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Rendering Outdoor Light Scattering in Real Time

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Introduction

Atmospheric scattering of light is important in outdoor scenes. It changes sunlight from the pale red of dawn to the bright yellow of midday and back again. It determines the color and brightness of the sky throughout the day, and it cues us to the distance of objects by shifting their colors. All these effects vary not only based on time of day, but also depending on weather, pollution and other factors. On planets with different atmospheric compositions, these effects would differ significantly from those seen on Earth.

In this paper, we will explain the ways in which atmosphere affects light, including the underlying theory. We will show the deficiencies of the commonly-used fog model, and describe models which are more physically accurate.

Interaction of Light with Particles

The interaction of light with particles is one of the most fundamental phenomena in graphics (surface reflectance is derived from it as a special case). Light, as an electromagnetic wave, is affected by the electromagnetic fields of particles of various kinds. When light interacts with a particle, the particle may absorb the light (becoming more energetic as a result) or scatter it in a new direction. Particles may also emit light on their own – we will ignore this possibility in this paper. We assume a basic understanding of the physical nature of light and quantities such as radiance and irradiance. If you wish to learn more about such topics there are introductions in [Hoffman2001] and [Yee2002].

Absorption

A particles' absorption of light can be quantified by its *absorption cross section* $\sigma_{ab}^{(\lambda)}$. This is measured in units of area (m^2) and is defined as the absorbed radiant flux (W) per unit incident irradiance (W/m^2). Note that it is dependent on wavelength – most particles absorb some wavelengths more readily than others. A more intuitive understanding of the absorption cross section can be obtained by looking at Fig. 1:

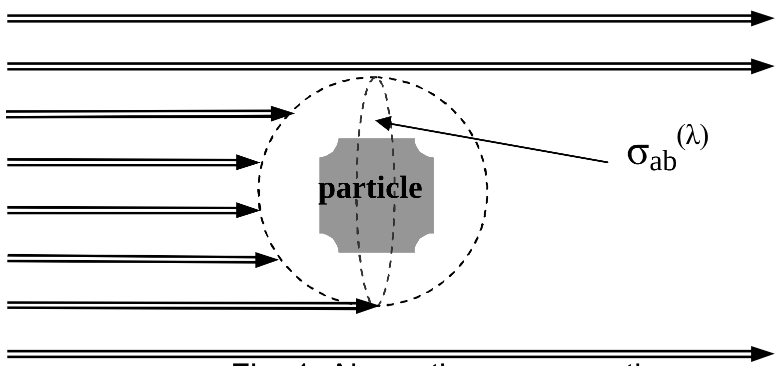

Fig. 1: Absorption cross section

For illustration purposes, we assume that the particle absorbs all light which hits a spherical volume around the particle. Note that this sphere may be larger or smaller than the actual geometric size of the particle. Looking at the cross section of this sphere, we have an area which absorbs the incident irradiance at each point. The total absorbed flux is the incident irradiance integrated over the cross section area, which matches the definition for $\sigma_{ab}^{(\lambda)}$.

An absorptive medium contains a certain volume density ρ_{ab} (m^{-3}) of particles, each with an absorption cross section $\sigma_{ab}^{(\lambda)}$. We define the *absorption coefficient*: $\beta_{ab}^{(\lambda)} = \rho_{ab}\, \sigma_{ab}^{(\lambda)}$, this is measured in units of inverse length (m^{-1}). $\beta_{ab}^{(\lambda)}$ is the absorption cross-section area per unit volume. To understand the significance of $\beta_{ab}^{(\lambda)}$, imagine that we are shooting rays in a fixed direction through the media (see Fig. 2.)

