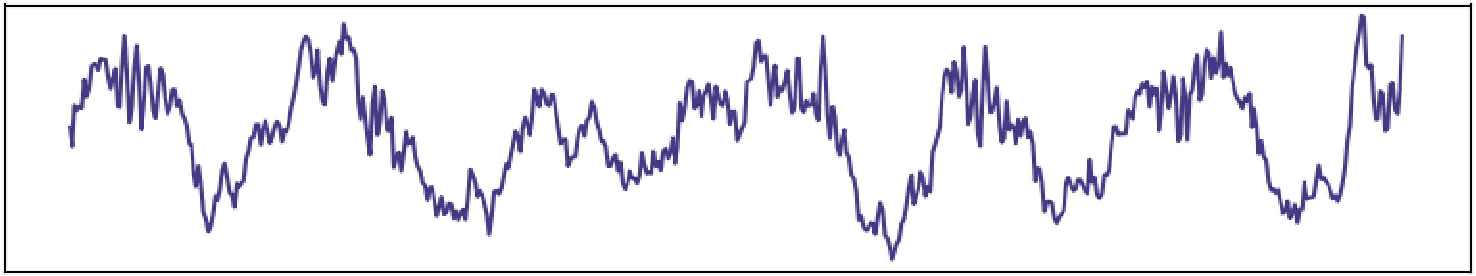
NON-LINEAR MODELS FOR NEUROPHYSIOLOGICAL TIME SERIES

Tom Dupré la Tour

PhD Defense – 26 Nov 2018



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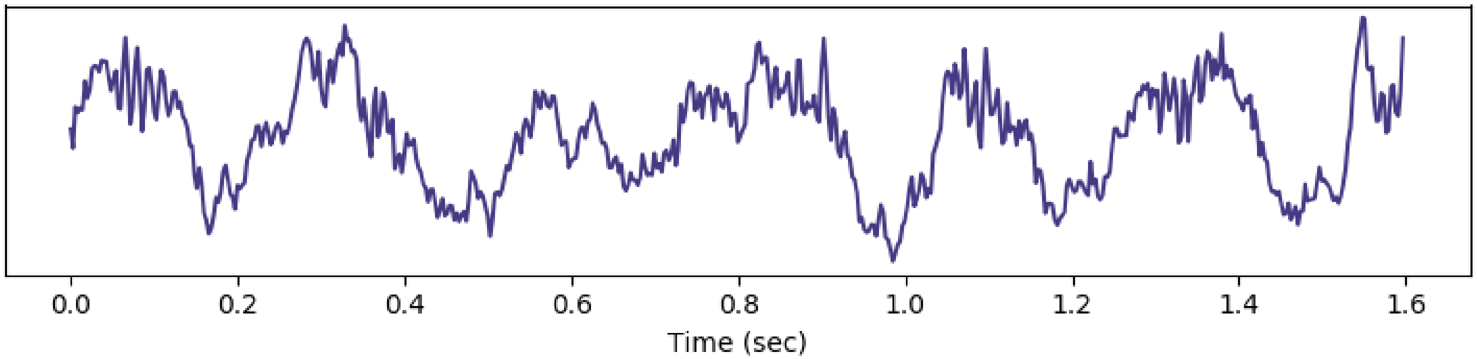
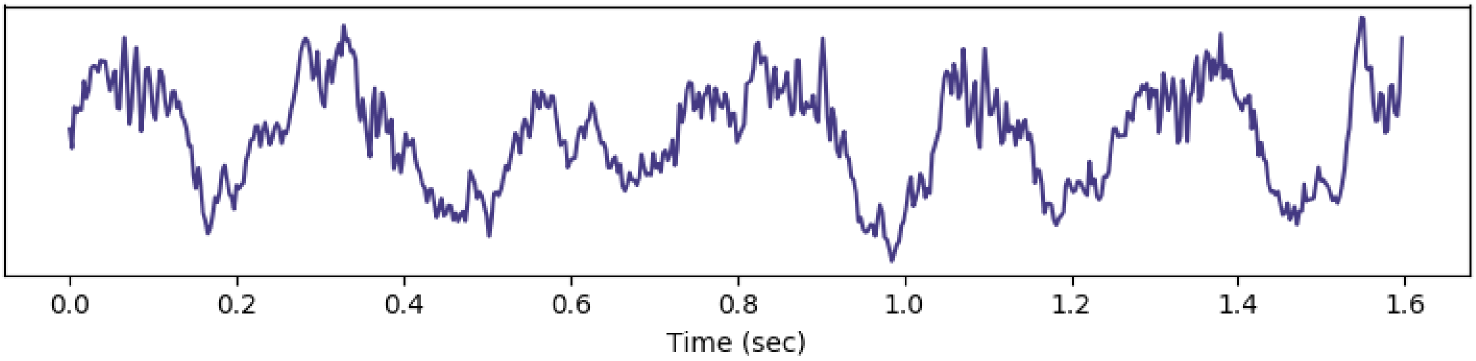


Neurophysiological time series

Local field potential (LFP)

Example: LFP in rodent striatum

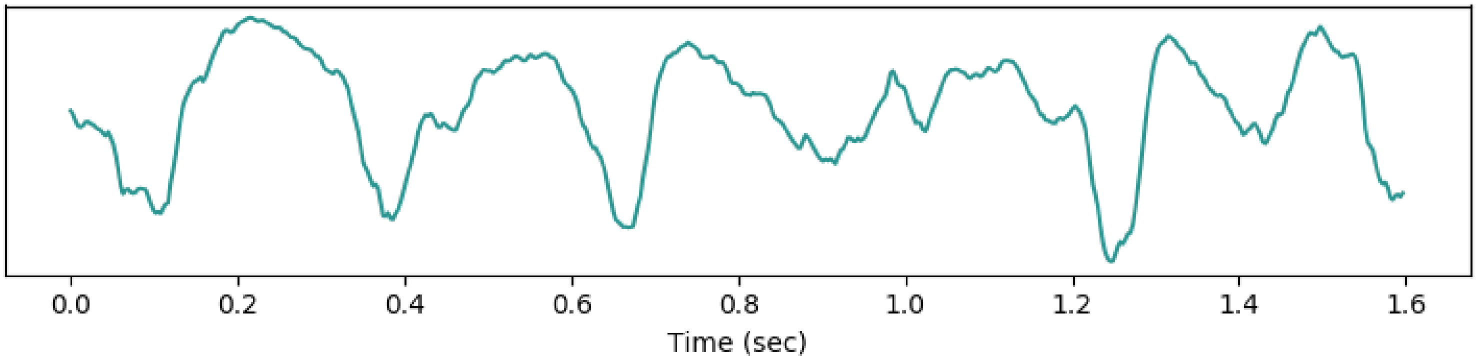


Neurophysiological time series

Local field potential (LFP)

Electro-corticogram (ECoG)

Example: ECoG in human auditory cortex



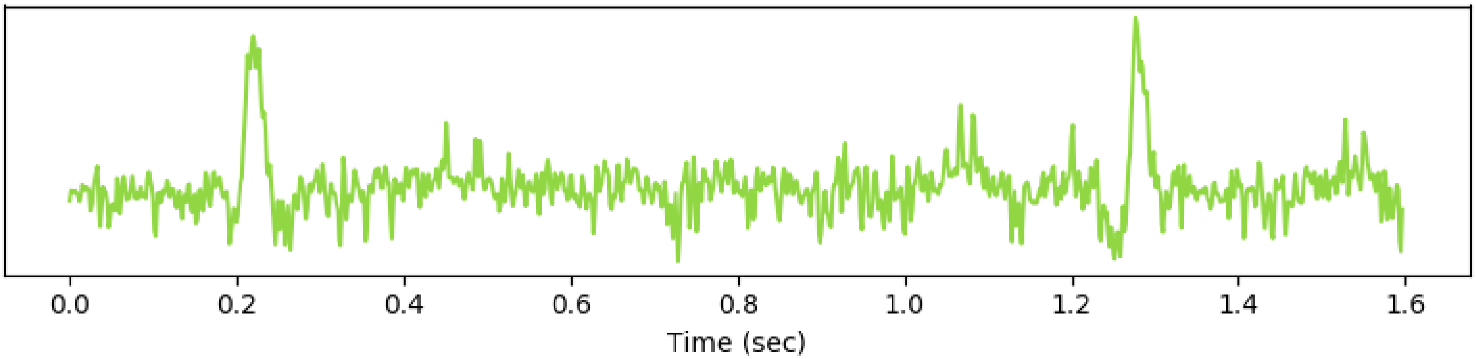
Neurophysiological time series

Local field potential (LFP)

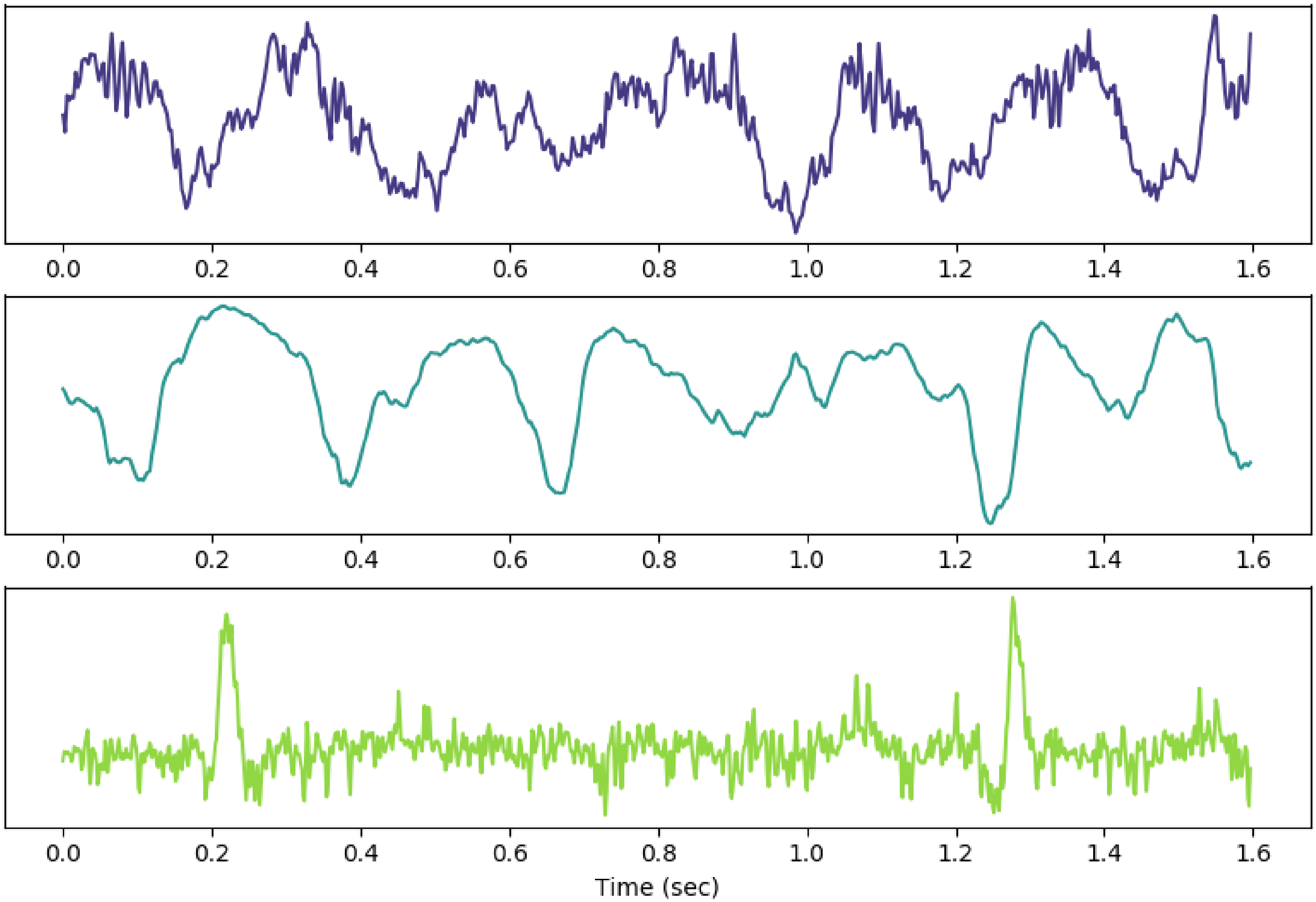
Electro-corticogram (ECoG)

Electro/Magneto-encephalogram (EEG/MEG)

Example: MEG in human

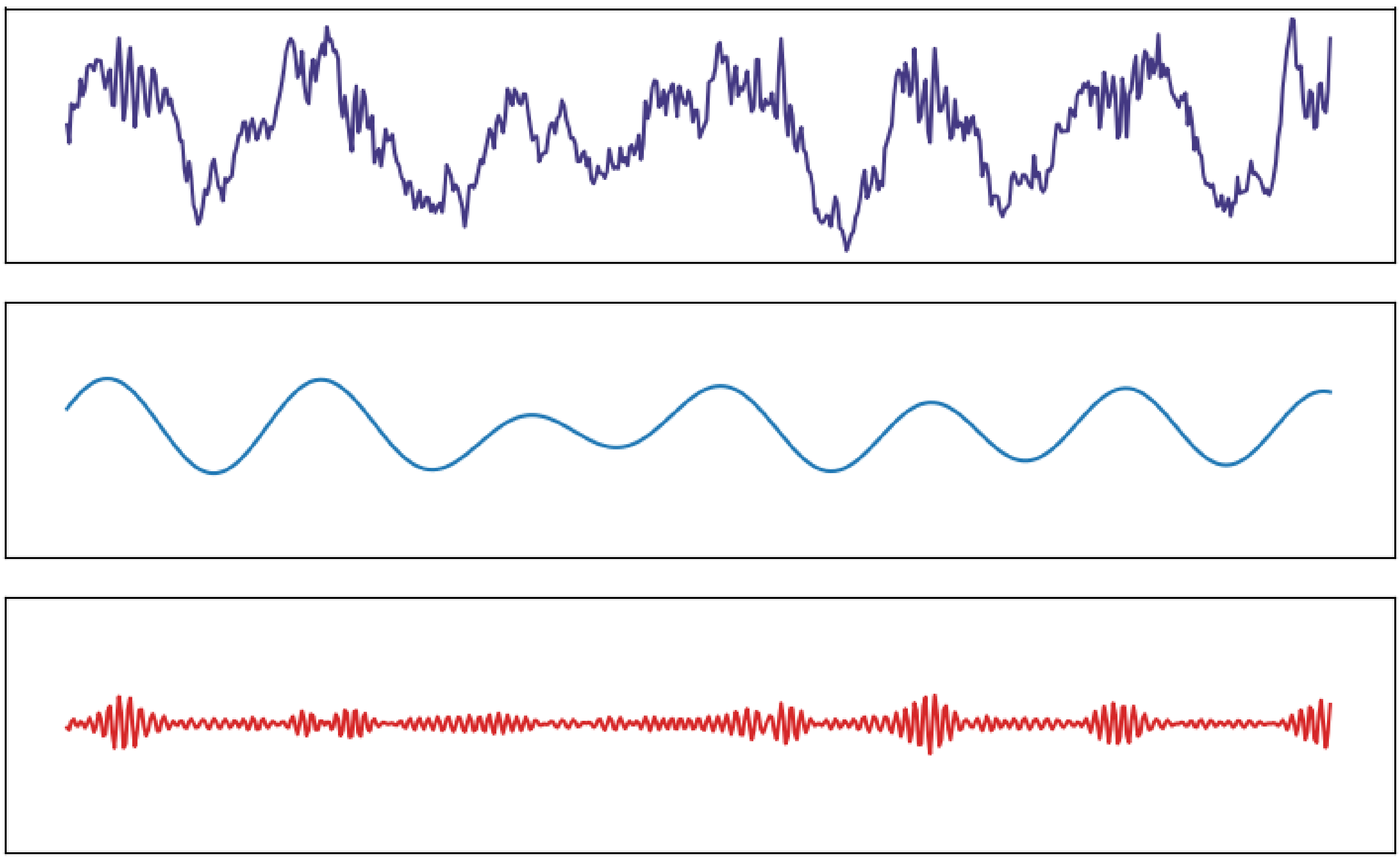


Neurophysiological time series



Narrow-band representation

LFP



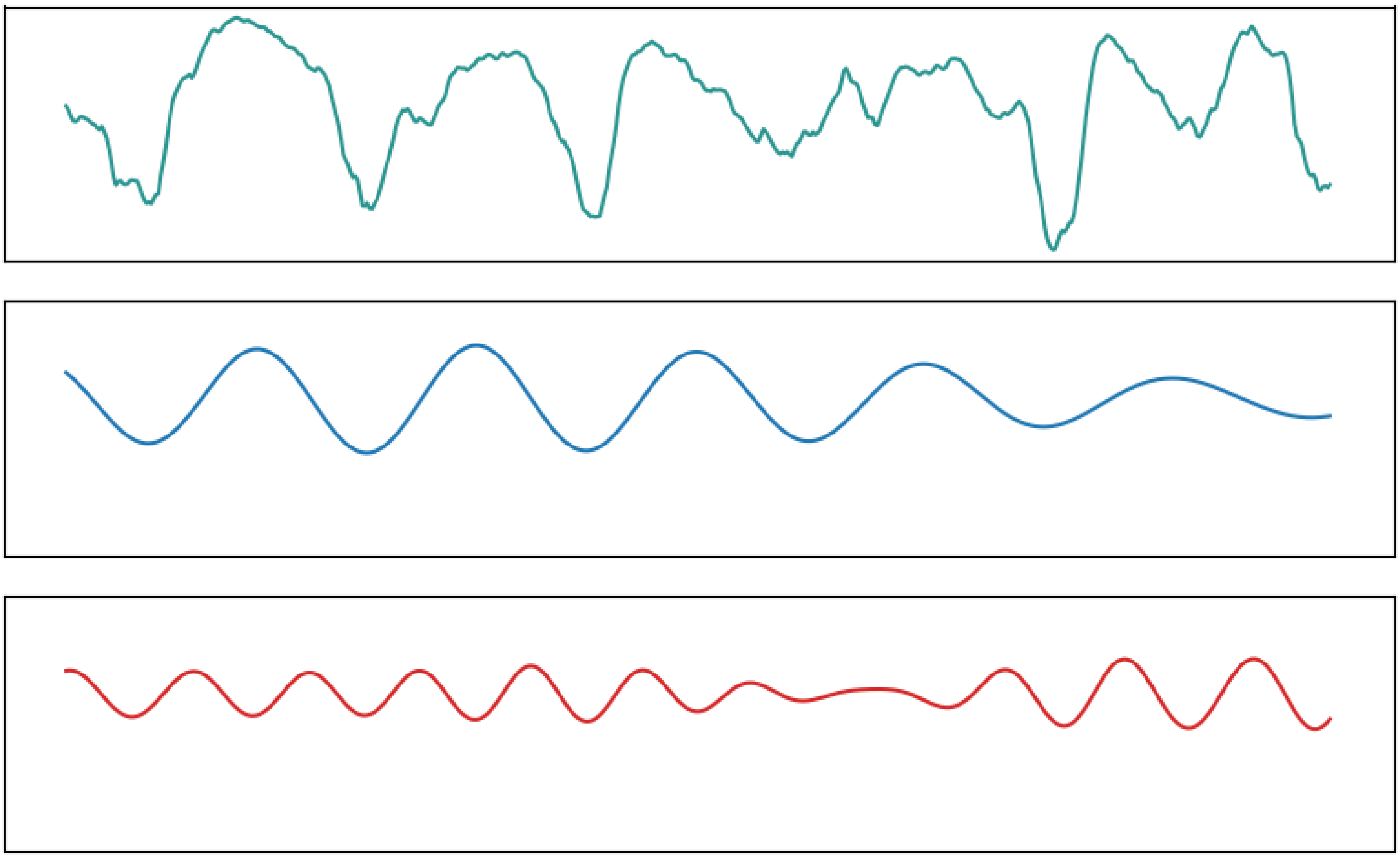
Fourier fallacy

«Even though it may be possible to analyze the complex forms of brain waves into **a number of different sine-wave** frequencies, this may lead only to what might be termed a **"Fourier fallacy"**, if one assumes ***ad hoc*** that all of the necessary frequencies actually occur as periodic phenomena in **cell groups** within the brain. »

Jasper, 1948

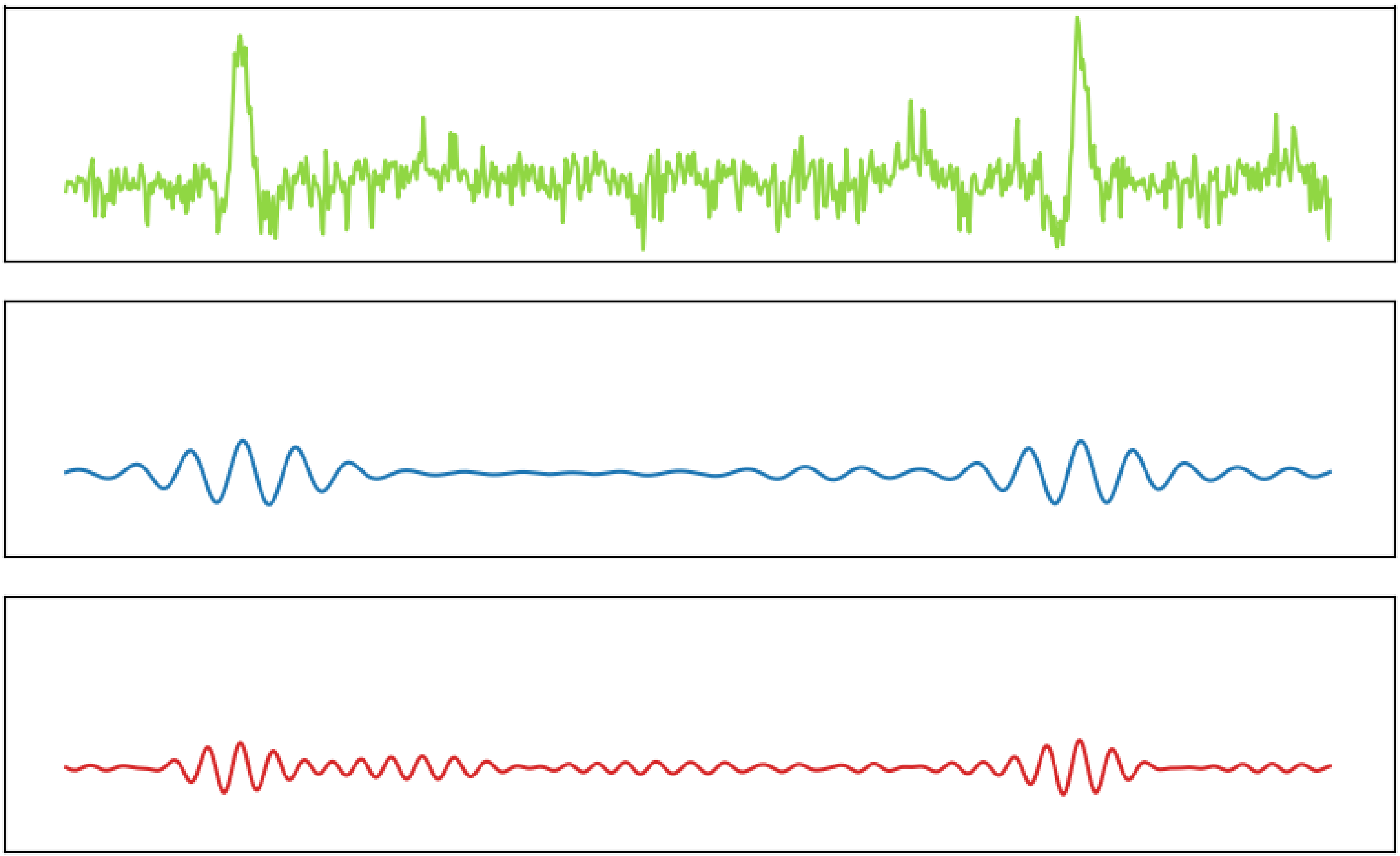
Narrow-band representation

ECoG



Narrow-band representation

MEG



Need to go beyond Fourier

- Fourier fallacy

(Jasper, 1948)

- Narrow-band linear filtering is too reductive

(Mazaheri and Jensen, 2008)

- Wide-band waveforms as key features

(Jones, 2016, Cole and Voytek, 2017)

In this work, we developed non-linear signal models, to go beyond narrow-band linear filtering.

