formation of a Schottky barrier between a metal and a n-doped semiconductor. (a) before being brought into contact the energy distribution in the metal and the semiconductor are independent with the Fermi-level (E_f, shown as the dashed line) sitting at the top of the occupied states in the metal (shown at $T = 0$ K) and just below the conduction band of the semiconductor. (b) Immediately after the metal and semiconductor are brought into contact the energy levels are unchanged however electrons (red circle) start to move from the conduction band of the semiconductor to the lower energy. These electrons leave behind positively charged ions or *donor states*. (c) The movement of the most energetic electrons from the semiconductor causes the Fermi-level to move, this continues until the Fermi-level in both the materials is the same. Away from the interface the band structure of the semiconductor move relative to the Fermi-level; at the interface however this is not the case, this causes the phenomenon know as *band-bending* in the semiconductor. The height of the Schottky barrier established is related to the difference between the vacuum level in the two materials and is given by Φ_B.

Roccaforte et al., 2003). Should the surface of the first material not be sufficiently cleaned contamination may either cause an insulating layer to form which, while itself acting as a tunnelling barrier, will inhibit or stop the development of a Schottky barrier; or alter the Fermi-level of the material and thus alter the Schottky barrier height (as discussed in the following paragraph).

One cannot simply combine any combination of semi-conductor and metal to create a Schottky barrier. As can be seen from the description above and Figure 5.2 there needs to be a difference between the inherent Fermi-levels in the two materials. If this is not the case, when the two materials are brought together there will be minimal movement of electrons from the semiconductor's conduction band to the metal. This will cause the barrier height, Φ_B, to be very small. A similar effect is observed when the Fermi-level of the metal is higher than that of the semiconductor, this results in the band-bending, seen in Figure 5.2c, to be downwards. This means that electrons do not encounter a barrier. Contacts of this type are known as *ohmic contacts*. A more detailed description of the formation and criteria for ohmic contacts is given by Rhoderick and Williams (1988) who also offer an excellent overview on the concepts relating to Schottky barriers.

5.3 The Tunnelling Current

In order to understand the behaviour of a Cold Electron Bolometer it is important to understand the movement of charges, at different energies, across the tunnelling barrier. To do this we need to consider four directions of charge transfer, these are*:

1. Charges in the superconductor with energies above the superconducting bandgap $\left(E > E_{f_S} + \Delta\right)$ tunnelling into the central, semiconductor, island.
2. Quasiparticles in the superconductor whose energies are below the Fermi-energy $\left(E < E_{f_S} - \Delta\right)$ tunnelling into the central island.
3. Charges in the central island with energy levels corresponding to the normal states in the superconductor $\left(E > E_{f_S} + \Delta\right)$.
4. Charges in the central island with energies below corresponding to the superconducting states in the superconductor $\left(E < E_{f_S} - \Delta\right)$.

Since the movement of charges in terms 3 and 4 is the opposite of those in the first two terms these act to suppress the total current.

Figure 5.3 shows these four possible forms of tunnelling when there is no bias across the structure. It can be seen that the tunnelling routes represented by numbers 1 and 3 are possible (providing there is sufficient thermal broadening of the density of states) since there are charges and vacant states on both sides of the Schottky barrier. The tunnelling shown by numbers 2 and 4 are less likely since there are very few vacant states for electrons to tunnel to.

*In the following list E_{f_S} is used to denote the Fermi-energy in the superconductor.

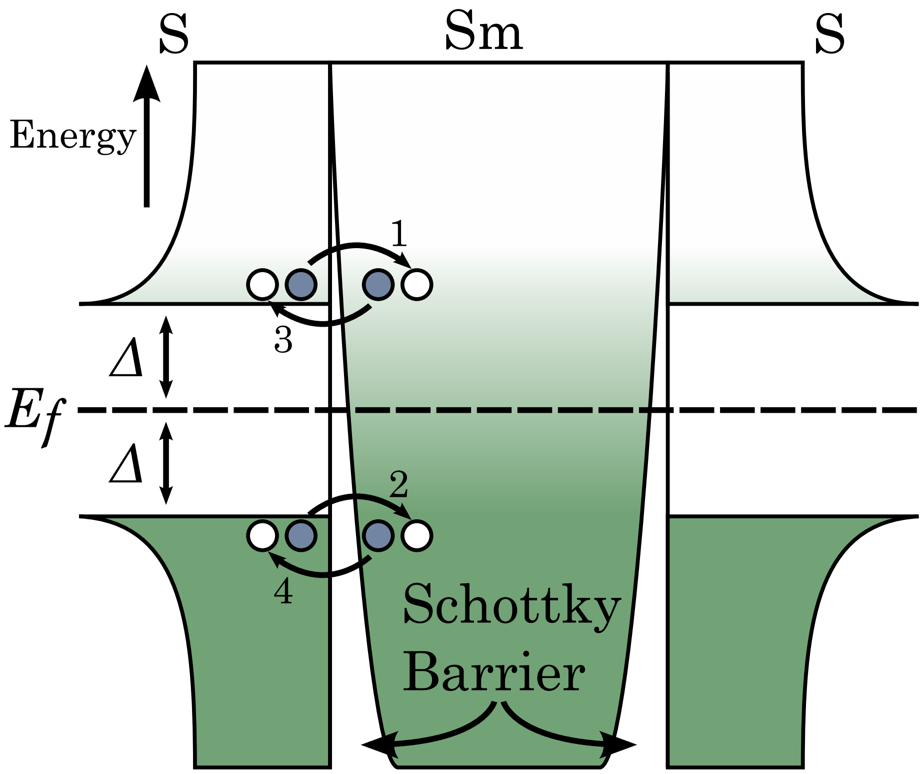

Figure 5.3: Possible tunnelling of charges in a SSmS structure, shown at non-zero temperature, without any external bias across this system and for only one junction. The density of the shading represents the number of occupied states (no shading - all states are vacant; fully shaded - all states are vacant) There are four different ways in which charges can tunnel across the structure (numbering as listed on Page 22).

By applying an external bias across the structure it is possible to shift the distribution of charges in the three layers relative to each other. This biasing causes the probability of tunnelling, via each of the routes to be altered. Figure 5.4 shows the effect of biasing a single junction structure such that the energy levels in the semiconductor (right) are raised above the energy levels in the superconductor (left). This has a notable effect to the probability of tunnelling via each of the described routes. Charges are less likely tunnel from superconductor into the semiconductor (routes one and two), since there are fewer states available in

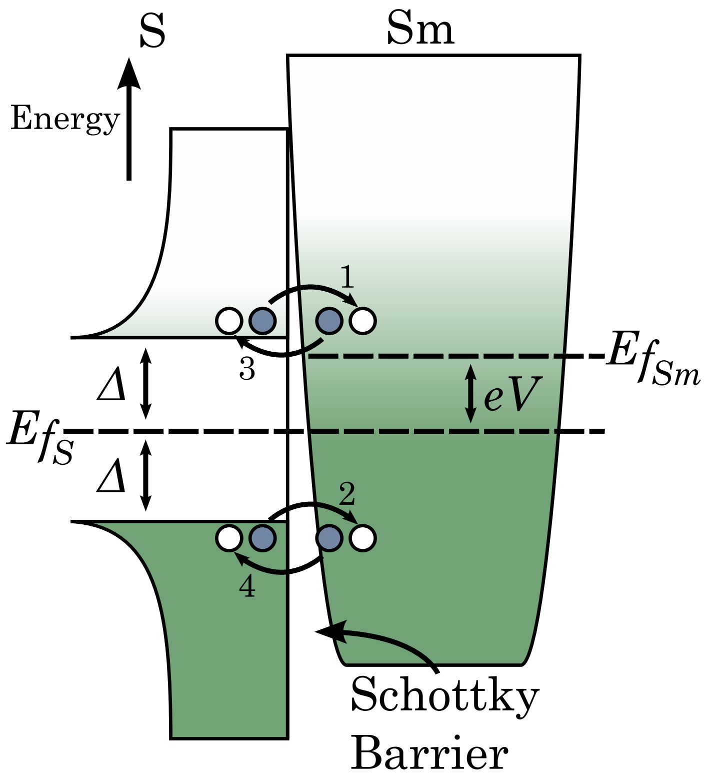

Figure 5.4: Tunnelling of charges across a single superconductor-semiconductor junction when biased by a voltage, V, such the Fermi level in the semiconductor, $E_{f_{Sm}}$, is raised above that of the superconductor, E_{f_S}.

the semiconductor at energies corresponding to the occupied in the superconductor. Conversely, charges are more likely to move from the semiconductor into the superconductor (routes three and four) as there are a greater number of occupied states in the semiconductor with energies corresponding to the vacant states in the superconductor.

