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AMS 578
Preliminary Report
4/20/2021

For my project I will be working in R. I began the preliminary part of this assignment by reading my three data files into R and the using a merge function to merge the three into a single data file by ID number. I then calculated the summary statistics of each variable in the data before imputation. Below is the summary output for each variable with its respective summary statistics. This includes mean, median, minimum and maximum value, lower and upper quartile, number of missing values (NA's), standard deviation, correlation matrix, and data counts.

Summary Statistics

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       E1              E2              E3              E4               E5              E6              R1              R2              R3       
 Min.   : 264.1   Min.   : 375.2   Min.   : 156.6   Min.   : -10.57   Min.   :  16.83   Min.   : 205.0   Min.   :0.0000   Min.   :0.0000   Min.   :0.000  
 1st Qu.: 639.4   1st Qu.: 767.7   1st Qu.: 557.6   1st Qu.: 441.54   1st Qu.: 451.02   1st Qu.: 593.8   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.000  
 Median : 748.4   Median : 879.7   Median : 655.7   Median : 544.49   Median : 563.45   Median : 703.4   Median :1.0000   Median :0.0000   Median :1.000  
 Mean   : 749.8   Mean   : 875.6   Mean   : 657.7   Mean   : 544.14   Mean   : 564.65   Mean   : 699.8   Mean   :0.5154   Mean   :0.4934   Mean   :0.506  
 3rd Qu.: 857.1   3rd Qu.: 986.0   3rd Qu.: 760.0   3rd Qu.: 648.85   3rd Qu.: 672.58   3rd Qu.: 806.5   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.000  
 Max.   :1233.9   Max.   :1329.5   Max.   :1178.6   Max.   :1069.37   Max.   :1072.95   Max.   :1247.2   Max.   :1.0000   Max.   :1.0000   Max.   :1.000  
 NA's   :20       NA's   :30       NA's   :20       NA's   :20        NA's   :30        NA's   :20       NA's   :30       NA's   :30       NA's   :30     
       R4              R5              R6              R7              R8              R9              R10             R11             R12       
 Min.   :0.0000   Min.   :0.0000   Min.   :0.000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000  
 1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000  
 Median :0.0000   Median :1.0000   Median :0.000   Median :0.0000   Median :0.0000   Median :0.0000   Median :0.0000   Median :1.0000   Median :0.0000  
 Mean   :0.4965   Mean   :0.5189   Mean   :0.491   Mean   :0.4898   Mean   :0.4861   Mean   :0.4917   Mean   :0.4929   Mean   :0.5079   Mean   :0.4802  
 3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.000   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000  
 Max.   :1.0000   Max.   :1.0000   Max.   :1.000   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000  
 NA's   :30                                                                                                          NA's   :30       NA's   :30     
      R13             R14             R15             R16             R17             R18             R19             R20             R21       
 Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000  
 1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000  
 Median :0.0000   Median :0.0000   Median :0.0000   Median :0.0000   Median :1.0000   Median :0.0000   Median :0.0000   Median :0.0000   Median :0.0000  
 Mean   :0.4923   Mean   :0.4972   Mean   :0.4694   Mean   :0.4842   Mean   :0.5003   Mean   :0.4787   Mean   :0.4947   Mean   :0.4954   Mean   :0.4997  
 3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000  
 Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000  
                                                                      NA's   :30                                                         NA's   :30     
      R22             R23             R24             R25               Y          
 Min.   :0.0000   Min.   :0.0000   Min.   :0.000   Min.   :0.0000   Min.   :-7.603e+08  
 1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.000   1st Qu.:0.0000   1st Qu.: 2.131e+10  
 Median :1.0000   Median :0.0000   Median :1.000   Median :1.0000   Median : 2.639e+10  
 Mean   :0.5133   Mean   :0.4833   Mean   :0.522   Mean   :0.5154   Mean   : 2.646e+10  
 3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.000   3rd Qu.:1.0000   3rd Qu.: 3.145e+10  
 Max.   :1.0000   Max.   :1.0000   Max.   :1.000   Max.   :1.0000   Max.   : 5.537e+10  
                  NA's   :30                       NA's   :30       NA's   :30          
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Standard Deviations

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         E1           E2           E3           E4           E5           E6           R1           R2           R3           R4           R5 
1.624432e+02 1.642357e+02 1.560307e+02 1.569808e+02 1.600738e+02 1.583657e+02 4.999191e-01 5.001138e-01 5.001218e-01 5.001456e-01 4.997987e-01 
         R6           R7           R8           R9          R10          R11          R12          R13          R14          R15          R16 
5.000742e-01 5.000505e-01 4.999610e-01 5.000850e-01 5.001041e-01 5.000955e-01 4.997634e-01 5.000949e-01 5.001469e-01 4.992164e-01 4.999058e-01 
         R17          R18          R19          R20          R21          R22          R23          R24          R25            Y 
5.001575e-01 4.996991e-01 5.001270e-01 5.001332e-01 5.001575e-01 4.999778e-01 4.998786e-01 4.996723e-01 4.999191e-01 7.799392e+09 
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Correlation Matrix (ignoring missing values)

