



Analysis of the hedging effect of financial futures based on improved machine learning algorithms

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Abstract

As a financial asset with multiple values, financial futures are widely welcomed by people, and many investors are willing to invest in and hold financial futures. However, although financial futures have good hedging characteristics, their prices will still be affected by external factors and fluctuate. The supplement to the existing mean spillover and volatility spillover effects enriches the research system on spillover effects in financial futures markets and helps to deepen the understanding of spillover effects in financial futures markets. Based on machine learning algorithms, this paper constructs an analysis model of the hedging effect of financial futures. Moreover, this paper combines financial theory for modeling and empirical analysis, and follows a simple to complex research strategy, and uses a combination of quantitative analysis and qualitative analysis to conduct research. In addition, based on the continuous quadratic variation theory, this paper combines the market microstructure noise model and the high-frequency financial measurement model to analyze the hedging effect of financial futures, and conducts research through examples. The research results show that the performance of the model constructed in this paper is good.

Keywords Machine learning · Improved algorithm · Financial futures · Value preservation effect

1 Introduction

In theory, if a company holds financial futures to hedge its existing risks, then as a way of risk management, the behavior of companies holding financial futures is beneficial to the value of the company. However, in practice, due to various reasons, the motivation of enterprises to hold financial futures has changed. Moreover, speculative behavior in financial futures is widespread, which has positive or negative impacts on enterprises, and increases the uncertainty of the effect of the use of company derivatives, such as the Barings Bank incident, the Procter & Gamble incident, the China Aviation Oil "movement" incident, and so on. Therefore, we can no longer simply attribute the motivation of companies to hold financial futures to hedging. However, under the constraints of the existing regulatory system, company managers can only trade financial futures in the name of hedging. Therefore, on the surface, we cannot

distinguish between hedging and speculation in enterprises. This requires us to find a way to explore whether the motivation of the company in the process of holding financial futures has changed (Siva et al. 2017).

With the development of my country's market economy and the continuous improvement of the derivatives market, more and more companies use derivatives to hedge their value will be the general trend. Moreover, my country is still in the process of exchange rate reform and interest rate marketization. With the advancement of exchange rate reform and interest rate reform, enterprises will also face the risk of more exchange rate and interest rate fluctuations. Compared with developed countries such as Europe and the United States, my country's derivatives market is still in its infancy, and companies' understanding and use of derivatives need to be continuously improved, and many companies have caused huge losses due to improper use of derivatives (Ansari et al. 2019).

The new Accounting Standards for Business Enterprises have made relevant provisions on the recognition and measurement of derivative financial instruments, hedging, the transfer of financial assets and the presentation of financial instruments. Moreover, the information on the use of

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derivatives by enterprises has also been included in the form of disclosure in the notes from the original voluntary principle, which is undoubtedly a huge improvement for the disclosure of information on derivatives. Different companies have different degrees of compliance with accounting standards, that is, the degree of disclosure of information on the use of derivatives by companies is different, which results in users of the company's annual report being unable to fully and accurately obtain the company's production and operation status. Therefore, we believe that to improve the quality of information disclosure of derivatives, the Ministry of Finance needs to issue detailed rules on the confirmation, measurement and recording of derivatives. At the same time, based on the problems found in the empirical research, we can put forward suggestions to adjust and refine the corporate accounting standards.

2 Related work

Based on the assumption that the direction and magnitude of changes in futures and spot prices are exactly the same, the literature (Bikmetov et al. 2017) proposed to conduct hedging transactions with the same amount in the futures market and in the opposite direction as in the spot market. In this way, the profit and loss of the two markets are offset, and systemic risks can be completely resolved.

The literature (Hussain and Beg 2019) put forward the theory of selective hedging from the perspective of arbitrage. The theory believes that the trading motives of hedgers and arbitrage traders are the same, that is, trying to obtain profits from changes in the prices of spot and futures. Moreover, they decide whether to hedge by judging whether the basis, that is, the price difference between spot and futures, will change. Therefore, they use hedging transactions to avoid systemic risks in the spot market only when the expected basis changes.

Literature (Mao et al. 2017) proposed the minimum variance theory. The variance is used in statistics to measure the degree of deviation of random variables from mathematical expectations. In economics, it can be used to measure risk. The smaller the variance, the smaller the fluctuation of the data and the smaller the risk.

The literature (Rabie et al. 2019) proposed the minimum variance model. This model refers to investors doing opposite trading operations in the futures and spot markets for hedging, and then treating the entire trading operation as an asset portfolio in order to minimize the variance of the portfolio. The final conclusion is that the best hedging ratio is equal to the covariance of spot futures divided by the variance of futures.

The literature (Saneja and Rani 2019) used the variance of the entire asset portfolio to measure the risk of the

portfolio, and regarded futures and spot as a complete portfolio. The goal is to minimize the variance of the return on the entire portfolio, that is, minimize risk. The dependent variable and independent variable in the model can be the level of return or the change of the return. Moreover, the model uses the least square method to regress the returns or changes in both spot and futures returns. The estimated value of the slope coefficient of the equation is the optimal hedging ratio, and the estimated value of the intercept term is the average change in the price difference between spot and futures. The literature (Tu et al. 2017) put forward a utility evaluation index to quantify hedging in the futures market on the basis of using the variance of the entire asset portfolio to measure the risk of the investment portfolio. This indicator is to measure the effect of hedging to avoid risk by comparing the level of variance reduction before and after hedging. This indicator can be used to judge whether the goal of risk aversion has been achieved. In order to eliminate the defect that the least squares method will be affected by the autocorrelation of the residual series, the literature (Vimala and Sharmili 2018) developed a bivariate autoregressive model (Bivariate-VAR model, B-VAR). The two groups of non-stationary time series data proposed in (Wosiak and Zakrzewska 2018) have a stationary linear combination, which is based on the existence of a cointegration relationship, and an error correction model is proposed. The model considers this long-term co-integration relationship between spot and futures prices, and adds an error correction term to the B-VAR model. In general, the error correction term is the difference between the lag value of the spot and futures. Moreover, the calculation method of the minimum risk hedge ratio of the model is the same as that of the bivariate autoregressive model. After adding the error correction term, the model obtains better hedging performance than the least square method. After comparing the hedging effects of the above three types of models, the literature (Yang et al. 2019) pointed out that the model with error correction term greatly improved the utility of hedging transactions. Therefore, the long-term co-integration relationship between spot and futures must be considered when calculating the optimal hedging ratio.