Figure 5.5 illustrates a single junction system biased in the opposite polarity to the structure shown in Figure 5.4. In this case, when compared to the unbiased state, charges are more likely to tunnel from the superconductor into the semiconductor (routes one and two) since the occupied states in the superconductor correspond to a greater number of vacant states in the semiconductor. Likewise, fewer charges will tunnel from the semiconductor into the superconductor (routes

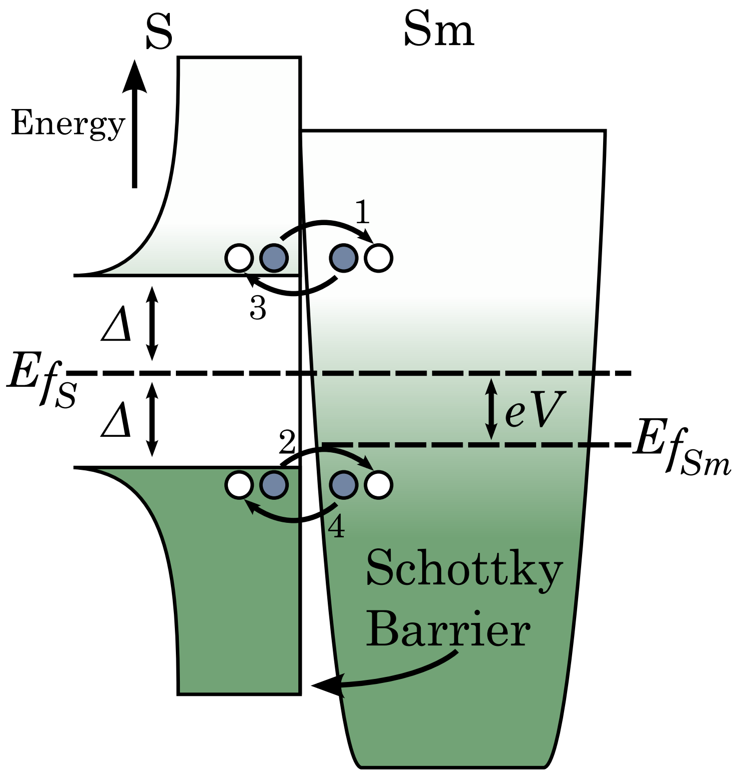

Figure 5.5: Tunnelling of charges across a single superconductor-semiconductor junction when biased in the opposite polarity to that shown in Figure 5.4. The biasing voltage, V, causes the Fermi level in the superconductor, E_{f_S}, to be raised above the Fermi level in the semiconductor, $E_{f_{Sm}}$.

three and four) as there are fewer occupied states in the semiconductor aligned with vacant sates in the superconductor.

When modelling the current in a two junction system it is useful to note that, since the two junctions can be thought of as resistors in series, we need only consider the current through one junction, since the current through each of the junction will be the same. This brings up two important definitions in the following derivation: the voltage used in these equations is defined to be the voltage dropped across a two junction system. The following equations assume there is no resistance to current flow from either the semiconductor or superconductor hence the voltage dropped across each junction will be $V/2$. The second definition to note is that the tunnelling resistance, R_N, is defined to be the resistance of a single junction.

For each of these movements of charge it is possibly to define a Fermi-distribution, q_n where the subscript n corresponds to the number of the term in the list on Page 22.

$$q_1 \sim \frac{1}{e^{\frac{|E|}{k_B T_S}} + 1}, \tag{5.1}$$

$$q_2 \sim \frac{1}{e^{\frac{-|E|}{k_B T_S}} + 1}, \tag{5.2}$$

$$q_3 \sim \frac{1}{e^{\frac{\left(|E| + eV/2\right)}{k_B T_e}} + 1}, \tag{5.3}$$

$$q_4 \sim \frac{1}{e^{\frac{-\left(|E| - eV/2\right)}{k_B T_e}} + 1}. \tag{5.4}$$

In these equations E is the energy of a carrier, k_B is Boltzmann's constant, T_S and T_e are the temperatures of the charge carries in the superconductor and the central island respectively, e is the electron charge and V is the voltage across structure. q_1 is the Fermi-distribution for charges in the superconductor with energy above the superconducting bandgap, q_2 relates to charges in the superconductor with energies below the bandgap, q_3 and q_4 are the distributions of charges in the central island with energies above and below the superconductor's bandgap respectively.

For each of the terms in the above list we can define a probability p_{1-4} that a charge will tunnel in the stated manner. This probability is related to the likelihood of an occupied state on one side of the tunnelling barrier corresponding to an empty state on the other side. For each of the forms of tunnelling defined above, this probability is:

$$p_1 = q_1 \times (1 - q_3), \tag{5.5}$$

$$p_2 = q_2 \times (1 - q_4), \tag{5.6}$$

$$p_3 = q_3 \times (1 - q_1), \tag{5.7}$$

$$p_4 = q_4 \times (1 - q_2). \tag{5.8}$$

The total movement of charges between the superconductor and the superconducting contact is related to the sum of these probabilities integrated over all energies and, if movement from the superconductor to the semiconductor is taken to be the positive direction, is given by:

$$p_T = \int_0^{\infty} [p_1 + p_2 - p_3 - p_4]\,\mathrm{d}E\,. \tag{5.9}$$

Substituting the terms for p_{1-4} from Equations 5.5–5.8 gives:

$$p_T = \int_{-\infty}^{\infty} [q_1 \times (1-q_3) + q_2 \times (1-q_4) - q_3 \times (1-q_1) - q_4 \times (1-q_2)]\,\mathrm{d}E\,. \tag{5.10}$$

Expanding the bracket and cancelling the like terms yields:

$$p_T = \int_{-\infty}^{\infty} [q_1 - q_1 q_3 + q_2 - q_2 q_4 - q_3 + q_3 q_1 - q_4 + q_4 q_2]\,\mathrm{d}E\,, \tag{5.11}$$

$$p_T = \int_{-\infty}^{\infty} [q_1 + q_2 - q_3 - q_4]\,\mathrm{d}E\,. \tag{5.12}$$

It is possible to simplify this result further by looking at the sum of various combinations of the q terms in Equation 5.12. Of most interest is the result of $q_1 + q_2$.

$$q_1 + q_2 = \frac{1}{\mathrm{e}^{\frac{|E|}{k_B T_S}} + 1} + \frac{1}{\mathrm{e}^{\frac{-|E|}{k_B T_S}} + 1}\,, \tag{5.13}$$

$$= \frac{\mathrm{e}^{\frac{-|E|}{k_B T_S}} + 1 + \mathrm{e}^{\frac{|E|}{k_B T_S}} + 1}{\left(\mathrm{e}^{\frac{|E|}{k_B T_S}} + 1\right) \times \left(\mathrm{e}^{\frac{-|E|}{k_B T_S}} + 1\right)}\,, \tag{5.14}$$

$$= \frac{\mathrm{e}^{\frac{|E|}{k_B T_S}} + \mathrm{e}^{\frac{-|E|}{k_B T_S}} + 2}{\mathrm{e}^{\frac{|E|}{k_B T_S}}\,\mathrm{e}^{\frac{-|E|}{k_B T_S}} + \mathrm{e}^{\frac{|E|}{k_B T_S}} + \mathrm{e}^{\frac{-|E|}{k_B T_S}} + 1}\,, \tag{5.15}$$

$$q_1 + q_2 = 1\,. \tag{5.16}$$

A useful result can also be found from examining the result of sum $q_1 + q_2 - q_3$ and using the result of Equation 5.16 above.

$$
\begin{aligned}
q_1 + q_2 - q_3 &= 1 - q_3\,, && (5.17)\\
&= 1 - \frac{1}{\mathrm{e}^{\frac{|E|+eV/2}{k_B T_e}} + 1}\,, && (5.18)\\
&= \frac{\mathrm{e}^{\frac{|E|+eV/2}{k_B T_e}}}{\mathrm{e}^{\frac{|E|+eV/2}{k_B T_e}} + 1}\,, && (5.19)\\
&= \frac{1}{\mathrm{e}^{-\frac{|E|+eV/2}{k_B T_e}}\left(\mathrm{e}^{\frac{|E|+eV/2}{k_B T_e}} + 1\right)}\,, && (5.20)\\
q_1 + q_2 - q_3 &= \frac{1}{\mathrm{e}^{\frac{-\left(|E|+eV/2\right)}{k_B T_e}} + 1}\,. && (5.21)
\end{aligned}
$$

Substituting this result into Equation 5.12 gives:

$$
\begin{aligned}
p_T &= \int_{-\infty}^{\infty} [q_1 + q_2 - q_3 - q_4]\,\mathrm{d}E\,, && \text{(5.12 revisited)}\\
&= \int_{-\infty}^{\infty} \left[\frac{1}{\mathrm{e}^{\frac{-\left(|E|+eV/2\right)}{k_B T_e}} + 1} - q_4\right]\mathrm{d}E\,, && (5.22)\\
&= \int_{-\infty}^{\infty} \left[\frac{1}{\mathrm{e}^{\frac{-\left(|E|+eV/2\right)}{k_B T_e}} + 1} - \frac{1}{\mathrm{e}^{\frac{-\left(|E|-eV/2\right)}{k_B T_e}} + 1}\right]\mathrm{d}E\,, && (5.23)\\
p_T &= \int_{-\infty}^{\infty} \left[\frac{1}{\mathrm{e}^{\frac{\left(|E|-eV/2\right)}{k_B T_e}} + 1} - \frac{1}{\mathrm{e}^{\frac{\left(|E|+eV/2\right)}{k_B T_e}} + 1}\right]\mathrm{d}E\,. && (5.24)
\end{aligned}
$$

The total number of charges tunnelling can be found by multiplying this probability by the density of states in the superconductor $N_S(E)$ which, from Bardeen, Cooper, and Schrieffer (1957), is given by:

$$
N_S(S) = \frac{E}{\sqrt{E^2 - \Delta^2}}\,, \qquad (5.25)
$$

where Δ is half the size of the superconducting energy gap. The energy gap is a function of the electron temperature, increasing from zero just above the superconducting critical temperature, T_c, to a maximum value of $1.764/k_B T_c$ at 0 K. The size of the energy gap with decreasing temperature is shown in Figure 5.6.

