An Extended Kalman Filter for Real-Time Estimation and Control of a Rigid-Link Flexible-Joint Manipulator

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***Abstract*—High performance tracking of an industrial robot depends on accurate expression of manipulator dynamics. When a robot has flexible joints efficient model parametrization will be difficult to achieve. In this course project report an extended Kalman filter (EKF) observer to estimate manipulator states is presented. In a computer simulation the estimation performance is demonstrated. Experimental setup for KUKA/DLR Light-Weight robot is also introduced. Lagrangian method is used for robot modeling and all maple files are included on github.**

I. Introduction

In industrial robots, the presence of transmission elements such as harmonic drives and transmission belts (typically, Scara arms) long shafts (e.g., last 3-dofs of Puma) introduce flexibility effects between actuating inputs and driven outputs (Fig. 1). In the flexible joint model, not only the motor position q, but also the joint torque τ, as well as their derivatives $\dot{q}$ and $\dot{\tau}$ are namely states of the system. The measurement of the former and the numerical computation of the latter provides the state estimation required for full state feedback. For the light-weight arm and hands, these methods were presented, e.g., in [1], [2], [3], and [4].

Taking into account the elasticity of the transmission, each joint becomes a mass-spring-damper system (Fig. 2) (and thus a fourth order system), so that the complete state is given by position and velocity (as for the second order rigid robot model), and additionally by the torque and its derivative.

In the paper which is chosen for this course [5], an EKF is proposed to estimate link and motor positions/velocities in a Rigid Link Flexible Joint (RLFJ) manipulator. The main contribution of this article is the design and implementation of a new observer-controller combination for accurate estimation and control of a RLFJ manipulator. The state vector in this work contains link and motor positions/velocities, unknown dynamic and kinematic parameter estimates, and link positions.

In this report main focus is on the observer design rather than the contributions on manipulator control approach.

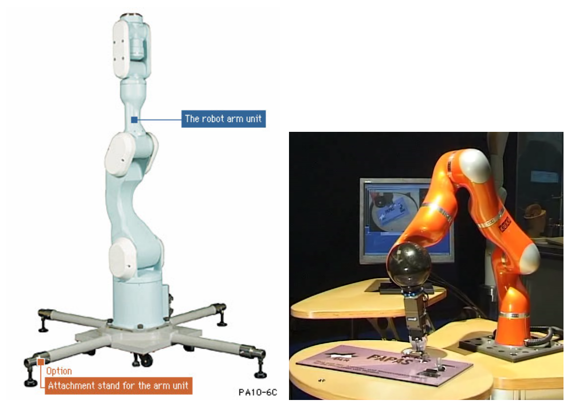

Figure 1. Mitsubishi PA10 arm and KUKA light-weight arm

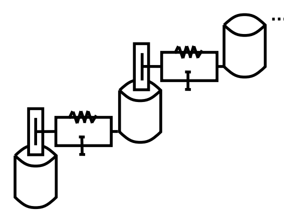

Figure 2. Flexible joint rigid link robot.

The adaptive RLFJ controller utilizes EKF-based estimates of link and motor positions/velocities.

This report is organised into six sections. Sections II and III outline the design of the EKF observer and its corresponding RLFJ controller. Section IV describes the simulation model and results. Section V details the experimental setup, procedure, and results. Section VI provides conclusions of the results.

II. Extended Kalman Filter Observer

A. Rigid-Link Flexible-Joint Model

The n-link rigid-link flexible-joint (RLFJ) robot dynamics can be expressed as

$$\begin{aligned} M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + G(q) + F_L(\dot{q}) + K_S(q - q_m) = 0 \\ J\ddot{q}_m + B\dot{q}_m + K_S(q_m - q) = u \end{aligned} \tag{1}$$

where $q(t), \dot{q}(t), \ddot{q}(t) \in R^n$ and $q_m(t), \dot{q}_m(t), \ddot{q}_m(t) \in R^n$ are the position, velocity, and acceleration of the link and motor angles, respectively, $M(q) \in R^{n\times n}$ is inertia matrix, $V_m(q,\dot{q}) \in R^{n\times n}$ is centripetal-Coriolis matrix, $G(q) \in R^n$ and $F_L(\dot{q}) \in R^n$ are the gravitational and frictional effects of the link dynamics, respectively, K_S, J, and $B \in R^n$ are constant, diagonal, positive-definite matrices representing joint stiffness, motor inertia, and motor viscous friction, respectively, and $u(t) \in R^n$ is the motor torque. There are also unknown and varying parameters in the dynamic model of the robot which can be shown by $\theta \in R^p$.

