PRESERVING THE STRUCTURE OF THE MOOG VCF IN THE DIGITAL DOMAIN

Federico Fontana
Università di Verona
Dipartimento di Informatica
15 Strada le Grazie, Verona 37134 - Italy

ABSTRACT

A discrete-time version of the Moog VCF structure is proposed based on the explicit computation of the delay-free loop contained in that filter. Compared to previous solutions, the model presented here shows accurate frequency responses when working in linear regimes meanwhile preserving good numerical stability and direct access to the parameters of cutoff frequency and feedback gain. These features make the proposed model a candidate for efficient implementation on fixed-point architectures as well as in normal PC's, in the form of software plug-in. The structural correspondence with the Moog VCF will allow future direct transpositions of the analog system in the discrete-time model.

1. INTRODUCTION

The Moog Voltage-Controlled Filter (VCF) has a recognized place in the history of electronic music [7]. By exploiting a clever interconnection of analog components it realizes the filtering scheme depicted in Figure 1, in which we note the existence of a loop containing a series of four identical filtering stages.

Every stage in this loop has the same lowpass characteristic. In linear regimes this characteristic is expressed with good approximation by the transfer function

$$G(s) = \frac{\omega_c}{\omega_c + s}. \tag{1}$$

In the Moog VCF both the parameter ω_c and the loopback gain k can be dynamically varied. The former sets the cutoff frequency. The latter ranges in the interval 0$\sim$4: by increasingly feeding back the output $y(t)$ to the input $u(t)$, the system's pure lowpass behavior (3 dB-cutoff at ω_c with $k = 0$) progressively changes into that of a resonant filter. At the stability limit $k = 4$ the Moog VCF oscillates at ω_c rad/s.

An excellent analysis of the Moog VCF, concerning many acoustically relevant aspects of this system, has been presented in 1996 by Stilson and Smith [8]. On top of that analysis, the authors propose some discrete-time models aiming at preserving the response of the Moog VCF without sacrificing the accessibility to its driving parameters. Controllability is in fact a crucial property that should be maintained in a good digital realization.

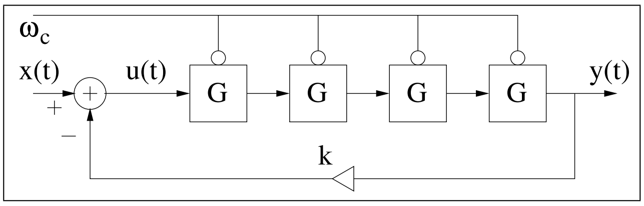

Figure 1. Structure of the Moog VCF.

After that work, other discrete-time models of the Moog VCF have been proposed [6]. In the meantime some commercial products have been launched into the market of virtual analog electronic musical instruments, especially in the form of software plug-ins. Whatever their quality and cost, their specification is obviously protected by nondisclosure terms.

The work presented here is mostly built on the research made by Stilson and Smith. Rather than designing digital filters that preserve the acoustical and control features of the Moog VCF, in this paper we will derive a discrete-time version that preserves the *structure* of the analog system. In practice, the one-pole analog filters are mapped into corresponding digital filter blocks. This design strategy can be pursued since a versatile formal tool capable of handling the delay-free loop, whose computation is known to be impossible using traditional sequential procedures, has been developed in the meantime. We will show that by means of this tool the digital system obtained by bilinear transformation of the Moog VCF has a magnitude response that matches that of the analog filter with good accuracy, once ω_c is preliminary bilinearly antitransformed to compensate for the frequency warping introduced by this transformation. Furthermore we will show that the resulting model preserves the direct access to the driving parameters to a good extent. As a consequence, it can be efficiently implemented in a fixed-point digital signal processing architecture at the cost of performing two table lookups and few multiply-and-add operations at every temporal step.

Besides these results, the proposed discrete-time model is numerically robust and prospectively versatile for future accurate digital transpositions of the Moog VCF. In fact, the one-by-one correspondence between the one-pole analog and digital filter blocks allows to transpose features of the analog filters directly into the discrete-time blocks.