Outline

1. Cross-frequency coupling analysis
   with driven autoregressive models

2. Temporal waveform analysis
   with convolutional sparse coding models

Part 1

1. Cross-frequency coupling analysis
with driven autoregressive models

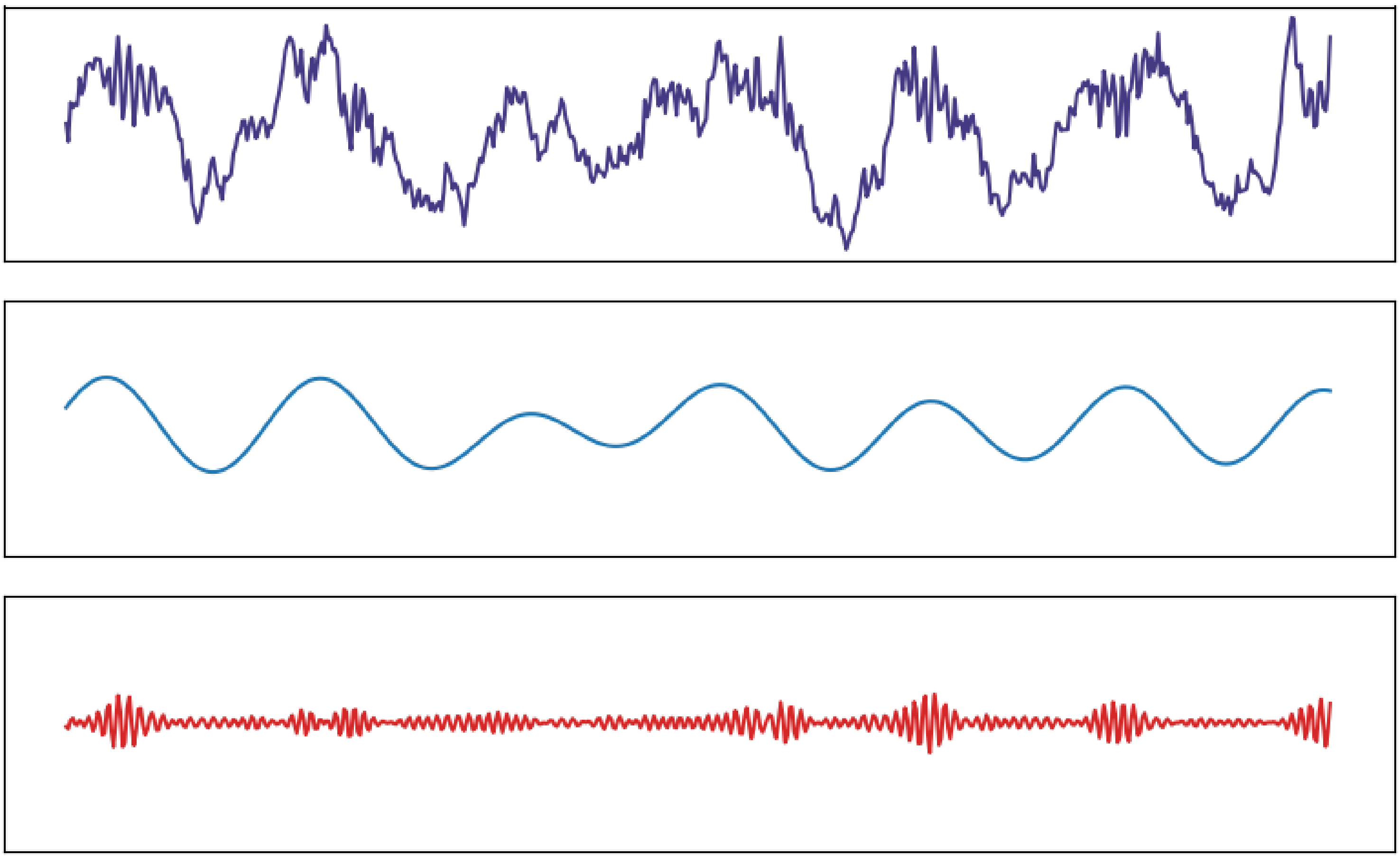
Non-linear autoregressive models for cross-frequency coupling in neural time series
T. Dupré la Tour, L. Tallot, L. Grabot, V. Doyère, V. van Wassenhove, Y. Grenier, A. Gramfort, *PLOS Computational Biology 2017*

Parametric estimation of spectrum driven by an exogenous signal
T. Dupré la Tour, Y. Grenier, A. Gramfort, *ICASSP 2017*

Driver estimation in non-linear autoregressive models
T. Dupré la Tour, Y. Grenier, A. Gramfort, *ICASSP 2018*

Cross-frequency coupling

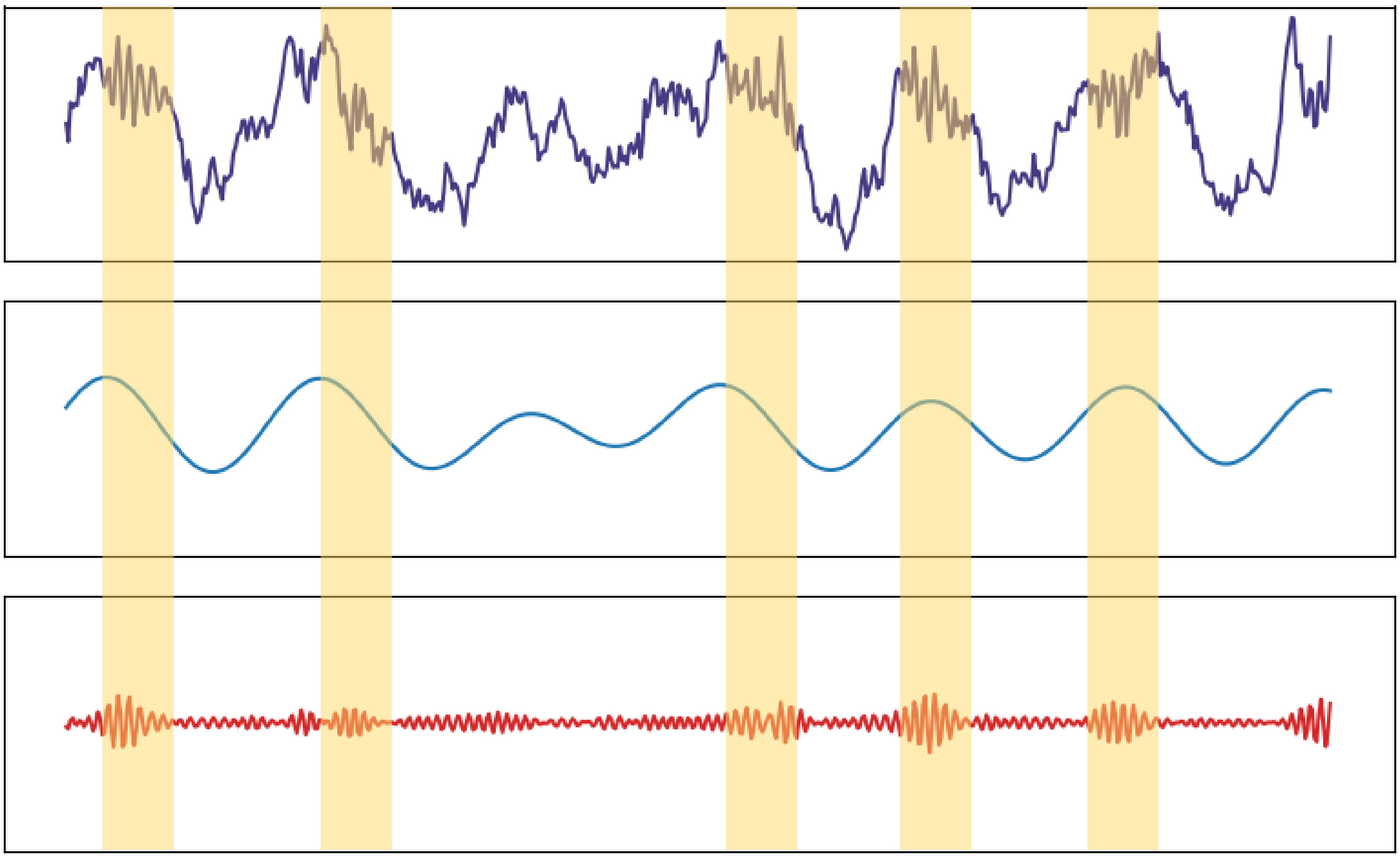
LFP



Cross-frequency coupling

Low-frequency phase and high-frequency amplitude

(Bragin et al 1995, Canolty et al, 2006)



What is the role of CFC?

- Multi-item/sequence representation

  (Penttonen et al, 1998, Gupta et al, 2012)

- Long-term and working memory

  (Lisman and Idiart, 1995, Jensen, 2006, Tort et al, 2009)

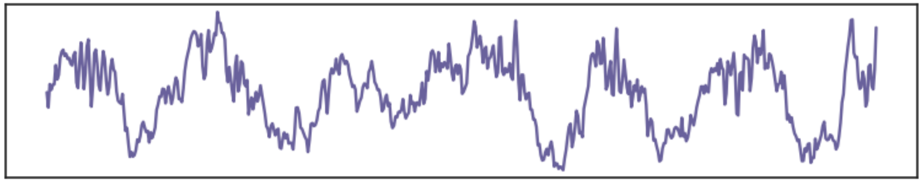
- Long distance synchronization

  (Canolty et al, 2006, Bonnefond et al, 2017)

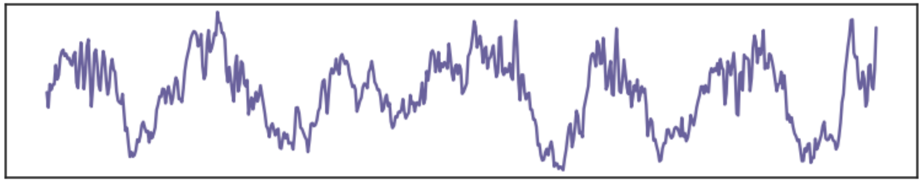
- A canonical neural syntax

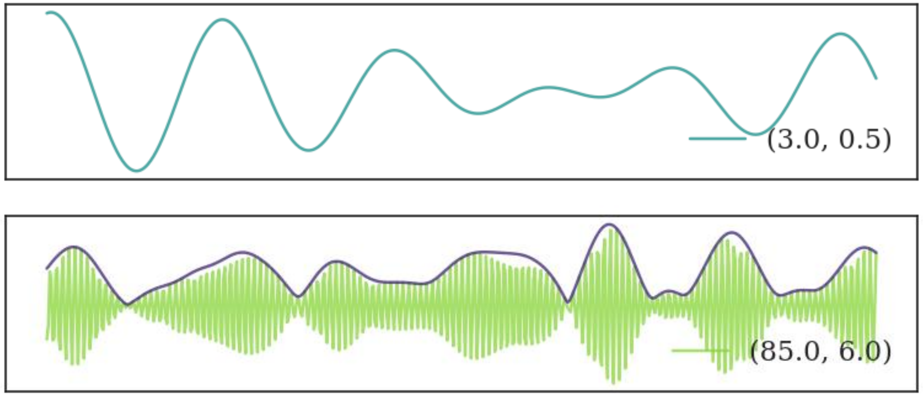
  (Buzsaki, 2010, Lisman and Jensen 2013, Hyafil et al, 2015)

Common approach to measure CFC…



Common approach to measure CFC...



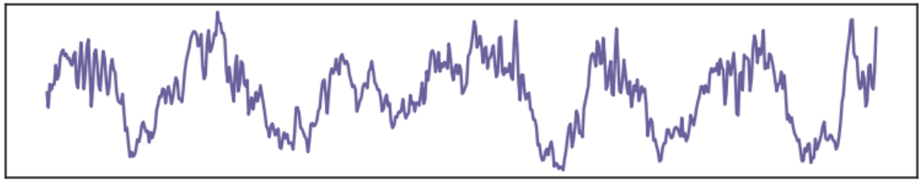


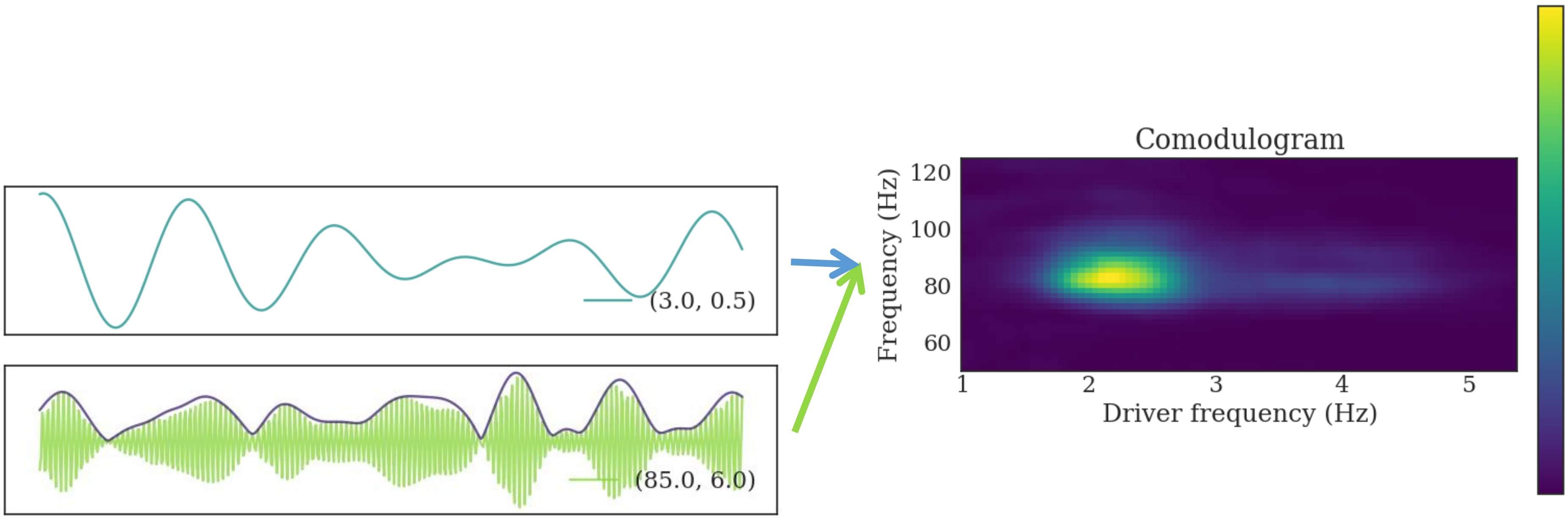
Correlation bewteen:

→ the low-frequency signal

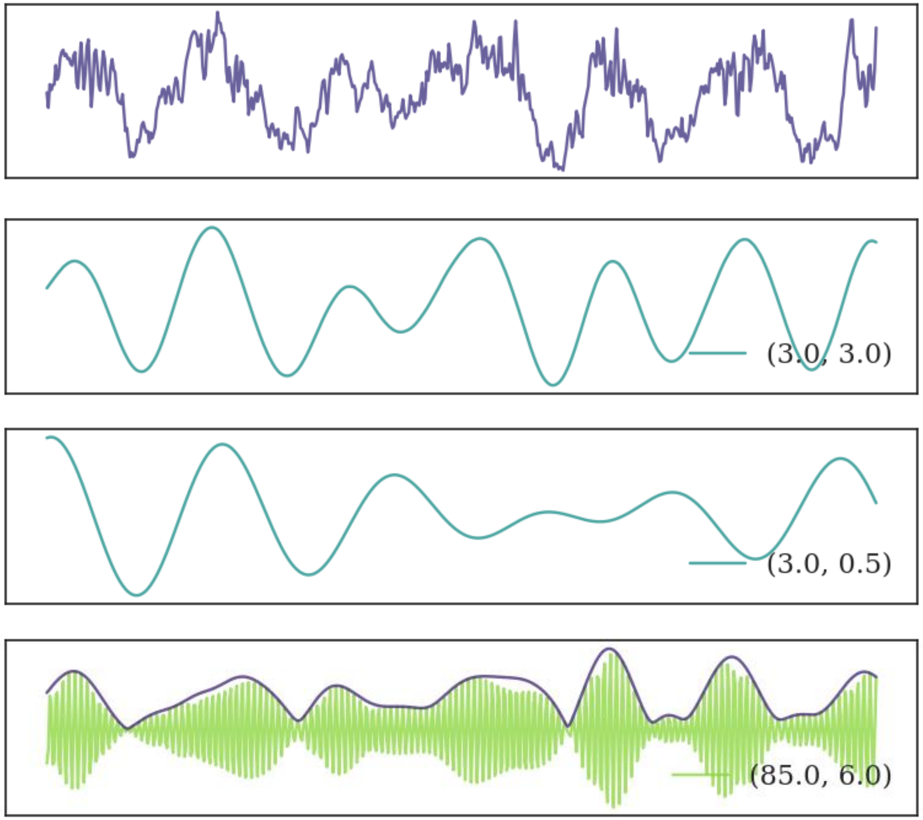
→ the high-frequency envelope

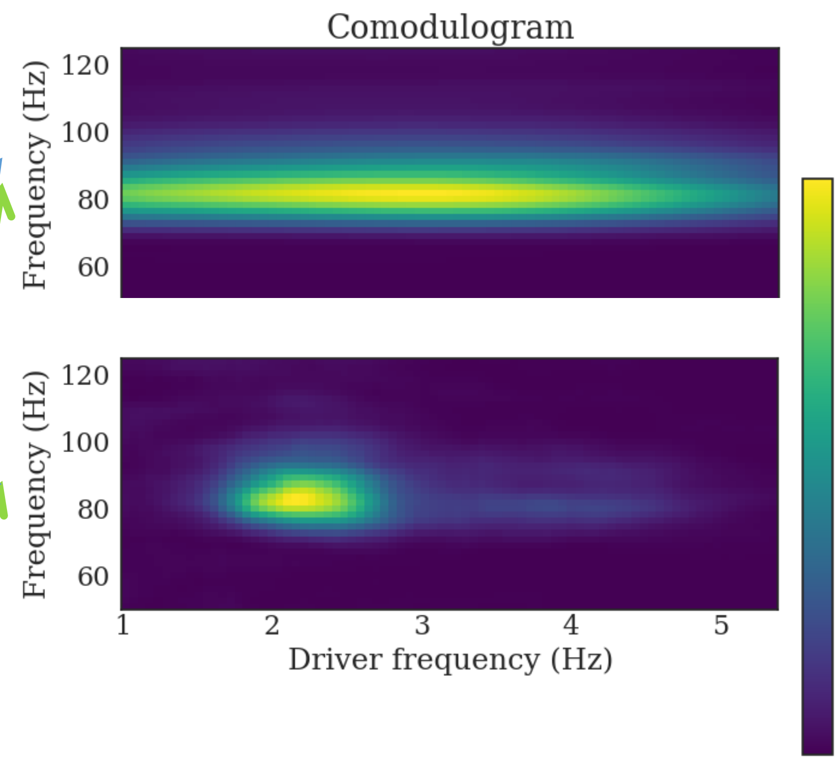
Common approach to measure CFC…





… but choosing filtering parameters is hard





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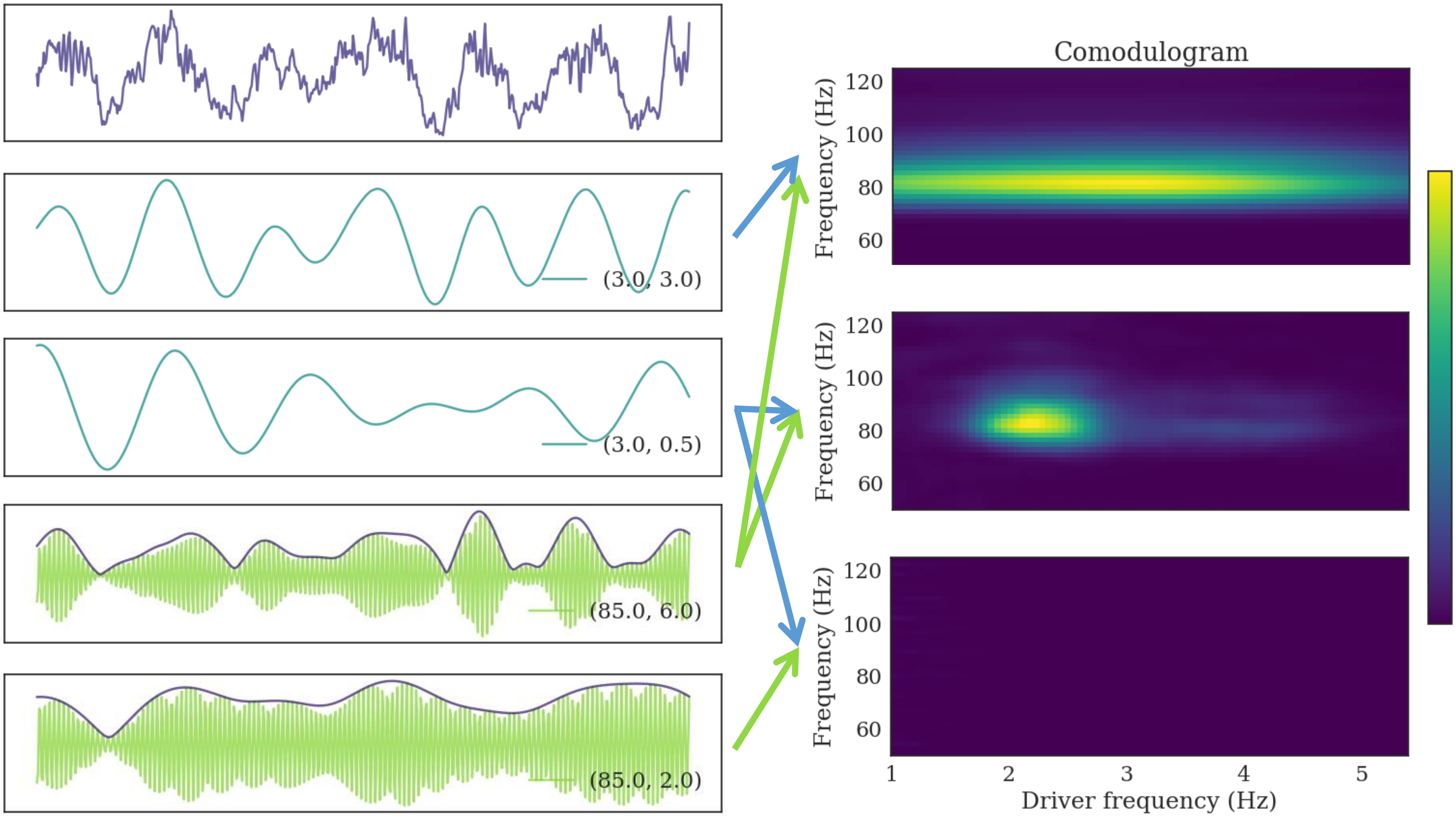


Autoregressive model

Autoregressive (AR) model

(Makhoul 1975)

$$y(t) + \sum_{i=1}^{p} a_i y(t-i) = \varepsilon(t) \qquad \varepsilon(t) \sim \mathcal{N}(0, \sigma^2)$$