Correlation Matrix

	E1	E2	E3	E4	E5	E6	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	R12	R13	R14	R15	R16	R17	R18	R19	R20	R21	R22	R23	R24	R25	Y
E1	1	0.0208	0.0404	0.0011	-0.005	-0.0334	0.0259	-0.0213	-0.0420	-0.0419	0.0046	0.0212	0.0500	0.0151	-0.0465	-0.0234	0.0053	-0.0150	0.0052	-0.0302	0.039	-0.035	-0.0193	-0.0689	0.0160	0.0027	-0.0142	-0.0525	-0.0008	-0.0176	0.0197	0.1137
E2	0.021	1	-0.0367	-0.0522	0.055	0.0474	-0.0227	0.0309	0.0558	-0.0433	0.0074	-0.0002	-0.0002	0.0118	-0.0160	0.0381	-0.0281	-0.0172	0.0205	-0.0041	0.002	-0.033	-0.0175	-0.0451	-0.0206	0.0384	-0.0235	0.0402	-0.0181	0.0386	-0.0466	0.2518
E3	0.0404	-0.0367	1	-0.0144	0.021	0.0009	-0.0128	0.0132	-0.0369	0.0048	-0.0021	0.0027	-0.0149	-0.0556	-0.0148	-0.0328	0.0138	-0.0250	0.0001	0.0165	0.027	0.075	-0.0212	0.0420	0.0465	-0.0326	-0.0002	-0.0145	-0.0125	-0.0306	-0.0406	0.3420
E4	0.0011	-0.0522	-0.0144	1	-0.031	0.0929	-0.0050	-0.0537	-0.0363	0.0653	0.0172	0.0014	0.0136	0.0098	-0.0170	-0.0227	0.0009	-0.0827	-0.0134	-0.0069	-0.015	0.067	-0.0133	-0.0094	-0.0255	0.0123	-0.0168	0.0077	0.0085	-0.0428	-0.0108	0.4278
E5	-0.0050	0.0548	0.0207	-0.0306	1	-0.0284	-0.0101	0.0271	-0.0107	0.0024	-0.0180	0.0143	-0.0319	-0.0096	0.0352	-0.0156	0.0026	-0.0014	-0.0013	0.0115	-0.020	0.013	-0.0256	0.0200	-0.0055	-0.0095	-0.0022	-0.0024	0.0165	0.0039	0.0161	0.0072
E6	-0.0334	0.0474	0.0009	0.0929	-0.028	1	0.0140	0.0043	-0.0676	0.0434	-0.0536	-0.0533	-0.0147	-0.0007	-0.0155	0.0172	-0.0179	-0.0248	0.0176	0.0484	0.026	0.017	-0.0116	0.0181	0.0156	0.0435	0.0248	0.0302	0.0025	0.0011	0.0308	0.0542
R1	0.0259	-0.0227	-0.0128	-0.0050	-0.010	0.014	1	0.0066	0.0181	-0.0431	0.0380	0.0226	0.0036	-0.0124	0.0093	0.0254	0.0359	-0.0015	0.0036	-0.0040	0.035	0.016	-0.0102	0.0166	0.0188	0.0287	-0.0127	-0.0455	-0.0021	-0.0301	-0.0076	-0.0246
R2	-0.0213	0.0309	0.0132	-0.0537	0.027	0.004	0.007	1	0.0363	-0.0024	-0.0106	0.0159	0.0291	-0.0113	0.0306	0.0160	-0.0590	-0.0517	-0.0178	-0.0109	-0.048	-0.005	0.0024	-0.0231	0.0475	-0.0007	-0.0360	-0.0086	0.0054	0.0198	-0.0490	-0.0244
R3	-0.0420	0.0558	-0.0369	-0.0363	-0.011	-0.068	0.018	0.036	1	0.0175	0.0187	0.0226	0.0096	0.0132	-0.0186	-0.0245	0.0370	0.0001	0.0096	0.0561	-0.007	0.034	-0.0006	0.0486	0.0046	-0.0245	-0.0394	-0.0035	0.0435	0.0050	0.0069	-0.0183
R4	-0.0419	-0.0433	0.0048	0.0653	0.002	0.043	-0.043	-0.002	0.017	1	-0.0194	0.0075	-0.0124	-0.0124	0.0496	-0.0427	0.0090	-0.0123	0.0210	-0.0326	-0.046	-0.026	-0.0024	-0.0274	-0.0007	-0.0160	0.0092	0.0156	-0.0123	0.0006	0.0124	-0.0109