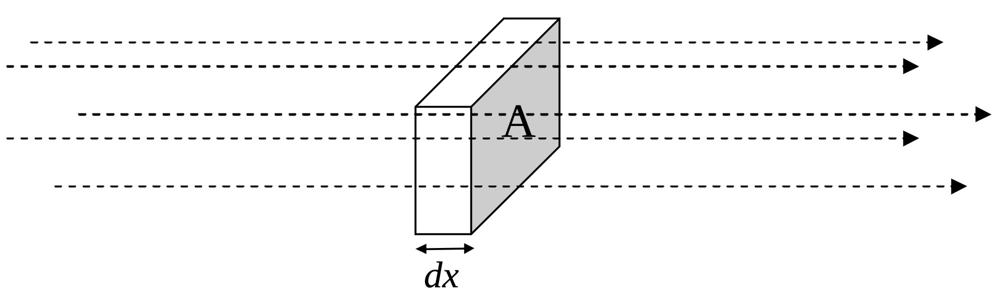

Fig. 2: Absorption coefficient

We define a box with an area A perpendicular to the ray direction, and a small depth dx in the ray direction. The volume of this box is $A dx$, so the aggregate absorption cross section area in the box is $\beta_{ab}^{(\lambda)} \mathrm{A} dx$. If we shoot a photon randomly through the box in the ray direction, the probability of it's being absorbed is the aggregate absorption cross section area divided by the box cross section area A, which is $\beta_{\mathrm{ab}}^{(\lambda)} dx$ (note that this is inverse length times length, which results in a dimensionless quantity). This means that a ray of light with radiance $L^{(\lambda)}$ traveling a distance dx through the media will lose a fraction $\beta_{\mathrm{ab}}^{(\lambda)} dx$ of itself to absorption. Another way of putting this is:

$$-\frac{dL^{(\lambda)}}{L^{(\lambda)}} = \beta_{\mathrm{ab}}^{(\lambda)} dx \tag{1}$$

The radiance $L^{(\lambda)}(x)$ after traveling for a total distance x can be found by solving the differential equation in Eq. 1:

$$L^{(\lambda)}(x) = L_0^{(\lambda)} e^{-\beta_{\mathrm{ab}}^{(\lambda)} x} \tag{2}$$

Where 0 is the starting point of the ray's passage through the absorptive media, x is the distance along the ray at which we are evaluating the radiance, and $L_0^{(\lambda)}$ is the radiance at point 0.

Eq. 2 assumes that $\beta_{\mathrm{ab}}^{(\lambda)}$ is constant. If it is spatially variant:

$$L^{(\lambda)}(x) = L_0^{(\lambda)} e^{-\int_0^x \beta_{\mathrm{ab}}^{(\lambda)}(x')dx'} \tag{3}$$

If we have n different kinds of particles, each with it's own absorption coefficient $\beta_{\mathrm{ab(i)}}^{(\lambda)}$, the total absorption coefficient $\beta_{\mathrm{ab}}^{(\lambda)}$ is equal to the sum of the absorption coefficients of all the particles:

$$\beta_{\mathrm{ab}}^{(\lambda)} = \sum_{i=1}^{n} \beta_{\mathrm{ab(i)}}^{(\lambda)} \tag{4}$$

The exact manner in which the absorption depends on wavelength is a property of the particle. For realism we can use empirical data (which is available for all common types of atmospheric particles),one possible source is [Preetham1999a].

Scattering (Out-Scattering)

Similarly to absorption, the degree by which a particle scatters light can be quantified by its *scattering cross section* $\sigma_{sc}^{(\lambda)}$, or the scattered radiant flux per unit incident irradiance.

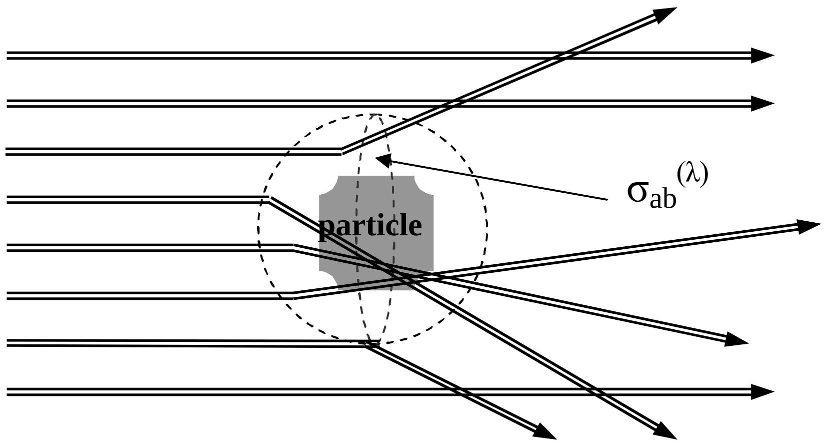

Fig. 3: Scattering cross section

Looking at Fig. 3, we can see that the incident irradiance integrated over the scattering cross section area gives us the total scattered radiant flux, which fits the definition. As we did for absorption, we define the *scattering coefficient*: $\beta_{sc}^{(\lambda)} = \rho_{sc}\, \sigma_{sc}^{(\lambda)}$ (m^{-1}), which is the scattering cross-section area per unit volume. Again similarly to the absorption case, a ray of light with radiance $L^{(\lambda)}$ traveling a distance dx through the media will lose a fraction $\beta_{sc}^{(\lambda)} dx$ of itself due to scattering out of the path of the ray (*out-scattering*):