Spectral estimation

Autoregressive (AR) model

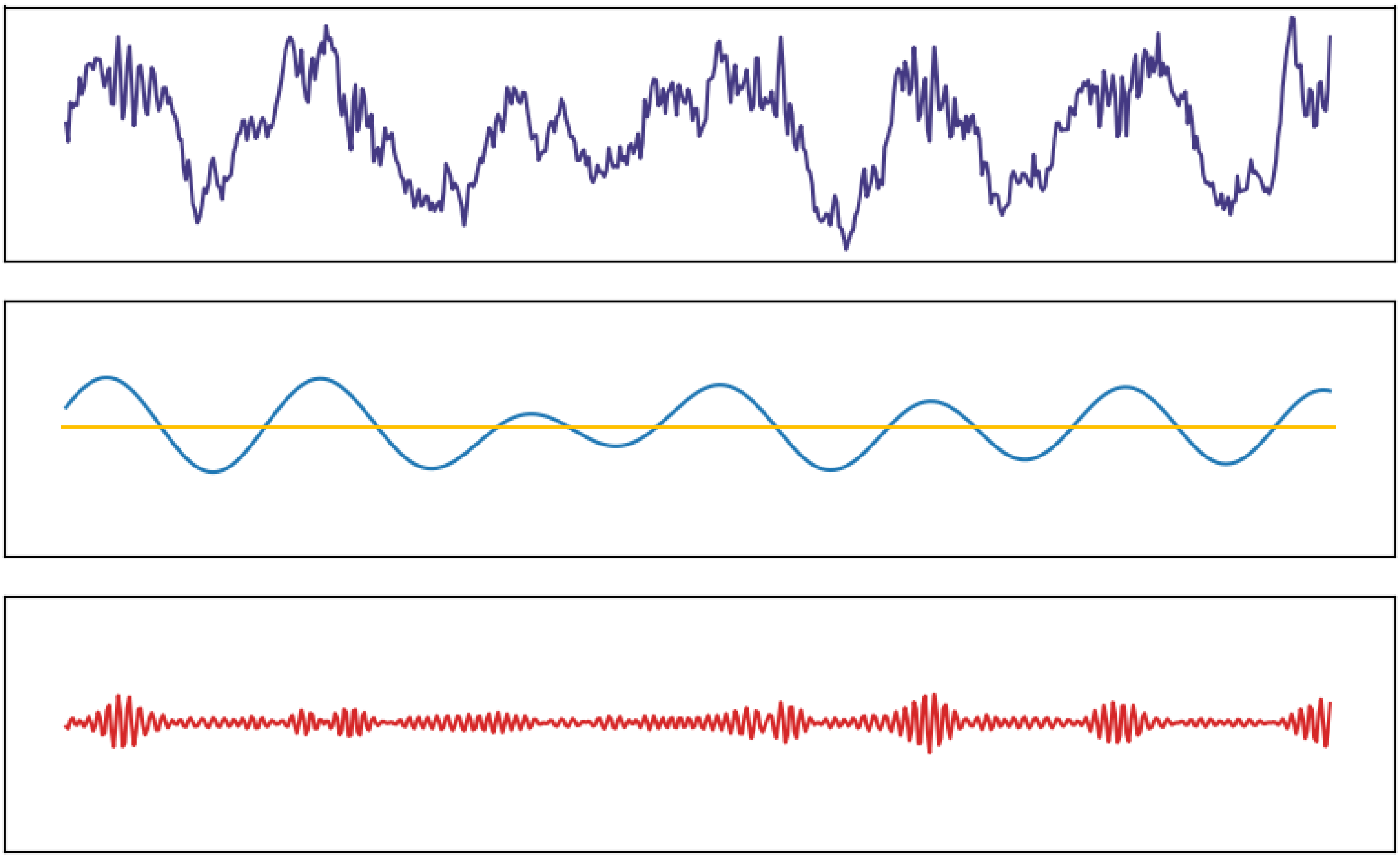
(Makhoul 1975)

$$y(t) + \sum_{i=1}^{p} a_i y(t-i) = \varepsilon(t) \qquad \varepsilon(t) \sim \mathcal{N}(0, \sigma^2)$$

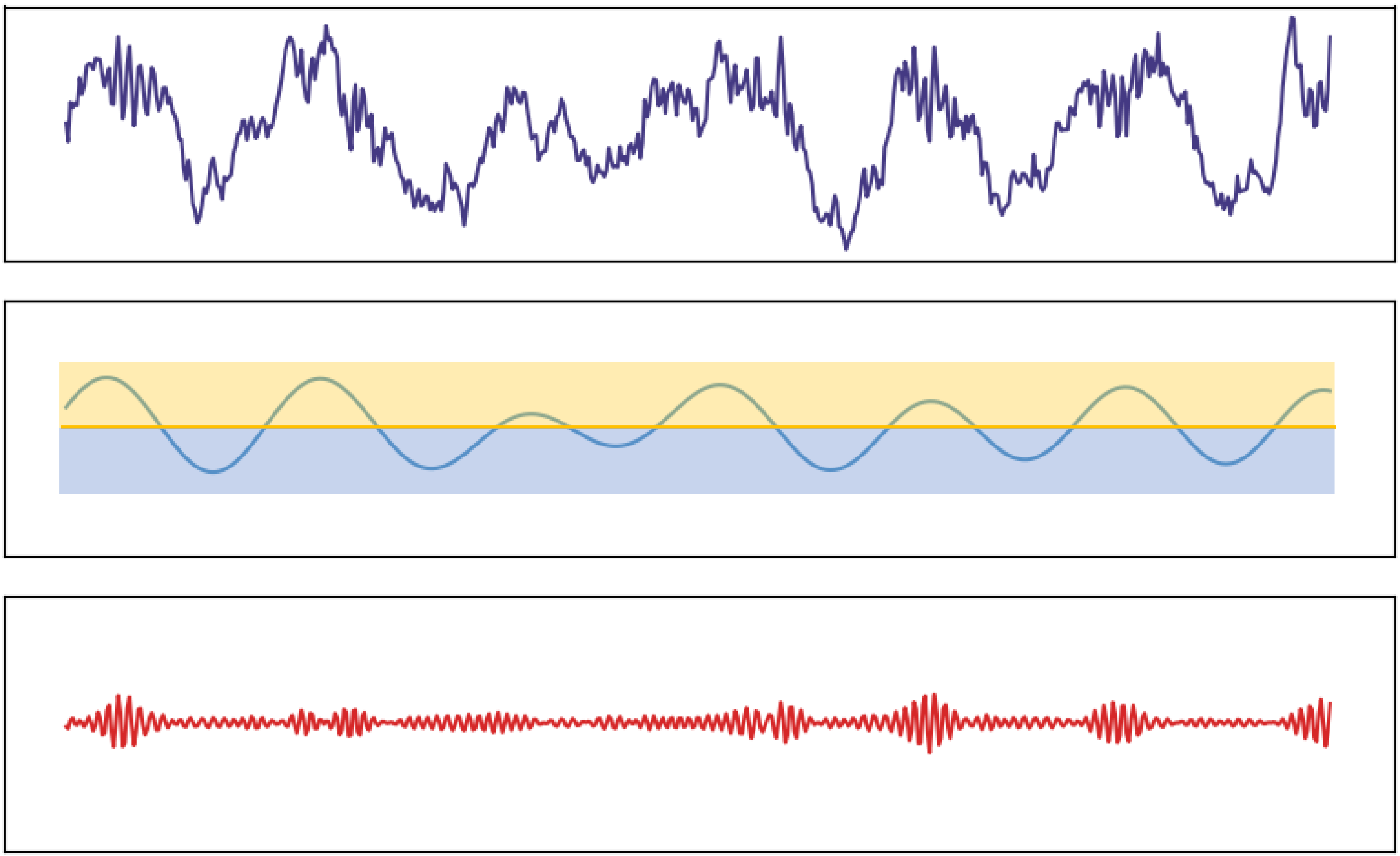
Power spectral density (PSD)

$$\mathrm{PSD}_y(f) = \sigma^2 \left| \sum_{i=0}^{p} a_i e^{-j2\pi f i} \right|^{-2}$$

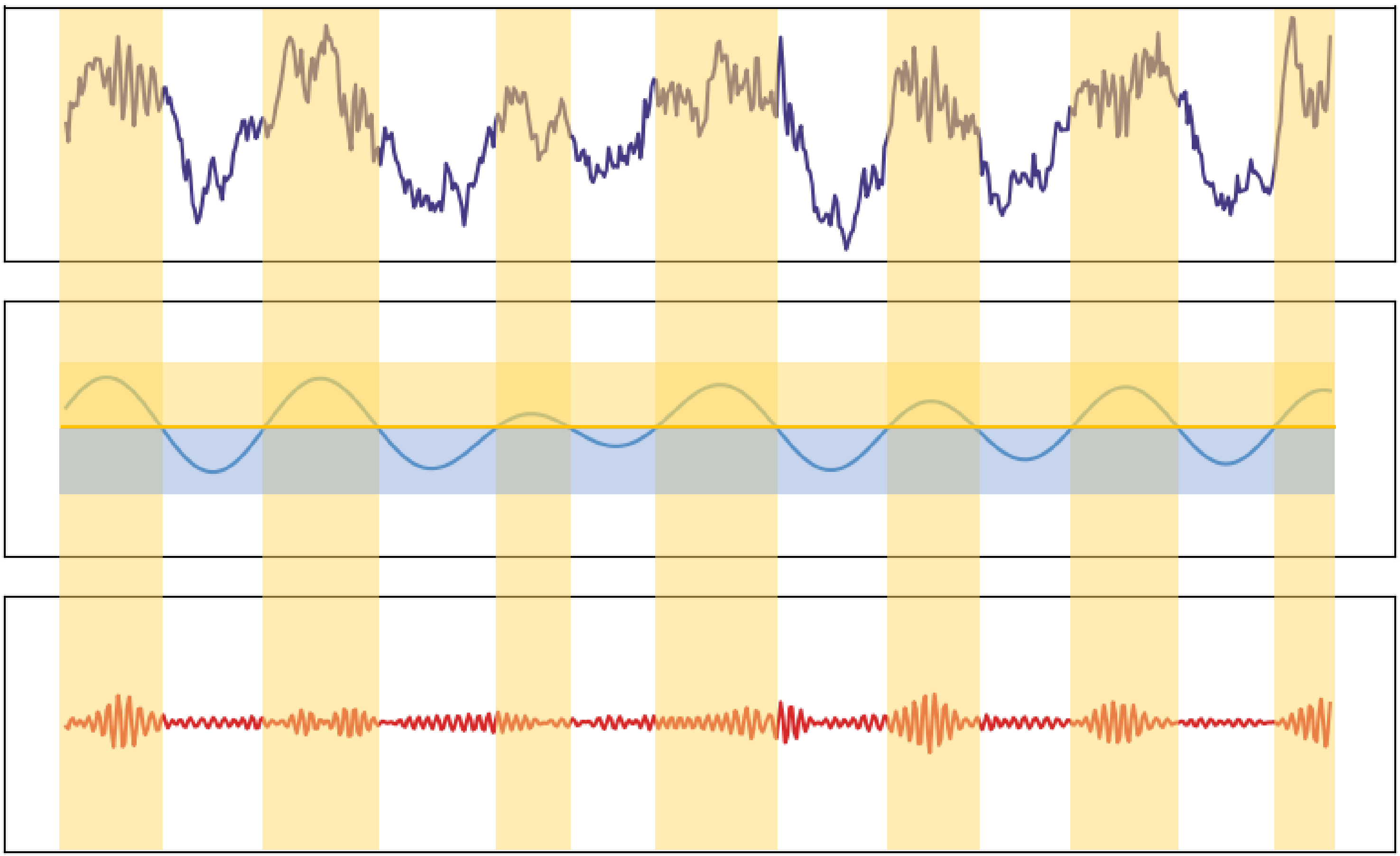
Cross-frequency coupling



Cross-frequency coupling

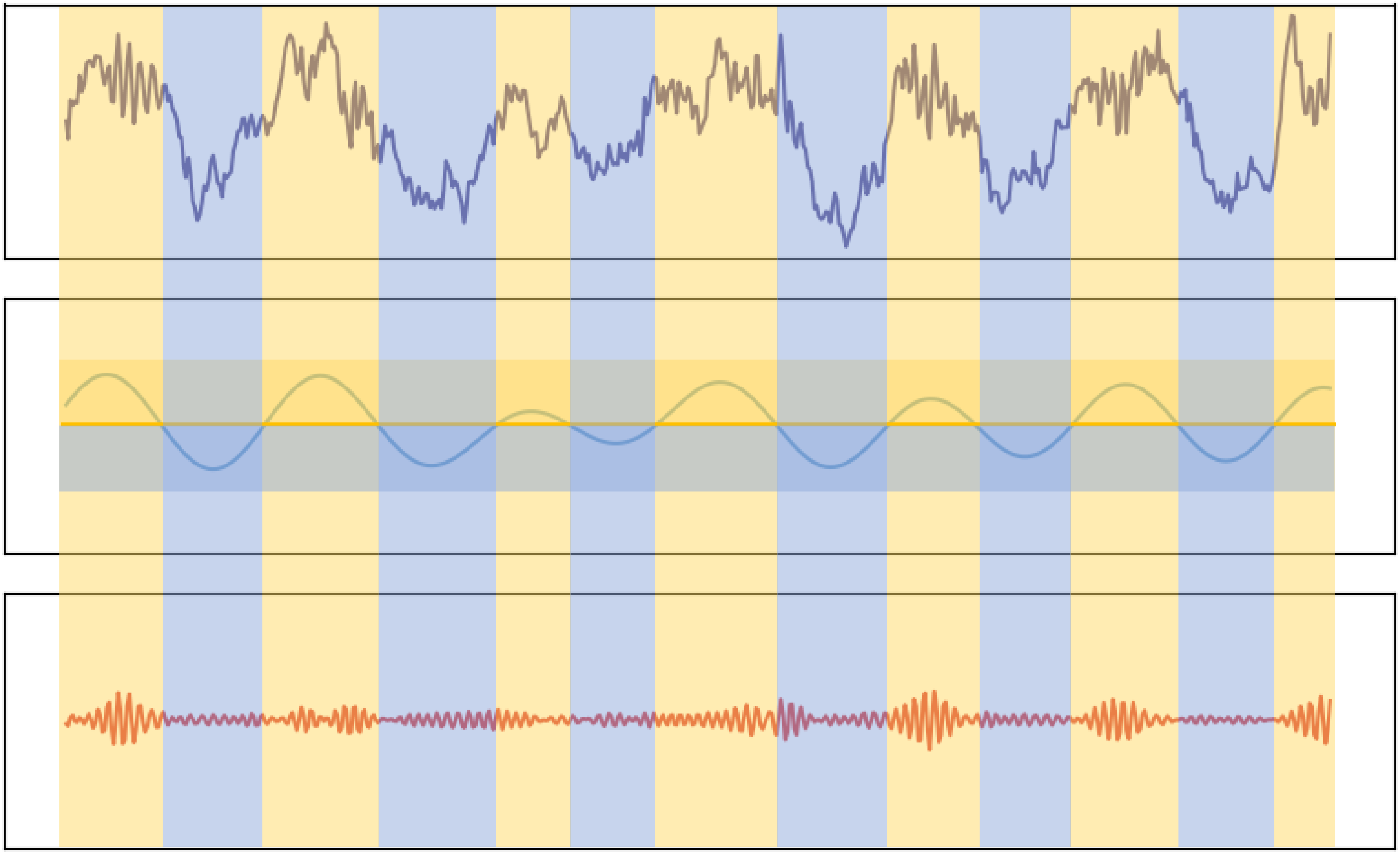


Cross-frequency coupling



Cross-frequency coupling

(Tong and Lim, 1980, Chan and Tong, 1986, Dijk et al, 2002)



Driven autoregressive (DAR) model

Autoregressive (AR) model

(Makhoul, 1975)

$$y(t) + \sum_{i=1}^{p} a_i y(t-i) = \varepsilon(t) \qquad \varepsilon(t) \sim \mathcal{N}(0, \sigma^2)$$

Driven AR (DAR) model

(Grenier, 1983, 2013)

$$a_i(t) = \sum_{j=0}^{m} a_{ij} x(t)^j \qquad \log(\sigma(t)) = \sum_{j=0}^{m} b_j x(t)^j$$

Driven autoregressive (DAR) model

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Driven AR (DAR) model

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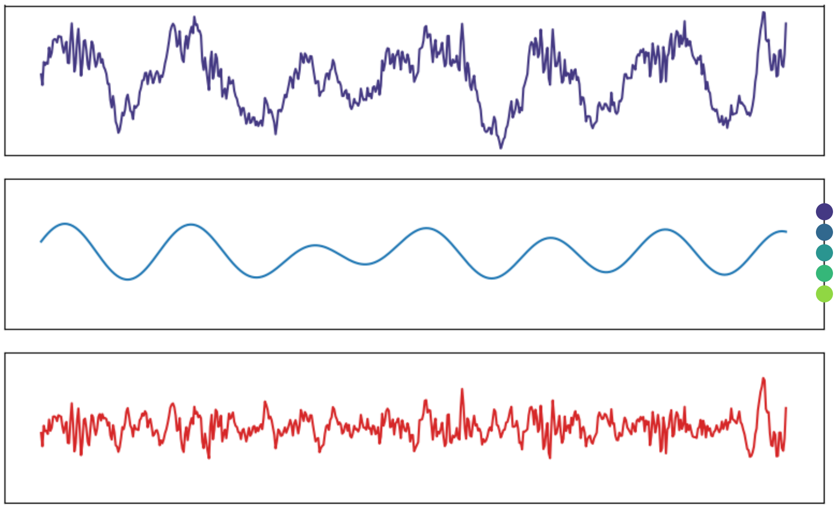
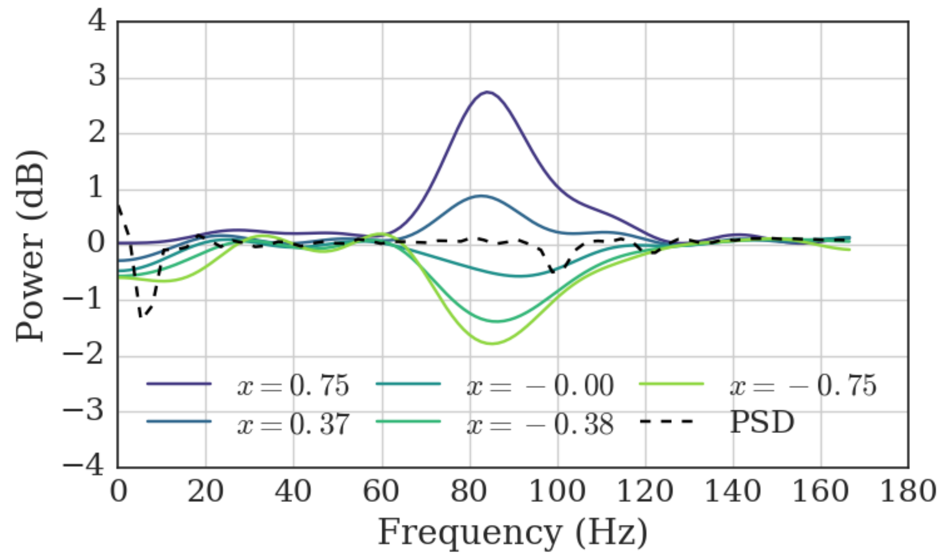
A different parametrization ensuring DAR model stability :

Parametric estimation of spectrum driven by an exogenous signal
T. Dupré la Tour, Y. Grenier, A. Gramfort, *ICASSP 2017*

Conditional PSD

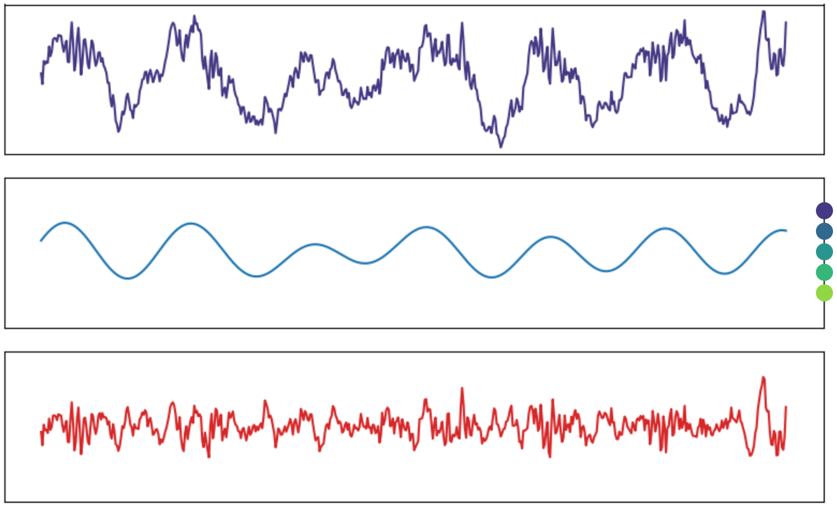


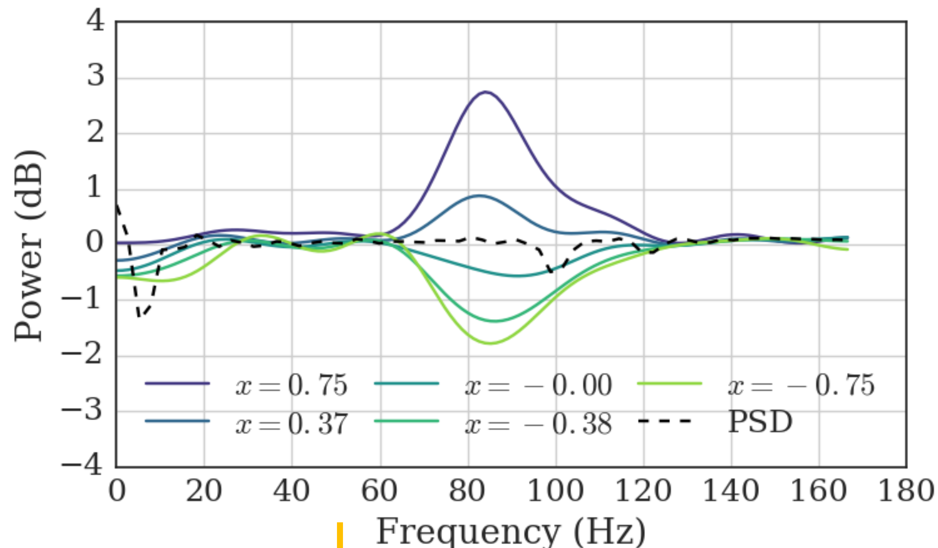


LFP

$$\mathrm{PSD}_y(x_0)(f) = \sigma(x_0)^2 \left| \sum_{i=0}^{p} a_i(x_0) e^{-j2\pi f i} \right|^{-2}$$