R5	0.0046	0.0074	-0.0021	0.0172	-0.018	-0.054	0.038	-0.011	0.019	-0.019	1	0.0728	-0.0205	0.0268	-0.0688	-0.0312	0.00004	0.0137	-0.0104	-0.0175	0.034	-0.020	0.0564	-0.0049	0.0549	-0.0078	-0.0126	0.0433	-0.0502	0.0183	0.0169	-0.0179
R6	0.0212	-0.0002	0.0027	0.0014	0.014	-0.053	0.023	0.016	0.023	0.008	0.0728	1	-0.0243	-0.0042	0.0009	-0.0042	-0.0394	-0.0109	0.0293	-0.0243	0.006	0.009	-0.0276	0.0478	0.0176	-0.0008	0.0109	-0.0126	-0.0243	0.0058	0.0209	0.0144
R7	0.0500	-0.0002	-0.0149	0.0136	-0.032	-0.015	0.004	0.029	0.010	-0.012	-0.02046	-0.02426	1	-0.0484	0.0136	-0.0308	-0.0019	-0.0151	-0.0581	-0.0009	-0.035	0.038	0.0391	-0.0201	-0.0130	0.0395	-0.0460	0.0118	-0.0618	-0.0269	-0.0187	0.0106
R8	0.0151	0.0118	-0.0556	0.0098	-0.010	-0.001	-0.012	-0.011	0.013	-0.012	0.02676	-0.00416	-0.0484	1	-0.0370	0.0498	-0.0249	0.0312	0.0220	0.0259	-0.029	-0.009	-0.0381	0.0196	0.0102	-0.0139	0.0512	-0.0345	-0.0020	0.0039	0.0051	0.0231
R9	-0.0465	-0.0160	-0.0148	-0.0170	0.035	-0.016	0.009	0.031	-0.019	0.050	-0.06876	0.00086	0.0136	-0.0370	1	-0.0156	0.0136	0.0095	-0.0165	0.0510	-0.034	-0.018	0.0205	0.0046	0.0220	-0.0223	0.0729	-0.0128	0.0500	-0.0247	-0.0368	-0.0369
R10	-0.0234	0.0381	-0.0328	-0.0227	-0.016	0.017	0.025	0.016	-0.025	-0.043	-0.03124	-0.00419	-0.0308	0.0498	-0.0156	1	0.0340	0.0064	-0.0241	0.0327	-0.031	0.007	0.0127	0.0314	0.0111	0.0292	-0.0461	0.0171	-0.0239	-0.0147	0.0139	-0.0205
R11	0.0053	-0.0281	0.0138	0.0009	0.003	-0.018	0.036	-0.059	0.037	0.009	0.00004	-0.03938	-0.0019	-0.0249	0.0136	0.0340	1	0.0325	0.0115	0.0612	-0.005	0.006	-0.0088	0.0675	0.0065	0.0038	-0.0311	0.0077	-0.0248	-0.0273	0.0451	-0.0133
R12	-0.0150	-0.0172	-0.0250	-0.0827	-0.001	-0.025	-0.002	-0.052	0.000	-0.012	0.01369	-0.01085	-0.0151	0.0312	0.0095	0.0064	0.0325	1	0.0083	0.0594	0.018	0.027	0.0086	-0.0346	-0.0538	0.0231	0.0110	-0.0641	-0.0226	-0.0191	0.0391	-0.0462
R13	0.0052	0.0205	0.0001	-0.0134	-0.001	0.018	0.004	-0.018	0.010	0.021	-0.01042	0.02931	-0.0581	0.0220	-0.0165	-0.0241	0.0115	0.0083	1	0.0125	-0.015	-0.009	-0.0145	-0.0134	-0.0498	0.0262	-0.0058	-0.0385	0.0219	-0.00005	0.0315	-0.0044
R14	-0.0302	-0.0041	0.0165	-0.0069	0.012	0.048	-0.004	-0.011	0.056	-0.033	-0.01752	-0.02427	-0.0009	0.0259	0.0510	0.0327	0.0612	0.0594	0.0125	1	0.006	0.012	0.0326	0.0242	-0.0595	0.0092	0.0008	0.0043	-0.0378	-0.0007	0.0478	0.0056
R15	0.0388	0.0016	0.0274	-0.0149	-0.020	0.026	0.035	-0.048	-0.007	-0.046	0.03354	0.00589	-0.0350	-0.0288	-0.0338	-0.0306	-0.0047	0.0176	-0.01493	0.00577	1	-0.006	-0.0448	0.0061	0.0436	-0.0004	0.0411	0.0159	0.0113	0.0308	0.0253	0.0124