2. COMPUTATION OF THE DELAY-FREE LOOP

Accurate continuous-to-discrete transformations which factor out a pure delay in the digital version of $G(s)$ cannot be found. Supporting this evidence is, first of all, the fact that $G(s)$ responds to an input without introducing temporal delay—however, this argument in general is not sufficient: see, for instance, the case of 2nd-order oscillators [2]. Holding this evidence, we soon realize that the Moog VCF cannot be computed in the digital domain at conventional audio sample rates using traditional sequential procedures due to the existence of a delay-free loop.

Graph topology-based methods exist which detect, then lump such loops into higher-order filter structures [9]. In the Moog VCF case it is immediate to detect the loop, and straightforward to derive such a "lumped" transfer function [8]:

$$H(s) = \frac{\{G(s)\}^4}{1 + k\{G(s)\}^4} = \frac{1}{k + \{1 + s/\omega_c\}^4}. \quad (2)$$

A 4th-order digital filter realizing $H(s)$ does not import the structural properties of the Moog VCF, hence loses the direct access to the driving parameters provided by the original filter. We have to expect that digital structures that are obtained by mere continuous-to-digital transformation of $H(s)$ are likely to suffer the same flaw concerning the dynamic control of the filter.

In 2000 Härmä has proposed an algorithm that computes, in the discrete time, a delay-free loop topology similar to that depicted in Figure 1 [5]. Later this algorithm has been generalized to linear [3] and nonlinear [4, 1] filter networks containing whatever delay-free loop topology. Peculiar, in this algorithm, is the fact that first the output $y[n]$ is computed, and then the states of the filter blocks forming the loop are updated. Since the first step solves the instantaneous interactions that take place within the loop, then all the calculations that are needed to update the filter states are performed aside of the loop computation.

The proposed algorithm comes particularly useful in the case of the Moog VCF. In this case we can in fact preserve the structure of Figure 1 during the computation of the discrete-time version of the filter. The algebra leading to a digital filter that is structurally identical to the Moog VCF is reported in the following of this section (the reader is referred to [3] for a detailed treatment of the algorithm).

The series containing the four lowpass filters is a linear system, whose output $y[n]$ can be seen as the superposition of the forced and free evolution: the former depends linearly on the signal $u[n]$, whereas the latter is a linear function s of the state of the four lowpass filters:

$$y[n] = bu[n] + s[n]. \quad (3)$$

Furthermore (see Figure 1) it is

$$u[n] = x[n] - ky[n]. \quad (4)$$

Substituting (4) in (3) we easily obtain

$$y[n] = \frac{bx[n] + s[n]}{1 + bk}. \quad (5)$$

At every step n the instantaneous contribution of the free evolution $s[n]$ to the output $y[n]$ can be figured out by imposing $u[n] = 0$. As a consequence, the following procedure can be implemented to compute $y[n]$: i) open the loop by cutting down the branch where u flows; ii) evaluate the output (that is, $s[n]$); iii) compute $y[n]$ using (5). From here, $u[n]$ is immediately computed by means of (4) and, hence, the state of the four digital lowpass blocks can be updated accordingly.

3. CONTINUOUS-TO-DISCRETE MAPPING

This digital realization of the Moog VCF will be as accurate, as much precisely the lowpass transfer function $G(s)$ is mapped into the discrete-time domain. Techniques exist to accurately map a continuous 1st-order lowpass into a digital filter of low order [10]. In this paper we make use of the well-known bilinear transformation, furthermore we will "warp" the discrete transfer function $G(z)$ obtained from $G(s)$ by means of this transformation by simply pre-compensating the cutoff pulsation to the value $\tilde{\omega}_c$ which is exactly bilinear transformed to ω_c.

By substituting s with $(2/T)(z-1)/(z+1)$ in $G(s)$ we obtain the corresponding (pre-compensated) bilinear transformed lowpass block

$$G(z) = b_0 \frac{1 + z^{-1}}{1 + a_1 z^{-1}}, \quad (6)$$

with $b_0 = T\tilde{\omega}_c/(T\tilde{\omega}_c + 2)$ and $a_1 = (T\tilde{\omega}_c - 2)/(T\tilde{\omega}_c + 2)$. By definition of bilinear transformation, the antitransformed pulsation $\tilde{\omega}_c$ of the digital filter is calculated by