Conditional PSD





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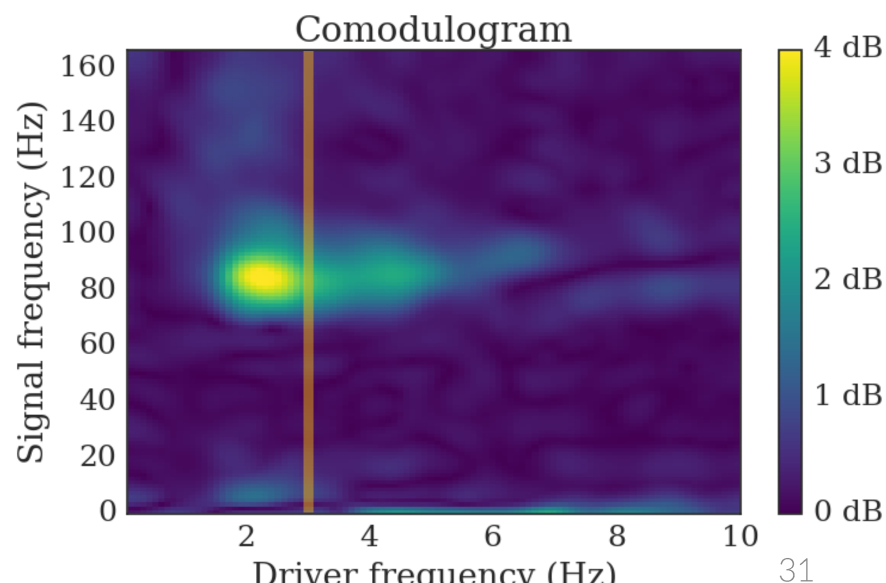
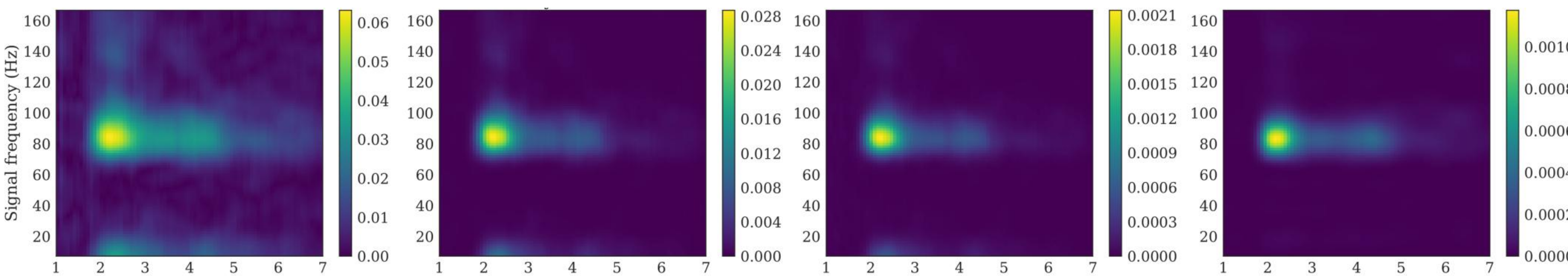


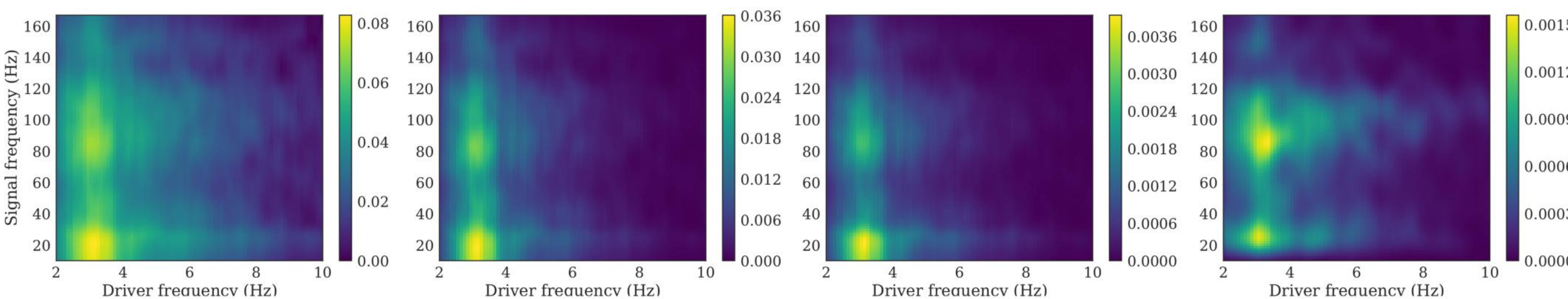
Comodulogram on two empirical signals

LFP

(Ozkurt et al, 2011) (Penny et al, 2008) (Tort et al, 2009) (Dupré la Tour et al, 2017)



ECoG



Model and parameter selection

$$L = \prod_{t=p+1}^{T} \frac{1}{\sqrt{2\pi\sigma(t)^2}} \exp\left(-\frac{\varepsilon(t)^2}{2\sigma(t)^2}\right)$$

$$-2\log(L) = T\log(2\pi) + \sum_{t=p+1}^{T} \frac{\varepsilon(t)^2}{\sigma(t)^2} + 2\sum_{t=p+1}^{T} \log(\sigma(t))$$

Main gain: A likelihood function → parameter selection

Model and parameter selection

$$L = \prod_{t=p+1}^{T} \frac{1}{\sqrt{2\pi\sigma(t)^2}} \exp\left(-\frac{\varepsilon(t)^2}{2\sigma(t)^2}\right)$$

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Example: Selection of AR order (p) and DAR polynomial order (m):

Model and parameter selection

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Main gain: A likelihood function → parameter selection

Example: Selection of AR order (p) and DAR polynomial order (m):

- Akaike information criterion (AIC), Bayesian information criterion (BIC), …

(Akaike, 1974)

(Schwartz, 1978)

Model and parameter selection

$$L = \prod_{t=p+1}^{T} \frac{1}{\sqrt{2\pi\sigma(t)^2}} \exp\left(-\frac{\varepsilon(t)^2}{2\sigma(t)^2}\right)$$

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Main gain: A likelihood function → parameter selection

Example: Selection of AR order (p) and DAR polynomial order (m):

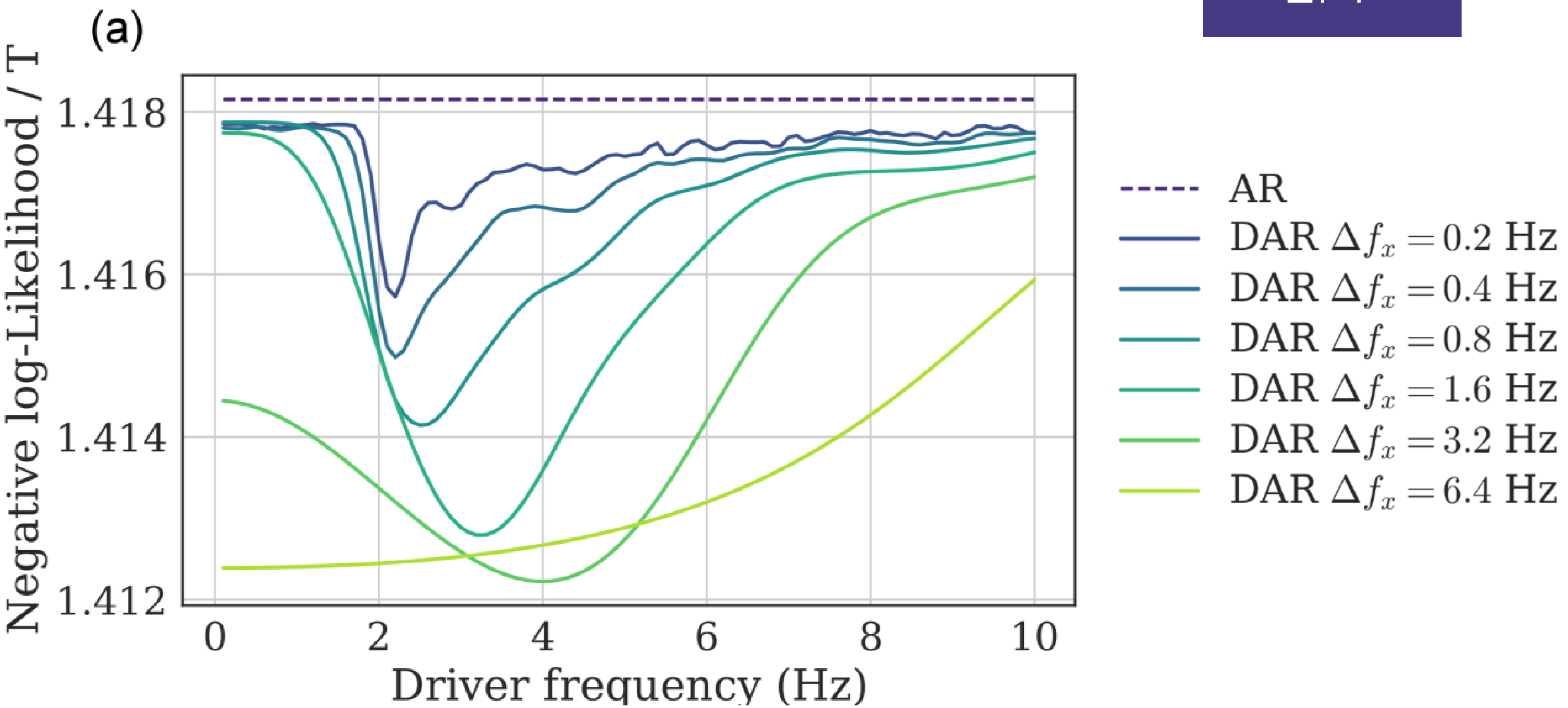
- Akaike information criterion (AIC), Bayesian information criterion (BIC), …
- Evaluation on left-out data, cross-validation

(Akaike, 1974)

(Schwartz, 1978)

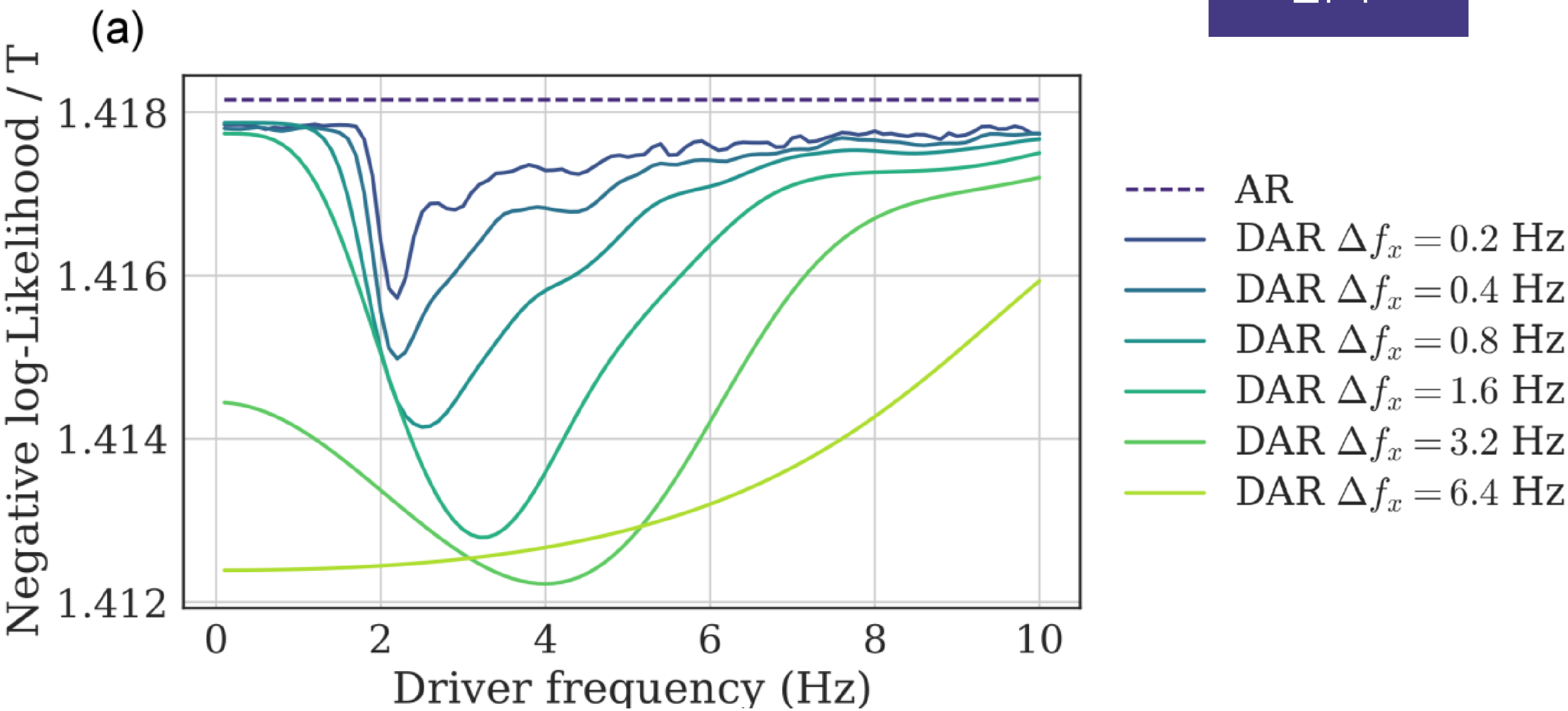
Driver selection

LFP



Driver selection

LFP



Further optimizing the driver filter with gradient descent :

Driver estimation in non-linear autoregressive models
T. Dupré la Tour, Y. Grenier, A. Gramfort, *ICASSP 2018*

Delay estimation

- DAR model between $y(t)$ and $x_\tau(t) = x(t - \tau)$
- Minimize the negative log-likelihood

Delay estimation

- DAR model between $y(t)$ and $x_\tau(t) = x(t - \tau)$
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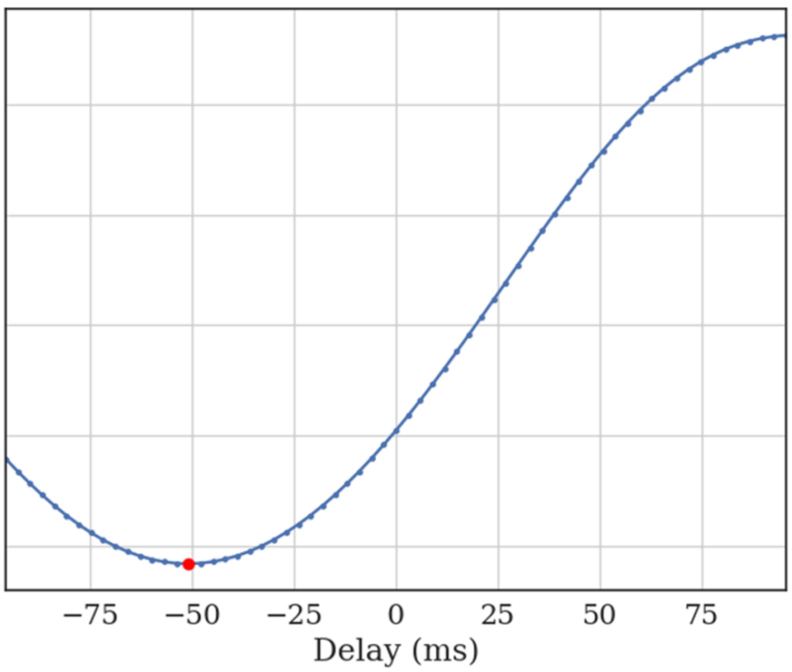
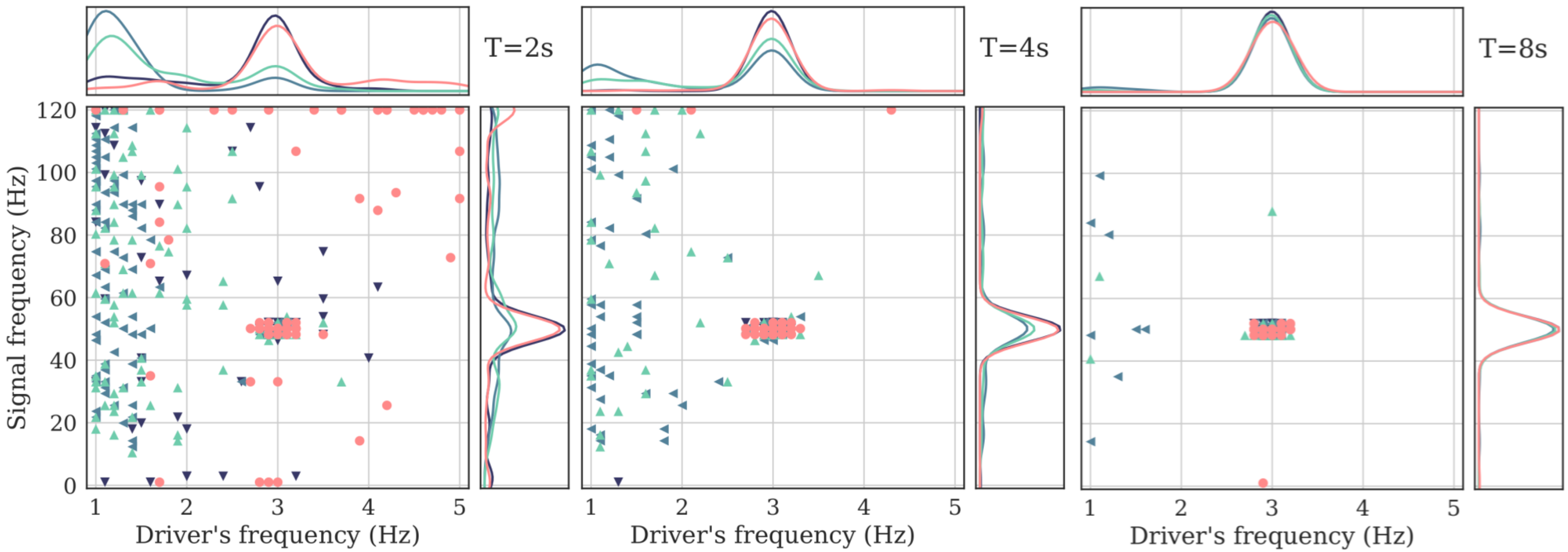


(Besserve et al , 2010)

(Jiang et al , 2015)

ECoG

Robustness to short signals



(Penny et al, 2008)

(Tort et al, 2009)

(Ozkurt et al, 2011)

(Dupré la Tour et al, 2017)

1. Cross-frequency coupling analysis *with driven autoregressive models*

- Estimation of spectral modulation → capture CFC
- Generative model → easy comparison of parameters
- Delay estimation → directionality of the coupling
- Parametric model → robust to short signals

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Outline

1. Cross-frequency coupling analysis

with driven autoregressive models

2. Temporal waveform analysis

with convolutional sparse coding models

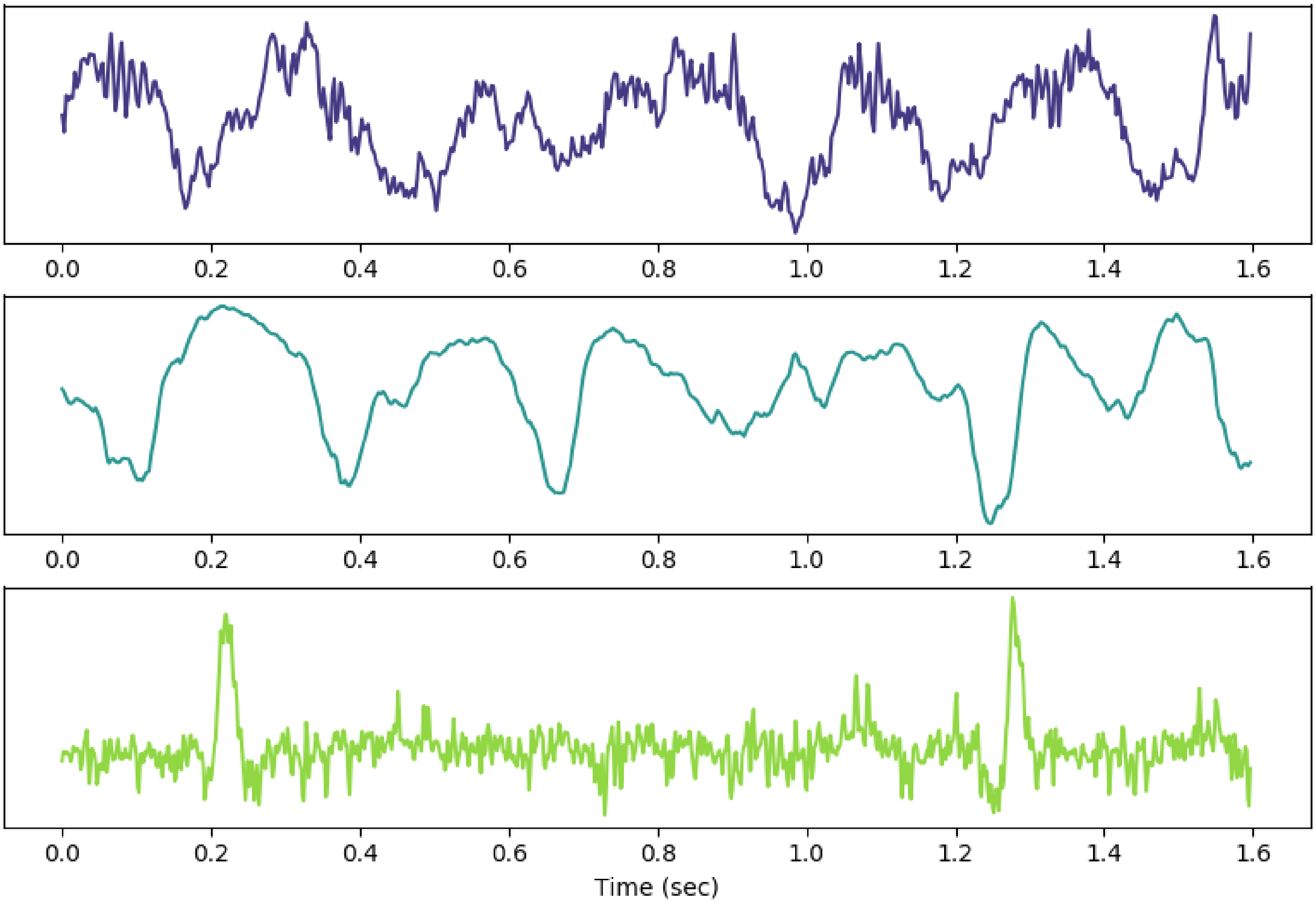
Part 2

2. Temporal waveform analysis
with convolutional sparse coding models

Learning the morphology of brain signals using alpha-stable convolutional sparse coding
M. Jas, T. Dupré la Tour, U. Şimşekli, A. Gramfort, *NeurIPS 2017*

Multivariate Convolutional Sparse Coding for Electromagnetic Brain Signals
T. Dupré la Tour*, T. Moreau*, M. Jas, A. Gramfort, *NeurIPS 2018*

Neurophysiological time series



Temporal waveform analysis

- Sparse representations: wavelet basis

  (Mallat and Zhang, 1993, Candès et al, 2006)

- Sparse coding / dictionary learning

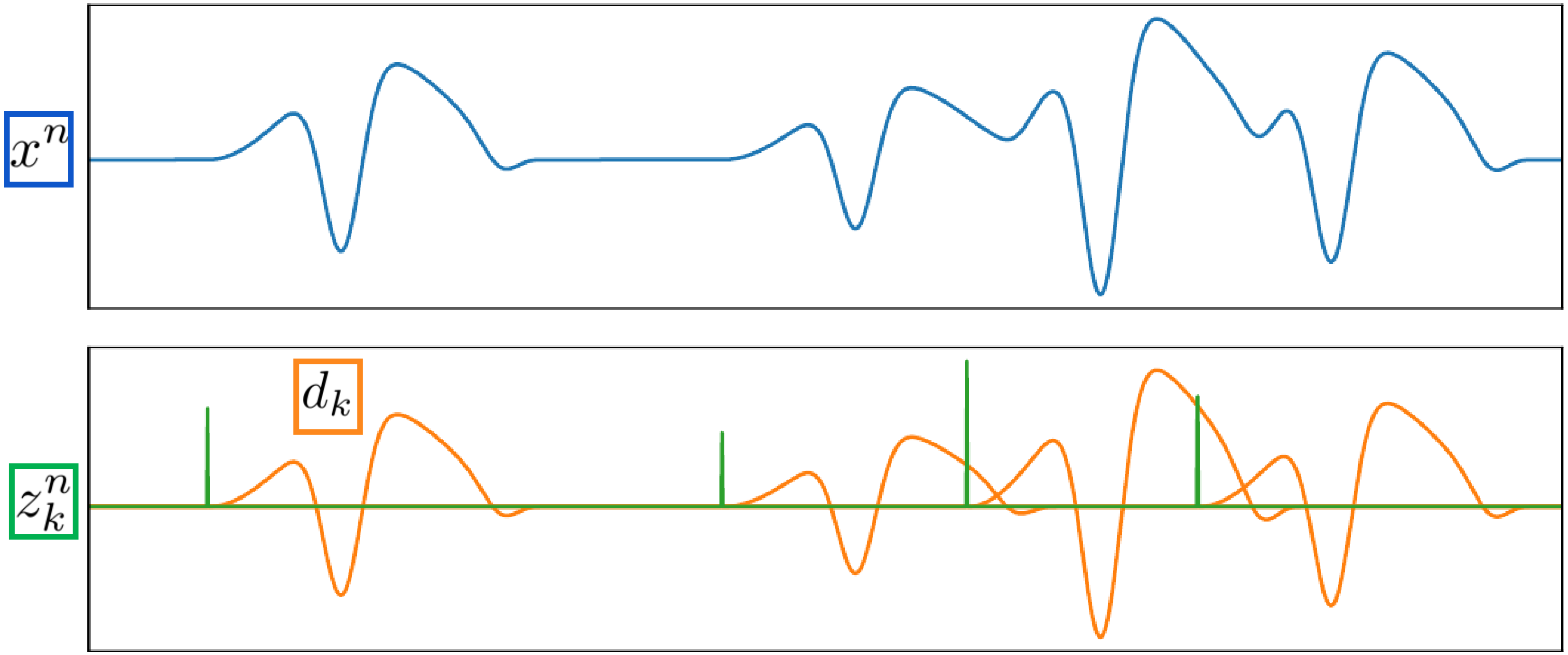
  (Olshausen and Field, 1996, Elad and Aharon, 2006)