R16	-0.0352	-0.0330	0.0747	0.0670	0.013	0.017	0.016	-0.005	0.034	-0.026	-0.01952	0.00927	0.0383	-0.0095	-0.0178	0.0065	0.0061	0.0266	-0.00869	0.01243	-0.006	1	-0.0251	-0.0150	0.0097	0.0032	0.0580	-0.0266	-0.0164	0.0285	-0.0209	0.0081
R17	-0.0193	-0.0175	-0.0212	-0.0133	-0.026	-0.012	-0.010	0.002	-0.001	-0.002	0.05639	-0.02761	0.0391	-0.0381	0.0205	0.0127	-0.0088	0.0086	-0.01447	0.03261	-0.0448	-0.0251	1	0.0137	-0.0363	0.0060	-0.0426	0.0081	0.0054	-0.0238	-0.0289	-0.0023
R18	-0.0689	-0.0451	0.0420	-0.0094	0.020	0.018	0.017	-0.023	0.049	-0.027	-0.00487	0.04777	-0.0201	0.0196	0.0046	0.0314	0.0675	-0.0346	-0.01336	0.02419	0.0061	-0.0150	0.0137	1	0.0486	-0.0289	0.0026	0.0480	0.0161	0.0528	0.0037	0.0060
R19	0.0160	-0.0206	0.0465	-0.0255	-0.005	0.016	0.019	0.047	0.005	-0.001	0.05493	0.01759	-0.0130	0.0102	0.0220	0.0111	0.0065	-0.0538	-0.04980	-0.05948	0.0436	0.0097	-0.0363	0.0486	1	-0.009	0.0327	-0.0133	0.0302	-0.0151	-0.0337	0.0363
R20	0.0027	0.0384	-0.0326	0.0123	-0.010	0.043	0.029	-0.001	-0.025	-0.016	-0.00780	-0.00085	0.0395	-0.0139	-0.0223	0.0292	0.0038	0.0231	0.02616	0.00923	-0.0004	0.0032	0.0060	-0.0289	-0.009	1	0.0142	0.0305	-0.0574	0.0456	0.0240	0.0624
R21	-0.0142	-0.0235	-0.0002	-0.0168	-0.002	0.025	-0.013	-0.036	-0.039	0.009	-0.01262	0.01088	-0.0460	0.0512	0.0729	-0.0461	-0.0311	0.0110	-0.00581	0.00084	0.0411	0.0580	-0.0426	0.0026	0.0327	0.0142	1	-0.024	0.0244	0.0342	0.0024	0.0050
R22	-0.0525	0.0402	-0.0145	0.0077	-0.002	0.030	-0.045	-0.009	-0.003	0.016	0.04329	-0.01260	0.0118	-0.0345	-0.0128	0.0171	0.0077	-0.0641	-0.03849	0.00430	0.0159	-0.0266	0.0081	0.0480	-0.0133	0.0305	-0.0244	1	-0.0243	0.0258	-0.0726	0.0174
R23	-0.0008	-0.0181	-0.0125	0.0085	0.017	0.002	-0.002	0.005	0.043	-0.012	-0.05022	-0.02425	-0.0618	-0.0020	0.0500	-0.0239	-0.0248	-0.0226	0.02192	-0.03777	0.0113	-0.0164	0.0054	0.0161	0.0302	-0.0574	0.0244	-0.0243	1	-0.0329	-0.0350	-0.0137
R24	-0.0176	0.0386	-0.0306	-0.0428	0.004	0.001	-0.030	0.020	0.005	0.001	0.01827	0.00583	-0.0269	0.0039	-0.0247	-0.0147	-0.0273	-0.0191	-0.00005	-0.00073	0.0308	0.0285	-0.0238	0.0528	-0.0151	0.0456	0.0342	0.0258	-0.0329	1	-0.0372	0.0045
R25	0.0197	-0.0466	-0.0406	-0.0108	0.016	0.031	-0.008	-0.049	0.007	0.012	0.01687	0.02090	-0.0187	0.0051	-0.0368	0.0139	0.0451	0.0391	0.03155	0.04779	0.0253	-0.0209	-0.0289	0.0037	-0.0337	0.0240	0.0024	-0.0726	-0.0350	-0.037	1	-0.0247
Y	0.1137	0.2518	0.3420	0.4278	0.007	0.054	-0.025	-0.024	-0.018	-0.011	-0.01787	0.01436	0.0106	0.0231	-0.0369	-0.0205	-0.0133	-0.0462	-0.00440	0.00561	0.0124	0.0081	-0.0023	0.0060	0.0363	0.0624	0.0050	0.0174	-0.0137	0.0045	-0.02471	1