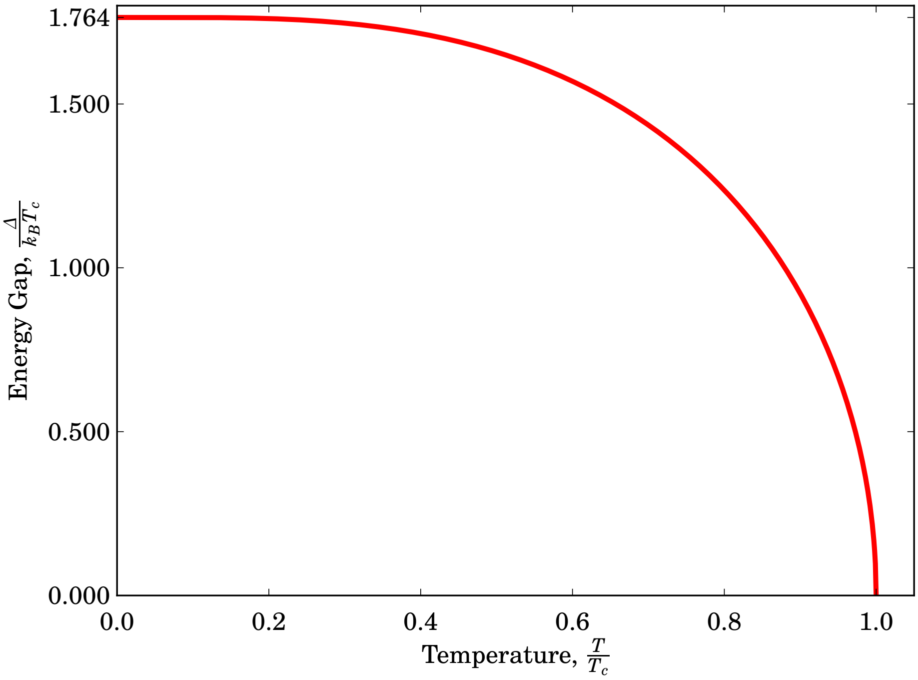

Figure 5.6: Increase in the superconducting energy gap with decreasing temperature as described by Bardeen, Cooper, and Schrieffer (1957).

Using Equation 5.25 in the result from Equation 5.24 gives the total number of charges, n, tunnelling across the barrier:

$$n = \int_{-\infty}^{\infty} \frac{|E|}{\sqrt{|E|^2 - \Delta^2}} \left[\frac{1}{\mathrm{e}^{\frac{\left(|E| - eV/2\right)}{k_B T_e}} + 1} - \frac{1}{\mathrm{e}^{\frac{\left(|E| + eV/2\right)}{k_B T_e}} + 1} \right] \mathrm{d}E\,. \tag{5.26}$$

Finally it is possible to convert this number of charges in to a tunnelling current, I, by converting Equation 5.26 to a voltage by dividing by the electron charge, e, and using Ohm's Law with the tunnelling resistance R_N.*

$$I = \frac{1}{eR_N} n\,. \tag{5.27}$$

*The subscript N denotes that this is the *normal state* resistance of an current-voltage (IV) curve.

Which, after substituting the term for n from Equation 5.26 gives the final result:

$$I = \frac{1}{eR_N} \int_{-\infty}^{\infty} \frac{E}{\sqrt{E^2 - \Delta^2}} \left[\frac{1}{\mathrm{e}^{\frac{(E - eV/2)}{k_B T_e}} + 1} - \frac{1}{\mathrm{e}^{\frac{(E + eV/2)}{k_B T_e}} + 1} \right] \mathrm{d}E\,. \tag{5.28}$$

Using this result, and if the current and the voltage of a particular device have been measured it is possible to using a parameter fitting program to calculate the electron temperature.

Inspection of Equation 5.28 shows that there is an exponential dependance on the electron temperature for the tunnelling current. It is this dependency which makes the tunnelling contacts described here high sensitive thermometers. The relationship between the electron temperature and the tunnelling current (at a constant voltage bias) is shown in Figure 5.7. The tunnelling current increases rapidly with the electron temperature until the temperature of the electrons is greater than the critical temperature; at which point the tunnelling current remains constant.

5.4 The Cooling Power

Each time a charge leaves the central island by tunnelling into one of the superconducting contacts, as described in Section 5.3, it must be replaced by a charge from one of the superconducting conducting. When the device is biased the most likely flow of charge will be from the semiconductor into the lower energy contact $\left(E_{f_S} - E_{f_{Sm}} = {-eV}/2\right)$ and for a charge from the superconductor at a higher energy $\left(E_{f_S} - E_{f_{Sm}} = {eV}/2\right)$ to fill this vacant state. This is illustrated in Figure 5.8. Since the charges which tunnel out are replaces by less energetic charges the overall energy (and thus temperature) of the changes in the central island is reduced. It is this process which is utilised to create the microrefrigerator type of device (Nahum, Eiles, and Martinis, 1994).

This cooling of charges in the central island of the structure can be expressed as a cooling or heating power, P, depending on the exact route by which charges pass through the structure this term will either be positive, meaning that energy is added to the semiconductor and there is net heating; or it will be negative, due to energy being removed and the temperature of the charges is lowered.

To derive an expression for this power it is possible to follow a similar derivation to that given in Section 5.3 to find the tunnelling current (Equation 5.28).

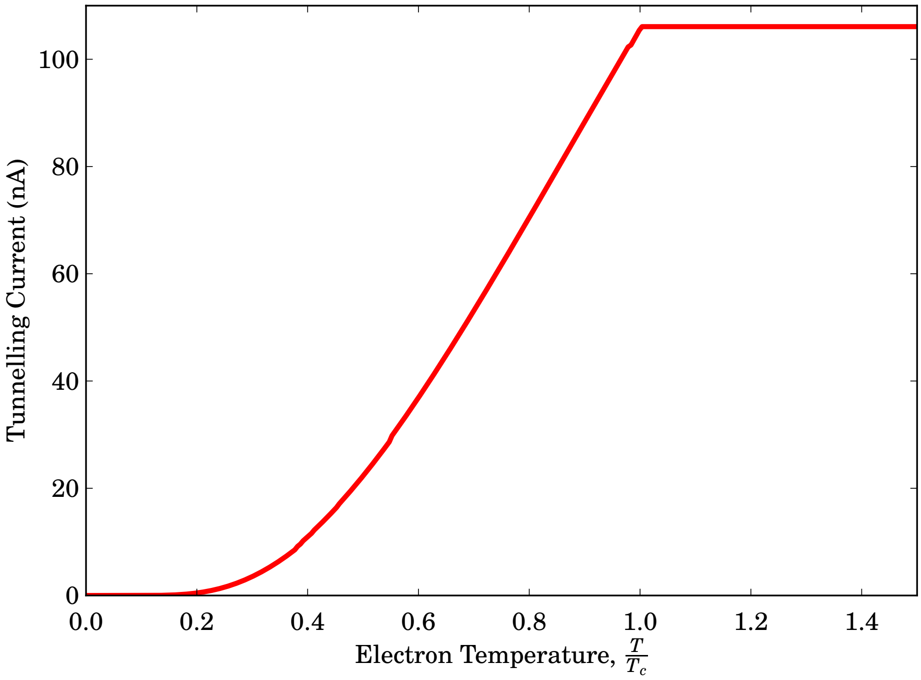

Figure 5.7: The relationship between the tunnelling current and the electron temperature. This was modelled using Equation 5.28 for a superconductor with a critical temperature of 1.2 K, biased by a voltage $V = \Delta_{T=0}$ and a tunnelling resistance, per junction, of 1 kΩ.