- Shift-invariant representations

  (Lewicki and Sejnowski, 1999, Grosse et al, 2007)

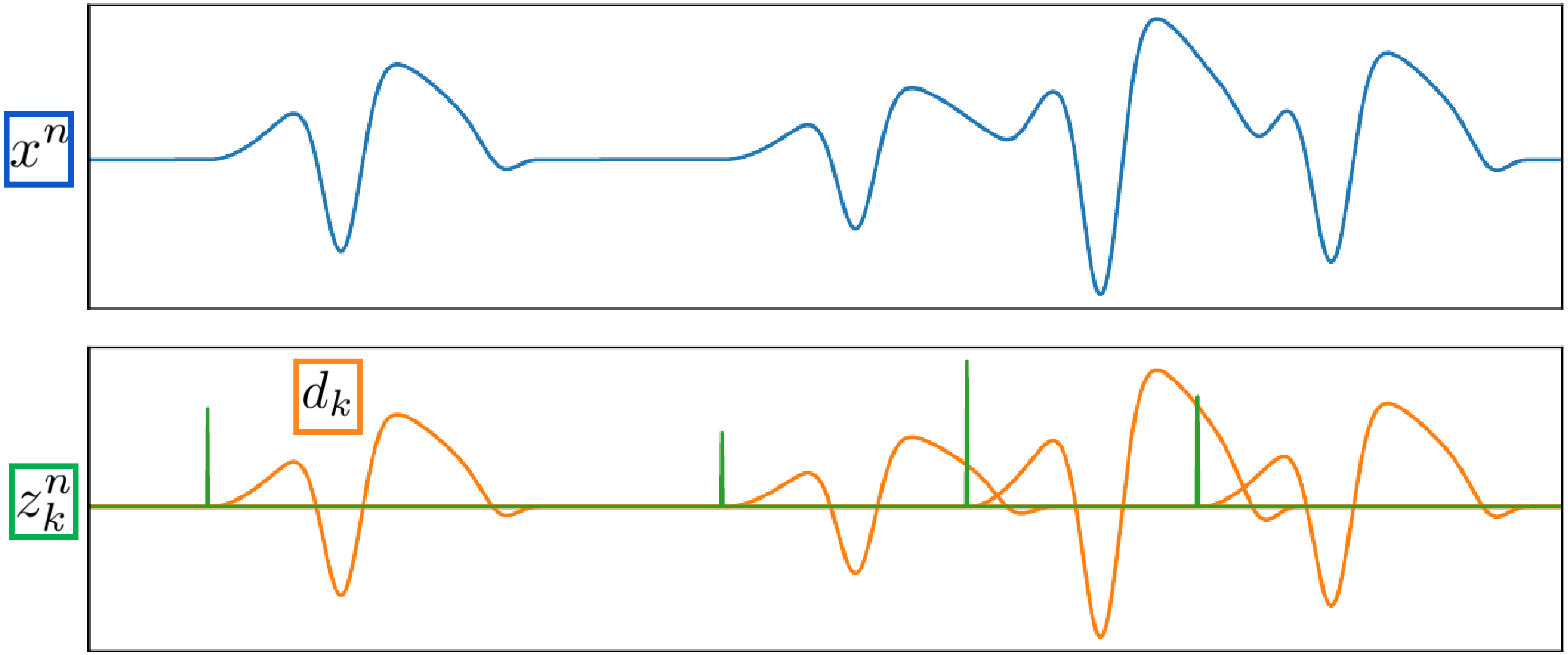
- In neurophysiology:
  - Matching of time-invariant filters (Jost et al, 2006)
  - Multivariate orthogonal matching pursuit (Barthélemy et al, 2012)
  - Matching pursuit and heuristics (Brokmeier and Principe, 2016)
  - Sliding window machine (Gips et al, 2017)
  - Adaptive waveform learning (Hitziger et al, 2017)

Convolutional sparse coding



(Grosse et al, 2007)

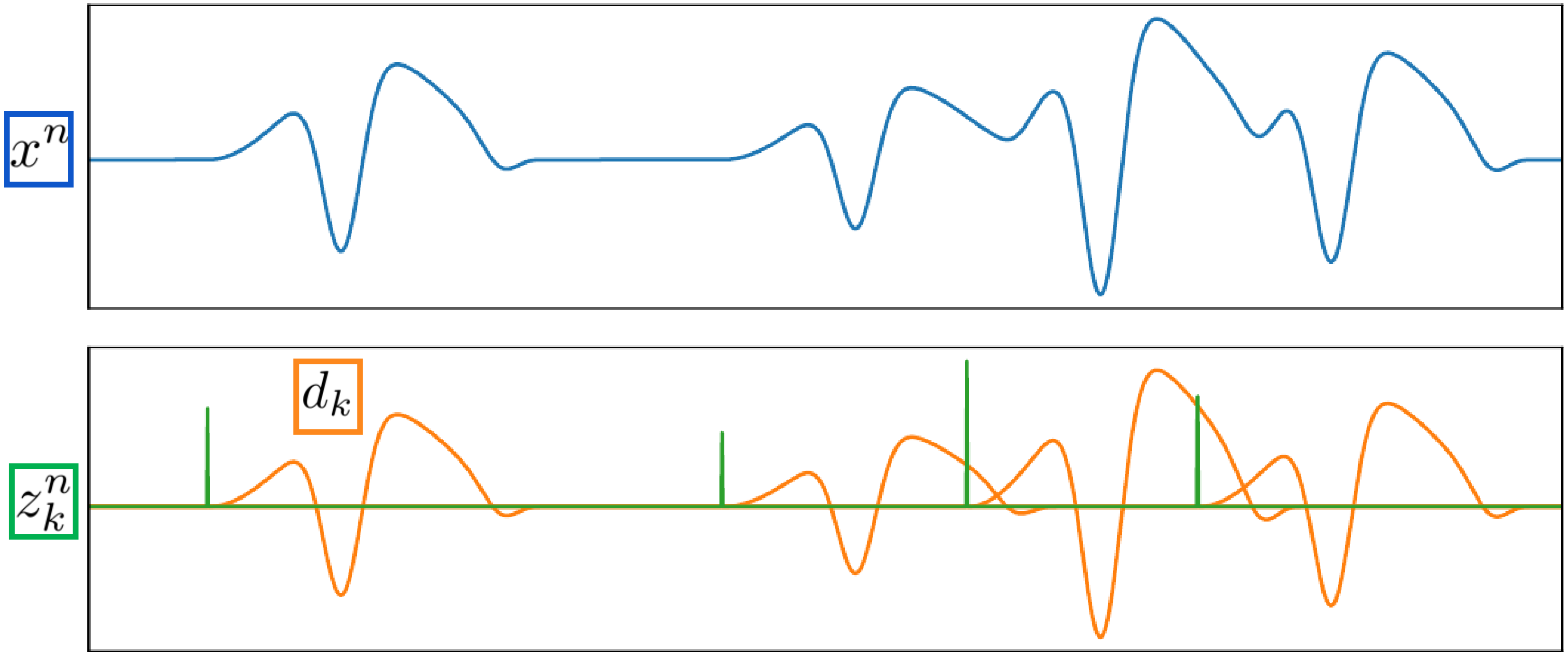
Convolutional sparse coding



$$\boxed{x^n}[t] = \sum_{k=1}^{K} \left(\boxed{z_k^n} * \boxed{d_k}\right)[t] + \varepsilon[t]$$

(Grosse et al, 2007)

Convolutional sparse coding

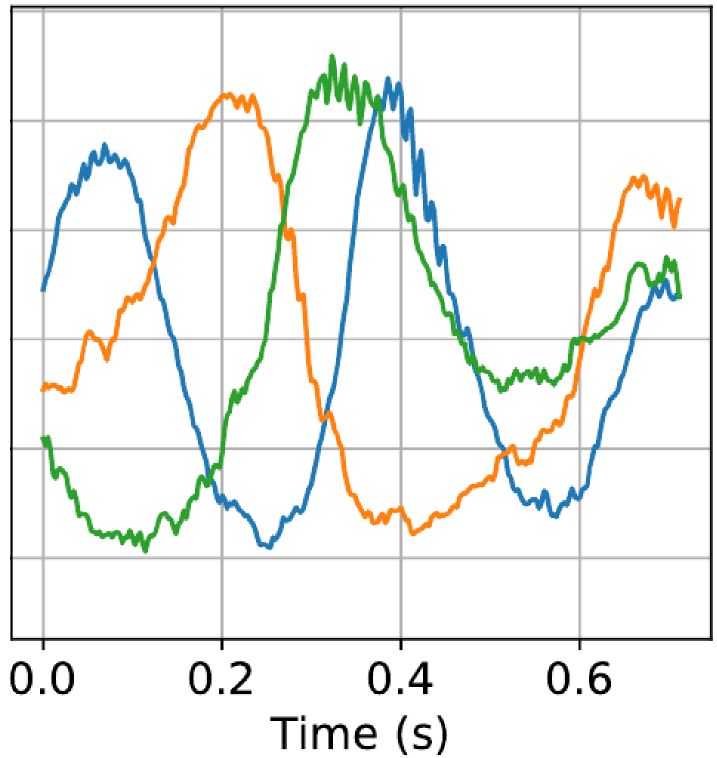


$$\min_{d,z} \sum_{n=1}^{N} \frac{1}{2} \left\| \boxed{x^n} - \sum_{k=1}^{K} \boxed{z_k^n} * \boxed{d_k} \right\|_2^2 + \lambda \sum_{k=1}^{K} \|\boxed{z_k^n}\|_1,$$

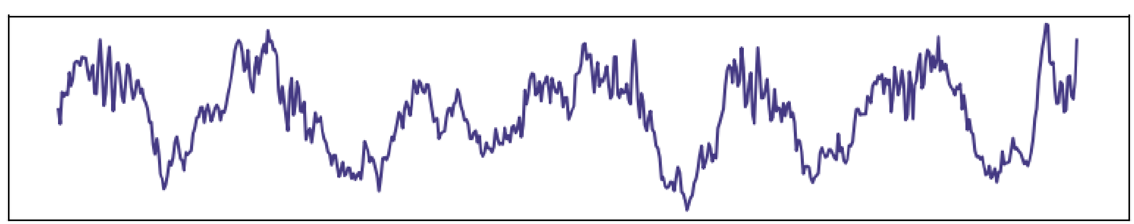
$$\text{s.t.} \quad \|\boxed{d_k}\|_2^2 \leq 1 \text{ and } \boxed{z_k^n} \geq 0.$$

(Grosse et al, 2007)

Learned atoms



LFP



First challenge: optimization speed

$$\min_{d,z} \sum_{n=1}^{N} \frac{1}{2} \left\| x^n - \sum_{k=1}^{K} z_k^n * d_k \right\|_2^2 + \lambda \sum_{k=1}^{K} \|z_k^n\|_1,$$
$$\text{s.t.} \quad \|d_k\|_2^2 \leq 1 \text{ and } z_k^n \geq 0.$$

Block-coordinate descent

- Z-step
- D-step

First challenge: optimization speed

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Block-coordinate descent

- Z-step
  - GCD (Kavukcuoglu et al, 2010)
  - FISTA (Chalasani et al, 2013)
  - ADMM (Bristow et al, 2013)
  - ADMM + FFT (Wohlberg, 2016)
  - L-BFGS (Jas et al, 2017)
  - LGCD (Dupré la Tour et al, 2018)
- D-step

First challenge: optimization speed

$$\min_{d,z} \sum_{n=1}^{N} \frac{1}{2} \left\| x^n - \sum_{k=1}^{K} z_k^n * d_k \right\|_2^2 + \lambda \sum_{k=1}^{K} \|z_k^n\|_1,$$
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Block-coordinate descent

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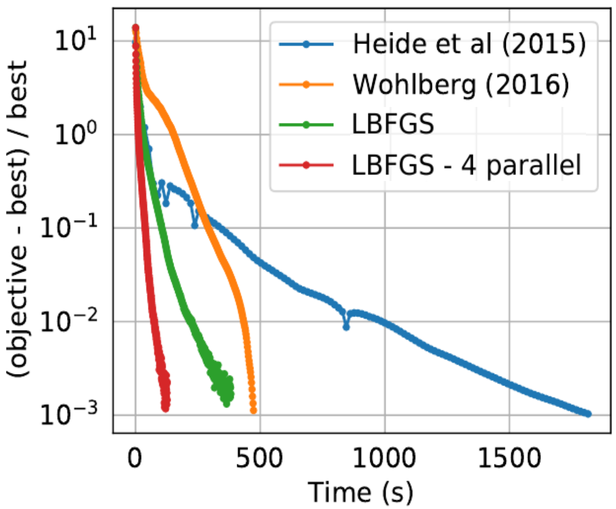
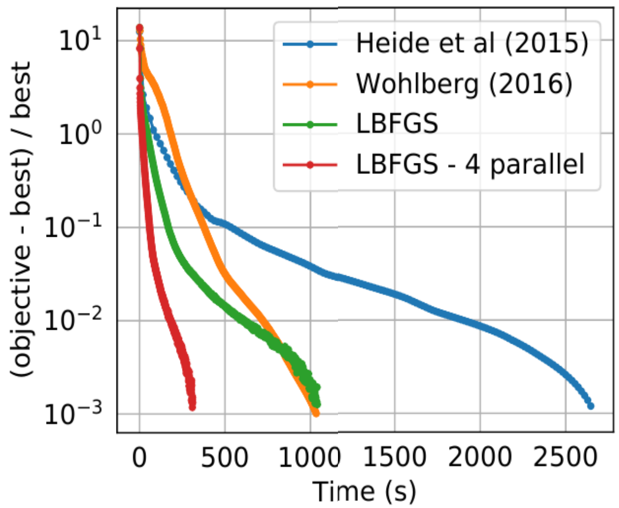
  - ADMM + FFT (Wohlberg, 2016)
  - L-BFGS (Jas et al, 2017)
  - LGCD (Dupré la Tour et al, 2018)
- D-step
  - FFT (Grosse et al, 2007)
  - ADMM + FFT (Heide et al, 2015)
  - ADMM + FFT (Wohlberg, 2016)
  - L-BFGS (dual) (Jas et al, 2017)
  - PGD (Dupré la Tour et al, 2018)

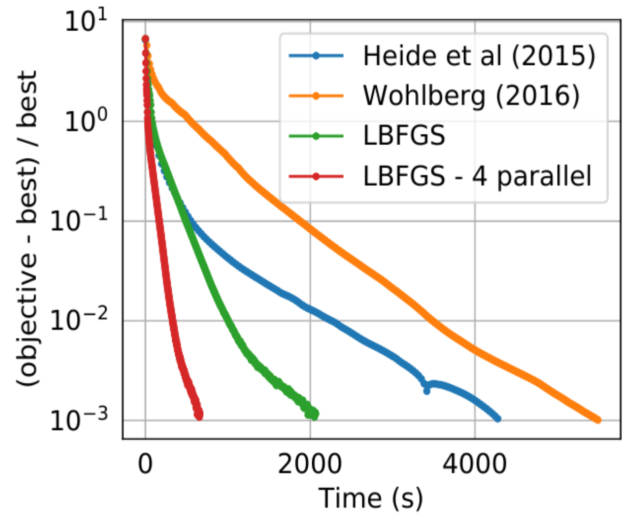
Speed benchmarks



(a) $K = 2,\ L = 32.$

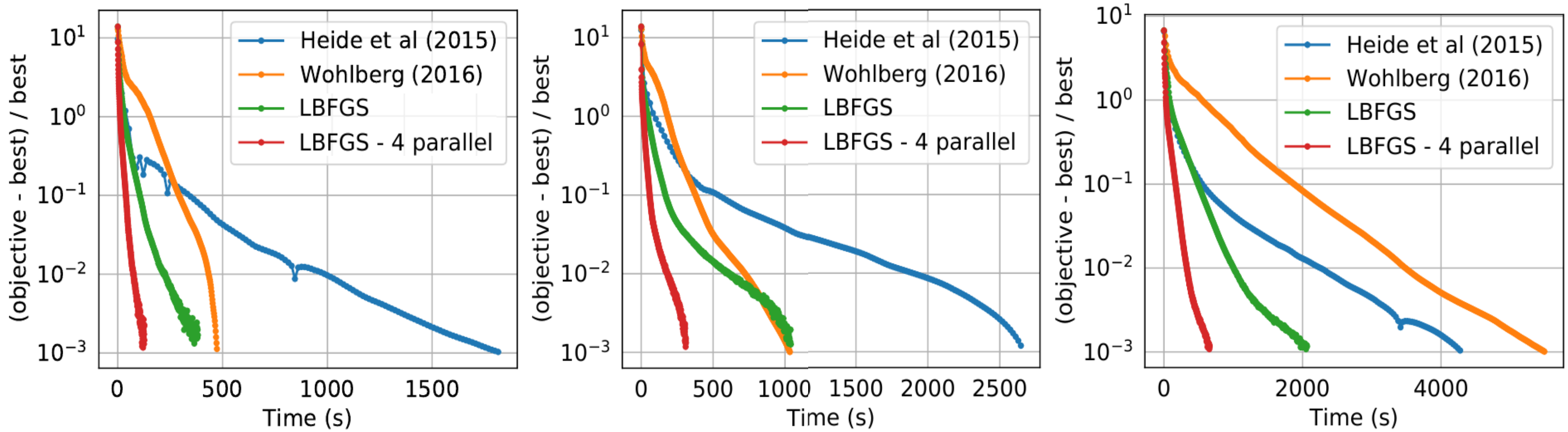


(b) $K = 2,\ L = 128.$

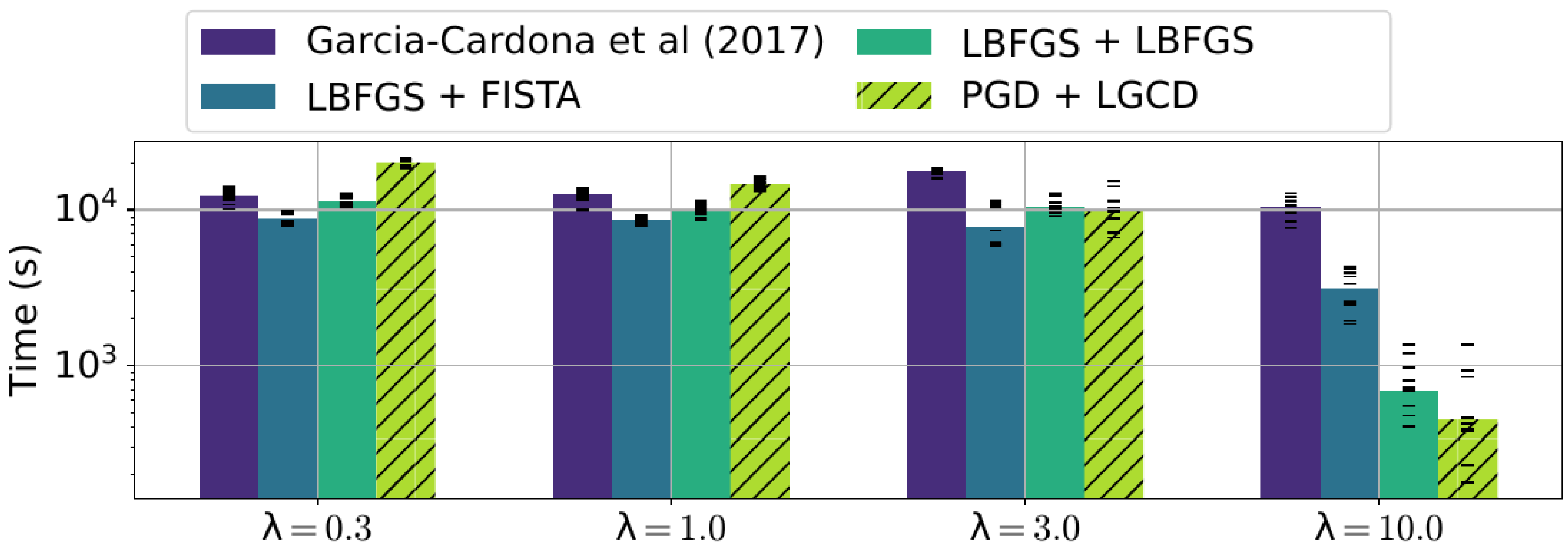


(c) $K = 10,\ L = 32.$

Speed benchmarks



(a) $K = 2,\ L = 32.$ (b) $K = 2,\ L = 128.$ (c) $K = 10,\ L = 32.$



Second challenge: strong artifacts

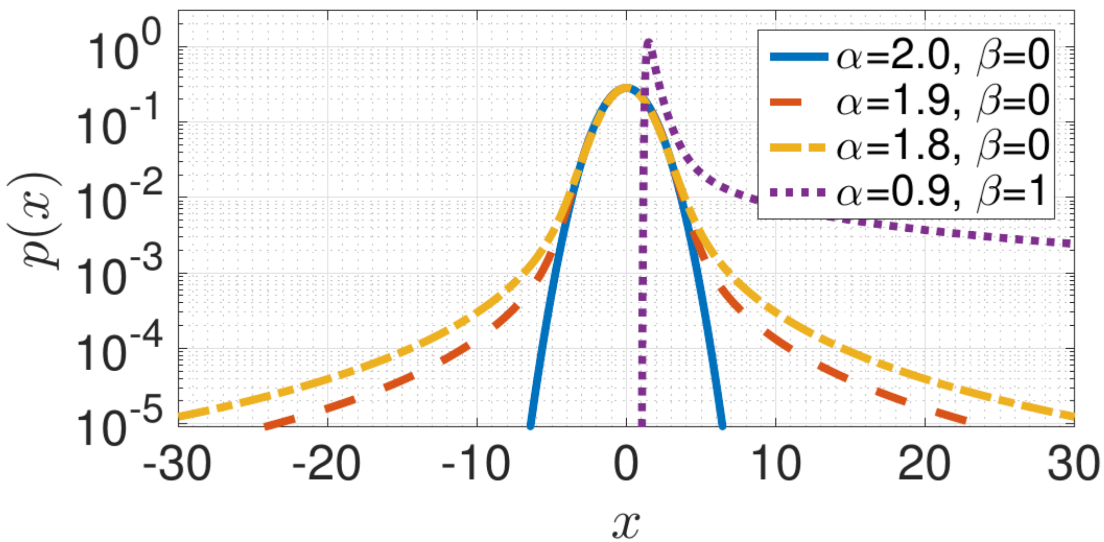
Gaussian CSC model

$$\hat{x}^n \triangleq \sum_{k=1}^{K} z_k^n * d_k$$

$$z_k^n[t] \sim \mathcal{E}(\lambda), \quad x^n[t] | z, d \sim \mathcal{N}(\hat{x}^n[t], 1),$$