Number of Observations

E1,E3,E4,E6	1,597 observations
E2,E5,R1,R2,R3,R4,R11,R12,R17,R21,R23,R25,Y	1,587 observations
R5,R6,R7,R8,R9,R10,R13,R14,R15,R16,R18,R19,R20,R22,R24	1,617 observations

Percentage of Missing Values by Variable

	Fraction Missing
ID	0.00000000
E1	0.01236858
E2	0.01855288
E3	0.01236858
E4	0.01236858
E5	0.01855288
E6	0.01236858
R1	0.01855288
R2	0.01855288
R3	0.01855288
R4	0.01855288
R5	0.00000000
R6	0.00000000
R7	0.00000000
R8	0.00000000
R9	0.00000000
R10	0.00000000
R11	0.01855288
R12	0.01855288
R13	0.00000000
R14	0.00000000
R15	0.00000000
R16	0.00000000
R17	0.01855288
R18	0.00000000
R19	0.00000000
R20	0.00000000
R21	0.01855288
R22	0.00000000
R23	0.01855288
R24	0.00000000
R25	0.01855288
Y	0.01855288

Since there are NA values in our data sets, I had to choose a method to fill in these missing values. I ultimately decided to use the Amelia package in R for imputing missing data. The Amelia package performs multiple imputation using the EMB algorithm. This means it creates as many imputations as requested, in my case five, and then fills the missing values using a bootstrap based Expectation-Maximization algorithm. The Expectation- Maximization (EM) algorithm is a method for finding the maximum- likelihood estimates for a given data set when there are missing values. It involves generating random values for missing data and then computing expected values based on this filled in data set. Then based off of these values, it estimates the value which maximize the likelihood function. This process is repeated until the sequence of new values converges. This EM algorithm combined with a bootstrap approach, which is a method for resampling, is what makes up the EBM algorithm which the Amelia

package uses to fill in missing data. This method requires two assumptions. One being that the missing values are randomly generated, which after analyzing the data I believe they are, and that the data approximately follows a multivariate normal distribution, which because of the size of the data I believe is also satisfied.

So as mentioned I then applied the Amelia function to my merged data set and set the twenty-five indictor variables to be nominal values. The Amelia function then returns five imputations of the data set with all missing values filled. I plan to use all five imputations in the second part of the project in which I will attempt to find the model that the TA used to create my data. After imputation all variables have 1,617 observations. Below, I have attached the summary statistics for each variable in the first imputation of completed data sets as well as the correlation matrix.

Summary Statistics

```
       E1              E2              E3              E4               E5              E6              R1              R2              R3       
 Min.   : 264.1   Min.   : 375.2   Min.   : 156.6   Min.   : -10.57   Min.   :  16.83   Min.   : 205.0   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000  
 1st Qu.: 639.4   1st Qu.: 768.1   1st Qu.: 558.5   1st Qu.: 441.45   1st Qu.: 450.27   1st Qu.: 593.8   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000  
 Median : 749.5   Median : 878.8   Median : 656.1   Median : 543.83   Median : 563.45   Median : 704.1   Median :1.0000   Median :0.0000   Median :1.0000  
 Mean   : 750.0   Mean   : 875.0   Mean   : 657.9   Mean   : 543.51   Mean   : 564.22   Mean   : 700.4   Mean   :0.5127   Mean   :0.4929   Mean   :0.5065  
 3rd Qu.: 857.0   3rd Qu.: 984.0   3rd Qu.: 759.7   3rd Qu.: 648.09   3rd Qu.: 672.31   3rd Qu.: 806.7   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000  
 Max.   :1233.9   Max.   :1329.5   Max.   :1178.6   Max.   :1069.37   Max.   :1072.95   Max.   :1247.2   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000  
       R4              R5              R6              R7              R8              R9              R10             R11             R12       
 Min.   :0.0000   Min.   :0.0000   Min.   :0.000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :0.000  
 1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.000  
 Median :0.0000   Median :1.0000   Median :0.000   Median :0.0000   Median :0.0000   Median :0.0000   Median :0.0000   Median :1.0000   Median :0.000  
 Mean   :0.4978   Mean   :0.5189   Mean   :0.491   Mean   :0.4898   Mean   :0.4861   Mean   :0.4917   Mean   :0.4929   Mean   :0.5077   Mean   :0.478  
 3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.000   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.000  
 Max.   :1.0000   Max.   :1.0000   Max.   :1.000   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :1.000  
      R13             R14             R15             R16             R17             R18             R19             R20             R21       
 Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000  
 1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000  
 Median :0.0000   Median :0.0000   Median :0.0000   Median :0.0000   Median :0.0000   Median :0.0000   Median :0.0000   Median :0.0000   Median :1.0000  
 Mean   :0.4923   Mean   :0.4972   Mean   :0.4694   Mean   :0.4842   Mean   :0.4985   Mean   :0.4787   Mean   :0.4947   Mean   :0.4954   Mean   :0.5015  
 3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000  
 Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000  
      R22             R23             R24             R25              Y          
 Min.   :0.0000   Min.   :0.0000   Min.   :0.000   Min.   :0.0000   Min.   :-7.603e+08  
 1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.000   1st Qu.:0.0000   1st Qu.: 2.132e+10  
 Median :1.0000   Median :0.0000   Median :1.000   Median :1.0000   Median : 2.640e+10  
 Mean   :0.5133   Mean   :0.4836   Mean   :0.522   Mean   :0.5152   Mean   : 2.649e+10  
 3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.000   3rd Qu.:1.0000   3rd Qu.: 3.144e+10  
 Max.   :1.0000   Max.   :1.0000   Max.   :1.000   Max.   :1.0000   Max.   : 5.537e+10
```