To do this the probability, p_T, of an occupied state on one side of the barrier corresponding to a vacant state on the other is again calculated. There are four possible routes by which charges can tunnel to or from the semiconductor. These were illustrated in Figures 5.3, 5.4 and 5.5. In order to calculate the total energy added to the semiconductor each of these probabilities needs to be multiplied by the energy of the charges tunnelling.

p_1 transfers charges with energy $E + {}^{eV}\!/_2$ from the superconducting contact into the semiconductor.

p_2 transfers charges with energy $-(E - {}^{eV}\!/_2)$ from the superconductor into the semiconductor.

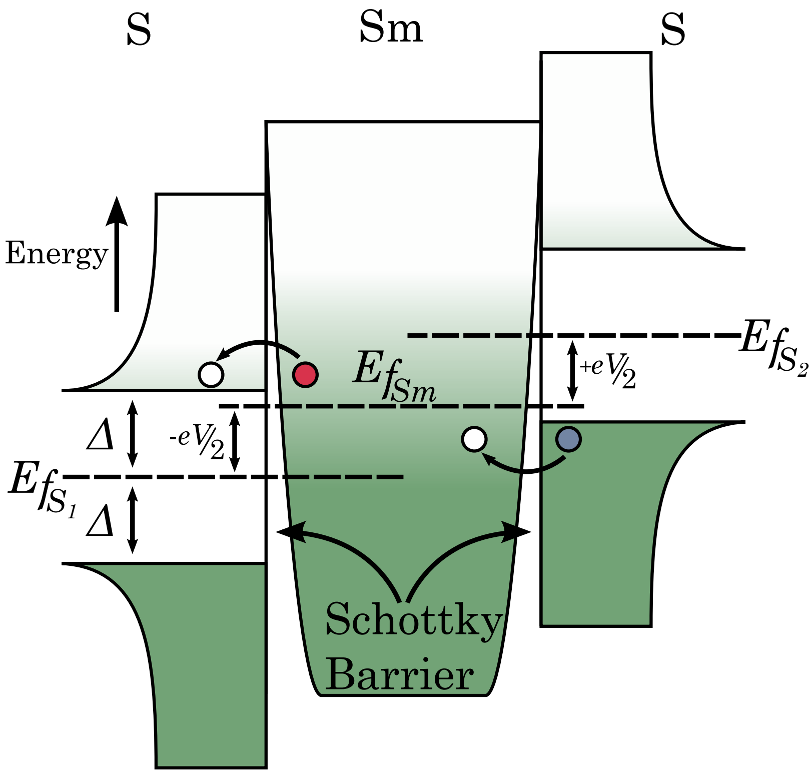

Figure 5.8: The most likely route for charges to tunnel in a biased two junction system. If the system is biased such that the energy levels in the left hand superconductor are lowered with respect the semiconductor and the states in the right hand contact are raised (as shown), the most likely movement of charges from the semiconductor will be for charges in the superconductor with energies corresponding to just above the energy gap in the left hand left hand superconductor to tunnel in to the left hand superconductor. This will create a vacant state which will be filled by a charge at the top of the superconducting state in the right hand superconductor.

p_3 removes charges with energy $E + {}^{eV}/_{2}$ from the central semiconductor, this means the energy contribution to the semiconductor is $-(E + {}^{eV}/_{2})$.

p_4 removes chagres with energy $-(E - {}^{eV}/_{2})$ from the semiconductor, this results in an energy change of $E - {}^{eV}/_{2}$ in the semiconductor.

To find the total energy transferred through this tunnelling the summation of these terms needs to be integrated over all energies and multiplied by the superconducting density of states, given by:

$$N_S(S) = \frac{E}{\sqrt{E^2 - \Delta^2}}, \tag{5.25 revisited}$$

This gives:

$$E = \int_{-\infty}^{\infty} \frac{E}{\sqrt{E^2 - \Delta^2}} \Big[E(p_1 - p_2 - p_3 + p_4) \\ + \frac{eV}{2}(p_1 + p_2 - p_3 - p_4) \Big] \, \mathrm{d}E\,. \tag{5.29}$$

One important definition which can be taken from this is that the change in energy, due movement of charges, is defined such that an increase in this term corresponds to the overall energy of the changes in the semiconductor increasing (i.e. heating of the charges).

To calculate the power, P, associated with this change in energy Equation 5.29 needs to be divide by the tunnelling resistance, R_N, (as defined on 29) and the square of the electron charge. This additional electron charge is the result of multiplying by the energy of the carries in Equation 5.29 as opposed to the charge of the carriers in Equation 5.26. One further observation needs to be made prior to arriving at a term for the tunnelling power. When deriving Equation 5.28 for the tunnelling current it sufficed to examine only a single junction. This was due to the fact that since the two junctions are in a series configuration the current through the two must be the same. In the case of the tunnelling power heat can flow through either of the two junctions, this means that for symmetrical junctions Equation 5.29 needs to be further multiplied by a factor of two. This gives:

$$P = \frac{2}{e^2 R_N} \int_{-\infty}^{\infty} \frac{E}{\sqrt{E^2 - \Delta^2}} \Big[E(p_1 - p_2 - p_3 + p_4) \\ + \frac{eV}{2}(p_1 + p_2 - p_3 - p_4) \Big] \, \mathrm{d}E\,. \tag{5.30}$$

It is useful to split this in two terms, P_1 and P_2, such that:

$$P_1 = \frac{2}{e^2 R_N} \int_{-\infty}^{\infty} \frac{E}{\sqrt{E^2 - \Delta^2}} \left[E \left(p_1 - p_2 - p_3 + p_4\right)\right] \mathrm{d}E\,, \tag{5.31}$$

$$P_2 = \frac{2}{e^2 R_N} \int_{-\infty}^{\infty} \frac{E}{\sqrt{E^2 - \Delta^2}} \left[\frac{eV}{2} \left(p_1 + p_2 - p_3 - p_4\right)\right] \mathrm{d}E\,, \tag{5.32}$$

$$P = P_1 + P_2\,. \tag{5.33}$$

Equation 5.32 for P_2 can be rewritten as:

$$P_2 = \frac{eV}{2} \frac{2}{e^2 R_N} \int_{-\infty}^{\infty} \frac{E}{\sqrt{E^2 - \Delta^2}} \left(p_1 + p_2 - p_3 - p_4\right) \mathrm{d}E\,,$$

$$P_2 = V \frac{1}{e R_N} \int_{-\infty}^{\infty} \frac{E}{\sqrt{E^2 - \Delta^2}} \left(p_1 + p_2 - p_3 - p_4\right) \mathrm{d}E\,. \tag{5.34}$$

By noting that tunnelling current, I, can be rewritten as:

$$I = \frac{1}{e R_N} \int_{-\infty}^{\infty} \frac{E}{\sqrt{E^2 + \Delta^2}} \left[p_1 + p_2 - p_3 - p4\right] \mathrm{d}E\,, \tag{5.28 rewritten}$$

and comparing this to Equation 5.34 it can be seen that the latter can be simply written as:

$$P_2 = IV\,. \tag{5.35}$$

It is possible to slightly simplify Equation 5.31, in a way similar to that performed to simplify Equation 5.9 to Equation 5.24 in Section 5.3. This starts by recalling that $p_{1\text{–}4}$ can be written as:

$$p_1 = q_1 \times (1 - q_3)\,, \tag{5.5 revisited}$$

$$p_2 = q_2 \times (1 - q_4)\,, \tag{5.6 revisited}$$

$$p_3 = q_3 \times (1 - q_1)\,, \tag{5.7 revisited}$$

$$p_4 = q_4 \times (1 - q_2)\,. \tag{5.8 revisited}$$

Thus, the term in square brackets in Equation 5.31 (which for the sake of tidiness we will temporarily call A) can be written as:

$$A = E(p_1 - p_2 - p_3 + p_4)\,, \tag{5.36}$$

$$\therefore P_1 = \frac{2}{e^2 R_N}\int_{-\infty}^{\infty} \frac{E}{\sqrt{E^2-\Delta^2}} \times A\,\mathrm{d}E\,, \tag{5.37}$$

$$A = E(q_1 \times (1-q_3) - q_2 \times (1-q_4) - q_3 \times (1-q_1) + q_4 \times (1-q_2))\,. \tag{5.38}$$

Multiplying out these terms gives:

$$A = E(q_1 - q_1 q_3 - q_2 + q_2 q_4 - q_3 + q_3 q_1 + q_4 - q_4 q_2)\,, \tag{5.39}$$

$$A = E(q_1 - q_2 - q_3 + q_4)\,. \tag{5.40}$$

This gives:

$$P_1 = \frac{2}{e^2 R_N}\int_{-\infty}^{\infty} \frac{E}{\sqrt{E^2-\Delta^2}} [E(q_1 - q_2 - q_3 + q_4)]\,\mathrm{d}E\,. \tag{5.41}$$