$$-\frac{dL^{(\lambda)}}{L^{(\lambda)}} = \beta_{sc}^{(\lambda)} dx \tag{5}$$

Which gives us (in the constant $\beta_{sc}^{(\lambda)}$ case):

$$L^{(\lambda)}(x) = L_0^{(\lambda)} e^{-\beta_{sc}^{(\lambda)} x} \tag{6}$$

And in the case where $\beta_{sc}^{(\lambda)}$ is spatially variant:

$$L^{(\lambda)}(x) = L_0^{(\lambda)} e^{-\int_0^x \beta_{sc}^{(\lambda)}(x')dx'} \tag{7}$$

As for absorption, the total scattering coefficient $\beta_{sc}^{(\lambda)}$ is equal to the sum of all particle types' individual scattering coefficients $\beta_{sc(i)}^{(\lambda)}$:

$$\beta_{\mathrm{sc}}^{(\lambda)} = \sum_{i=0}^{n-1} \beta_{\mathrm{sc(i)}}^{(\lambda)} \tag{8}$$

In addition, we can add the coefficients for absorption and scattering, since both cause light to be removed from a ray. The result is the extinction coefficient (extinction refers to both absorption and out-scattering):

$$\beta_{\mathrm{ex}}^{(\lambda)} = \beta_{\mathrm{ab}}^{(\lambda)} + \beta_{\mathrm{sc}}^{(\lambda)} \tag{9}$$

Scattering (In-Scattering)

Although we have handled scattering very similarly to absorption, there is one important difference; after a photon is absorbed we can forget about it, but a scattered photon has to go somewhere. We have seen two phenomena which cause light to be removed from a ray, however there are also phenomena which can add light to a ray. One of these is *in-scattering*, where light which was originally headed in a different direction is scattered <u>into</u> the path of a light ray and adds to its radiance.

We have looked at the scattering coefficient, which tells us how much light is scattered by a medium. It does not tell us how much light is scattered in each direction, however. For this we define the scattering *phase function* $\Phi(\theta, \varphi)$ which gives the probability of scattered light going in the direction (θ, φ). This is probability per solid angle, which is measured in units of inverse solid angle (sr^{-1}). Note that if the scattering particles are either spherical or very small compared to the wavelength (which is true for the particles we are concerned with), then the phase function only depends on the angle θ between the original direction and the new direction. In this case we will write it as $\Phi(\theta)$. Since this is a probability function, when integrated over the entire sphere of directions the result is 1:

$$\int_{\Omega} \Phi(\theta)\, d\omega = 1 \tag{10}$$

To ensure this we may need normalization factors in some cases; for example the isotropic phase function is equal to $1 / 4\pi$. This is not surprising, since the number of steradians in a sphere is 4π. Note that here we assume that the phase function itself is not dependent on the wavelength; this is true for the phase functions we will be using.

Now we can define the *angular scattering coefficient* $\beta_{sc}^{(\lambda)}(\theta) = \beta_{sc}^{(\lambda)}\Phi(\theta)$, in which case:

$$\int_{\Omega} \beta_{\mathrm{sc}}^{(\lambda)}(\theta)\, d\omega = \beta_{\mathrm{sc}}^{(\lambda)} \tag{11}$$

Given that $\beta_{sc}^{(\lambda)}$ is measured in units of inverse length (m^{-1}), it is clear from Eq. 11 that $\beta_{sc}^{(\lambda)}(\theta)$ is measured in units of inverse length times inverse solid angle ($m^{-1}sr^{-1}$).

To understand the significance of $\beta_{sc}^{(\lambda)}(\theta)$, let's look at a single inscattering event (Fig. 4). We define this as an event where light was scattered into the path of the ray at a particular point.