$$\tilde{\omega}_c = \frac{2}{T}\tan\Big(\frac{\omega_c T}{2}\Big). \quad (7)$$

Since the loop contains four lowpass blocks in series, then in (3) it is $b = b_0^4$. If we realize $G(z)$ in canonic form, then $s[n]$ is obtained by multiplying the state value of every lowpass block times the b_0 gains that this value must pass through across the way to the output, and finally summing up the results of such multiplications. By calling these states s_1, s_2, s_3, s_4, respectively, then it is

$$s[n] = b_0^3 s_1[n] + b_0^2 s_2[n] + b_0 s_3[n] + s_4[n]. \quad (8)$$

Equations (8), (5), and (4), can be used in this order to perform the proposed procedure. Finally, the states s_1, s_2, s_3, s_4 of the lowpass blocks are updated.

4. RESULTS

Figures 2, 3, 4, 5 show, in dashed lines, magnitude responses of the analog Moog VCF obtained by plotting $|H(j2\pi f)|$ as in (2) and, in solid lines, spectra of impulse responses computed by implementing the aforementioned procedure in Matlab, respectively for gains k equal to 1, 2, 3, 4. All responses have been plotted for cutoff frequencies equal to 100, 1000 and 10000 Hz.

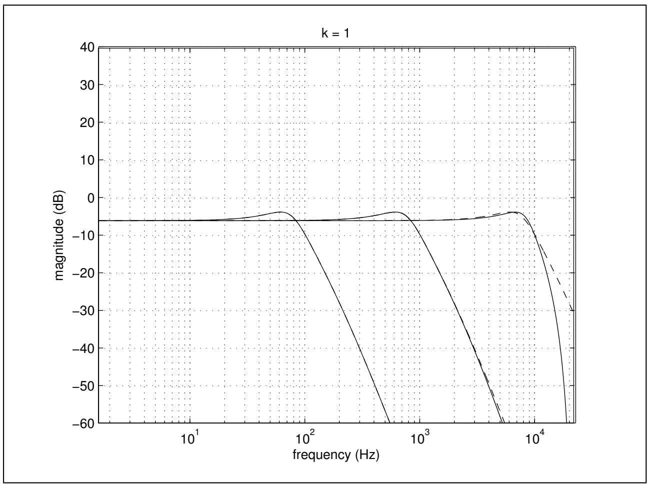

Figure 2. $k = 1$: magnitude response of the digital Moog VCF (solid line); magnitude response of the analog Moog VCF (dashed line). Cutoff frequencies equal to 100, 1000 and 10000 Hz are plotted.

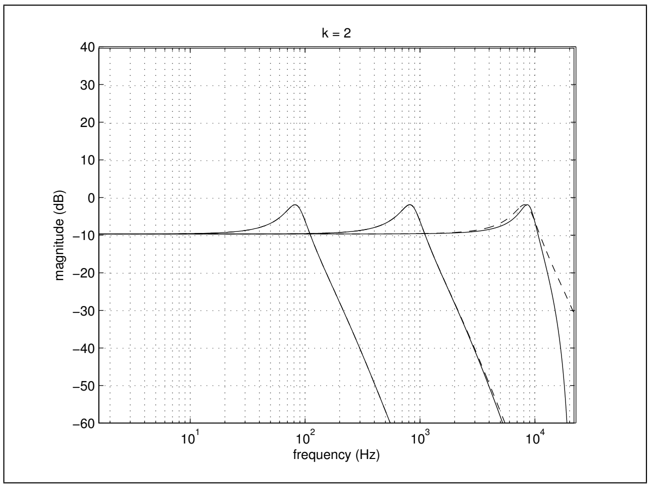

Figure 3. $k = 2$: magnitude response of the digital Moog VCF (solid line); magnitude response of the analog Moog VCF (dashed line). Cutoff frequencies equal to 100, 1000 and 10000 Hz are plotted.

Accurate responses are achieved for cutoff frequencies of 100 and 1000 Hz by the digital realization if ω_c is pre-compensated using (7). At 10 kHz the frequency warping effect affects the responses of the discrete-time system, as their magnitude rapidly drops beyond that frequency. Noticeable discrepancies between the analog and the digital responses arise only when the filter is at the stability limit, i.e., for $k = 4$. (Recall that for this value of the feedback gain the filter is unstable!) However, such discrepancies appear well below the amplitude of the peak frequency and their audibility should be systematically assessed by listening experiments, in which the digital model is compared against the analog filter.