Since it is not possible to further simplify this, the final form of P_1 can be written as:

$$\begin{aligned} P_1 = \frac{2}{e^2 R_N}\int_{-\infty}^{\infty} \frac{E^2}{\sqrt{E^2-\Delta^2}} &\left[\frac{1}{\mathrm{e}^{\frac{E}{k_B T_S}}+1} - \frac{1}{\mathrm{e}^{\frac{-E}{k_B T_S}}+1}\right. \\ &\left. - \frac{1}{\mathrm{e}^{\frac{(E+eV/2)}{k_B T_e}}+1} + \frac{1}{\mathrm{e}^{\frac{-(E-eV/2)}{k_B T_e}}+1}\right]\mathrm{d}E\,. \end{aligned} \tag{5.42}$$

Using this result, along with that of Equation 5.35, in Equation 5.33 gives the final result of the tunnelling power as:

$$\begin{aligned} P = IV + \frac{2}{e^2 R_N}\int_{-\infty}^{\infty} \frac{E^2}{\sqrt{E^2-\Delta^2}} &\left[\frac{1}{\mathrm{e}^{\frac{E}{k_B T_S}}+1} - \frac{1}{\mathrm{e}^{\frac{-E}{k_B T_S}}+1}\right. \\ &\left. - \frac{1}{\mathrm{e}^{\frac{(E+eV/2)}{k_B T_e}}+1} + \frac{1}{\mathrm{e}^{\frac{-(E-eV/2)}{k_B T_e}}+1}\right]\mathrm{d}E\,. \end{aligned} \tag{5.43}$$

5.5 The Responsivity

Like all bolometric detectors it is possible to bias a Cold Electron Bolometer with either a voltage or a current. In either case the quantity which is not providing the bias is monitored and it is in this signal that the response to a change in incident optical power will be measured. The responsivity, S, of a detector is defined as the ratio of the change in the measured signal to the change in the power absorbed in the detector. This is written as:

$$S = \frac{\mathrm{d}signal}{\mathrm{d}P_{abs}}, \tag{5.44}$$

where $signal$ is the quantity being monitored and P_{abs} is the absorbed power. Since a bolometric detector is biased by either a voltage or a current we can define two types of responsivity. In the case of a voltage biased detector, where the current flowing through the detector is monitored, the current responsivity, S_I, is given by:

$$S_I = \frac{\mathrm{d}I}{\mathrm{d}P_{abs}}, \tag{5.45}$$

where I is the current being measured. For a detector biased by a current, the voltage responsivity, S_V is given by:

$$S_V = \frac{\mathrm{d}V}{\mathrm{d}P_{abs}}, \tag{5.46}$$

where V is the measured voltage.

These terms are general expressions and need to be rewritten such that their values can be calculated. In order to derive useful expressions for the responsivity it is important to understand the relative contributions to the heating (or cooling) of the electrons in the detector's absorber. Along with tunnelling power (Equation refres:tunnellingP) which adds or removed heat via the tunnelling current the electrons in the absorber are also affected by various other sources of heating or cooling. The most significant of which are: Joule heating, P_J, due the resistance experienced by the current within the absorber; the energy from radiative source which is absorbed by the detector, P_{abs}; and the flow of energy directly between the electron and the phonon systems, $P_{e\text{-}ph}$. Joule heating was first described by Joule (1837), it is caused the collisions between the charges flowing in the current and the atomic ions in the conductor. The heating power from these collisions is given by:

$$P_J = I^2 R\,, \tag{5.47}$$

where I is the current flowing through a resistance R.

Unless the electron and phonon systems are at thermal equilibrium there will be a flow of heat between the two due to the thermal link between the systems. The heating (or cooling) power resulting from this flow of heat is given by:

$$P_{e\text{-}ph} = \Sigma\Omega\left(T_e^{\beta} - T_{ph}^{\beta}\right), \tag{5.48}$$

where Σ is a material constant relating to how strong the thermal link between the electrons and phonons is, Ω is the volume in which the electrons and phonons are not in thermal equilibrium, T_e and T_{ph} are the temperatures of the electrons and the phonons respectively, and β is the power dependance on the temperature of the heat flow. Unlike all of the other terms mentioned (including the tunnelling power) this term is positive for the removal of energy (cooling) from the electrons and negative for heating of the electrons.

For the absorber to be at a constant temperature these terms must add up to zero. This is referred to as the heat balance equation (or the heat balance condition) and can be expressed as:

$$P + P_{abs} + P_J - P_{e\text{-}ph} = 0\,, \tag{5.49}$$

where P is the tunnelling power given in Equation 5.43, P_{abs} is the power absorbed in the detector due to incident optical power, I is the current flowing through the absorber of the detector, R_{abs} is the resistance of the detector's absorber, Σ is the material constant of the absorber, Ω is the volume of the absorber, T_e and T_{ph} are the temperatures of the electrons and phonons respectively and β is the power of temperature dependency of electron-phonon cooling power, this has been found by Prest et al. (2011) to be 6. This equation is simply the equilibrium condition for temperature of the electrons in the absorber, the first three terms are defined as being positive for heating of the electrons whereas the final term is defined as being positive for cooling of the charges. The meaning of the first two terms has already been covered, the third term is simply the Joule heating of the electrons in the absorber due to the current flowing through the absorber; the final term is the cooling of the electrons due to their thermal link to the phonons, this is positive when the temperature of the phonons is less than that of the electrons and is thus a heating power as opposed to a cooling power.

In the case of a voltage biased detector the current responsivity, S_I, can be derived by noting that Equation 5.45 can be rewritten as:

$$S_I = \frac{\mathrm{d}I}{\mathrm{d}P_{abs}} = \frac{\frac{\partial I}{\partial T_e}}{\frac{\partial P_{abs}}{\partial T_e}}. \tag{5.50}$$

The denominator of this can be found by differentiating the heat balance equation (Equation 5.49).

$$0 = \frac{\partial P}{\partial T_e} + \frac{\partial P_{abs}}{\partial T_e} + \frac{\partial}{\partial T_e} I^2 R_{abs} - \beta \Sigma \Omega T_e^{\beta-1}, \tag{5.51}$$

$$\frac{\partial P_{abs}}{\partial T_e} = \beta \Sigma \Omega T_e^{\beta-1} - \frac{\partial P}{\partial T_e} - \frac{\partial}{\partial T_e} I^2 R_{abs}. \tag{5.52}$$

Substituting this result into Equation 5.50 gives the final result:

$$S_I = \frac{\frac{\partial I}{\partial T_e}}{\beta \Sigma \Omega T_e^{\beta-1} - \frac{\partial P}{\partial T_e} - \frac{\partial}{\partial T_e} I^2 R_{abs}}. \tag{5.53}$$

This differs slightly from Golubev and Kuzmin (Equation 10 of 2001) who do not consider the Joule heating of the absorber in their model but who consider the effects of operating in an AC regime*.

It is possible to derive the voltage responsivity, S_V, similarly by starting with:

$$S_V = \frac{\mathrm{d}V}{\mathrm{d}P_{abs}} = \frac{\frac{\partial V}{\partial T_e}}{\frac{\partial P_{abs}}{\partial T_e}}. \tag{5.54}$$

Since in the current biased case the current across the detector cannot change it is possible to write:

$$\frac{\mathrm{d}I}{\mathrm{d}T_e} = 0 = \frac{\partial I}{\partial V}\frac{\partial V}{\partial T_e} + \frac{\partial I}{\partial T_e}, \tag{5.55}$$

$$\frac{\partial V}{\partial T_e} = \frac{-\frac{\partial I}{\partial T_e}}{\frac{\partial I}{\partial V}}. \tag{5.56}$$

*Also note that Golubev and Kuzmin define the tunnelling power to be positive for cooling of the electrons.