Based on the autoregressive conditional heteroscedasticity (ARCH) model, literature (Bouteraa and Abdallah 2017) proposed a generalized autoregressive conditional heteroscedasticity (GARCH) model, which later became the most widely used dynamic hedging model in the later period. Literature (Chen et al. 2018) proposed that the perturbed variance stability in the autoregressive conditional heteroscedastic model is worse than commonly assumed. Moreover, the prediction error is relatively small in one period and relatively large in another period, indicating that there is heteroscedasticity in the time series data, and there is a certain correlation in the variance of this prediction error.

Therefore, the autoregressive conditional heteroscedasticity model is to describe the correlation between the variances. Subsequently, the improved GARCH model in the literature (Delavari 2017) considered the dependence on all historical values and established a more extensive model. With the proposal and continuous expansion of these two types of models, the literature (Elgammal and El-Naggar 2017) used the binary GARCH model to calculate the optimal hedging ratio of futures, adding the effect of time-varying variance in the forecasting process. Therefore, the coefficient Beta obtained after regression is the optimal hedging ratio. By comparing it with the hedging performance of the static model, it is found that the GARCH model with the time-varying variance effect improves the utility of the traditional static hedging model. The literature (Guo et al. 2018) considered the transaction costs incurred during dynamic hedging, and compared with other static hedging models, it was concluded that the performance of GARCH model is better than other static hedging models. The literature (He et al. 2018) found that most of the financial time series showed sharp peaks and thick tails, with different edge distributions. Therefore, researchers began to integrate mathematical functions into the model for hedging research. The literature (Jung 2018) added the mathematical function copula to the GARCH model to estimate the hedge ratio. It first used the GARCH model to fit the marginal distribution, and then used the copula function to obtain the joint distribution of the two time series of spot and futures according to the marginal distribution, thereby obtaining the correlation coefficient. The two-variable autoregressive model takes the returns of spot and futures or the change of returns as dependent variables and uses all lagging terms as independent variables to establish the regression equations of spot returns and futures returns respectively. After that, it obtains two random error terms from the regression equation. Then, the optimal hedging ratio is the covariance of two random error terms divided by the variance of the futures random error term (Le et al. 2017).

3 Implementation of futures hedging strategy

Generally, the parameters that characterize investors' decision-making can be roughly estimated through specially designed questionnaires. The distribution parameters of futures prices and spot prices are different due to different econometric models. These econometric models are usually estimated based on historical time series data of spot portfolio and stock index futures yields. In order to eliminate the influence of decision parameter estimation errors, the empirical analysis mainly considers the minimum variance hedging problem.

The single-period minimum variance hedge ratio is:

$$h_t^* = \frac{Cov(R_{s,t}, R_{f,t})}{Var_t(R_{f,t})} \quad (1)$$

Therefore, when estimating the single-period minimum variance hedge ratio, the simple rates of return $R_{s,t}$ and $R_{f,t}$ are optional modeling variables. In addition, due to:

$$R_{s,t} = \frac{S_{t+1} - S_t}{S_t}, R_{f,t} = \frac{F_{t+1} - F_t}{F_t} \quad (2)$$

Marked as

$$\Delta S_t = S_{t+1} - S_t, \Delta F_t = F_{t+1} - F_t \quad (3)$$

Then (1) can be reduced to

$$h_t^* = \frac{S_t}{F_t} \frac{Cov(\Delta S_t, \Delta F_t)}{Var(\Delta S_t)} \quad (4)$$

Therefore, the price changes ΔS_t and ΔF_t can also be used as modeling variables (Panwar 2017).

Although in theory there is an equivalent change relationship between different modeling variables, in empirical research, we need to make appropriate choices based on the available data and the statistical characteristics of the modeling variables. Except for a small number of literatures that choose $R_{s,t}$ and $R_{f,t}$ as modeling variables, very few literatures choose ΔS_t and ΔF_t as modeling variables, and most of the literatures choose the modeling variables for logarithmic returns $r_{s,t} = \ln(1 + R_{s,t})$ and $r_{f,t} = \ln(1 + R_{f,t})$. The main reasons for choosing $r_{s,t}$ and $r_{f,t}$ as modeling variables are: The value range of $r_{s,t}$ and $r_{f,t}$ does not need to be limited to the interval $[-1, \infty)$ like the simple rate of return, and $r_{s,t}$ and $r_{f,t}$ in actual data have better statistical characteristics. In addition, in the analysis of multi-period problems, the multi-period logarithmic rate of return of a single asset can be obtained by simply summing the single-period logarithmic rate of return. The multi-period simple return of a single asset needs to be multiplied by the single-period simple rate of return. It should be noted that when $r_{s,t}$ and $r_{f,t}$ are selected as modeling variables, the optimal hedge ratio according to the above formula is:

$$h_t^* = \frac{Cov(e^{r_{s,t}}, e^{r_{f,t}})}{Var_t(e^{r_{f,t}})} \quad (5)$$

However, most literature simply replaces $R_{s,t}$ and $R_{f,t}$ with $r_{s,t}$ and $r_{f,t}$, and calculates the hedge ratio by the following formula:

$$h_t = \frac{Cov(r_{s,t}, r_{f,t})}{Var_t(r_{f,t})} \quad (6)$$

The reason is that the method assumes that $r_{s,t}$ and $r_{f,t}$ can approximate $R_{s,t}$ and $R_{f,t}$ numerically. When the time span corresponding to a single period is small, usually when the values of $R_{s,t}$ and $R_{f,t}$ are small, $r_{s,t}$ and $r_{f,t}$ can indeed approximate $R_{s,t}$ and $R_{f,t}$ better. When $R_{s,t}$ and $R_{f,t}$ are used as random variables with small fluctuations, the distribution of $r_{s,t}$ and $r_{f,t}$ can also approximate the distribution of $R_{s,t}$ and $R_{f,t}$ well. However, when the time span corresponding to a single period is large, or when the rate of return on assets fluctuates drastically during this period, the error caused by this approximation will not be negligible.