Standard Deviations

```
       E1        E2        E3        E4        E5        E6        R1        R2        R3        R4        R5        R6        R7 
0.1803265 0.1834427 0.1854879 0.1935964 0.1781835 0.1788435 0.1776255 0.1788931 0.1781915 0.1789711 0.1783317 0.1782825 0.1799099 
       R8        R9       R10       R11       R12       R13       R14       R15       R16       R17       R18       R19       R20 
0.1786242 0.1798127 0.1792670 0.1787964 0.1800707 0.1775635 0.1776377 0.1785216 0.1778850 0.1783678 0.1778199 0.1783356 0.1771319 
       R21       R22       R23       R24       R25         Y 
0.1785904 0.1789218 0.1795489 0.1775849 0.1792758 0.1997602
```

Correlation Matrix

Correlation Matrix

	E1	E2	E3	E4	E5	E6	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	R12	R13	R14	R15	R16	R17	R18	R19	R20	R21	R22	R23	R24	R25	Y
E1	1	0.020	0.044	-0.021	-0.006	-0.030	0.016	-0.025	-0.005	-0.050	0.016	-0.008	0.039	-0.013	-0.035	-0.044	0.000	-0.016	0.008	-0.011	0.020	-0.008	-0.014	-0.053	-0.010	0.001	-0.007	-0.051	-0.004	-0.005	-0.004	0.108
E2	0.020	1	-0.026	-0.047	0.058	0.035	-0.011	0.010	0.050	-0.017	0.002	-0.012	0.000	0.037	-0.030	0.029	-0.030	-0.029	0.015	0.002	-0.022	-0.037	-0.007	-0.024	-0.026	0.009	-0.050	0.027	-0.003	0.030	-0.033	0.247
E3	0.044	-0.026	1	-0.047	0.006	-0.019	-0.015	0.000	-0.034	0.032	-0.0003	-0.002	0.005	-0.030	-0.010	-0.008	0.003	-0.024	0.000	0.032	0.014	0.058	-0.008	0.036	0.018	-0.024	0.011	-0.005	-0.010	-0.022	-0.035	0.312
E4	-0.021	-0.047	-0.047	1	-0.033	0.088	0.013	-0.036	-0.044	0.053	0.023	-0.025	0.027	0.021	-0.017	-0.024	0.011	-0.040	-0.007	0.011	0.028	0.039	-0.014	-0.016	-0.018	0.026	-0.004	0.005	-0.011	-0.010	-0.017	0.435
E5	-0.006	0.058	0.006	-0.033	1	-0.028	-0.004	0.018	-0.013	-0.014	-0.021	-0.008	-0.025	-0.008	0.020	-0.041	-0.001	0.018	0.029	0.007	-0.009	0.017	-0.022	0.034	-0.004	0.006	-0.023	-0.013	0.021	0.008	0.013	-0.007
E6	-0.030	0.035	-0.019	0.088	-0.028	1	0.003	-0.006	-0.042	0.046	-0.055	-0.032	-0.005	-0.020	-0.019	-0.005	-0.012	-0.040	0.001	0.025	0.037	0.029	-0.013	0.029	0.009	0.036	0.028	0.026	-0.012	0.025	-0.001	0.030
R1	0.016	-0.011	-0.015	0.013	-0.004	0.003	1	0.003	-0.011	-0.016	0.013	0.026	-0.001	-0.004	-0.015	0.047	0.011	-0.020	0.001	-0.009	0.018	-0.005	-0.009	0.002	0.033	0.017	0.009	-0.032	-0.009	-0.023	0.000	-0.027
R2	-0.025	0.010	0.000	-0.036	0.018	-0.006	0.003	1	0.019	-0.014	-0.013	0.003	0.020	0.003	0.031	0.012	-0.047	-0.057	-0.005	-0.024	-0.024	0.009	-0.004	-0.025	0.043	-0.003	-0.008	-0.016	-0.002	0.009	-0.011	-0.012