Sets of retro-reflective markers can be attached to the surface of the robot to improve the accuracy of the link position measurement. The position of the ith marker in link coordinate system L_i can be identified as $p_i^{L_i}$, with $i = 1,\ldots,m$, and the collection of all markers in their respective local coordinate systems as $p^L \in R^{3m}$. Let us also define a kinematic parameter vector $\phi \in R^6$ to represent the position and orientation of the robot coordinate system in the motion capture coordinate system. The the state vector can be defined as

$$x = \begin{bmatrix} q^T & \dot{q}^T & q_m^T & \dot{q}_m^T & \theta^T & \phi^T & (p^L)^T \end{bmatrix}^T \tag{2}$$

such that $\dot{x} = f(x,u)$ where

$$f(x,u) = \begin{bmatrix} \dot{q} \\ \ddot{q} = M^{-1}(q,\theta)(K_S(q_m - q) - V_m(q,\dot{q},\theta)\dot{q} - G(q,\theta) - F_L(\dot{q},\theta)) \\ \dot{q}_m \\ J^{-1}(u - B\dot{q}_m - K_S(q_m - q)) \\ 0 \end{bmatrix} \tag{3}$$

The random vaiable regarding the uncertainties can be grouped as $\omega = [\ \omega_L^T \quad \omega_M^T \quad \omega_\theta^T \quad \omega_\phi^T \quad \omega_P^T\ ]^T$ where ω_L and ω_M are random variables related to uncertainties in the link and motor dynamics respectively. The unknown dynamic and kinematic parameter estimates are modeled as random walk processes, such that $\dot{\theta} = \omega_\theta$, $\dot{\phi} = \omega_\phi$, and $\dot{p}^L = \omega_P$. we can write

$$\dot{x} = f(x,u) + G(x)\omega \tag{4}$$

$$G(x) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ M^{-1}(q,\theta) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & J^{-1} & 0 & 0 & 0 \\ 0 & 0 & I_{Param} & 0 & 0 \\ 0 & 0 & 0 & I_{Trans} & 0 \\ 0 & 0 & 0 & 0 & I_{Geom} \end{bmatrix} \tag{5}$$

and its first-order Taylor expansion is

$$\dot{x}_0 = f(x_0, u_0) \tag{6}$$

$$\delta\dot{x} = \frac{\partial f(x,u)}{\partial x}\delta x + G(x_0)\omega + \varphi(x, x_0, u_0) \tag{7}$$

where $\varphi(x, x_0, u_0)$ is the higher order terms. and

$$\begin{aligned} \frac{\partial f}{\partial x} = F(t) &= \begin{bmatrix} 0 & I & 0 & 0 & 0 & 0 & 0 \\ F_{21} & F_{22} & F_{23} & 0 & F_{25} & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 & 0 \\ F_{41} & 0 & F_{43} & F_{44} & F_{45} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ F_{21} &= -M^{-1}(\tfrac{\partial M}{\partial q}\ddot{q} + \tfrac{\partial V_m}{\partial q}\dot{q} + \tfrac{\partial G}{\partial q} + K_S) \\ F_{22} &= -M^{-1}(\tfrac{\partial V_m}{\partial \dot{q}}\dot{q} + V_m + \tfrac{\partial F_L}{\partial \dot{q}}) \\ F_{23} &= M^{-1}K \\ F_{25} &= -M^{-1}(\tfrac{\partial M}{\partial \theta}\ddot{q} + \tfrac{\partial V_m}{\partial \theta}\dot{q} + \tfrac{\partial G}{\partial \theta} + \tfrac{\partial F_L}{\partial \theta} + \tfrac{\partial K_S}{\partial \theta}(q - q_m)) \\ F_{41} &= J^{-1}K_S \\ F_{43} &= -J^{-1}K_S \\ F_{44} &= -J^{-1}B \\ F_{45} &= -J^{-1}(\tfrac{\partial J}{\partial \theta}\ddot{q}_m + \tfrac{\partial B}{\partial \theta}\dot{q}_m + \tfrac{\partial K_S}{\partial \theta}(q_m - q)) \end{aligned} \tag{8}$$

