Lyon 1

Mesh and Computational Geometry

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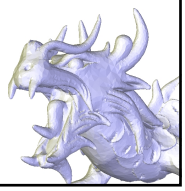
M2 ID3D
Image, Développement
et Technologie 3D
et 3A Centrale



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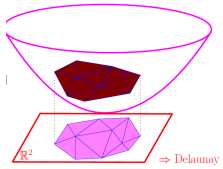
Interpretation of 2D Voronoi in the space of the spheres

- By lifting the points in a higher dimensional space, there is another geometric interpretation of Voronoi
- Parabolic lift?
  - Of what?
  - For the interpretation of Delaunay we lifted the points, and we used the fact that the lift of the points of a circle were coplanar



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Space of spheres

- Voronoi cell of one site P : locus of the center of the empty circles passing through P
- How to represent 2D circles by 3D points?
  - The circle C of center c and radius R will be represented by the point (c, c^2-R^2)
    - Lift the center point $c(x_c, y_c)$ to the altitude c^2-R^2 ie. at the coordinate point $(x_c, y_c, x_c^2+y_c^2-R^2)$
  - Note that 2D points alone correspond to circles of radius 0
    - Where are they located in the space of spheres?

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Demonstration

- 2D points correspond to circles of radius 0
  - Where are they located in the space of the spheres?
  - On the paraboloid $z = x^2 + y^2$

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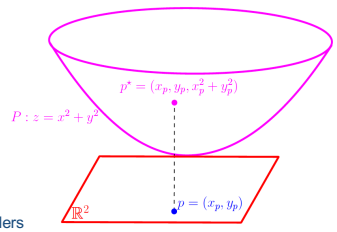
Demonstration

- 2D points correspond to circles of radius 0
  - Where are they located in the space of spheres?
  - On the paraboloid

$$z = x^2 + y^2$$



Images by O. Devillers

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Nice interpretation of Voronoi in the space of spheres

- Representation of a circle of center c and radius R by the point (c, c^2-R^2)
- What is the lift of all the circles passing through a point P?

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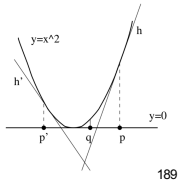
Nice interpretation of Voronoi in the space of the spheres

- Representation of a circle of center c and radius R by the point (c, c^2-R^2)
- All circles passing through a point P : hyperplane tangent in Φ(P) to the paraboloid (Φ(P) lift of P on the paraboloid)



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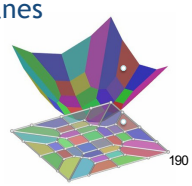
Nice interpretation of Voronoi in the space of the spheres

- We consider the lifting $\Phi(P_i)$ of all the input points P_i on the paraboloid
- Correspondence between Voronoi and the intersection of the half spaces located above the previous hyperplanes



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