5. DISCUSSION AND CONCLUSIONS

The plots in Figures 2–5 are obtained by a digital filter structure that, although computed using a specific procedure, ultimately realizes the bilinearly transformed version of (2). Once ω_c is pre-compensated as it has been done here, every equivalent structure is expected to provide the same responses. Especially in architectures enabling floating point operations in real time, as it happens in a normal PC running pd or Max/MSP, it can be questioned whether the given model is competitive or not compared to a fourth-order digital filter directly realizing the bilinear transformation of $H(s)$.

A first observation around this question is that the proposed structure splits the fourth-order filter into a series of first-order blocks, for this reason it maximizes the robustness of the discrete-time realization independently of the architecture implementing it. Robustness is especially desirable in the case of filters whose coefficients are dynamically varied. The Moog VCF in fact is often exposed to rapid, large variations of its driving parameters, for instance when ω_c is controlled by a Low Frequency Oscillator (LFO) and k is changed contextually.

Furthermore the same structure resembles the analog system, hence it inherently exhibits higher flexibility compared to other digital realizations. This flexibility comes useful if specific customizations of the discrete-time model are needed, e.g., to introduce nonlinearities in the one-pole analog blocks or to simulate analog circuitry to which time and age has conferred unique sound features [6].

Besides this, the proposed model can be efficiently implemented on fixed-point digital signal processing architectures providing instructions such as the $\mathrm{MAC}(a, x, y)$ (multiply a by x and accumulate into y). In these architectures the larger amount of operations that is needed to compute the four first-order blocks across the delay-free loop is payed off by the stability of the procedure and the compactness of the lookup tables containing the filter coefficients. In fact, we have to store pre-computed values of b_0 and a_1 as functions of ω_c (notice that it is $|b_0| < 1$ and $|a_1| < 1$), plus a third lookup table containing values of the function $1/r$, with $1 < r < 5$: this table will be accessed to look up the value $1/(1 + bk)$.

At every step the filter can be updated using the following indicative procedure:

1. compute s in (8) by looking up b_0 and iterating the instruction $\mathrm{MAC}(s, b_0, s_i)_{i=2,3,4}$ three times, with s initially set to be equal to s_1;

2. compute y in (5) by figuring out $b = b_0^2 \cdot b_0^2$, then by computing bk, and by finally looking up the reciprocal of $1 + bk$ that is used to multiply the result of $\mathrm{MAC}(b, x, s)$;

3. compute u in (4) as $\mathrm{MAC}(k, y, x)$;

4. look up a_1 and update s_1, s_2, s_3, and s_4.

A reduction in memory occupation is obtained at a minimal additional computational cost by noticing that b_0 and a_1 can be respectively written as $1 - 2/(T\tilde{\omega}_c + 2)$ and

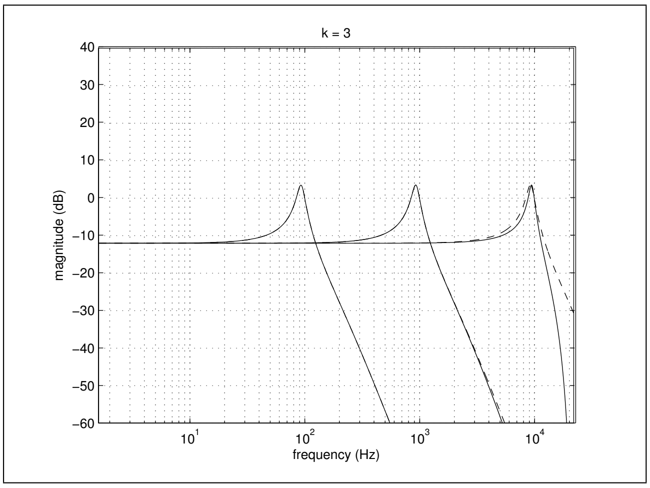

Figure 4. $k = 3$: magnitude response of the digital Moog VCF (solid line); magnitude response of the analog Moog VCF (dashed line). Cutoff frequencies equal to 100, 1000 and 10000 Hz are plotted.