According to formula (5) and formula (6), there is obviously $h_t^* \neq h_t$. In order to compare h_t^* and h_t more clearly, we assume that $r_{s,t}$ and $r_{f,t}$ obey the following price process:

$$r_{s,t} = \mu_{s,t} + \sigma_{s,t} \varepsilon_{s,t} \quad (7)$$

$$r_{f,t} = \mu_{f,t} + \sigma_{f,t} \varepsilon_{f,t} \quad (8)$$

Among them,

$$\begin{pmatrix} \varepsilon_{s,t} \\ \varepsilon_{f,t} \end{pmatrix} \sim N \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho_t \\ \rho_t & 1 \end{pmatrix} \right) \quad (9)$$

The settings of $r_{s,t}$ and $r_{f,t}$ in Eqs. (8) and (9) are universal, and they include most models involved in empirical analysis. Under the above model settings, according to the properties of the first two moments of the normal distribution and the lognormal distribution, we can get:

$$h_t = \rho_t \frac{\sigma_{s,t}}{\sigma_{f,t}} \quad (10)$$

$$h_t^* = \frac{e^{\mu_{s,t} + \frac{\sigma_{s,t}^2}{2}} (e^{\rho_t \sigma_{s,t} \sigma_{f,t}} - 1)}{e^{\mu_{f,t} + \frac{\sigma_{f,t}^2}{2}} (e^{\frac{\sigma_{f,t}^2}{2}} - 1)} \quad (11)$$

Comparing Eqs. (10) and (11), it is not difficult to find that even if the conditional distributions of $r_{s,t}$ and $r_{f,t}$ are normal distributions, there is no simple connection between h_t^* and h_t , and even the numerical characteristics that affect the two are different: h_t depends on the digital feature $\sigma_{s,t} \sigma_{f,t} \rho_t$, while h_t^* also depends on the conditional expectations $\mu_{s,t}$ and $\mu_{f,t}$. When the digital feature $\mu_{s,t} \mu_{f,t} \sigma_{s,t} \sigma_{f,t} \rho_t$ of the conditional distribution is time-varying, the difference between h_t^* and h_t may be large in some periods. In addition, when the conditional distribution of $r_{s,t}$ and $r_{f,t}$ is non-normal, h_t still only depends on $\sigma_{s,t} \sigma_{f,t} \rho_t$, but since the correspondence between $r_{s,t}$ and $r_{f,t}$ and $R_{s,t}$ and $R_{f,t}$ is nonlinear, the analytical expression of h_t^* is usually not available. h_t^* cannot be determined by the first

second-order conditional moments of $r_{s,t}$ and $r_{f,t}$. When calculating h_t^* , it is necessary to specify the conditional distribution of $r_{s,t}$ and $r_{f,t}$ and then use numerical integration or Monte Carlo simulation.

Based on the above analysis, for the single-period minimum variance hedging problem, three different modeling variables can be selected, that is, simple yields $R_{s,t}$ and $R_{f,t}$, logarithmic yields $r_{s,t}$ and $r_{f,t}$, and price changes ΔS_t and ΔF_t . However, after selecting the modeling variables, the calculation formula of the optimal hedge ratio needs to be adjusted accordingly (Zhao et al. 2018).

4 Estimation model of hedge ratio

In fact, most relevant measurement models are mainly used to describe and estimate the variance and covariance involved in the minimum variance hedge ratio. Because the variance and covariance involved in formula (1) are based on the conditional variance and conditional covariance of investor information set F_t . Therefore, for different hedging periods $[t, t+1]$, the minimum variance hedging ratio is a function of the information set, which is random and time-varying. According to the assumption of whether the hedging ratio of $[t, t+1]$ during hedging period will change with time t , corresponding econometric models can be designed respectively.

1. The estimation model of the hedging ratio from time to time

According to formula (1), when estimating the hedging ratio from time to time, $R_{s,t}$ and $R_{f,t}$ are suitable modeling variables. In fact, when the optimal hedge ratio has nothing to do with the utility function, the correct estimation model is set as the following formula (12).

(1) Simple linear regression model

$$R_{s,t} = \alpha + \beta R_{f,t} + \varepsilon_t \quad (12)$$

Among them, ε_t is the error term, the mean is 0, and the variance exists. Using the least squares method to estimate formula (12), the estimated value of β is the minimum variance hedge ratio during $[t, t+1]$ period.

Least squares estimation has the advantages of robustness and convenience, but the premise of its good properties is strict. The setting of model (12) is relatively simple, and does not take into account the impact of changes in past spot prices and futures market prices on current price changes. The hedge ratio estimated by model (12) may be biased. In addition, the error term ε_t is likely to be heteroscedastic, and the least square method cannot effectively estimate the model (12).

(2) Vector autoregressive model (VAR)

$$R_{s,t} = \alpha_s + \beta_{s1}^s R_{s,t-1} + \beta_{f1}^s R_{f,t-1} + \varepsilon_{s,t} \quad (13)$$

$$R_{f,t} = \alpha_f + \beta_{s1}^f R_{s,t-1} + \beta_{f1}^f R_{f,t-1} + \varepsilon_{f,t} \quad (14)$$

Among them, $(\varepsilon_{s,t}, \varepsilon_{f,t})$ is the error vector, the mean value is 0, and the covariance matrix \sum_t is the positive definite constant matrix $\begin{pmatrix} \sigma_s^2 & \sigma_{sf} \\ \sigma_{sf} & \sigma_f^2 \end{pmatrix}$, so the minimum variance hedge ratio during the $[t, t+1]$ period is σ_{sf}/σ_f .

Model (12) is the VAR(1,1) model, which can be extended to a more general VAR(m, n) model in practical applications. If there are no special instructions, if the models appearing below have components similar to VAR(1,1), they can be extended to VAR(m, n) type. Compared with model (9), model (10) (11) considers the possibility that changes in past spot prices and futures market prices may affect current price changes, and weakens the problem of missing variables and the correlation problem of residual sequence in model (9).

(3) Error correction model (ECM).

Models (12) (13) (14) describe the behavior of futures returns and spot returns. The rate of return is obtained based on the price difference, which may lose long-term information in the price data, such as the long-term equilibrium relationship between futures prices and spot prices. The error correction model of the futures rate of return and the spot rate of return is considered.

$$R_{s,t} = \alpha_s + \beta_{s1}^s R_{s,t-1} + \beta_{f1}^s R_{f,t-1} + \lambda_s z_{t-1} + \varepsilon_{s,t} \quad (15)$$

$$R_{f,t} = \alpha_f + \beta_{s1}^f R_{s,t-1} + \beta_{f1}^f R_{f,t-1} + \lambda_f z_{t-1} + \varepsilon_{f,t} \quad (16)$$

Among them, $z_t = S_t - \phi F_t$ is the error correction term, $S_t - \phi F_t$ is the cointegration relationship between the futures price and the spot price, and $(\varepsilon_{s,t}, \varepsilon_{f,t})$ is the error vector, with the mean value being 0. At the same time, the covariance matrix \sum_t is a positive constant moment $\begin{pmatrix} \sigma_s^2 & \sigma_{sf} \\ \sigma_{sf} & \sigma_f^2 \end{pmatrix}$, so the minimum variance hedge ratio during $[t, t+1]$ period is σ_{sf}/σ_f .