B. Measurement Model

Measurement vector can be defined as

$$h(x) = \begin{bmatrix} q_m^T & \dot{q}_m^T & (p^G)^T \end{bmatrix}^T \tag{9}$$

where $p^G(q,\phi,p^L) \in R^{3m}$ is the position of all markers on the links in the global coordinate system. We can consider the vector $v = [\ v_1^T \quad v_2^T \quad v_3^T\ ]^T$ as random variable, such that

$$y = h(x) + v \tag{10}$$

We can also write

$$y_0 = h(x_0) \tag{11}$$

$$\delta y = \frac{\partial h(x_0)}{\partial x}\delta x + v + \xi(x, x_0) \tag{12}$$

where $\xi(x, x_0)$ is the higher order terms. and

$$\begin{aligned} \frac{\partial h}{\partial x} = H(t) &= \begin{bmatrix} 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 & 0 \\ H_{31} & 0 & 0 & 0 & 0 & H_{36} & H_{37} \end{bmatrix} \\ H_{31} &= \begin{bmatrix} \frac{\partial p_1^G}{\partial q}^T & \ldots & \frac{\partial p_M^G}{\partial q}^T \end{bmatrix} \\ H_{36} &= \begin{bmatrix} \frac{\partial p_1^G}{\partial \phi}^T & \ldots & \frac{\partial p_M^G}{\partial \phi}^T \end{bmatrix} \\ H_{37} &= \begin{bmatrix} \frac{\partial p_1^G}{\partial p_1^{L_1}}^T & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{\partial p_1^G}{\partial p_M^{L_M}}^T \end{bmatrix} \end{aligned} \tag{13}$$

C. Continuous-Time Extended Kalman Filter

In the reviewed paper [5], it attempts a slight modification of the extended Kalman filter to adjust the dynamics with an aim of increasing the domain of attraction and reducing the time for error decay. The basic method consists of determining the observer gain for a fictitious more instable system. This is done by supplying the system matrix, which is set-up by a standard linearisation, with an additive unstable term. For the case of linear stochastic systems the design is known as exponential data weighting.

Given the system and measurement models in (4) and (10), having zero-mean Gaussian noise covariance matrices $Q(t)$ and $R(t)$, the state estimate and error covariance update equations are

$$\dot{\hat{x}} = f(\hat{x}, u) + K(t)[y(t) - h(\hat{x})] \tag{14}$$

$$\begin{aligned}\dot{P}(t) &= (F(\hat{x}, t) + \alpha I_n)P(t) + P(t)(F(\hat{x}, t) + \alpha I_n)^T \\ &+ G(\hat{x}, t)Q(t)G^T(\hat{x}, t) - K(t)H(t)P(t)\end{aligned} \tag{15}$$

with $\alpha > 0$ and the Kalman gain $K(t) = P(t)H^T(t)R^{-1}$.

There are three assumptions which must be satisfied in order to prove exponential stability of the EKF which can be found in [5] and [6].

D. Discrete-Time Extended Kalman Filter

The discrete-time EKF is an extension of the EKF to discrete-time systems. The updated state estimate and error covariance matrix are obtained from the discrete-time EKF equations

$$x_k^+ = x_k^- + K_k(z_k - H_k x_k^-) \tag{16}$$

$$P_k^+ = (I - K_k H_k)P_k^-, \tag{17}$$

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1} \tag{18}$$

III. Controller

Since the main focus of this report is on the state observer part not control contributions readers are referred to part III of [5] for details on the proposed adaptive back-stepping controller.

IV. Simulation

Simulations has been done for 2DOF planar robot (SCARA). Matrix F which is the partial derivative of the robot dynamics with respect to the states is calculated for SCARA using maple. The maple file is attached to this report.

Fig.(3), Fig.(5), Fig.(7), and Fig.(9) show the estimation of q_1, q_2, $\dot{q}_1$, and $\dot{q}_2$ respectively. Fig.(4), Fig.(6), Fig.(8), and Fig.(10) show the error of estimation of q_1, q_2, $\dot{q}_1$, and $\dot{q}_2$ respectively.