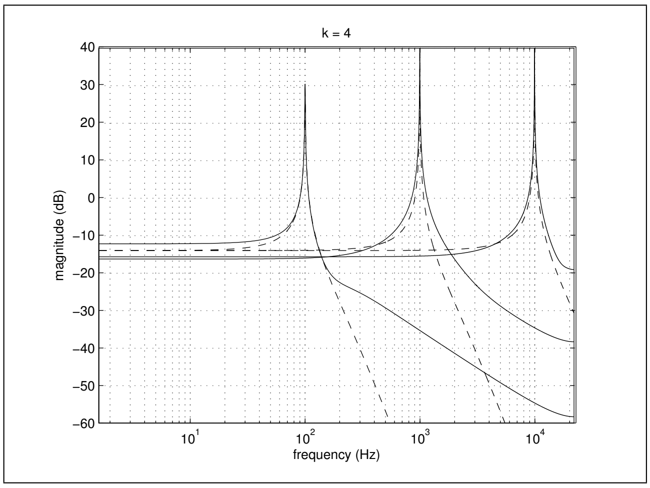

Figure 5. $k = 4$: magnitude response of the digital Moog VCF (solid line); magnitude response of the analog Moog VCF (dashed line). Cutoff frequencies equal to 100, 1000 and 10000 Hz are plotted.

$1 - 4/(T\tilde{\omega}_c + 2)$. Thus, their values are figured out by looking up values of $1/(T\tilde{\omega}_c + 2)$ followed by single or double bit-shift and complementation to unity.

In conclusion, the proposed digital model of the Moog VCF is certainly attractive concerning fixed-point implementations, and, due to its robusness, it is likely to be competitive against other digital structures even if programmed as a software plug-in running on floating-point architectures such as normal PC's.

Further work is ongoing to implement this model as a platform-independent Java external for Max/MSP and pd, to evaluate its robustness and quality against available plug-ins of the Moog VCF.

6. ACKNOWLEDGMENTS

The author would like to acknowledge Vittorio Martinelli for making preliminary simulations of the proposed model in his B.Sc. thesis made in this Department.

This work has been supported by the EU project FP6-NEST-29085 *CLOSED - Closing the Loop Of Sound Evaluation and Design* under the path “Measuring the impossible”.

7. REFERENCES

[1] F. Avanzini, F. Fontana, and D. Rocchesso. Efficient computation of nonlinear filter networks with delay-free loops and applications to physically-based sound models. In *Proc. of ”The Fourth International Workshop on Multidimensional Systems, 2005 (NDS 2005)*, pages 110–115, Wuppertal, Germany, July 2005. IEEE.

[2] B. Bank. *Physics-based Sound Synthesis of String Instruments Including Geometric Nonlinearities*. PhD thesis, Budapest University of Technology and Economics, Budapest, Hungary, 2006.

[3] F. Fontana. Computation of linear filter networks containing delay-free loops, with an application to the waveguide mesh. *IEEE Trans. on Speech and Audio Processing*, 11(6):774–782, Nov. 2003.

[4] F. Fontana, F. Avanzini, and D. Rocchesso. Computation of nonlinear filter networks containing delay-free paths. In *Proc. Conf. on Digital Audio Effects (DAFX-04)*, pages 113–118, Naples, Italy, Oct. 2004.

[5] A. Härmä. Implementation of frequency-warped recursive filters. *EURASIP Signal Processing*, 80(3):543–548, Mar. 2000.

[6] A. Huovilainen. Nonlinear digital implementation of the moog ladder filter. In *Proc. Conf. on Digital Audio Effects (DAFX-04)*, pages 61–64, Naples, Italy, Oct. 2004.

[7] R. A. Moog. A voltage-controlled low-pass high-pass filter for audio signal processing. In *Audio Engineering Society Convention*. AES, Oct. 1965. Preprint 413.

[8] T. Stilson and J. O. Smith. Analyzing the Moog VCF with considerations for digital implementation. In *Proc. Int. Computer Music Conf.*, Hong Kong, 1996.

[9] J. Szczupak and S. K. Mitra. Detection, location, and removal of delay-free loops in digital filter configurations. *IEEE Trans. on Acoustics, Speech and Signal Processing*, ASSP-23(6):558–562, 1975.

[10] C. Wan and A. M. Schneider. Further improvements in digitizing continuous-time filters. *IEEE Trans. on Signal Processing*, 45(3):533–542, Mar. 1997.