R3	-0.005	0.050	-0.034	-0.044	-0.013	-0.042	-0.011	0.019	1	0.009	0.018	0.024	0.007	0.012	-0.012	-0.007	0.028	-0.010	-0.003	0.056	-0.016	0.048	0.002	0.034	-0.003	-0.022	-0.034	0.007	0.022	0.026	-0.001	-0.026
R4	-0.050	-0.017	0.032	0.053	-0.014	0.046	-0.016	-0.014	0.009	1	-0.030	-0.004	-0.014	0.003	0.054	-0.008	-0.015	-0.020	0.023	-0.027	-0.051	-0.028	0.003	-0.012	0.010	-0.013	0.002	0.028	0.003	-0.009	0.011	0.000
R5	0.016	0.002	0.000	0.023	-0.021	-0.055	0.013	-0.013	0.018	-0.030	1	0.077	-0.002	0.010	-0.026	-0.038	0.005	0.009	0.017	-0.033	0.018	-0.001	0.034	-0.004	0.037	-0.006	0.001	0.055	-0.032	0.040	-0.004	-0.003
R6	-0.008	-0.012	-0.002	-0.025	-0.008	-0.032	0.026	0.003	0.024	-0.004	0.077	1	-0.052	0.003	0.029	-0.006	-0.025	0.000	0.023	-0.017	-0.002	0.001	-0.022	0.037	0.010	-0.001	0.002	0.011	-0.030	-0.013	0.028	0.006
R7	0.039	0.000	0.005	0.027	-0.025	-0.005	-0.001	0.020	0.007	-0.014	-0.002	-0.052	1	-0.049	-0.006	-0.016	0.010	-0.008	-0.059	-0.002	-0.027	0.041	0.030	-0.018	-0.019	0.061	-0.020	0.011	-0.045	-0.033	-0.028	0.007
R8	-0.013	0.037	-0.030	0.021	-0.008	-0.020	-0.004	0.003	0.012	0.003	0.010	0.003	-0.049	1	-0.043	0.044	-0.032	0.019	0.003	0.028	-0.027	-0.009	-0.009	0.019	0.008	-0.006	0.024	-0.056	-0.013	0.007	0.021	0.032
R9	-0.035	-0.030	-0.010	-0.017	0.020	-0.019	-0.015	0.031	-0.012	0.054	-0.026	0.029	-0.006	-0.043	1	-0.037	0.001	0.008	-0.018	0.022	-0.040	-0.012	-0.001	-0.006	0.021	-0.022	0.045	-0.013	0.048	-0.017	-0.047	-0.028
R10	-0.044	0.029	-0.008	-0.024	-0.041	-0.005	0.047	0.012	-0.007	-0.008	-0.038	-0.006	-0.016	0.044	-0.037	1	0.008	-0.026	-0.035	0.041	-0.003	-0.010	0.012	0.031	-0.001	0.030	-0.039	0.002	-0.023	-0.002	0.005	-0.007
R11	0.0002	-0.030	0.003	0.011	-0.001	-0.012	0.011	-0.047	0.028	-0.015	0.005	-0.025	0.010	-0.032	0.001	0.008	1	0.027	-0.005	0.056	0.006	-0.029	-0.023	0.057	-0.008	-0.009	-0.039	0.006	-0.025	-0.011	0.039	-0.010
R12	-0.016	-0.029	-0.024	-0.040	0.018	-0.040	-0.020	-0.057	-0.010	-0.020	0.009	0.000	-0.008	0.019	0.008	-0.026	0.027	1	0.012	0.033	-0.001	0.008	-0.002	-0.044	-0.032	0.026	0.012	-0.050	-0.001	-0.005	0.047	-0.042
R13	0.008	0.015	0.000	-0.007	0.029	0.001	0.001	-0.005	-0.003	0.023	0.017	0.023	-0.059	0.003	-0.018	-0.035	-0.005	0.012	1	0.005	-0.004	-0.001	0.003	0.005	-0.037	0.026	-0.003	-0.001	0.022	-0.006	0.028	0.005
R14	-0.011	0.002	0.032	0.011	0.007	0.025	-0.009	-0.024	0.056	-0.027	-0.033	-0.017	-0.002	0.028	0.022	0.041	0.056	0.033	0.005	1	0.006	-0.008	0.023	0.020	-0.059	0.022	-0.006	-0.009	-0.029	-0.016	0.026	0.026
R15	0.020	-0.022	0.014	0.028	-0.009	0.037	0.018	-0.024	-0.016	-0.051	0.018	-0.002	-0.027	-0.027	-0.040	-0.003	0.006	-0.001	-0.004	0.006	1	-0.014	-0.050	-0.003	0.053	0.005	0.028	0.021	-0.005	0.007	-0.009	0.030