2. Estimation model of time-varying hedge ratio

By expanding the estimation model of the hedging ratio from time to time, the error vector $(\varepsilon_{s,t}, \varepsilon_{f,t})$ is set to an appropriate time-varying volatility model, and the time-varying conditional minimum variance hedging ratio can be obtained. Among them, the time-varying models to choose from are mainly autoregressive conditional heteroscedasticity models (GARCH) and stochastic volatility models (SV). GARCH-type models are widely used due to their advantages of flexible setting, easy estimation and application. In this section, only GARCH-type models will be considered.

To ensure the positive definiteness of the conditional covariance matrix, it is necessary to limit the setting form of the multivariate GARCH model. In addition, the multivariate GARCH model usually contains a large number of parameters to be estimated. In practice, it is often necessary to reasonably limit the parameters to simplify the model. The BEKK-GARCH model proposed by Engle and Kroner (1995) can better overcome the above-mentioned problems. It can ensure the positive definiteness of the conditional covariance matrix. It also considers the cross-sectional correlation of multivariate random variables and requires fewer parameters to estimate. Subsequently, Kroner and Ng (1998) considered the asymmetric impact of returns on volatility and proposed the asymmetric BEKK-GARCH model. Although the BEKK-GARCH model has many good properties, the parameters of the model have no intuitive economic explanation, and the correlation between the variables contained in the model is random. In order to better understand the volatility of variables themselves and to more clearly and concisely describe the change pattern of correlation between variables, scholars have begun to model the volatility of variables and the correlation between variables separately. Bollerslev (1990) assumes that the conditional correlation coefficient matrix of variables is time-invariant and introduces CCC-GARCH model.

Taking into account the nature of the model and its ability to describe, this paper considers the use of binary BEKK-GARCH, CCC-GARCH, DCC-GARCH and their extended models to model the volatility of futures returns and spot returns. The rate of return vector $R_t = (R_{s,t}, R_{f,t})$ and its conditional mean vector $\mu_t = (E_t(R_{s,t}), E_t(R_{f,t}))$ are recorded. $\varepsilon_t = (\varepsilon_{s,t}, \varepsilon_{f,t})$ is the rate of return new interest vector, then there is $R_t = \mu_t + \varepsilon_t$. At the same time, the conditional mean vector of innovation ε_t is $(0, 0)$, and the covariance matrix of ε_t is set to:

$$\sum_t = \begin{pmatrix} \sigma_{s,t}^2 & \sigma_{sf,t} \\ \sigma_{sf,t} & \sigma_{f,t}^2 \end{pmatrix} \quad (17)$$

The model of μ_t can be set similarly to the static model, and the binary GARCH model is used to model \sum_t . Obviously, the r_t conditional covariance matrix is \sum_t , and the minimum conditional variance hedge ratio during $[t, t+1]$ is $\sigma_{sf,t}/\sigma_{f,t}$.

(1) BEKK-GARCH

For the sake of simplicity, only BEKK-GARCH(1,1) is given, and it is not difficult to expand to BEKK-GARCH(m, n). Similar simplifications are also done for CCC-GARCH and DCC-GARCH. The BEKK-GARCH(1,1) model is set to

$$\sum_t = \Omega' \Omega + A' \sum_{t-1} A + B' \varepsilon_{t-1} \varepsilon_{t-1}' B \quad (18)$$

Among them, Ω is the lower triangular matrix, and A and B are square matrices of order 2.

(2) Asymmetric BEKK–GARCH

$$\sum_i \varepsilon_i = \Omega' \Omega + A' \sum_{t-1} A + B' \varepsilon_{t-1} \varepsilon'_{t-1} B + D' \xi_{t-1} \xi'_{t-1} D \quad (19)$$

Among them, Ω is the lower triangular matrix, A , B , and D are square matrices of order 2, and $\xi_t = \min(\varepsilon_t, 0)$.

(3) CCC–GARCH

Since the volatility of a single variable and the correlation between variables are modeled separately in CCC–GARCH, various univariate GARCH models can be used to model volatility. For the sake of simplicity, we assume that the univariate volatility model is $GARCH(1, 1)$, and only $CCC - GARCH(1, 1)$ is given. It is not difficult to extend to $CCC - GARCH(m, n)$ considering other univariate volatility models. DCC–GARCH is similarly simplified.

$$R_{i,t} = \mu_{i,t} + \varepsilon_{i,t} \mu_{i,t} + \sigma_{i,t} \eta_{i,t} \quad (20)$$

$$\sigma_{i,t}^2 = \omega_i + \alpha_i \sigma_{i,t-1}^2 + \beta_i \varepsilon_{i,t-1}^2, i = s, f. \quad (21)$$

Among them, $\eta_{i,t}$ is a random variable with a mean of 0 and a variance of 1. The correlation coefficient between $\eta_{s,t}$ and $\eta_{f,t}$ is a constant ρ . Therefore, the conditional correlation coefficient of $r_{s,t}$ and $r_{f,t}$ is ρ . The minimum variance hedge ratio during $\rho \frac{\sigma_{s,t}}{\sigma_{f,t}}$ period is $[t, t + 1]$.

(4) DCC–GARCH

The settings of DCC–GARCH and CCC–GARCH are very similar, the only difference is the setting of the correlation coefficient between $\eta_{s,t}$ and $\eta_{f,t}$. The correlation coefficient ρ_t between $\eta_{s,t}$ and $\eta_{f,t}$ in DCC–GARCH is time-varying. The minimum variance hedge ratio during $[t, t + 1]$ period is $\rho_t \frac{\sigma_{s,t}}{\sigma_{f,t}}$. Regarding the specific setting of ρ_t , Tse and Tsai (2002) and Engle (2002) gave similar settings. Tse and Tsai (2002) adopt the $ARMA(1, 1)$ setting.

$$\rho_t = (1 - \theta_1 - \theta_2) \bar{\rho} + \theta_1 \rho_{t-1} + \theta_2 \tilde{\rho}_{t-1}^m \quad (22)$$