Using this in the numerator of Equation 5.54 yields:

$$S_V = \frac{\frac{-\frac{\partial I}{\partial T_e}}{\frac{\partial I}{\partial V}}}{\frac{\partial P_{abs}}{\partial T_e}}. \tag{5.57}$$

As in the case for the current responsivity, the numerator can be found by differentiating the heat balance equation. In the current biased case there are two subtle difference to that of Equation 5.52. This first is that the Joule heating term no longer depends on the electron temperature since the current is constant and thus:

$$\left(\frac{\partial}{\partial T_e} I^2 R_{abs}\right)_I = 0. \tag{5.58}$$

The second is the that the tunnelling power (given by Equation 5.43) is a function of both the electron temperature and the voltage (along with the temperature of the charges in the superconductor). In the voltage biased case the voltage was not a function of the electron temperature. In the current bias case since the current is fixed the voltage across the tunnelling contacts must vary with the temperature of the charges, this means that:

$$\frac{\mathrm{d}P}{\mathrm{d}T_e} = \frac{\partial P}{\partial V}\frac{\partial V}{\partial T_e} + \frac{\partial P}{\partial T_e}. \tag{5.59}$$

Substituting the result of Equation 5.56 gives:

$$\frac{\mathrm{d}P}{\mathrm{d}T_e} = \frac{\partial P}{\partial T_e} - \frac{\frac{\partial I}{\partial T_e}}{\frac{\partial I}{\partial V}}\frac{\partial P}{\partial V}. \tag{5.60}$$

Noting this along with the differential of the Joule heating power from Equation 5.58 the differential of the heat balance equation is now:

$$0 = \frac{\partial P_{abs}}{\partial T_e} + \frac{\partial P}{\partial T_e} - \frac{\frac{\partial I}{\partial T_e}}{\frac{\partial I}{\partial V}}\frac{\partial P}{\partial V} - \beta \Sigma \Omega T_e^{\beta-1}, \tag{5.61}$$

giving:

$$\frac{\partial P_{abs}}{\partial T_e} = \beta \Sigma \Omega T_e^{\beta-1} + \frac{\frac{\partial I}{\partial T_e}}{\frac{\partial I}{\partial V}}\frac{\partial P}{\partial V} - \frac{\partial P}{\partial T_e}. \tag{5.62}$$

Substituting this for the denominator of Equation 5.57 gives the final form of the voltage responsivity to be:

$$S_V = \frac{\frac{-\frac{\partial I}{\partial T_e}}{\frac{\partial I}{\partial V}}}{\beta \Sigma \Omega T_e^{\beta-1} + \frac{\frac{\partial I}{\partial T_e}}{\frac{\partial I}{\partial V}} \frac{\partial P}{\partial V} - \frac{\partial P}{\partial T_e}}. \tag{5.63}$$

This is the same, in the DC limit, as Golubev and Kuzmin (Equation 30 of 2001).

5.6 The Sources of Electrical Noise

In order for incoming optical radiation to be measured by a detector system the output signal produced must be greater than the noise signal of the detector or the readout system used. This means that a detector with a high level of inherent noise is less sensitive than a detector with lower level of noise. When developing a detector there are two main goals. The first is to prove that the underlying principles are in fact valid and that a functioning detector can be made. The second is to make the detector as sensitive as possible*. Because of this it is important to have a strong understanding of the noise processes involved in a detector.

There are several physical phenomena that cause noise. These can be roughly grouped into two categories: noise due to some form of fluctuation and noise resulting from the contamination of one signal by another. It is usually possible to shield electrical wiring and components to eliminate contamination of a signal however some sources may be more problematic to remove; for example 50 Hz or *mains noise* is caused by the switching AC voltage used to power electrical equipment, since it is most likely essential to use some form of mains powered equipment to monitor a detector this noise may be more problematic to remove. Noise due to fluctuations in the energy of the charges within the detector are inevitable and can, at best, only be partially reduced through cleverly designing the detector or the materials used.

In deriving a term for the expected noise measured for a Silicon Cold Electron Bolometer three types of noise will be considered: Firstly, internal noise within the detector, these terms are due to various internal factors which cause the energy of the charges to fluctuate. Secondly, noise which is the result of incident power

*It is worth mentioning that, depending upon the desired application for a detector, improvements to the detector's speed may be as desirable (or even more so) as improved sensitivity.

absorbed by the device. Finally, amplifier or readout noise which is added to the signal by the electronic systems used to monitor the detector.

For the internal noise two contributions will be considered, these are the current noise $\langle \delta I \rangle$ and the power or heat flow noise $\langle \delta P \rangle$. As can be seen from Section 5.4 (and particularly Equation 5.43) these two quantities are not uncorrelated, indeed the tunnelling power depends heavily on the current (as one might expect). To address this correlation of the two quantities a third term, the correlated noise, $\langle \delta P \delta I \rangle$, is introduced.

For these terms the fluctuations that cause the corresponding noise will be taken to be governed by Poisson statistics, meaning that:

$$\sigma_x = \sqrt{\bar{x}}\,, \tag{5.64}$$

where σ_x is the standard deviation of the quantity x and $\bar{x}$ is the mean value of x.

In the case of the current noise, the generated noise is due to the fluctuations in the number of electrons tunnelling across the Schottky contacts. The total number of charges, n, tunnelling (in any direction) for a two junction system is given by:

$$n = 2\int_{-\infty}^{\infty} \frac{E}{\sqrt{E^2 - \Delta^2}}\left[p_1 + p_2 + p_3 + p_4\right] \mathrm{d}E\,, \tag{5.65}$$

where $p_{1\text{–}4}$ are the terms defined on Page 26, the factor of two comes from the fact that charges can tunnel through two junctions. This can be converted to a total noise current, I_N by dividing by the electron charge and the tunnelling resistance to give:

$$I_N = \frac{2}{eR_N}\int_{-\infty}^{\infty} \frac{E}{\sqrt{E^2 - \Delta^2}}\left[p_1 + p_2 + p_3 + p_4\right] \mathrm{d}E\,. \tag{5.66}$$

Schottky (1918) provides the general formula for the shot noise due to the flow of current to be:

$$\langle \delta I \rangle = \sqrt{2eI}\,. \tag{5.67}$$

Substituting the noise current from Equation 5.66 gives the final equation for the shot noise due to current flow to be:

$$\langle \delta I^2 \rangle = \frac{4}{R_N}\int_{-\infty}^{\infty} \frac{E}{\sqrt{E^2 - \Delta^2}}\left[p_1 + p_2 + p_3 + p_4\right] \mathrm{d}E\,, \tag{5.68}$$

or more completely:

$$\langle \delta I^2 \rangle = \frac{4}{R_N} \int_{-\infty}^{\infty} \frac{E}{\sqrt{E^2 - \Delta^2}} \left[\frac{1}{e^{\frac{E}{k_B T_S}} + 1} + \frac{1}{e^{\frac{-E}{k_B T_S}} + 1} + \frac{1}{e^{\frac{(E + eV/2)}{k_B T_e}} + 1} \right.$$
$$+ \frac{1}{e^{\frac{-(E - eV/2)}{k_B T_e}} + 1} - 2 \frac{1}{e^{\frac{E}{k_B T_S}} + 1} \frac{1}{e^{\frac{(E + eV/2)}{k_B T_e}} + 1}$$
$$\left. -2 \frac{1}{e^{\frac{-E}{k_B T_S}} + 1} \frac{1}{e^{\frac{-(|E - eV/2)}{k_B T_e}} + 1} \right] \mathrm{d}E \,. \tag{5.69}$$

The noise due the flow of heat, either into or from the semiconducting absorber, is derived similarly. As was the case when deriving Equation 5.43 for the tunnelling power, the power noise is essentially the current noise multiplied by the energy of the charges tunnelling. Each of the routes shown in Figure 5.3 contributes a different amount of energy to the semiconductor.

As previously covered on Page page 31 of Section 5.4, each of the tunnelling routes (shown on Page page 23) contributes a different amount of energy (per charge tunnelling) to the semiconductor. To recap the energy change in the semiconductor due to each of the four routes is:

Route 1 causes an energy change of $E + eV/2$,

Route 2 causes an energy change of $-(E - eV/2)$,

Route 3 causes an energy change of $-(E + eV/E)$,

Route 4 causes an energy change of $E - eV/2$.

As was the case when calculating the current shot noise, it is only the magnitude of the fluctuations in the tunnelling power that are of interest when calculating the noise. This means that to find to power noise, P_N, Equation 5.30 can be rewritten as:

$$P_N = \frac{2}{e^2 R_N} \int_{-\infty}^{\infty} \frac{E}{\sqrt{E^2 - \Delta^2}} \left[E\,(p_1 + p_2 + p_3 + p_4) \right.$$
$$\left. + \frac{eV}{2}(p_1 - p_2 + p_3 - p_4) \right] \mathrm{d}E \,. \tag{5.70}$$

As explained by Golubev and Kuzmin (2001) the heat flow noise, $\langle \delta P \rangle$, is given by:

$$\langle \delta P \rangle = \sqrt{2EP_N}\,. \tag{5.71}$$

This is the same as Equation 5.67 for the current noise but the electron charge (e) is replaced by the energy (E) which is fluctuating. Combining these two equations give the heat flow noise to be:

$$\begin{aligned}\langle \delta P^2 \rangle = \frac{4}{e^2 R_N} \int_{-\infty}^{\infty} \frac{E}{\sqrt{E^2 - \Delta^2}} \Bigg[& E^2 (p_1 + p_2 + p_3 + p_4) \\ & + \left(\frac{eV}{2}\right)^2 (p_1 - p_2 + p_3 - p_4) \Bigg] \mathrm{d}E\,. \end{aligned} \tag{5.72}$$