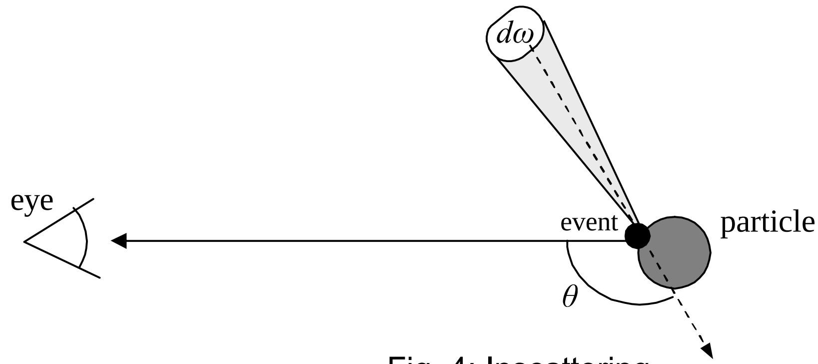

Fig. 4: Inscattering

We are looking at an infinitesimal solid angle patch of incoming directions $d\omega$. Given the definition of $\Phi(\theta)$, we can see that $\Phi(\theta)\, d\omega$ is equal to the probability of light being scattered from that patch into the ray (probability per solid angle times solid angle). If we multiply this probability times the incoming radiance $L_i^{(\lambda)}$ from the patch, we get $L_i^{(\lambda)}(\theta, \varphi)\Phi(\theta)\, d\omega$ which is the inscattered radiance from the patch. We can also look at it as the irradiance from the patch ($E_{(d\omega)} = L_i^{(\lambda)}(\theta, \varphi)d\omega$) times $\Phi(\theta)$. Integrating over the sphere, we get the total radiance added by this inscattering event which is equal to:

$$\int_{\Omega} L_i^{(\lambda)}(\theta, \varphi)\Phi(\theta)d\omega \tag{12}$$

To get the total inscattered radiance over a path length dx, we need to multiply this by the probability of an inscattering event happening over that path length, which is $\beta_{sc}^{(\lambda)}\, dx$ (note that this is the same as the probability of an outscattering event happening). The result is that the radiance added due to inscattering over an infinitesimal path dx is:

$$dL_{\text{inscatter}}^{(\lambda)} = \beta_{sc}^{(\lambda)} dx \int_{\Omega} L_i^{(\lambda)}(\theta, \varphi)\Phi(\theta)d\omega = dx \int_{\Omega} L_i^{(\lambda)}(\theta, \varphi)\beta_{sc}^{(\lambda)}(\theta)d\omega \tag{13}$$

When we consider in-scattering, we have to consider extinction as well. The reason is that the in-scattered light undergoes extinction on its way to the eye. For this reason, we will not solve Eq. 13 alone – instead we will combine it with Eq. 1, Eq. 5 and Eq. 9 and solve the combined differential equation:

$$\frac{dL^{(\lambda)}}{dx} = -\beta_{ex}^{(\lambda)} L^{(\lambda)} + \int_{\Omega} L_i^{(\lambda)}(\theta, \varphi)\beta_{sc}^{(\lambda)}(\theta)d\omega \tag{14}$$

The solution to which is:

$$L^{(\lambda)}(x) = L_0^{(\lambda)} e^{-\int_0^x \beta_{ex}^{(\lambda)}(x')dx'} + \int_0^x \left(e^{-\int_{x'}^{x} \beta_{ex}^{(\lambda)}(x'')dx''} \int_\Omega L_i^{(\lambda)}(\theta,\varphi)\beta_{sc}^{(\lambda)}(\theta)d\omega \right) dx' \quad (15)$$

Recall that 0 is the point where the ray starts to pass through the media (usually the emission point of the ray on the surface of an object), and x is the distance along the ray at which we are evaluating the radiance (usually the viewpoint). Eq. 15 may seem intimidating, but in many cases it reduces to much simpler forms. The important thing to remember is that the participating media has two effects on an object's perceived color - one multiplicative (extinction) and one additive (in-scattering):

$$L^{(\lambda)}(x) = L_0^{(\lambda)} f_{\text{extinction}} + f_{\text{inscattering}} \quad (16)$$

Scattering Coefficients

What does $\beta_{sc}^{(\lambda)}(\theta)$ look like? This depends of course on the particle doing the scattering. For particles much smaller than the wavelength of light ($r < 0.05\,\lambda$) the scattering coefficient was published by Lord Rayleigh [Rayleigh1871]. The phase function $\Phi_R(\theta)$ for *Rayleigh scattering* is:

$$\Phi_R(\theta) = \frac{3}{16\pi}\left(1 + \cos^2\theta\right) \quad (17)$$

Note that the phase function is symmetrical about the forward and backward directions.