R16	-0.008	-0.037	0.058	0.039	0.017	0.029	-0.005	0.009	0.048	-0.028	-0.001	0.001	0.041	-0.009	-0.012	-0.010	-0.029	0.008	-0.001	-0.008	-0.014	1	0.007	-0.029	0.011	0.000	0.043	-0.025	-0.011	0.028	-0.017	-0.001
R17	-0.014	-0.007	-0.008	-0.014	-0.022	-0.013	-0.009	-0.004	0.002	0.003	0.034	-0.022	0.030	-0.009	-0.001	0.012	-0.023	-0.002	0.003	0.023	-0.050	0.007	1	0.008	-0.004	0.002	-0.018	-0.004	-0.019	-0.007	-0.012	-0.012
R18	-0.053	-0.024	0.036	-0.016	0.034	0.029	0.002	-0.025	0.034	-0.012	-0.004	0.037	-0.018	0.019	-0.006	0.031	0.057	-0.044	0.005	0.020	-0.003	-0.029	0.008	1	0.052	-0.013	-0.013	0.051	0.019	0.030	-0.005	0.005
R19	-0.010	-0.026	0.018	-0.018	-0.004	0.009	0.033	0.043	-0.003	0.010	0.037	0.010	-0.019	0.008	0.021	-0.001	-0.008	-0.032	-0.037	-0.059	0.053	0.011	-0.004	0.052	1	-0.018	0.022	0.013	0.035	-0.014	-0.046	0.017
R20	0.001	0.009	-0.024	0.026	0.006	0.036	0.017	-0.003	-0.022	-0.013	-0.006	-0.001	0.061	-0.006	-0.022	0.030	-0.009	0.026	0.026	0.022	0.005	0.000	0.002	-0.013	-0.018	1	-0.002	0.024	-0.045	0.022	0.012	0.048
R21	-0.007	-0.050	0.011	-0.004	-0.023	0.028	0.009	-0.008	-0.034	0.002	0.001	0.002	-0.020	0.024	0.045	-0.039	-0.039	0.012	-0.003	-0.006	0.028	0.043	-0.018	-0.013	0.022	-0.002	1	-0.025	0.002	-0.003	0.012	-0.010
R22	-0.051	0.027	-0.005	0.005	-0.013	0.026	-0.032	-0.016	0.007	0.028	0.055	0.011	0.011	-0.056	-0.013	0.002	0.006	-0.050	-0.001	-0.009	0.021	-0.025	-0.004	0.051	0.013	0.024	-0.025	1	-0.021	0.027	-0.020	0.002
R23	-0.004	-0.003	-0.010	-0.011	0.021	-0.012	-0.009	-0.002	0.022	0.003	-0.032	-0.030	-0.045	-0.013	0.048	-0.023	-0.025	-0.001	0.022	-0.029	-0.005	-0.011	-0.019	0.019	0.035	-0.045	0.002	-0.021	1	-0.013	-0.038	-0.022
R24	-0.005	0.030	-0.022	-0.010	0.008	0.025	-0.023	0.009	0.026	-0.009	0.040	-0.013	-0.033	0.007	-0.017	-0.002	-0.011	-0.005	-0.006	-0.016	0.007	0.028	-0.007	0.030	-0.014	0.022	-0.003	0.027	-0.013	1	-0.023	0.012
R25	-0.004	-0.033	-0.035	-0.017	0.013	-0.001	-0.0002	-0.011	-0.001	0.011	-0.004	0.028	-0.028	0.021	-0.047	0.005	0.039	0.047	0.028	0.026	-0.009	-0.017	-0.012	-0.005	-0.046	0.012	0.012	-0.020	-0.038	-0.023	1	-0.036
Y	0.108	0.247	0.312	0.435	-0.007	0.030	-0.027	-0.012	-0.026	0.000	-0.003	0.006	0.007	0.032	-0.028	-0.007	-0.010	-0.042	0.005	0.026	0.030	-0.001	-0.012	0.005	0.017	0.048	-0.010	0.002	-0.022	0.012	-0.036	1

References:

Honaker, J., King, G., & Blackwell, M. (2019, November 24). *Amelia II: A Program for Missing Data*. https://cran.r-project.org/web/packages/Amelia/vignettes/amelia.pdf.