Substituting for $p_{1\text{–}4}$ gives the complete result:

$$\begin{aligned}\langle \delta P^2 \rangle = \frac{4}{e^2 R_N} \int_{-\infty}^{\infty} \frac{E}{\sqrt{E^2 - \Delta^2}} \Bigg[E^2 \Bigg(& \frac{1}{\mathrm{e}^{\frac{E}{k_B T_S}} + 1} + \frac{1}{\mathrm{e}^{\frac{-E}{k_B T_S}} + 1} + \frac{1}{\mathrm{e}^{\frac{(E + eV/2)}{k_B T_e}} + 1} \\ & + \frac{1}{\mathrm{e}^{\frac{-(E - eV/2)}{k_B T_e}} + 1} + 2 \frac{1}{\mathrm{e}^{\frac{E}{k_B T_S}} + 1} \frac{1}{\mathrm{e}^{\frac{(E + eV/2)}{k_B T_e}} + 1} \\ & + 2 \frac{1}{\mathrm{e}^{\frac{-E}{k_B T_S}} + 1} \frac{1}{\mathrm{e}^{\frac{-(|E - eV/2)}{k_B T_e}} + 1} \Bigg) \\ + \left(\frac{eV}{2}\right)^2 \Bigg(& \frac{1}{\mathrm{e}^{\frac{E}{k_B T_S}} + 1} - \frac{1}{\mathrm{e}^{\frac{-E}{k_B T_S}} + 1} + \frac{1}{\mathrm{e}^{\frac{(E + eV/2)}{k_B T_e}} + 1} \\ & - \frac{1}{\mathrm{e}^{\frac{-(E - eV/2)}{k_B T_e}} + 1} + 2 \frac{1}{\mathrm{e}^{\frac{E}{k_B T_S}} + 1} \frac{1}{\mathrm{e}^{\frac{(E + eV/2)}{k_B T_e}} + 1} \\ & - 2 \frac{1}{\mathrm{e}^{\frac{-E}{k_B T_S}} + 1} \frac{1}{\mathrm{e}^{\frac{-(|E - eV/2)}{k_B T_e}} + 1} \Bigg) \Bigg] \mathrm{d}E\,. \end{aligned} \tag{5.73}$$

When combining the noise sources it is important to note that the current shot noise and the heat flow noise are correlated since the tunnelling power depends heavily on the current flowing through the junctions. As such the total noise resulting from these two source is not simply given by adding them in quadrature, instead a third term, the cross-correlator, needs to be added. For two correlated noise source (e_1 and e_2) the total noise, e_T can be found by:

$$e_T^2 = e_1^2 + e_2^2 + 2Ce_1e_2\,, \tag{5.74}$$

where the dimensionless constant C is the *correlation coefficient* which varies between -1 (if the two sources are anti-correlated) and $+1$ (if the two sources are perfectly correlated). This means the correlator (the final term in Equation 5.74) of the current and heat flow noise, $\langle\delta P\delta I\rangle$, is given by:

$$\langle\delta P\delta I\rangle = 2C\langle\delta P\rangle\langle\delta I\rangle\,. \tag{5.75}$$

The current shot noise and the power noise have been shown by Golwala, Jochum, and Sadoulet (1997) to be anti-correlated so $C = -1$. Using this results and Equations 5.67 and 5.71 gives the correlator of the current shot and power noise to be:

$$\langle\delta P\delta I\rangle = -4\sqrt{eEI_N P_N}\,. \tag{5.76}$$

Golubev and Kuzmin (2001) show that in the case $T_e = T_S$ then this can be simplified to :

$$\langle\delta P\delta I\rangle = -4eP\,. \tag{5.77}$$

Substituting Equation 5.43 for tunnelling power gives the result:

$$\langle\delta P\delta I\rangle = 4eIV + \frac{8}{eR_N}\int_{-\infty}^{\infty}\frac{E^2}{\sqrt{E^2-\Delta^2}}\left[\frac{1}{e^{\frac{E}{k_BT}}+1} - \frac{1}{e^{\frac{-E}{k_BT}}+1} - \frac{1}{e^{\frac{(E+eV/2)}{k_BT}}+1} + \frac{1}{e^{\frac{-(E-eV/2)}{k_BT}}+1}\right]\mathrm{d}E\,. \tag{5.78}$$

The quantised flow of heat between the electron and phonon systems causes thermal fluctuations which result in further electrical noise. This noise is given by the well known expression:

$$\langle\delta P\rangle_{e\text{-}ph} = \sqrt{4k_BT^2G_{e,ph}}\,. \tag{5.79}$$

$G_{e,ph}$ is the non direction specific thermal conductance between the two systems and is given by:

$$G_{e,ph} = |G_e| + \left|G_{ph}\right|\,, \tag{5.80}$$

This is true since noise results from heat flow to or from either system and is always positive. G_e and G_{ph} are given by:

$$G_e = \frac{\mathrm{d}P_{e\text{-}ph}}{\mathrm{d}T_e}, \tag{5.81}$$

$$G_{ph} = \frac{\mathrm{d}P_{e\text{-}ph}}{\mathrm{d}T_{ph}}, \tag{5.82}$$

where $P_{e\text{-}ph}$ is the heating or cooling power of the electron-phonon link, given by Equation 5.48. Substituting these into the above gives:

$$G_{e,ph} = \left|\frac{\mathrm{d}}{\mathrm{d}T_e}\Sigma\Omega\left(T_e^\beta - T_{ph}^\beta\right)\right| + \left|\frac{\mathrm{d}}{\mathrm{d}T_{ph}}\Sigma\Omega\left(T_e^\beta - T_{ph}^\beta\right)\right|, \tag{5.83}$$

$$G_{e,ph} = \beta\Sigma\Omega\left(T_e^{\beta-1} + T_{ph}^{\beta-1}\right). \tag{5.84}$$

Substituting this into Equation 5.79 gives the final form of the heat flow noise:

$$\langle\delta P\rangle_{e\text{-}ph} = \sqrt{2\beta k_B \Sigma\Omega\left(T_e^{\beta+1} + T_{ph}^{\beta+1}\right)}. \tag{5.85}$$

The factor of $\sqrt{2}$ difference between the above and Equation 5.79 can be explained by setting $T_e = T_{ph} = T$ in which case the two equations are the same.

The final source of noise to be considered is the system used to readout the detector. Inevitably this will involve some for of amplifier. Any amplifier will add a certain level of noise to a signal, this is usually the result of Johnson noise from the resistors used to set the amplifier's gain but can also be the result of shot noise due to the currents flowing in the amplifier. While intelligent design of an amplifier system (described by Horwitz and Hill, 1989) can result in amplifiers with very low noise levels (often of the order of $\mathrm{nVHz}^{-1/2}$), it is not possible to completely remove this noise source and as such it should be included when considering the fundamental limits of a system.

5.7 The Noise Equivalent Power

The electrical noise is a useful quantity in that it corresponds to what one would measure when characterising a detector however the eventual goal of any detector is not to itself be an object of study but instead be used to study other objects. As such the Noise Equivalent Power (NEP) is a more useful figure of merit since it gives the minimum power that can be detected with a signal-to-noise ratio of one and an integration time of half a second* this allows someone using the detector to calculate if it is appropriate for an application given restrictions such as: signal power, measurement time or acceptable signal-to-noise ratio.

To derive the noise equivalent power it is perhaps most logical to start by specifying exactly what is meant by the *signal-to-noise ratio*. The signal-to-noise ratio is (as its name implies) simply the ratio of the amplitude of a signal to the amplitude of any noise on the signal. As such the signal-to-noise ratio, SNR, can be expressed as:

$$SNR = \frac{V_{signal}}{V_{noise}}, \tag{5.86}$$

where V_{signal} and V_{noise} are the Root Mean Square (RMS) voltages of the signal and the noise respectively.

The physical issues associated with a low signal to noise ratio are illustrated in Figure 5.9. It is clear that with $SNR = 0.1$ (upper-left in Figure 5.9) the signal can barely be seen and neither the width or amplitude can be ascertained. When the signal-to-noise ratio is increased to unity (upper tight is Figure 5.9) the presence of a signal is clear but there are still significant uncertainties in both the amplitude and the width of the signal. When the signal-to-noise ratio is increased to 10 these uncertainties are greatly reduced and the pulse can be well characterised. Finally if the ratio is increased further to 100 the fluctuations are reduced to such a level that, except under close examination, they are not noticeable.

The units of noise equivalent power are $\mathrm{WHz}^{-1/2}$. In order to calculate the noise equivalent power in the presence of noise sources, such as Johnson noise, which are most commonly measured or calculated as a voltage these quantities need to be converted into units of Watts. This is done by dividing the noise voltage (units: $\mathrm{VHz}^{-1/2}$) by the voltage responsivity (Equation 5.63, units: VW^{-1}). Should the noise be measured or calculated in units of amperes (as is the case for Equations 5.66 and 5.77) then the noise needs to be divided by the differential of the IV-curve ($\partial I/\partial V$, units AV^{-1}) as well as the responsivity†.