An exact expression for the total scattering coefficient $\beta_{sc}^{(\lambda)}$ for Rayleigh scattering can be found in [Preetham1999a]. The important thing about it is that it is proportional to $1 / \lambda^4$, so shorter wavelengths are scattered to a much larger extent than longer wavelengths.

A theoretical model for larger (spherical) particles was developed by Mie [Mie1908]. *Mie scattering* is much more complex in the general case, however we can make some simplifying assumptions. The Henyey / Greenstein phase function $\Phi_{HG}(\theta)$ [Henyey1941] may be used as an approximation to the Mie phase function:

$$\Phi_{HG}(\theta) = \frac{(1-g)^2}{4\pi\left(1 + g^2 - 2g\cos(\theta)\right)^{3/2}} \quad (18)$$

The use of this function was discussed in [Blinn1982] and [Klassen1987]. Eq. 18 is simply the polar form for an ellipse (centered at one of the foci), where g is the eccentricity. Negative values of g will cause most of the light to be scattered in the forward direction, and positive values will cause most of it to be scattered backwards (for most particles, g should be negative and increase in magnitude when the particle size increases). Another option is to use available tables which have been calculated by the direct use of Mie theory.

To understand the wavelength dependence we can look at empirical data. Fig. 5 contains a graph, adapted from [Klassen1987], which shows the *scattering efficiency* Q_{sc} for non-absorbing spherical particles as a function of r / λ. Q_{sc} is defined as the ratio of the scattering cross section of the particle and it's geometric cross section ($Q_{sc} = \sigma_{sc}^{(\lambda)} / 2\pi r^2$).