Alpha-stable CSC model

$$z_k^n[t] \sim \mathcal{E}(\lambda), \quad x^n[t] | z, d \sim \mathcal{S}(\alpha, 0, 1/\sqrt{2}, \hat{x}^n[t])$$



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Alpha-stable CSC model

$$z_k^n[t] \sim \mathcal{E}(\lambda), \quad x^n[t] | z, d \sim \mathcal{S}(\alpha, 0, 1/\sqrt{2}, \hat{x}^n[t])$$

Conditional formulation (Samorodnitsky and Taqqu, 1994)

$$z_k^n[t] \sim \mathcal{E}(\lambda), \quad \phi^n[t] \sim \mathcal{S}\left(\frac{\alpha}{2}, 1, 2(\cos\frac{\pi\alpha}{4})^{2/\alpha}, 0\right)$$

$$x^n[t] | z, d, \phi \sim \mathcal{N}\left(\hat{x}^n[t], \frac{1}{2}\phi^n[t]\right)$$

Alpha CSC estimation

Monte Carlo Expectation-Maximization algorithm

- E-step: MCMC estimation (Chib and Greenberg, 1995)

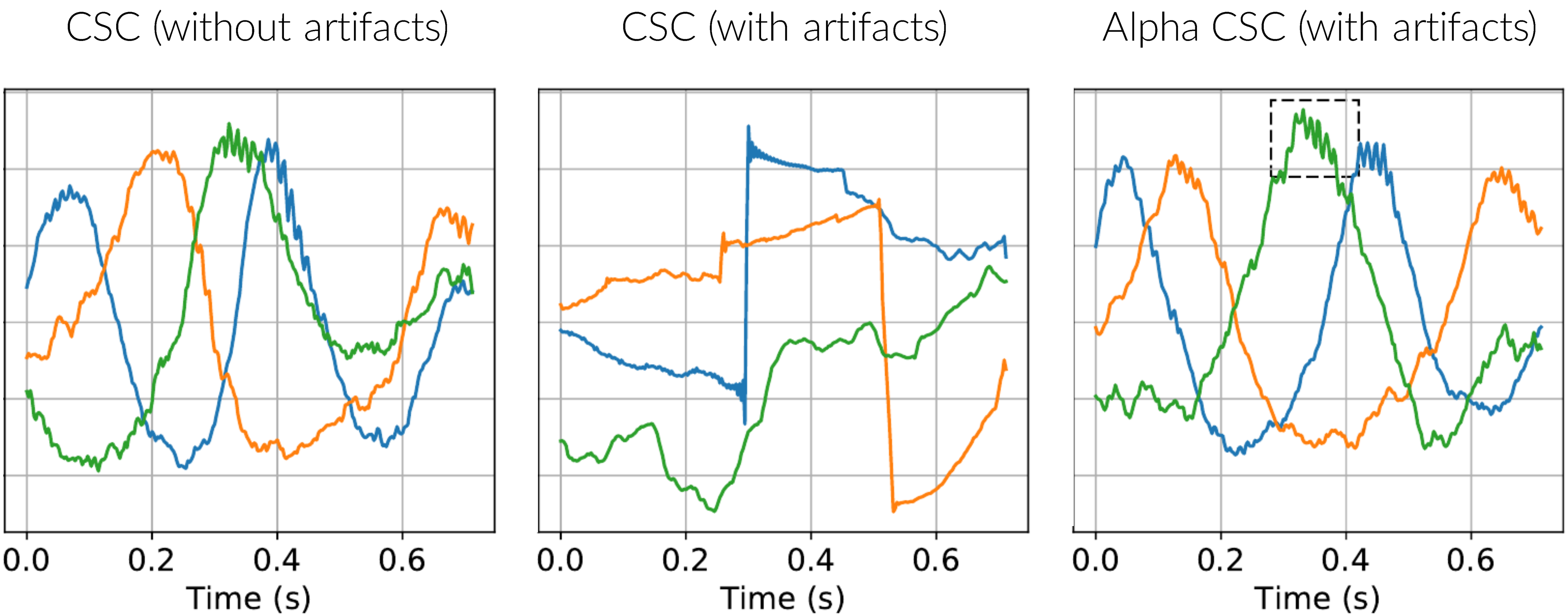
$$w^n[t]^{(i)} \triangleq \mathbb{E}\left[1/\phi^n[t]\right]_{p(\phi|x,z^{(i)},d^{(i)})}$$

- M-step: weighted CSC

$$\min_{d,z} \sum_{n=1}^{N} \frac{1}{2} \left\| \sqrt{w^n} \odot (x^n - \sum_{k=1}^{K} z_k^n * d_k) \right\|_2^2 + \lambda \sum_{k=1}^{K} \|z_k^n\|_1$$

$$\text{s.t.} \quad \|d_k\|_2^2 \leq 1, \text{ and } z_k^n \geq 0, \quad \forall k, n.$$

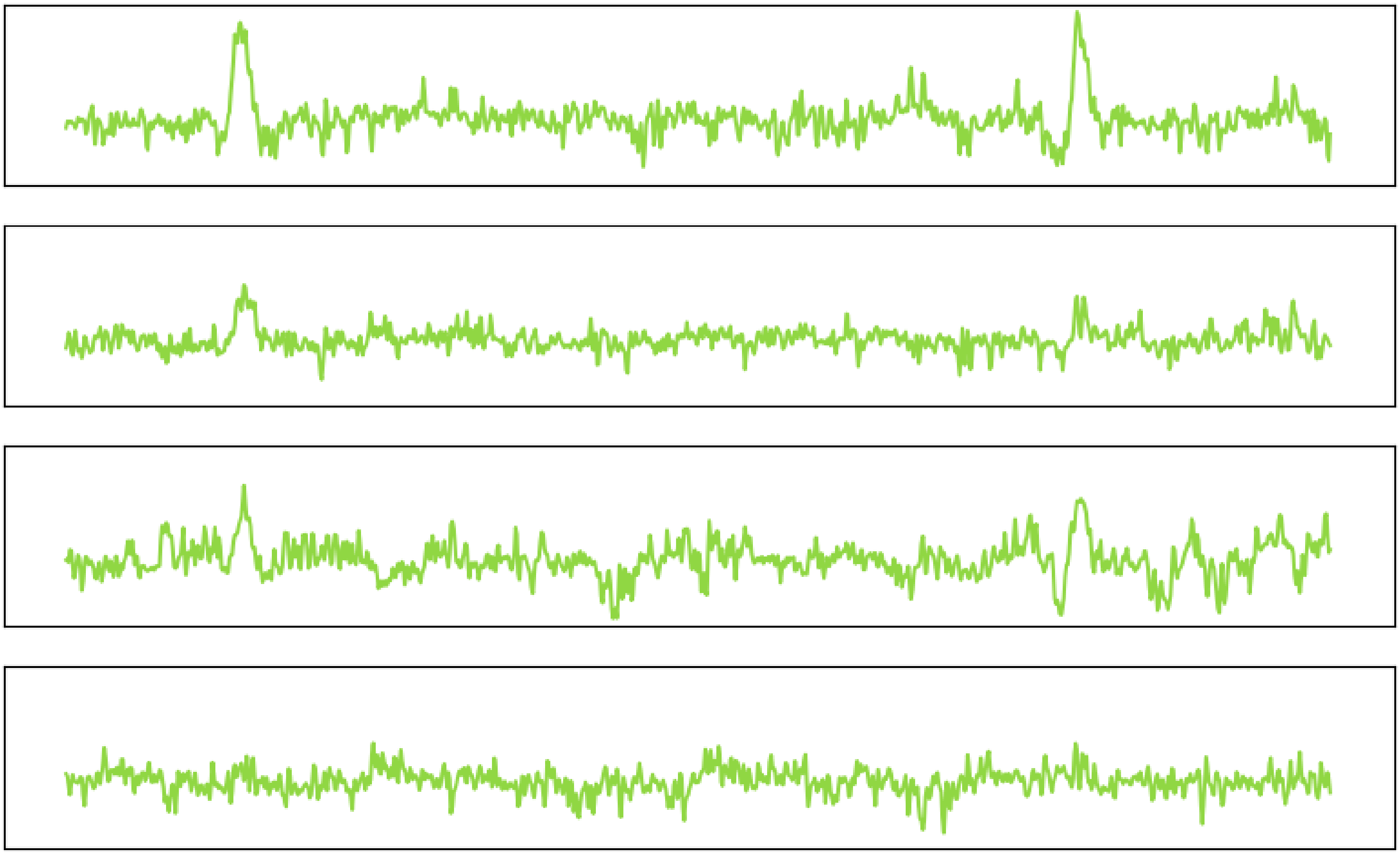
Learned atoms



Learning the morphology of brain signals using alpha-stable convolutional sparse coding
M. Jas, T. Dupré la Tour, U. Şimşekli, A. Gramfort, *NeurIPS 2017*

Third challenge: multivariate models

MEG



Third challenge: multivariate models

$$\min_{D,z} \sum_{n=1}^{N} \frac{1}{2} \left\| X^n - \sum_{k=1}^{K} z_k^n * D_k \right\|_2^2 + \lambda \sum_{k=1}^{K} \|z_k^n\|_1,$$
$$\text{s.t.} \quad \|D_k\|_2^2 \leq 1 \text{ and } z_k^n \geq 0.$$

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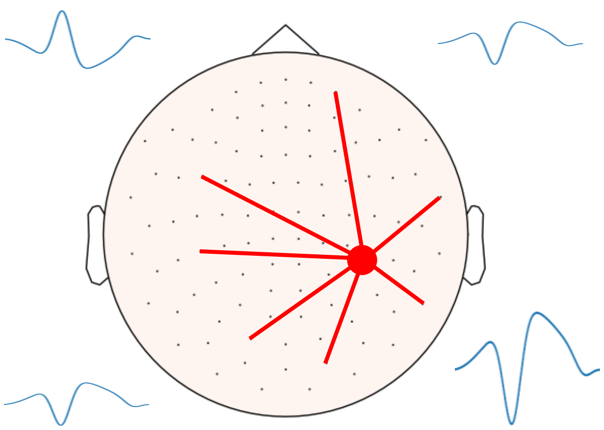
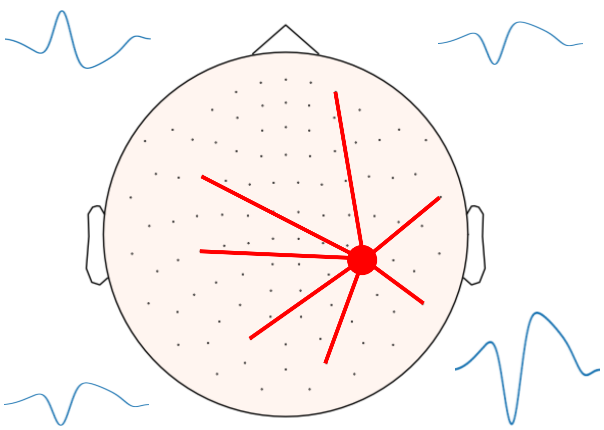


Third challenge: multivariate models

$$\min_{u,v,z} \sum_{n=1}^{N} \frac{1}{2} \left\| X^n - \sum_{k=1}^{K} z_k^n * (u_k v_k^\top) \right\|_2^2 + \lambda \sum_{k=1}^{K} \|z_k^n\|_1 ,$$
$$\text{s.t.} \quad \|u_k\|_2^2 \leq 1 \ , \ \|v_k\|_2^2 \leq 1 \text{ and } z_k^n \geq 0.$$



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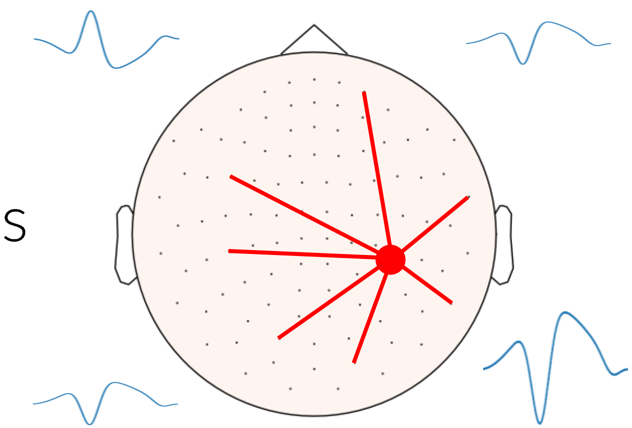
$$\text{s.t.} \quad \|u_k\|_2^2 \leq 1 \ , \ \|v_k\|_2^2 \leq 1 \text{ and } z_k^n \geq 0.$$

Rank-1 constraint

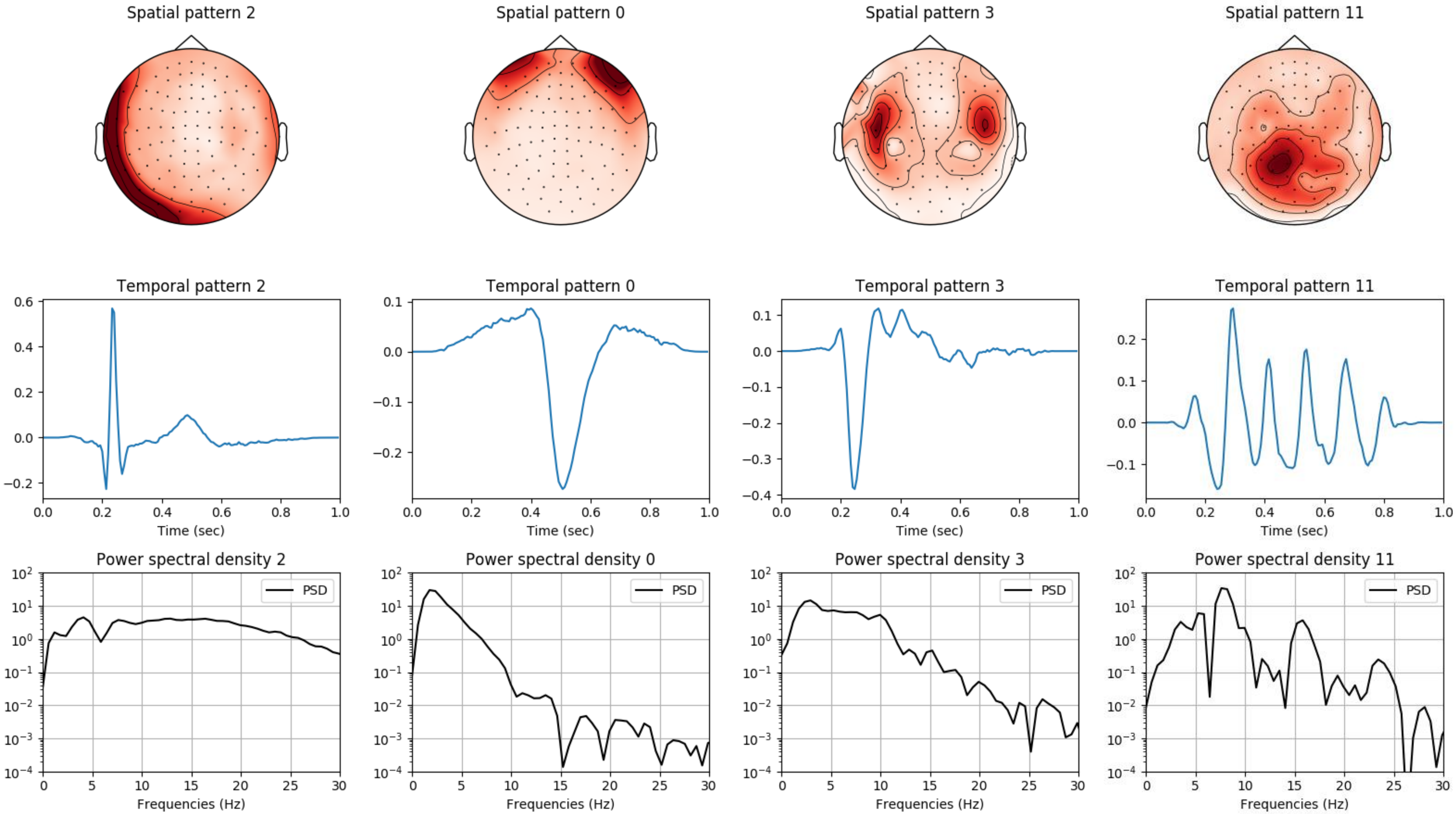
- Consistent with Physics of EM waves
- Scales in (L+P) instead of (LP)



Multivariate Convolutional Sparse Coding for Electromagnetic Brain Signals

T. Dupré la Tour*, T. Moreau*, M. Jas, A. Gramfort, *NeurIPS 2018*

Multivariate atoms



2. Temporal waveform analysis

with convolutional sparse coding models

- CSC well-posed optimization problem
- State-of-the-art optimization speed
- Alpha CSC model for robustness to strong artifacts
- Multivariate CSC model, rank-1 constraint

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M. Jas, T. Dupré la Tour, U. Şimşekli, A. Gramfort, *NeurIPS 2017*

Multivariate Convolutional Sparse Coding for Electromagnetic Brain Signals
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Conclusion

1. Cross-frequency coupling analysis
with driven autoregressive models

2. Temporal waveform analysis
with convolutional sparse coding models

Dissemination of our work

We published our code online, with:

- Tests, and continuous integration
- Extensive documentation
- Gallery of examples

1. PAC metrics, DAR models

`https://pactools.github.io`

2. CSC, alpha-stable CSC, multivariate CSC, rank-1 constraint

`https://alphacsc.github.io`

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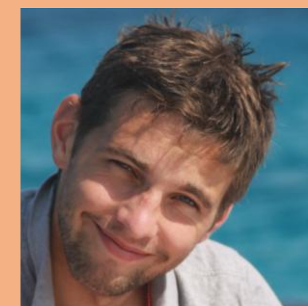



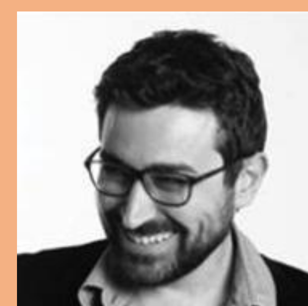


matplotlib

python™

NumPy

scikit learn

MNE MEG + EEG ANALYSIS & VISUALIZATION

SciPy

Thank you for your attention

Non-linear autoregressive models for cross-frequency coupling in neural time series
T. Dupré la Tour, L. Tallot, L. Grabot, V. Doyère, V. van Wassenhove, Y. Grenier, A. Gramfort, *PLOS Computational Biology 2017*

Parametric estimation of spectrum driven by an exogenous signal
T. Dupré la Tour, Y. Grenier, A. Gramfort, *ICASSP 2017*

Driver estimation in non-linear autoregressive models
T. Dupré la Tour, Y. Grenier, A. Gramfort, *ICASSP 2018*

`https://pactools.github.io`

Learning the morphology of brain signals using alpha-stable convolutional sparse coding
M. Jas, T. Dupré la Tour, U. Şimşekli, A. Gramfort, *NeurIPS 2017*

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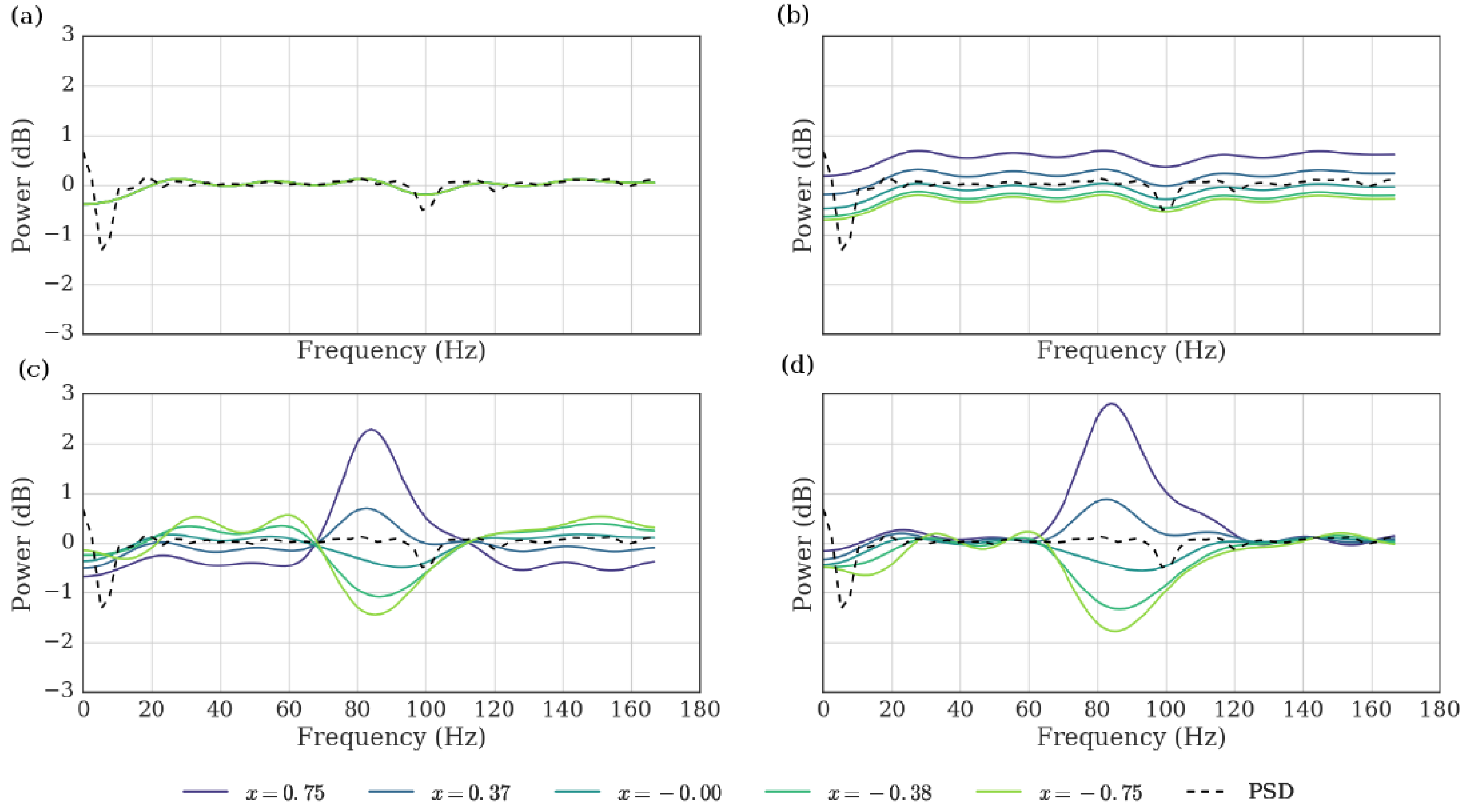
`https://alphacsc.github.io`

DAR models conditional PSD



Guarantee local stability

- AR model

$$y(t) + \sum_{i=1}^{p} a_i y(t-i) = \varepsilon(t)$$

- Non-stationary AR model

$$a_i(t) = \sum_{j=0}^{m} a_{ij} x(t)^j$$

- Lattice parameterization

$$a_p^{(p)} = k_p; \qquad \forall i \in [1, p-1],\ a_i^{(p)} = a_i^{(p-1)} + k_p a_{p-i}^{(p-1)}$$

- Local stability criterion

$$-1 < k_i < 1$$

- Log Area Ratio

$$\gamma_i = \log\left(\frac{1+k_i}{1-k_i}\right) \iff k_i = \frac{e^{\gamma_i}-1}{e^{\gamma_i}+1}$$