Among them, $\theta_1, \theta_2, \bar{\rho}$ is a constant. In model estimation, $\bar{\rho}$ is designated as the correlation coefficient between $r_{s,t}$ and $r_{f,t}$ samples. $\tilde{\rho}_{t-1}^m$ is the sample correlation coefficient obtained according to the standardized innovation item $\eta_{i,t-m}, \dots, \eta_{i,t-1}, i = s, f$ at time $t - m, \dots, t - 1$,

$$\tilde{\rho}_{t-1}^m = \frac{\sum_{i=t-m}^{t-1} \eta_{s,i} \eta_{f,i}}{\sqrt{\sum_{i=t-m}^{t-1} \eta_{s,i}^2 \sum_{i=t-m}^{t-1} \eta_{f,i}^2}} \quad (23)$$

The setting of Tse and Tsai (2002) is easy to estimate, but it cannot guarantee that the value of ρ_t falls within the interval $[-1, 1]$, while the setting of Engle (2002) can ensure that the value of ρ_t falls within the interval $[-1, 1]$. Specifically,

$Q = \begin{pmatrix} q_{s,t} & q_{sf,t} \\ q_{sf,t} & q_{f,t} \end{pmatrix}$ is recorded, and the $J_t = \begin{pmatrix} \sqrt{q_{s,t}} & \\ & \sqrt{q_{f,t}} \end{pmatrix}^{-1}$ correlation coefficient matrix is set as

$$\begin{pmatrix} 1 & \rho_t \\ \rho_t & 1 \end{pmatrix} = J_t Q_t J_t \quad (24)$$

Among them, Q_t satisfies:

$$Q_t = (\tau \tau' - A - B) \circ \bar{Q} + A \circ Q_{t-1} + B \circ \eta_{t-1} \eta'_{t-1} \quad (25)$$

Here is $\tau = (1, 1)$. A and B are second-order square matrices, representing the Hadamard product of the matrix, and \bar{Q} is a symmetric matrix. \bar{Q} is designated as the sample covariance matrix of $r_{s,t}$ and $r_{f,t}$ during model estimation.

5 Model checking

This paper selects the daily closing prices of financial futures on the London Metal Exchange, the New York Mercantile Exchange, the Tokyo Mercantile Exchange, and the Shanghai Financial Futures Exchange and the fixed price of financial futures at 15:00 on the London OTC financial futures trading market as the daily financial futures prices in the corresponding markets. Considering that the 2007–2009 financial crisis may have an impact on the financial futures market, this paper mainly uses data from the subsequent financial crisis period as an empirical sample.

After MODWT, the return rate sequence of each market can be decomposed to obtain wavelet sequences under the scales d1, d2, d3 and d4. The time window corresponding to each wavelet sequence is d1: 2 to 4 days, d2: 4 to 8 days, d3: 8 to 16 days, and d4: 16 to 32 days. Tables 1, 2, 3, 4 are the descriptive statistical results of each sequence, and the corresponding statistical diagrams are shown in Fig. 1, 2, 3, 4. The average returns of the 4 markets are all greater than zero with little difference, indicating that the overall returns of the 4 markets are similar during the sample period. The kurtosis of the original sequence of returns and the wavelet sequence of the four markets is greater than 3, and the

Table 1 The rate of return of Shanghai market

	S	$d1$	$d2$	$d3$	$d4$
Mean	0.012	0.000	0.000	0.000	0.000
Standard deviation	0.930	0.669	0.460	0.321	0.233
Skewness	−0.637	0.059	−0.035	0.096	0.168
Kurtosis	12.833	6.520	9.367	9.125	5.812
Jargue-Beta	9821.240	1225.130	4035.960	3738.010	788.810
ARCH-LM	132.674	129.108	105.050	297.768	338.976

Table 2 The rate of return of London market

	<i>S</i>	<i>d1</i>	<i>d2</i>	<i>d3</i>	<i>d4</i>
Mean	0.020	0.000	0.000	0.000	0.000
Standard deviation	0.948	0.668	0.481	0.327	0.245
Skewness	-0.417	0.163	-0.013	0.045	0.067
Kurtosis	11.638	7.915	6.662	9.010	4.788
Jargue-Beta	7517.430	2408.850	1326.130	3595.600	313.100
ARCH-LM	101.475	105.141	95.021	296.112	339.037

Table 3 The rate of return of New York market

	<i>S</i>	<i>d1</i>	<i>d2</i>	<i>d3</i>	<i>d4</i>
Mean	0.011	0.000	0.000	0.000	0.000
Standard deviation	1.022	0.744	0.495	0.341	0.261
Skewness	-0.753	-0.087	0.089	0.008	0.174
Kurtosis	10.003	6.371	4.659	4.913	5.136
Jargue-Beta	5114.640	1125.140	269.670	356.530	457.530
ARCH-LM	77.114	95.172	143.208	305.495	339.320

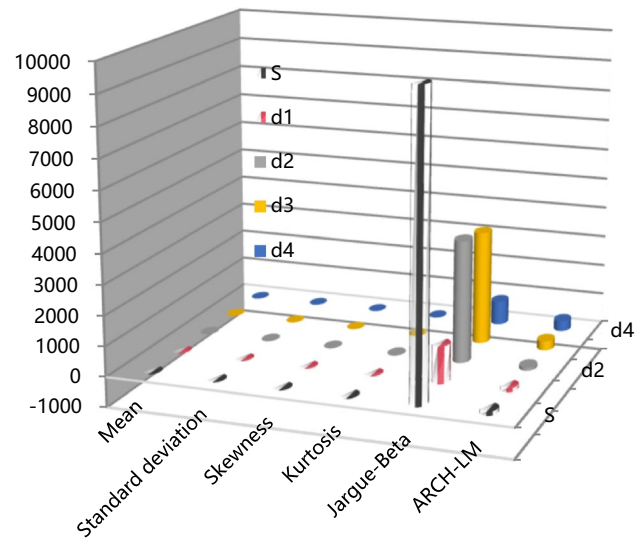
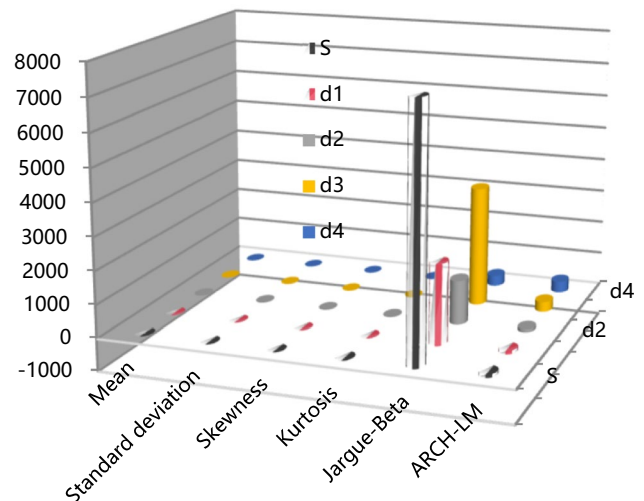
Table 4 The rate of return of Tokyo market