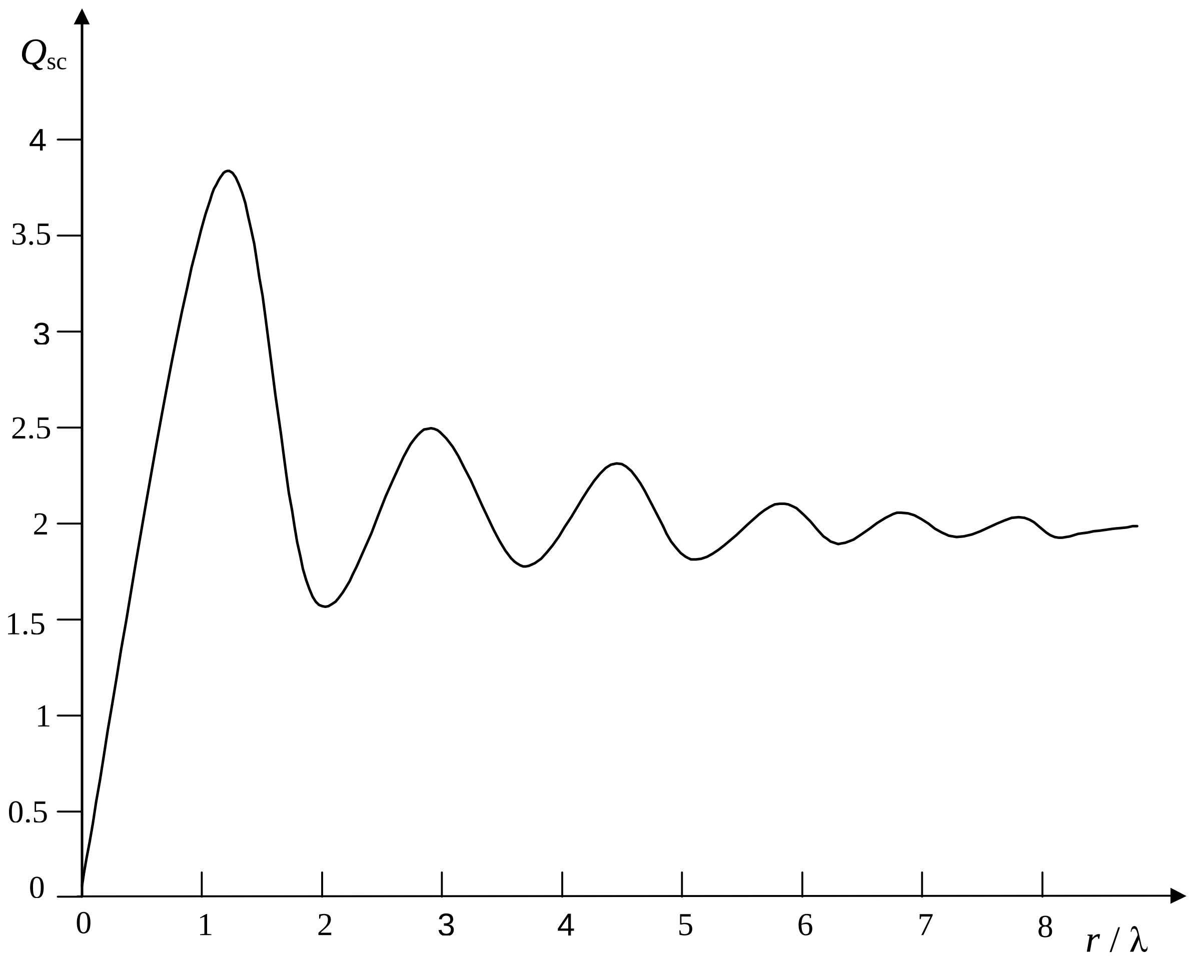

Fig. 5: Scattering efficiency

For very small particles, Q_{sc} varies with the square of r / λ. This combined with the r^2 dependence of the geometric cross-section gives us the Rayleigh $1 / \lambda^4$ dependence for fixed r. As the particle size increases, the dependency on wavelength becomes weaker, until at around $r \approx \lambda$, Q_{sc} starts actually increasing with wavelength. Q_{sc} then oscillates a few times before settling down to a limit value of 2 (for larger particles, there is no significant wavelength dependency). In most cases, Mie scatterers are not present in a single size but in a continuous distribution of sizes. Under these circumstances, the oscillations tend to average out and we can approximate $Q_{sc} = 2$ with no dependence on wavelength.

Earth's Atmosphere

Earth's atmosphere has two main classes of scatterers: the gas molecules present in clean, dry air, which are Rayleigh scatterers; and aerosols of various types which are Mie scatterers. Some aerosols also absorb light. The range of aerosol size is large enough so the assumption of constant Q_{sc} is usually valid. Numeric data on these is available from many sources – of which [Preetham1999a] may be the most accessible.

Applying the Theory - Aerial Perspective

As we look at distant objects, their color varies based on the distance and the time of day. This is due to the interaction of the light with the particles in the atmosphere. In Fig. 6, we can see the camera looking at a distant mountain. The mountain is reflecting light of spectral radiance $L_0^{(\lambda)}$ towards the camera. This will undergo extinction on the way to the camera, as well as in-scattering from the sun and sky. If the air is reasonably clean, Rayleigh scattering will cause the extinction predominantly of the bluer (shorter) wavelengths so the extinction factor will have a reddish color. During daytime, there will also be significant in-scattering which will be mostly blue (again due to Rayleigh scattering), so the in-scattering factor will have a bluish color, or bright white for directions adjacent to the sun (due to Mie scattering).