- Driven AR model

$$\gamma_i(t) = \sum_{j=0}^{m} \gamma_{ij} x(t)^j$$

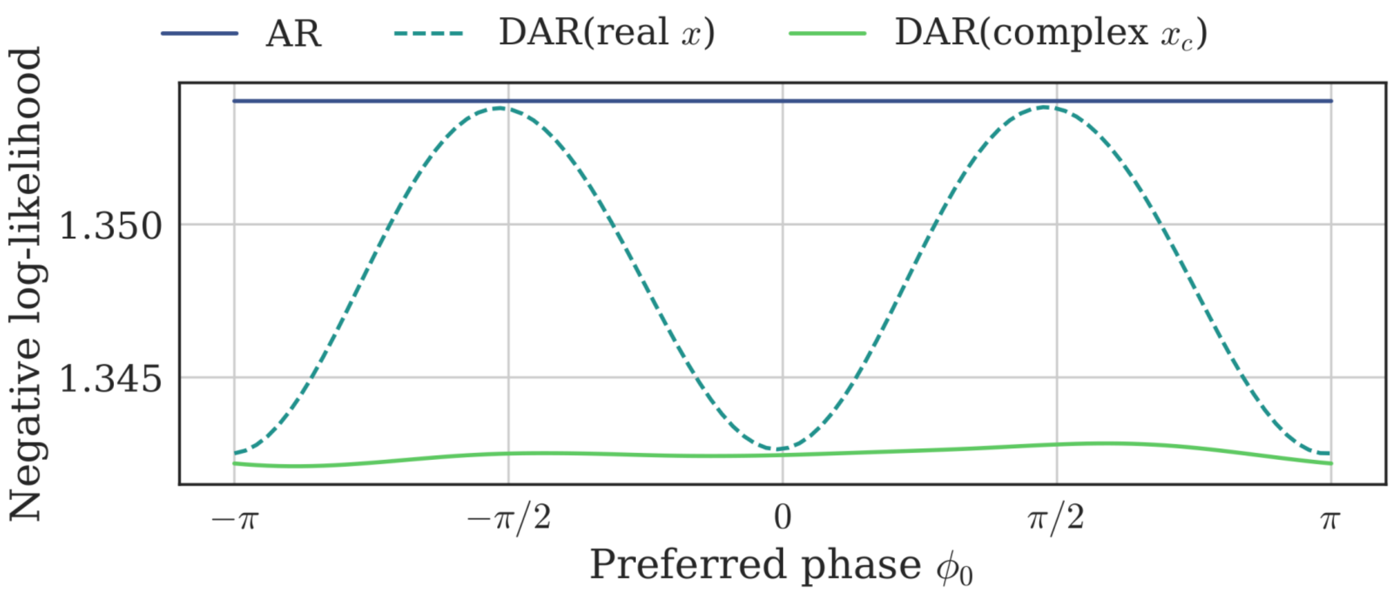
Using a complex driver

- With a real driver

$$a_i(t) = \sum_{k=0}^{m} a_{ik} x(t)^k$$

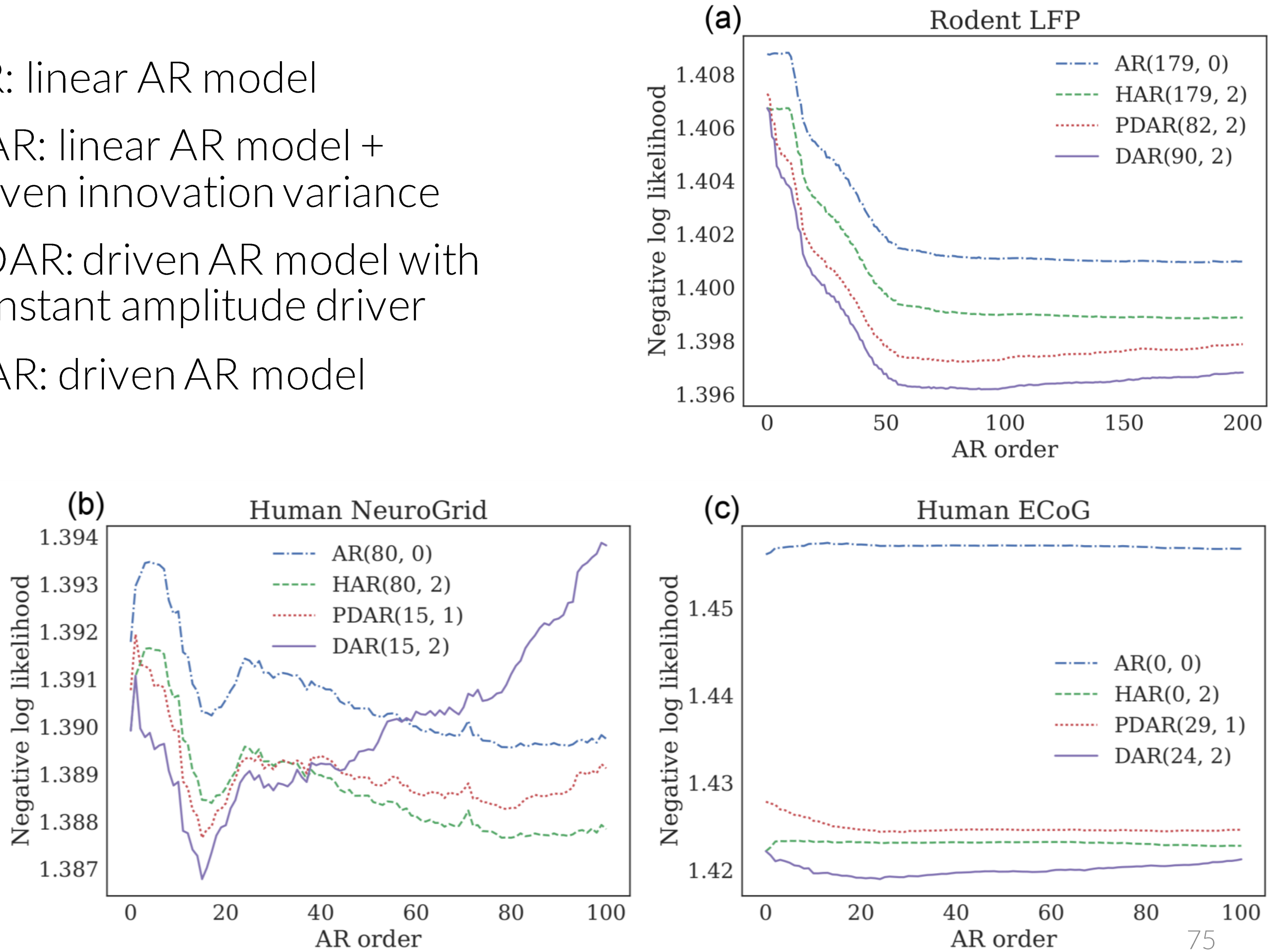
- With a complex driver

$$a_i(t) = \sum_{0 \leq k+l \leq m} a_{ikl}\, x(t)^k \bar{x}(t)^l$$

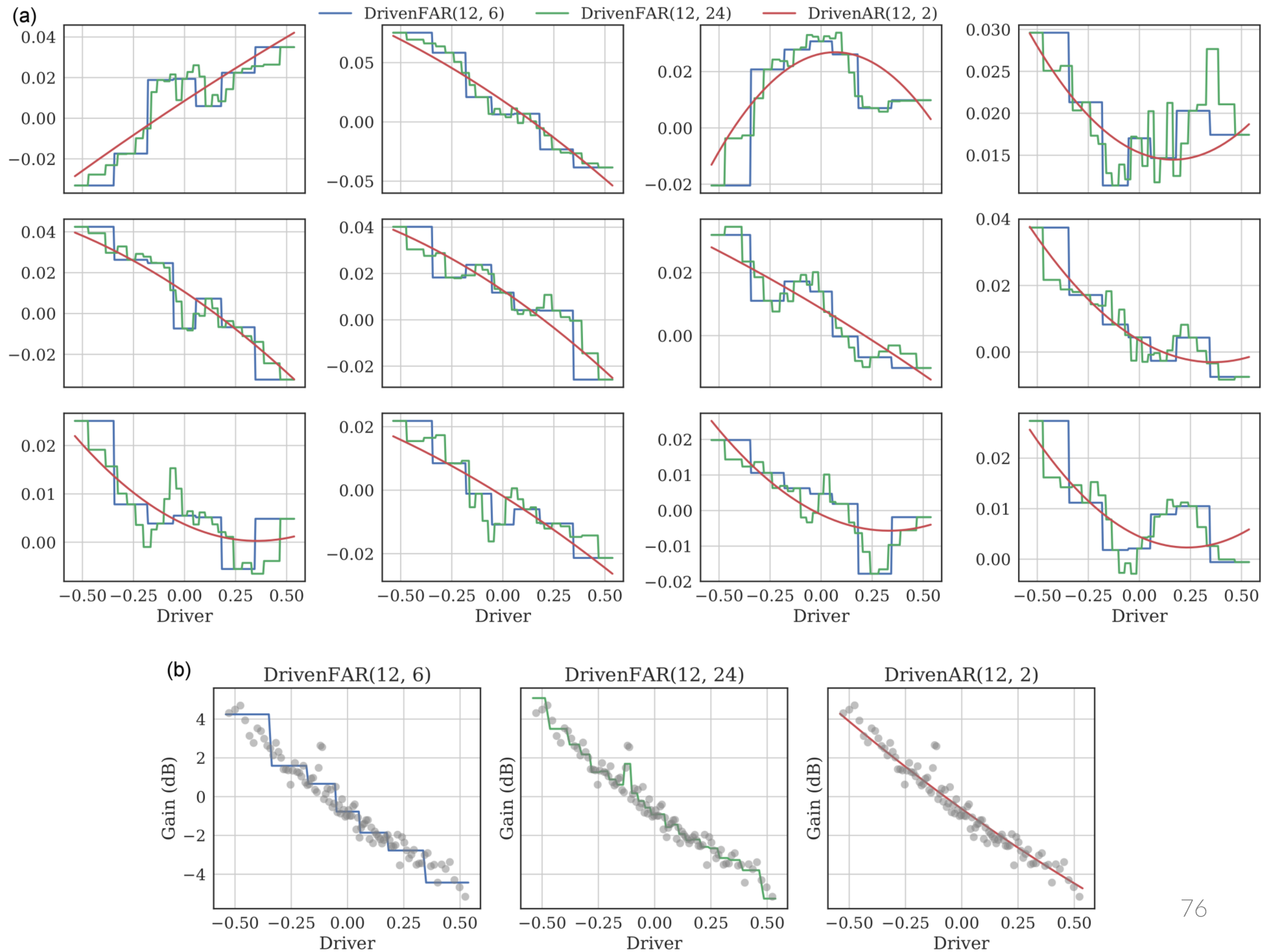


Model variants and cross-validation

- AR: linear AR model
- HAR: linear AR model + driven innovation variance
- PDAR: driven AR model with constant amplitude driver
- DAR: driven AR model



The polynomial basis is good enough



Model and parameter selection

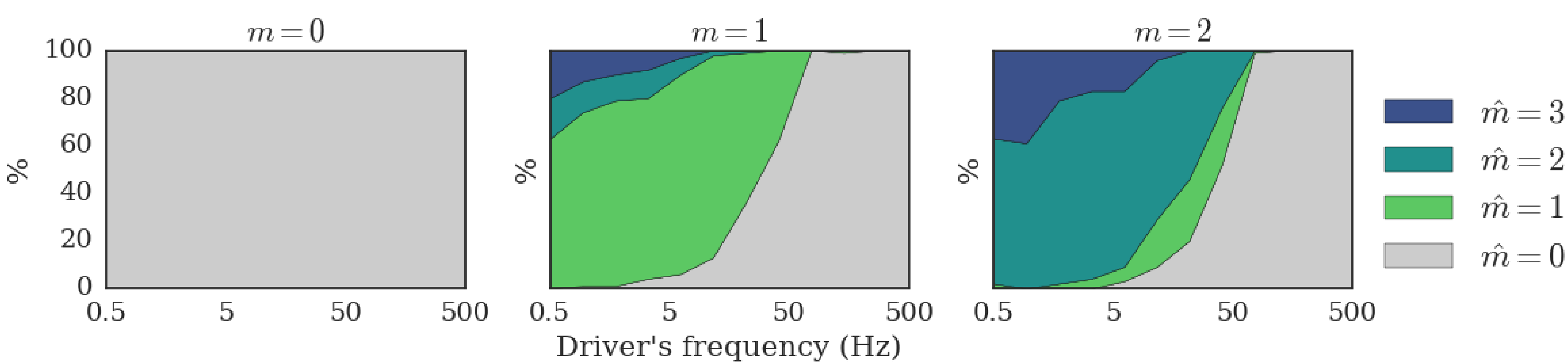
- Likelihood function

$$L = \prod_{t=p}^{T} \frac{1}{\sqrt{2\pi\sigma(t)^2}} \exp\left(-\frac{\varepsilon(t)^2}{2\sigma(t)^2}\right)$$

- BIC selection

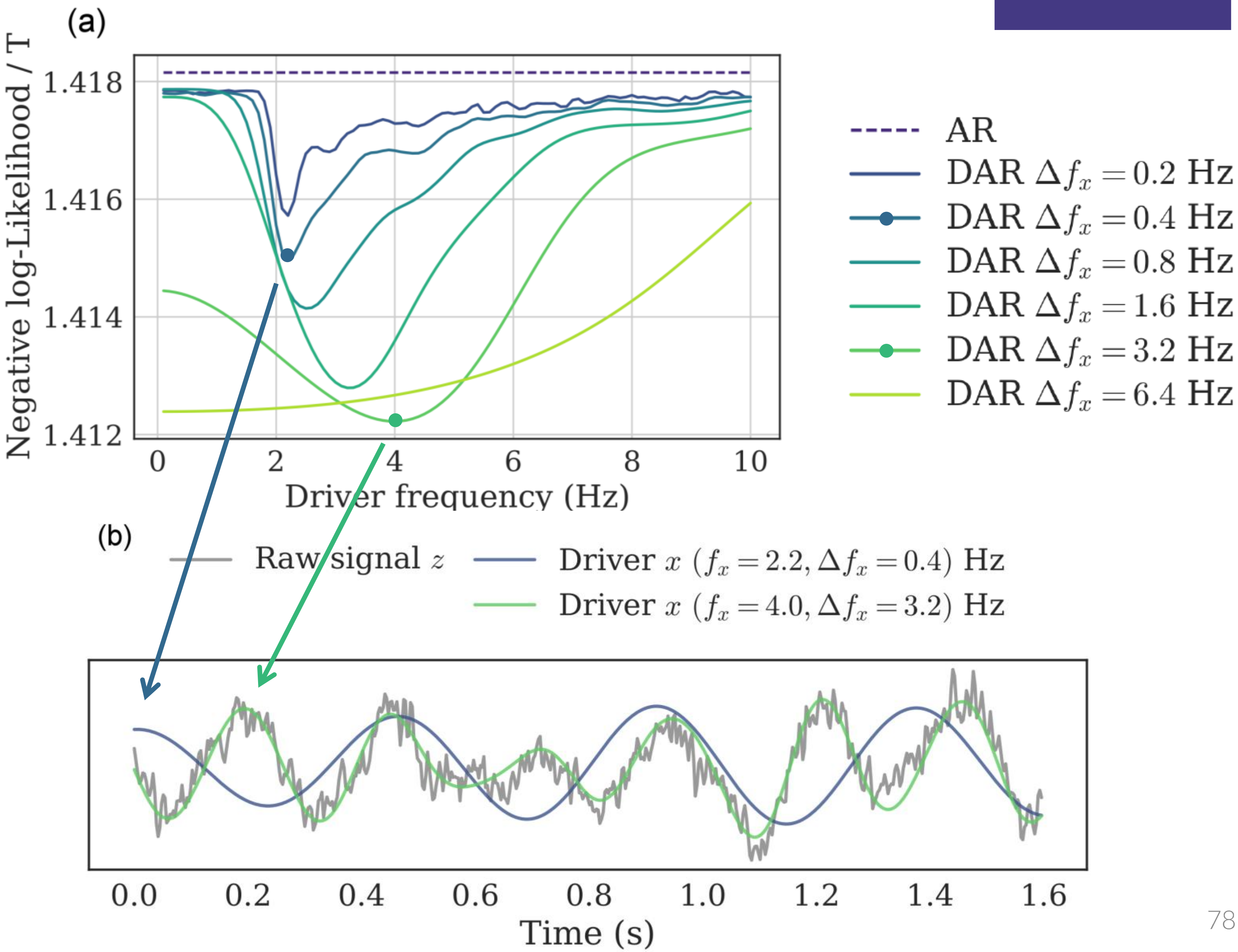
$$BIC = -2\log(L) + d\log(T)$$
$$d = (p+1)(m+1)$$