	<i>S</i>	<i>d1</i>	<i>d2</i>	<i>d3</i>	<i>d4</i>
Mean	0.017	0.000	0.000	0.000	-0.001
Standard deviation	0.992	0.711	0.492	0.366	0.234
Skewness	-1.258	-0.063	-0.097	0.092	0.112
Kurtosis	17.958	9.399	11.487	10.671	5.918
Jargue-Beta	2302.800	4077.370	7190.190	5869.110	843.350
ARCH-LM	77.134	119.847	106.838	299.213	339.340

skewness is not zero. It shows that the income distribution of each market has the characteristic of obvious peak and fat tail, and is skewed, and does not obey the normal distribution. By comparing the kurtosis values of the four markets, it can be found that the obvious peak and fat tail characteristics of the Tokyo Financial Futures Market are the most significant. The ARCH-LM test shows that there are volatility clusters in each return series at a significance level of 1%, so the GARCH model can be considered.

Since the value of the dynamic hedging ratio is negative at certain points in time, in order to more accurately represent the average cost of hedging, the absolute value of the hedging ratio is taken before the hedging average is counted. The statistical results of the mean are shown in Table 5 and Fig. 5.

It can be seen that between the same pair of markets, as the time scale increases, the value of the average hedging

**Fig. 1** Statistical diagram of the rate of return of Shanghai market**Fig. 2** Statistical diagram of the rate of return of London market

ratio increases. This shows that compared with shorter investment cycles, hedging in longer investment cycles requires more asset positions and higher costs. In addition, under the same scale, the average hedging ratio of the London and New York financial futures markets is smaller than that of the Tokyo financial futures market. Moreover, from a cost point of view, the cost of hedging financial futures in the first two markets is lower and more efficient.

Compared with GARCH model hedging which calculates the ratio at each time point, hedging based on wavelet correlation only calculates one value. The calculation results are shown in Table 6 and Fig. 6. The change of the hedge ratio with the scale is similar to the change of the wavelet correlation with the scale. Compared with the average hedge

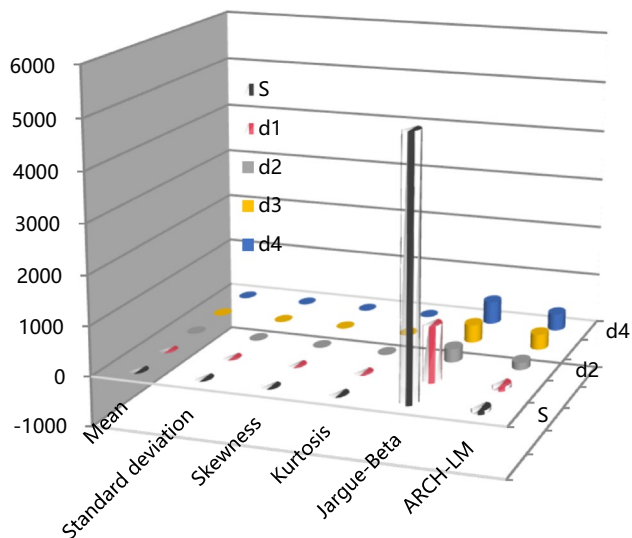


Fig. 3 Statistical diagram of the rate of return of New York market

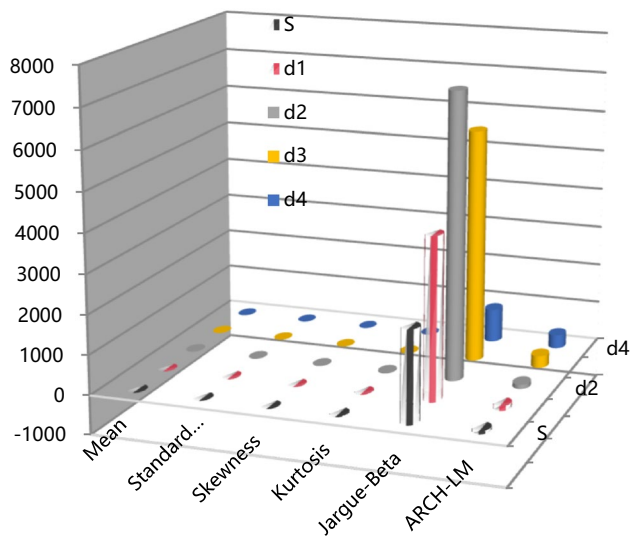


Fig. 4 Statistical diagram of the rate of return of Tokyo market

Table 5 Mean values of the hedge ratios under different scales based on the GARCH model

	$d1$	$d2$	$d3$	$d3$
London	0.272	0.387	0.535	0.773
new York	0.242	0.378	0.679	0.779
Tokyo	0.470	0.602	1.035	1.243

ratio based on the GARCH model, the hedge ratio calculated by the wavelet correlation method is lower. Therefore, without considering the validity, the use of wavelet correlation method for hedging requires a lower cost to invest in futures

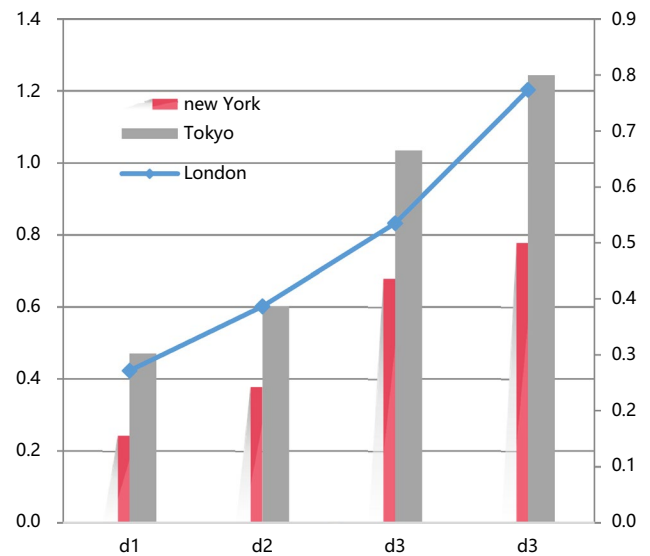


Fig. 5 Statistical diagram of mean values of the hedge ratios under different scales based on the GARCH model