*The factor of a half comes from the formal definition of NEP being the power needed to achieve a signal-to-noise ratio (SNR) of one with a 1 Hz bandwidth. Because of the Nyquist-Shannonsampling theorem (Nyquist, 1928a; Shannon, 1949) the bandwidth, Δv, is defined as $1/(2T)$, where T is the integration time.

†This paragraph assumes the detector is being current biased. Should the biasing signal be a voltage then sources measured in amperes do not need to be divided by the differential, whereas those measured in volts need to be divided by $\partial V/\partial I$ and the current responsivity is used.

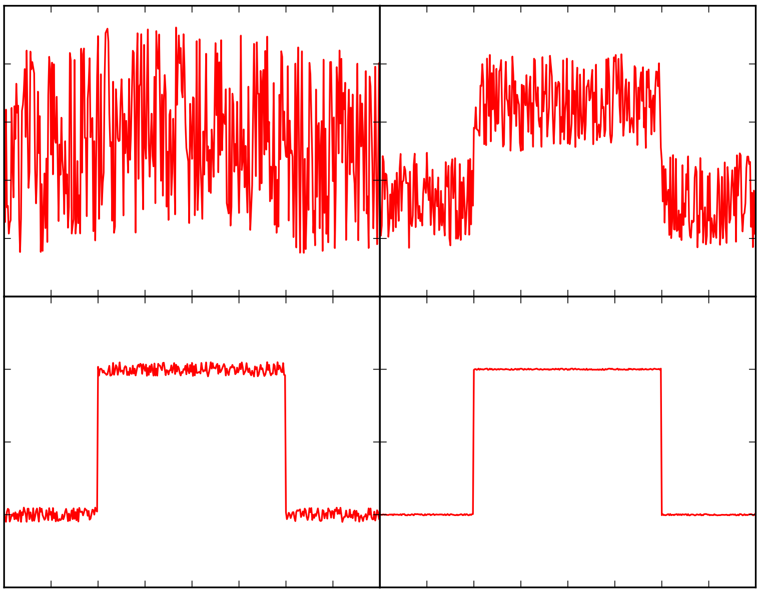

Figure 5.9: The effect of increasing the signal-to-noise ratio of a measurement. Left to right, top to bottom: $SNR = 0.1$, 1, 10, 100.

The NEP is given by the total noise, e_{tot} divided by the responsivity (i.e. the incident power that would produce a signal equal in amplitude to the noise). This means it can be written as:

$$NEP = \frac{e_{tot}}{S_V.SNR}. \tag{5.87}$$

The simplest example of calculating the noise equivalent power is to take the case where the measured noise is dominated by a single source. In the real world this is most commonly the case when the amplifier noise in the readout is not low enough. If we take the case where the amplifier noise, $\langle \delta V \rangle_{amp}$, is very large, i.e.:

$$\langle \delta V \rangle_{amp} = 100 \text{ nVHz}^{-1/2}. \tag{5.88}$$

Provided that this is significantly larger than any other noise source which contaminates the measurement then:

$$e_{tot} \approx \langle \delta V \rangle_{amp} \,. \tag{5.89}$$

If in this example the signal-to-noise ratio was 10^5 and the voltage responsivity was 1000 $\mathrm{V W^{-1}}$, then the noise equivalent power would be:

$$NEP = \frac{\langle \delta V \rangle_{amp}}{S.SNR} \,, \tag{5.90}$$

$$NEP = \frac{100 \times 10^{-9}}{1000 \times 10^{5}} \,, \tag{5.91}$$

$$NEP = 1 \times 10^{-15}\ \mathrm{W Hz^{-1/2}} \,. \tag{5.92}$$

In order to arrive at a term for total noise equivalent power of a detector system which includes a non-perfect amplifier, the various noise terms need to be converted into a noise equivalent power and then added.

Taking the noise terms individually it is possibly to see the specific conversions needed to change them to units of NEP ($\mathrm{W Hz^{-1/2}}$).

For a voltage amplifier (as was always for the experiments in this thesis) the noise is in units of $\mathrm{V Hz^{-1/2}}$ so division by the responsivity (which converts between volts and watts) is the only required step to convert the amplifier noise into a noise equivalent power.

$$NEP_{amp} = \frac{\langle \delta V \rangle_{amp}}{S_V} \,. \tag{5.93}$$

A brief inspection of the units of the heat flow noise ($\langle \delta P \rangle$) shows that this term, as expected, is already a noise equivalent power:

$$\langle \delta P \rangle = \sqrt{2 E P_N} \,, \tag{5.71 revisited}$$

$$= \sqrt{\mathrm{J W}} \,, \tag{5.94}$$

$$= \sqrt{\mathrm{W s W}} \,, \tag{5.95}$$

$$= \sqrt{\mathrm{W^2 s}} \,, \tag{5.96}$$

$$= \sqrt{\mathrm{W^2 Hz^{-1}}} \,, \tag{5.97}$$

$$\therefore \quad \langle \delta P \rangle = \mathrm{W Hz^{-1/2}} \,, \tag{5.98}$$

$$\therefore \quad NEP_P = \langle \delta P \rangle \,. \tag{5.99}$$

A similar treatment reveals that the electron-phonon heat flow noise ($\langle \delta P \rangle_{e\text{-}ph}$) is also already in units of noise equivalent power:

$$\langle \delta P \rangle_{e\text{-}ph} = \sqrt{2\beta k_B \Sigma \Omega \left(T_e^{\beta+1} + T_{ph}^{\beta+1} \right)}, \qquad \text{(5.85 revisited)}$$

$$= \sqrt{\mathrm{J\,K^{-1}\,W\,K^{-\beta}\,m^{-3}\,m^{3}\,K^{\beta+1}}}, \qquad (5.100)$$

$$= \sqrt{\mathrm{J\,W}}, \qquad (5.101)$$

which is the same as Equation 5.94. Therefore:

$$\langle \delta P \rangle_{e\text{-}ph} = \mathrm{W\,Hz^{-1/2}}, \qquad (5.102)$$

$$\therefore \quad NEP_{e\text{-}ph} = \langle \delta P \rangle_{e\text{-}ph} \, . \qquad (5.103)$$

By dimensional analysis the units of the tunnelling current noise are shown to be $\mathrm{A\,Hz^{-1/2}}$:

$$\langle \delta I \rangle = \sqrt{2eI}, \qquad \text{(5.67 revisited)}$$

$$= \sqrt{\mathrm{C\,A}}, \qquad (5.104)$$

$$= \sqrt{\mathrm{A\,s\,A}}, \qquad (5.105)$$

$$\therefore \quad = \mathrm{A\,Hz^{-1/2}}. \qquad (5.106)$$

this means that the tunnelling shot noise equivalent power NEP_S can be found by dividing the current noise by both the differential of the current by the voltage and voltage responsivity, i.e.:

$$NEP_S = \frac{\sqrt{2eI}}{\frac{\partial I}{\partial V} S_V}. \qquad (5.107)$$

The final noise term to inspect is the correlator of the noise due to the tunnelling power and current ($\langle \delta P \delta I \rangle$). This is found to have units of $\mathrm{A\,W\,Hz^{-1}}$:

$$\langle \delta P \delta I \rangle = -4eP, \qquad \text{(5.77 revisited)}$$

$$= \mathrm{C\,W}, \qquad (5.108)$$

$$= \mathrm{A\,s\,W}, \qquad (5.109)$$

$$\therefore \quad \langle \delta P \delta I \rangle = \mathrm{A\,W\,Hz^{-1}}. \qquad (5.110)$$

Which makes sense considering that dimensionally this is just the multiplication of $\langle\delta P\rangle$ and $\langle\delta I\rangle$. This means that the noise equivalent power due to this correlation of terms, NEP_{PI}, is given by:

$$NEP_{PI} = 2C\sqrt{\frac{eP}{\frac{\partial I}{\partial V}S_V}}, \tag{5.111}$$

$$NEP_{PI} = -2\sqrt{\frac{eP}{\frac{\partial I}{\partial V}S_V}} \tag{5.112}$$

Having converted the various noise sources into units of NEP it is possible to arrive at a final equation for the total noise equivalent power of a CEB detector, NEP_{CEB}. This is found by simply adding the uncorrelated noise terms in quadrature with the addition of the cross-correlator of the power and current shot noise. This gives the final result for a voltage biased device to be:

$$NEP^2_{CEB} = \frac{\langle\delta V^2\rangle_{amp}}{S_V^2} + 2\beta k_B \Sigma\Omega\left(T_e^{\beta+1} + T_{ph}^{\beta+1}\right) + \langle\delta P^2\rangle + \frac{\langle\delta P\delta I\rangle}{\frac{\partial I}{\partial V}S_V} + \frac{\langle\delta I^2\rangle}{\left(\frac{\partial I}{\partial V}S_V\right)^2}, \tag{5.113}$$

(5.114)

It is not advisable to write this equation more thoroughly (as was done for Equations 5.69 and 5.73) as this would be several pages long.