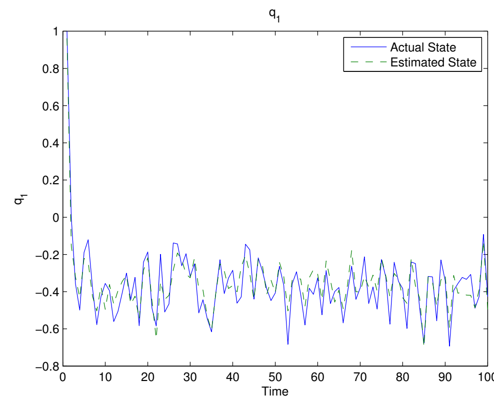

Figure 3. Estimation of q_1

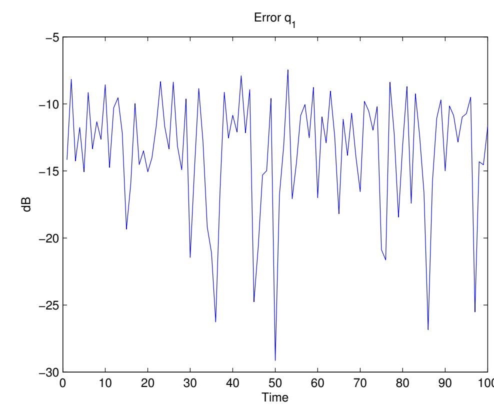

Figure 4. Estimation error of q_1

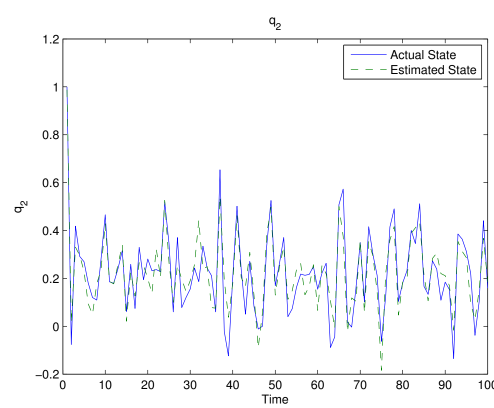

Figure 5. Estimation of q_2

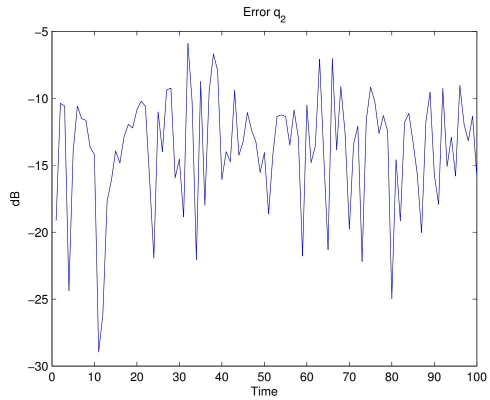

Figure 6. Estimation error of q_2

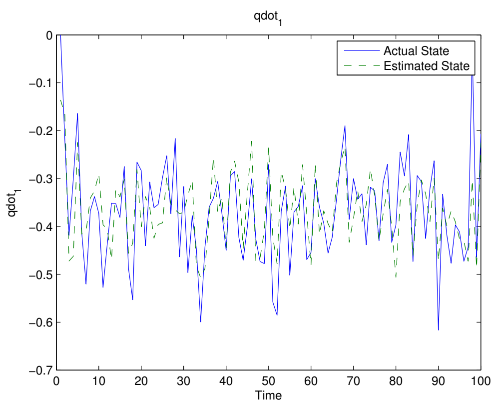

Figure 7. Estimation of $\dot{q}_1$

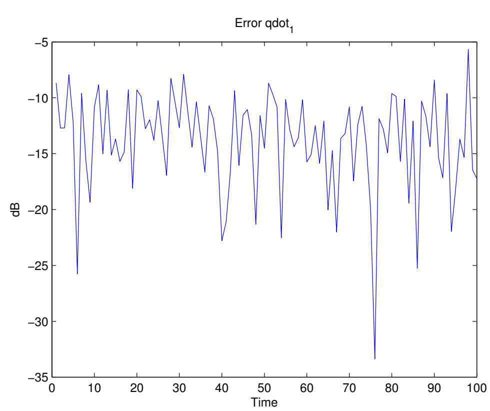

Figure 8. Estimation error of $\dot{q}_1$

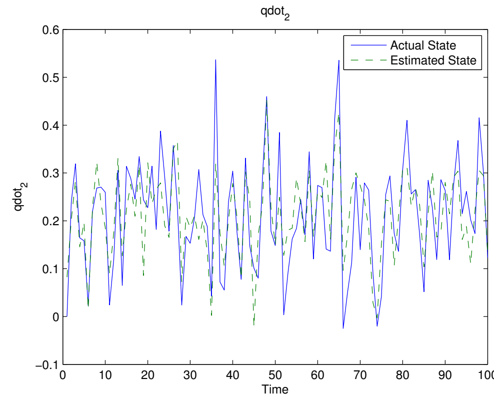

Figure 9. Estimation of $\dot{q}_2$

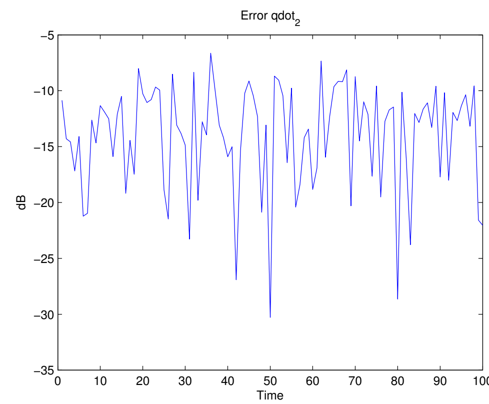

Figure 10. Estimation error of $\dot{q}_2$

V. Experiment

A. Experimental Setup

DLR/KUKA Light Weight Robot III in Human Robot Interaction Lab. at the University of Western Ontario is considered to be used as a test bed for the proposed controller and state observer. This robot is shown in Fig.11.

Figure 11. DLR/KUKA Light Weight Robot III, Human robot Interaction Lab., University of Western Ontario.

B. Implementations Issues

Since DLR/KUKA Light Weight Robot III is a commercial product, the company provides very little information regarding the robot modeling. For this report modeling procedures has been carried out however without any success. Lagrangian method has been used for robot modeling and all maple files are on github.

VI. CONCLUSION

This report discusses a novel observer-controller combination for accurate estimation and control of a RLFJ manipulator. The EKF-RLFJ controller does not require direct measurements of link positions, critical for control of industrial manipulators lacking link position