Table 6 Hedging ratios under different scales based on wavelet correlation

	$d1$	$d2$	$d3$	$d3$
London	0.057	0.238	0.531	0.750
new York	0.099	0.233	0.523	0.751
Tokyo	0.362	0.525	0.289	0.377

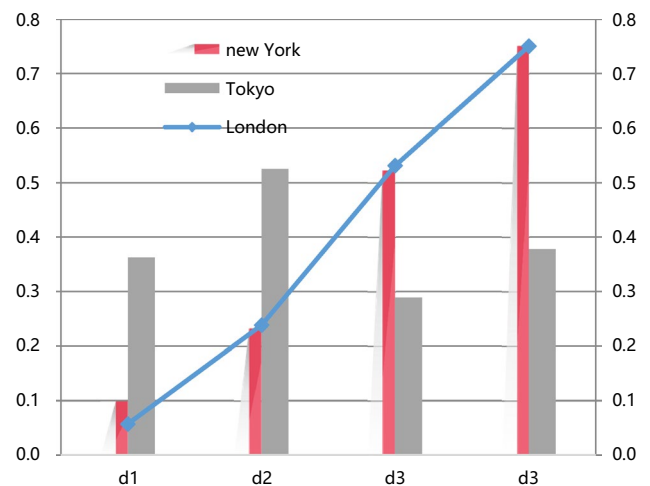


Fig. 6 Statistical diagram of hedge ratios under different scales based on wavelet correlation

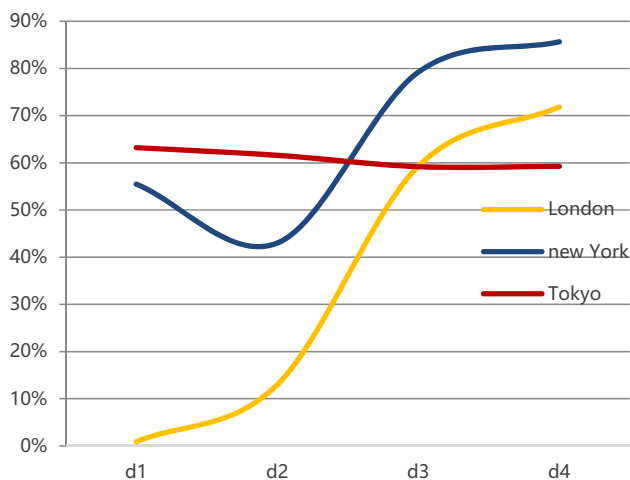
contracts. In addition, this method reduces transaction costs without requiring frequent adjustments to the ratio. Therefore, the overall cost of hedging based on wavelet correlation is lower.

Table 7 Comparison of hedging effectiveness of wavelet correlation model

	London	New York	Tokyo
<i>d1</i>	0.90%	55.49%	63.22%
<i>d2</i>	12.99%	43.01%	61.59%
<i>d3</i>	59.28%	79.20%	59.19%
<i>d4</i>	71.80%	85.65%	59.26%

Table 8 Comparison of hedging effectiveness of GARCH model

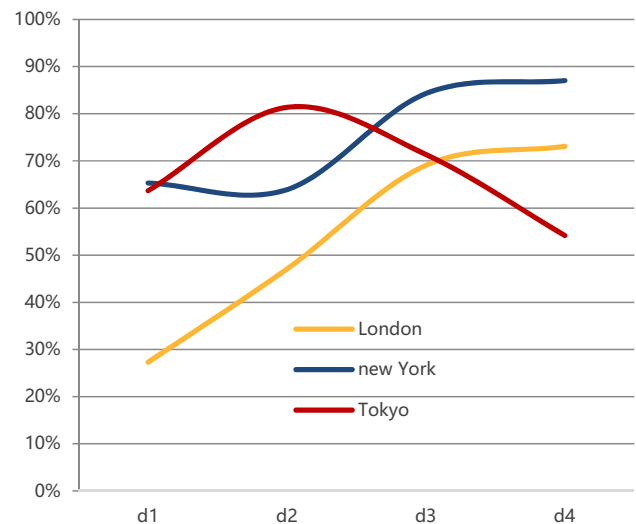
	London	New York	Tokyo
<i>d1</i>	27.28%	65.31%	63.67%
<i>d2</i>	47.08%	63.90%	81.36%
<i>d3</i>	69.06%	84.28%	71.36%
<i>d4</i>	73.07%	87.03%	54.18%

**Fig. 7** Statistical diagram of comparison of hedging effectiveness of wavelet correlation model

In order to test and compare the effectiveness of hedging based on GARCH model and wavelet correlation, the variance reduction ratio evaluation method can be used for evaluation. First, the hedging ratios calculated by the two methods are used to construct hedging asset portfolios, and then the variances before and after hedging are calculated respectively, so as to obtain the hedging effectiveness of the two models under different scales. The results are shown in Tables 7, 8, and Figs. 7, 8.

Based on the semivariance and regret indicators, the hedging effects of financial futures in the three markets of London, New York and Tokyo are shown in Tables 9, 10, Figs. 9, and 10.

By combining the two indicators, it can be seen that under the *d1* and *d2* scales, Tokyo market financial futures have a

**Fig. 8** Statistical diagram of comparison of hedging effectiveness of the GARCH model**Table 9** Comparison of the hedging effect of futures based on semi-variance financial futures

	London	New York	Tokyo
<i>d1</i>	3.400	3.924	3.514
<i>d2</i>	1.414	1.478	0.890
<i>d3</i>	0.367	0.350	0.664
<i>d4</i>	0.298	0.179	0.817

Table 10 Comparison of the hedging effect of futures based on regret index financial futures

	London	New York	Tokyo
<i>d1</i>	10.047	10.210	9.280
<i>d2</i>	6.545	6.688	5.276
<i>d3</i>	3.184	2.960	4.154
<i>d4</i>	2.047	1.959	3.529

better hedging effect on Shanghai market financial futures assets. Under the *d3* and *d4* scales, financial futures on the New York market have a better hedging effect on financial futures assets on the Shanghai market.