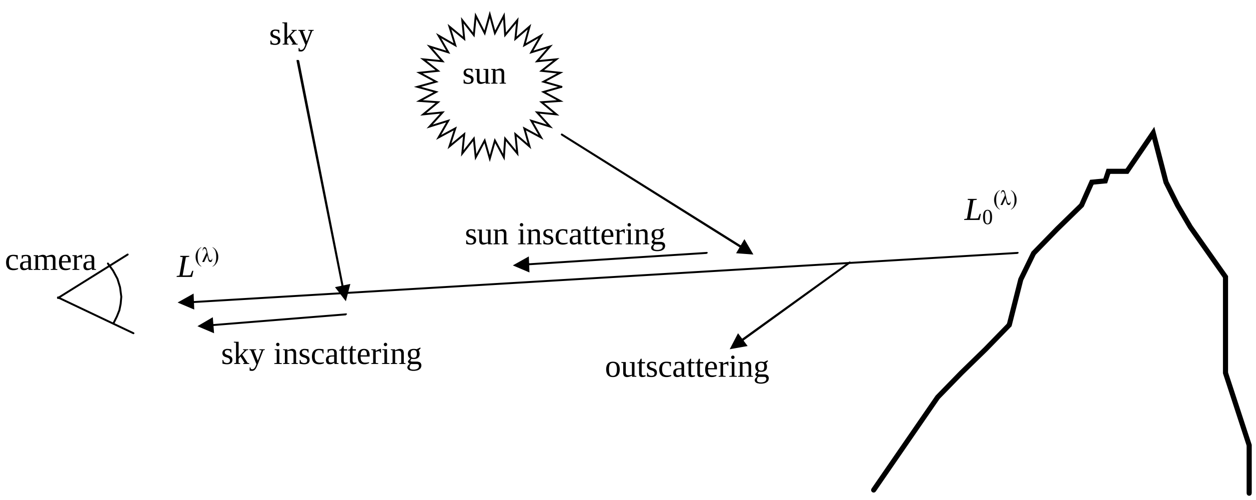

Fig. 6:

Hardware fog, which is the most common model used for aerial perspective, can be summarized as $L^{(\lambda)} = L_0^{(\lambda)}\,(1 - f) + \mathrm{C_{fog}}\;f$, where f is the fog factor and $\mathrm{C_{fog}}$ is the fog color. This is clearly wrong: the multiplicative factor is monochrome and the additive factor's color or intensity does not change based on viewing direction.

To calculate the physically correct extinction and inscattering factors, we can use Eq. 15 directly. Various simplifications and assumptions can be used to evaluate Eq. 15 in real-time for all the objects in the scene. One of the most important sources of simplification is the density function of the various scattering particles. In most cases both the camera and target object are low in the atmosphere, so we can make major simplifications here.

Applying the Theory - Sunlight

Sunlight undergoes extinction on its way from outer space to the surface. We can use the extinction factor from Eq. 15 to calculate the sun's radiance based on the sun's position in the sky and atmospheric factors. The amount of blue scattered out by Rayleigh scattering increases as the sun moves lower on the horizon, so the sunlight reddens.

Sunlight would seem to be a subset of aerial perspective, but actually there are some additional complications introduced by the fact that we need to use more accurate density functions – at the very least we need to take account of the curvature of the atmosphere.

Applying the Theory - Sky Color

The sky color is the result of in-scattering. Rayleigh scattering causes the bright blue color of the sky during daytime, while Mie scattering explains the gray color of a polluted or overcast sky. Mie scattering also causes the reddish tinge in the sky around the sun at sunset. The sky opposite from the sun darkens at sunset due to the Earth's shadow being cast through the air and decreasing the amount of in-scattering from those directions. In the case of sky color $L_0^{(\lambda)}$ is usually 0 (black), which removes the extinction factor. Again, sky color is not purely a subset of aerial perspective due to the fact that to achieve reasonable results fairly accurate density functions are needed.

Conclusion

In this paper, we have presented the basic background material for understanding the effect of atmosphere on light and the applications to outdoor scenes.

Implementation Note

If we assume a constant density atmosphere and low camera and target objects, we can simplify the general equation and render scenes with light scattering in real-time. Vertex and pixel shaders available in the RADEON™ 9500/9700 family of products can implement the following equations:

$$L(s,\theta) = L_0 F_{ex}(s) + L_{in}(s,\theta) \tag{19}$$

where L_0 is the input color, s is the distance to the camera, and θ is the angle to the sun.

In the vertex shader we compute $F_{ex}(s)$ and $L_{in}(s,\theta)$ using the following equations:

$$F_{ex}(s) = e^{-(\beta_R + \beta_M)s}$$

$$L_{in}(s,\theta) = \frac{\beta_R(\theta) + \beta_M(\theta)}{\beta_R + \beta_M} E_{sun}(1 - e^{-(\beta_R + \beta_M)s})$$

$$\beta_R(\theta) = \frac{3}{16\pi}\beta_R(1 + \cos^2\theta)$$

$$\beta_M(\theta) = \frac{1}{4\pi}\beta_M \frac{(1-g)^2}{(1+g^2-2g\cos(\theta))^{3/2}}$$

where β_R is the Rayleigh coefficient, β_M is the Mie coefficient, g is Henyey/Greenstein phase function eccentricity, and E_{sun} is the irradiance of the sun. All that's left for the pixel shader is the multiplication and addition as described in Eq. 19.

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