encoders. Simulations regarding this state observer were provided. The EKF-RLFJ controller has the potential to further improve real-time tracking performance via inclusion of more dynamic parameters in the EKF model. In addition, this observer-controller should provide an even greater performance improvement in manipulators with greater joint flexibility.

REFERENCES

[1] A. Albu-Schäffer, C. Ott, and G. Hirzinger, "A unified passivity-based control framework for position, torque and impedance control of flexible joint robots," *The International Journal of Robotics Research*, vol. 26, pp. 23 – 39, 2007.

[2] C. Ott, A. Kugi, and G. Hirzinger, "On the passivity-based impedance control of flexible joint robots," *Robotics, IEEE Transactions on*, vol. 24, pp. 416 – 429, 2008.

[3] A. Albu-Schäffer, S. Haddadin, C. Ott, A. Stemmer, T. Wimböck, and G. Hirzinger, "The dlr lightweight robot: design and control concepts for robots in human environments," *Industrial Robot: An International Journal*, vol. 34, pp. 376 – 385, 2007.

[4] T. Wimboeck, C. Ott, and G. Hirzinger, "Passivity-based object-level impedance control for a multifingered hand," in *Intelligent Robots and Systems, 2006 IEEE/RSJ International Conference on*, 2006.

[5] C. A. Lightcap and S. A. Banks, "An extended kalman filter for real-time estimation and control of a rigid-link flexible-joint manipulator," *Control Systems Technology, IEEE Transactions on*, vol. 18, pp. 91 – 103, 2010.

[6] K. Reif, F. Sonnemann, and R. Unbehauen, "An ekf-based nonlinear observer with a prescribed degree of stability," *Automatica*, vol. 34, pp. 1119 – 1123, 1998.