6 Conclusion

Based on the quadratic variation theory of continuous-time Ito Semimartingale, this paper combines high-frequency data and low-frequency data modeling to characterize asset returns. In addition, this paper focuses on analyzing the path characteristics of asset price processes such as stock indexes

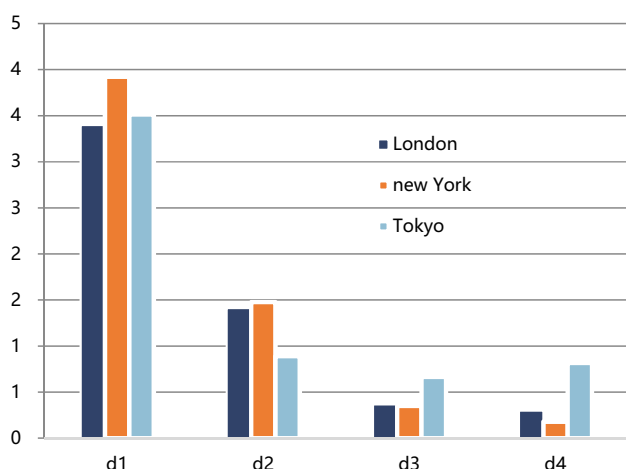


Fig. 9 Statistical diagram of the hedging effect of futures based on semi-variance financial futures

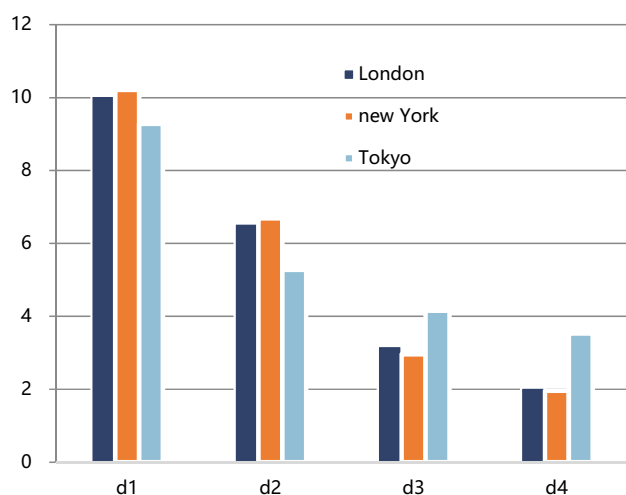


Fig. 10 Statistical diagram comparison of the hedging effect of futures based on the regret index financial futures

and stock index futures, and the interaction between asset returns and volatility. On this basis, according to economics and financial theories, this paper conducts theoretical analysis and empirical research on important financial decision-making issues such as the implementation and evaluation of single-period stock portfolio hedging strategies, multi-period futures hedging issues, and multi-period portfolio optimization issues.

Taking into account the volatility clustering characteristics and heteroscedasticity of financial time series data, this paper chooses the most general machine learning model among the dynamic hedging models for research. On this basis, this paper calculates the optimal hedging ratio of the model under two different risk measures, minimum variance and minimum semivariance, and compares them. In general,

investment strategies that consider the predictability of asset returns can create significant economic value for investors.

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References

- Ansari M, Vakili V, Bahrak B (2019) Evaluation of big data frameworks for analysis of smart grids. *J Big Data* 6(109):1–14
- Bikmetov R, Raja M, Sane T (2017) Infrastructure and applications of Internet of Things in smart grids: a survey. In: *Proceedings of the 2017 North American power symposium (NAPS)*. IEEE, Morgantown, pp 1–6
- Bouteraa Y, Abdallah IB (2017) A gesture-based telemanipulation control for a robotic arm with biofeedback-based grasp. *Ind Robot* 44(5):575–587
- Chen G, Song Y, Guan Y (2018) Terminal sliding mode-based consensus tracking control for networked uncertain mechanical systems on digraphs. *IEEE Trans Neural Netw Learn Syst* 29(3):749–756
- Delavari H (2017) A novel fractional adaptive active sliding mode controller for synchronization of non-identical chaotic systems with disturbance and uncertainty. *Int J Dyn Control* 5(1):102–114
- Elgammal AAA, El-Naggar MF (2017) MOPSO-based optimal control of shunt active power filter using a variable structure fuzzy logic sliding mode controller for hybrid (FC-PV-Wind-Battery) energy utilisation scheme. *IET Renew Power Gener* 11(8):1148–1156
- Guo X, Liang Z, Li C (2018) Finite time tracking control of mobile robot based on non-singular fast terminal sliding mode. *Syst Sci Control Eng Open Access J* 6(1):492–500
- He W, Dong Y (2018) Adaptive fuzzy neural network control for a constrained robot using impedance learning. *IEEE Trans Neural Netw Learn Syst* 29(4):1174–1186
- Hussain M, Beg M (2019) Fog computing for internet of things (IoT)-aided smart grid architectures. *Big Data Cogn Comput* 3(8):1–29
- Jung S (2018) Improvement of tracking control of a sliding mode controller for robot manipulators by a neural network. *Int J Control Autom Syst* 16(2):937–943
- Le AT, Joo YH, Le QT et al (2017) Adaptive neural network second-order sliding mode control of dual arm robots. *Int J Control Autom Syst* 15(5):1–9
- Mao J, Wang T, Jin C, Zhou A (2017) Feature grouping-based outlier detection upon streaming trajectories. *IEEE Trans Knowl Data Eng* 29(12):2696–2709
- Panwar V (2017) Wavelet neural network-based H_{∞} trajectory tracking for robot manipulators using fast terminal sliding mode control. *Robotica* 35(7):1–16
- Rabie A, Ali S, Saleh A, Ali H (2019) A new outlier rejection methodology for supporting load forecasting in smart grids based on big data. *Cluster Comput*. <https://doi.org/10.1007/s10586-019-02942-0>
- Saneja B, Rani R (2019) A scalable correlation-based approach for outlier detection in wireless body sensor networks. *Int J Commun Syst* 32(7):1–15
- Siva SL, M S, M S (2017) In: 2017 4th international conference on advanced computing and communication systems (ICACCS) (IEEE, 2017), pp 1–5

- Tu C, He X, Shuai Z, Jiang F (2017) Big data issues in smart grid—a review. *Renew Sustain Energy Rev* 79:1099–1107
- Vimala S, Sharmili K (2018) Prediction of loan risk using Naive Bayes and support vector machine. *Int Conf Adv Comput Technol* 4(2):110–113
- Wosiak A, Zakrzewska D (2018) Integrating correlation-based feature selection and clustering for improved cardiovascular disease diagnosis. *Complexity Hindawi* 2018:1–11
- Yang C, Chen S, Liu J, Liu R, Chang C (2019) On construction of an energy monitoring service using big data technology for the smart campus. *Cluster Comput*. <https://doi.org/10.1007/s10586-019-02921-5>
- Zhao B, Yu H, Yu J et al (2018) Port-controlled hamiltonian and sliding mode control of gantry robot based on induction motor drives. *IEEE Access* 6:43840–43849

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