- Testing the limits

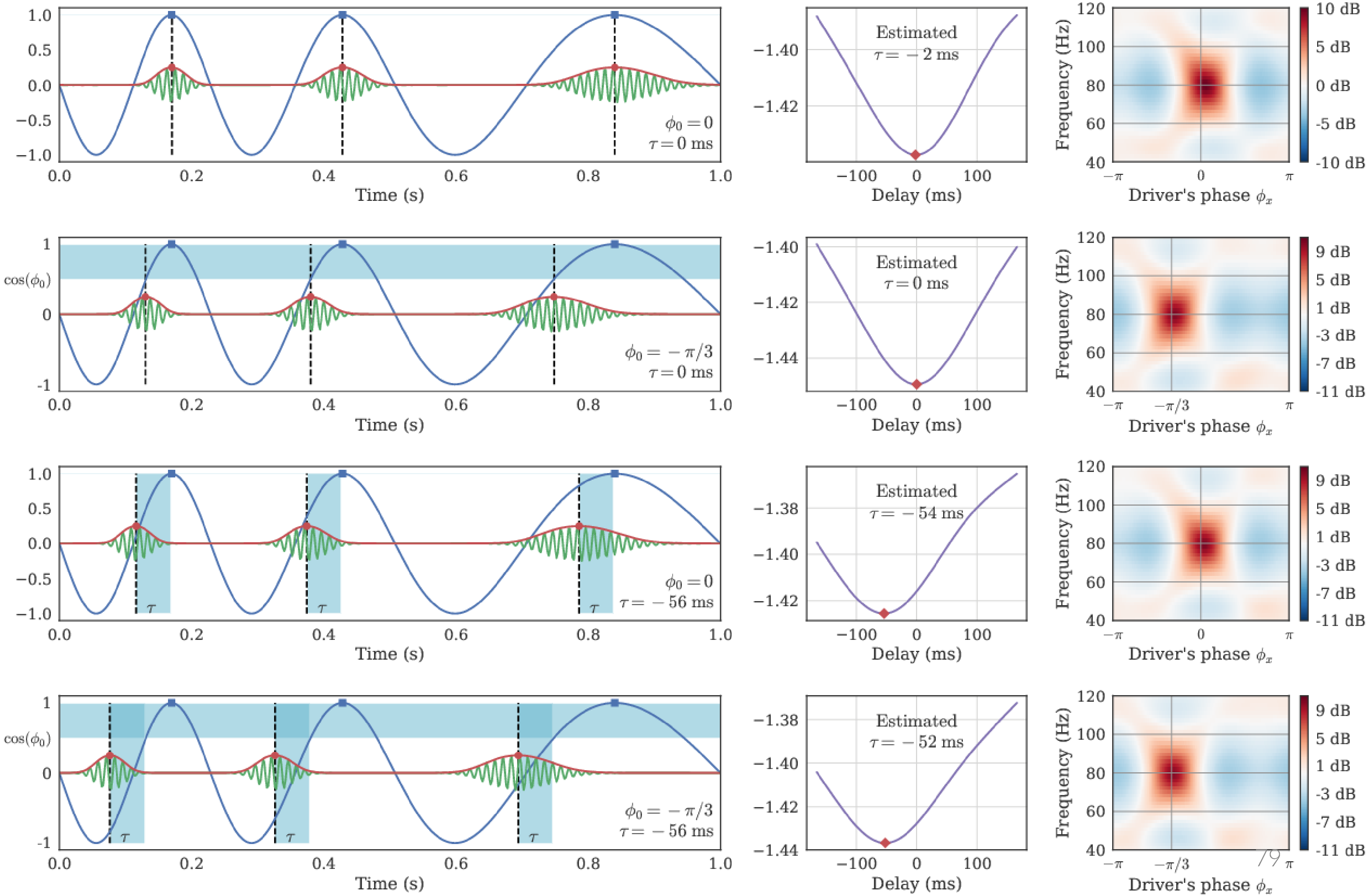


Driver selection

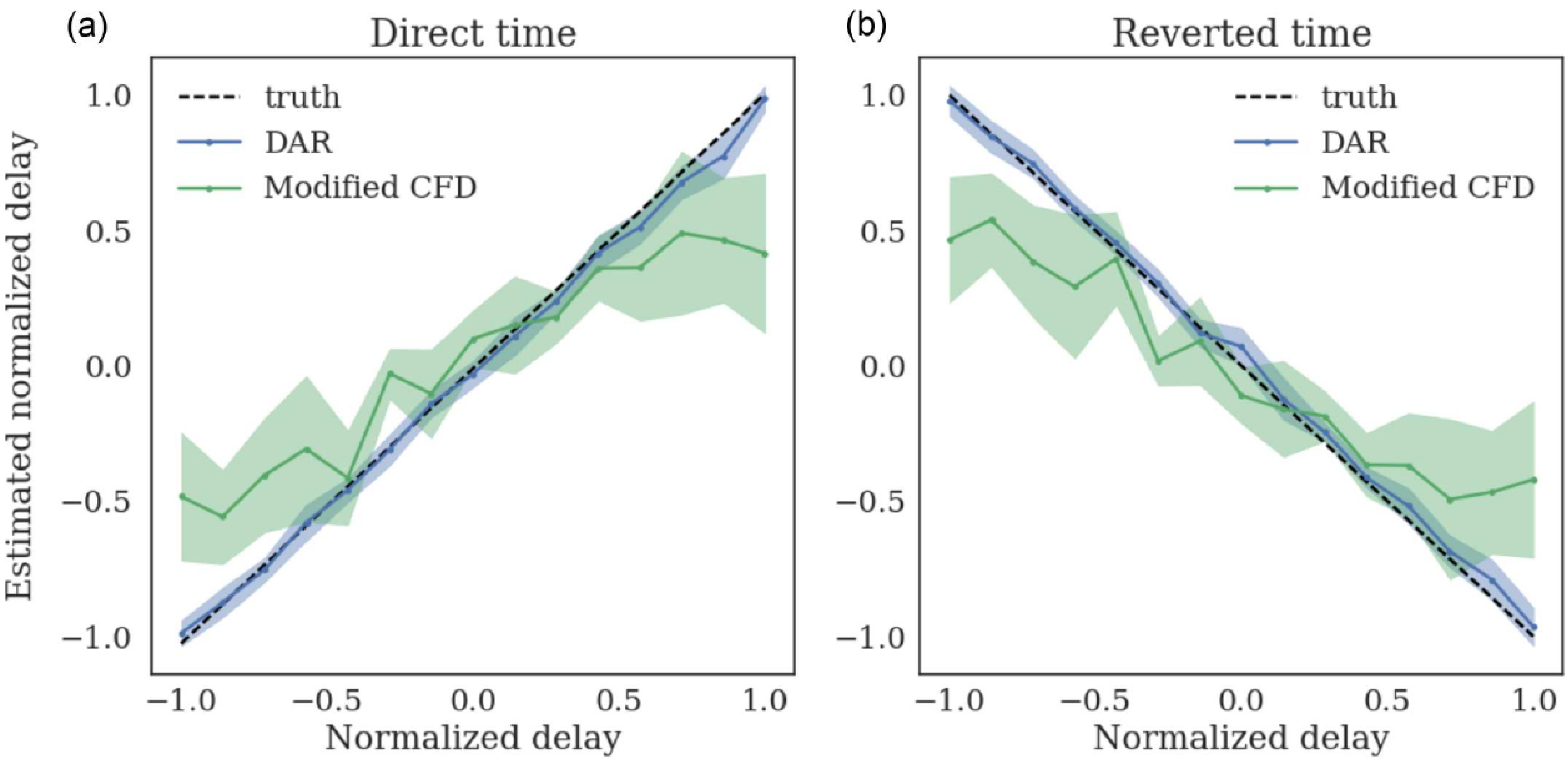
LFP



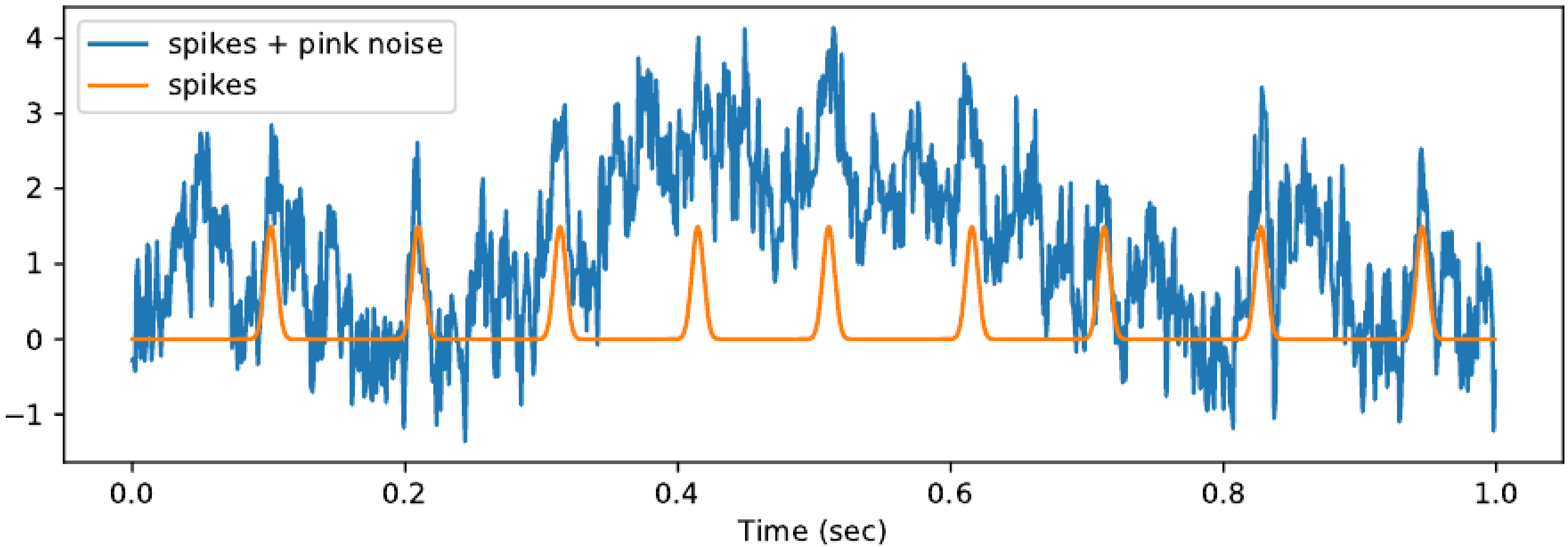
Time delay and preferred phase

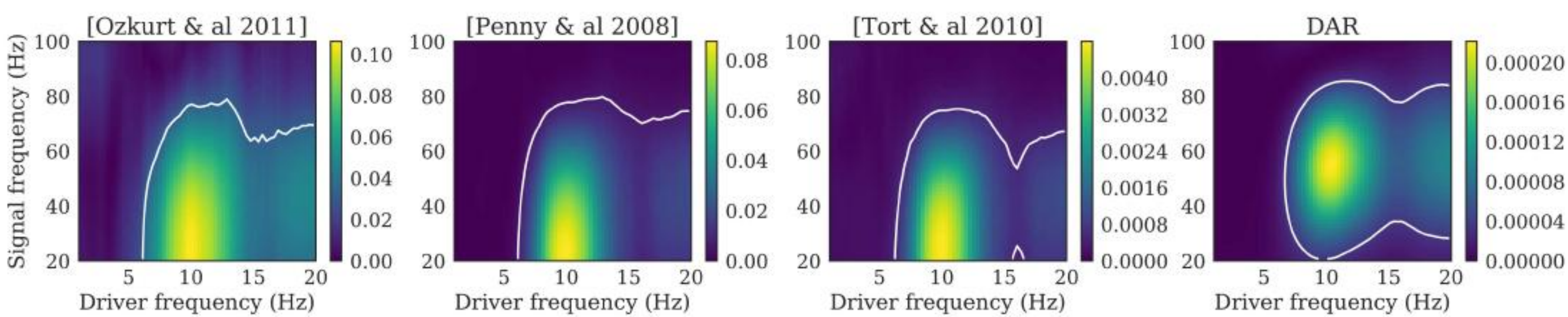


Delay estimation on simulations

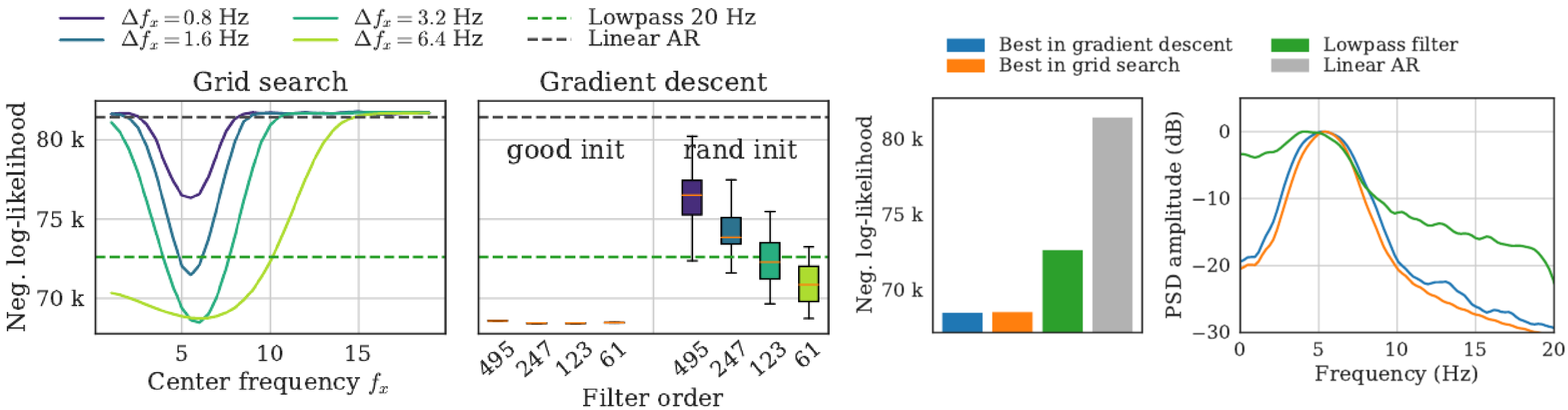


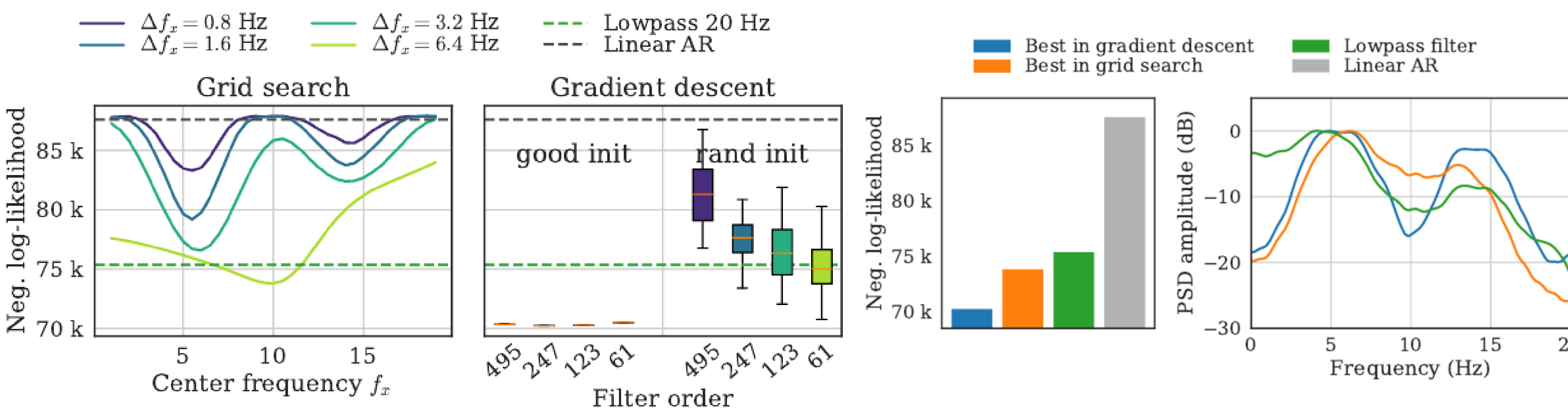
Spurious CFC simulation





Driver filter estimation



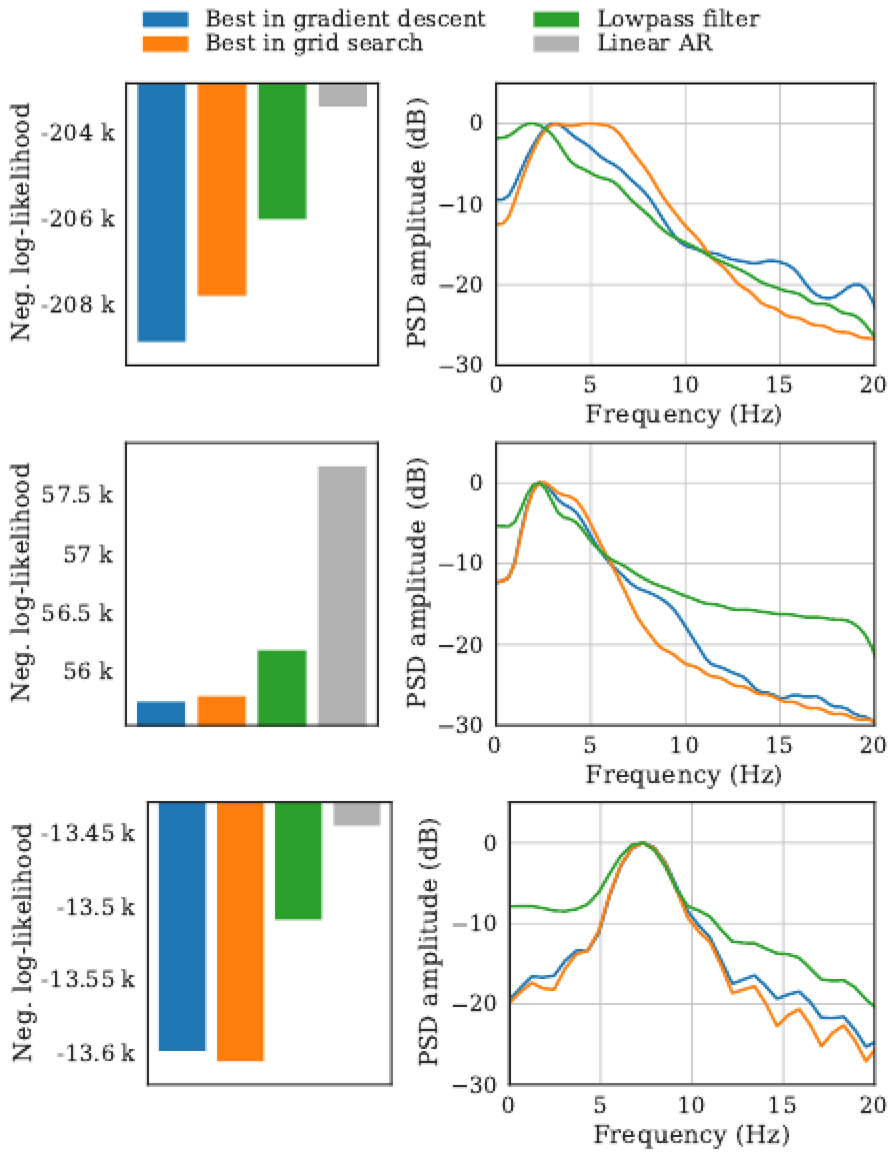


Driver filter estimation

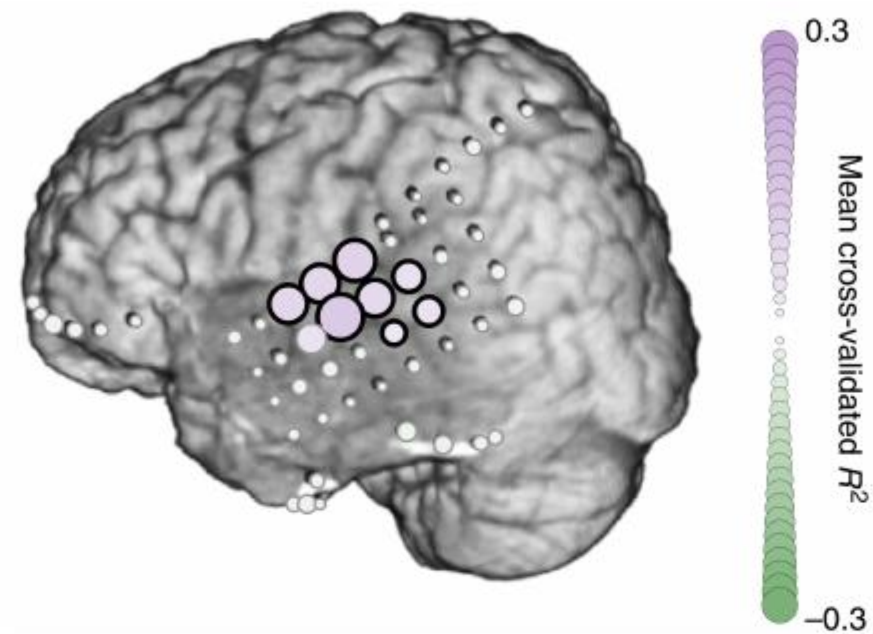
ECoG

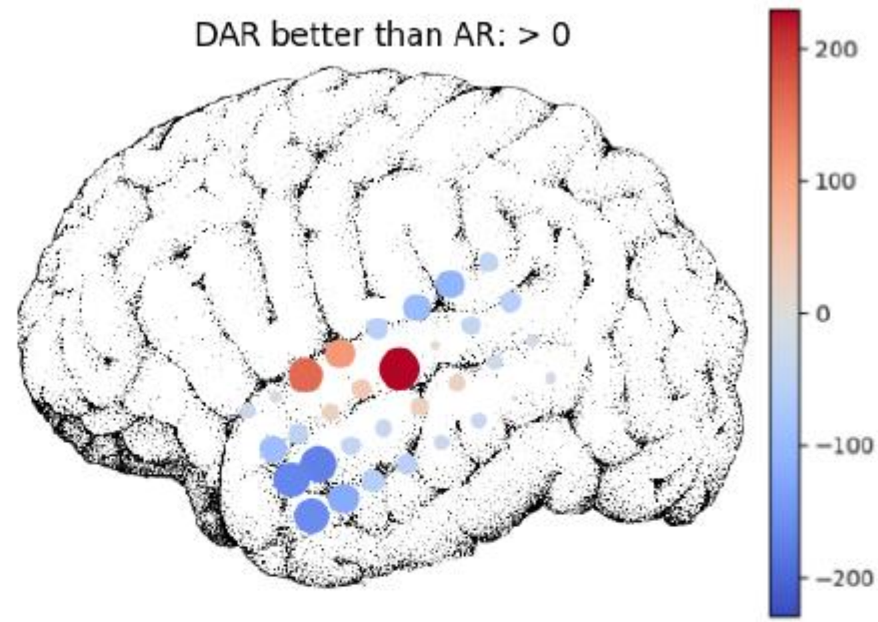
LFP

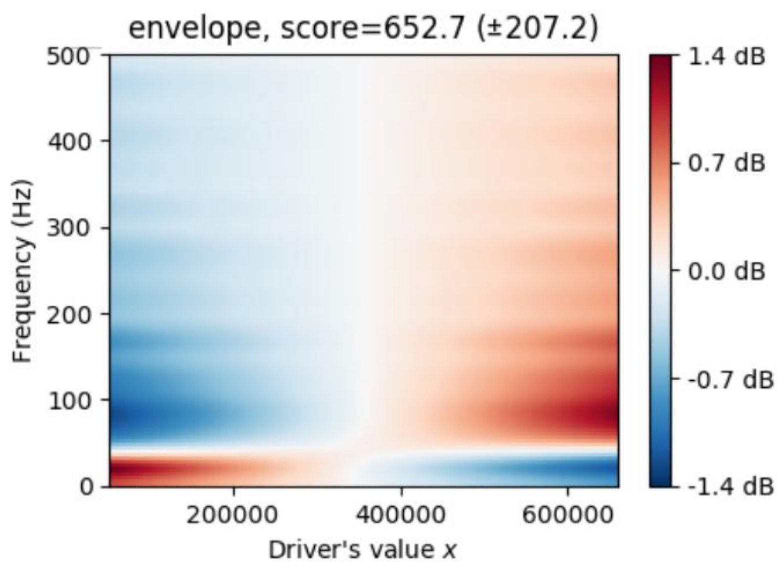
LFP



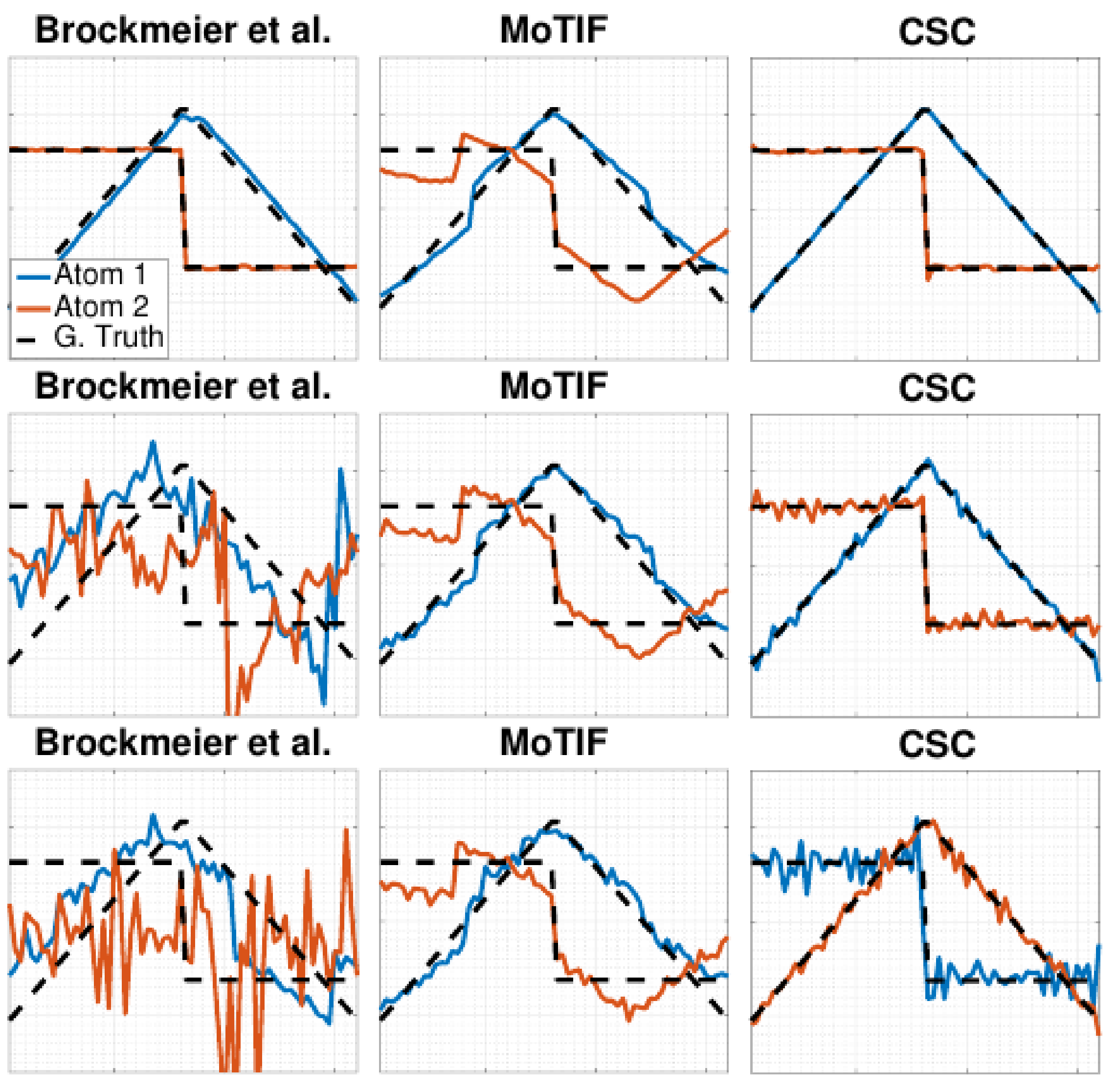
Encoding DAR model







Shift-invariant sparse coding comparison

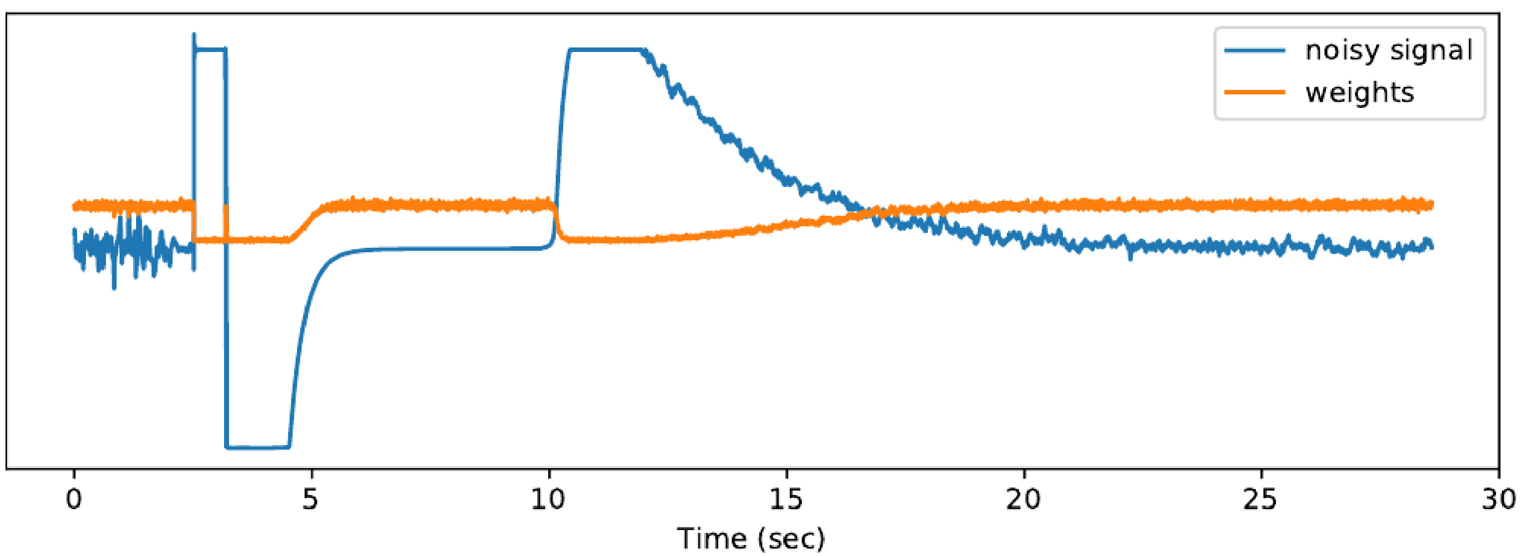


Weights in alpha CSC

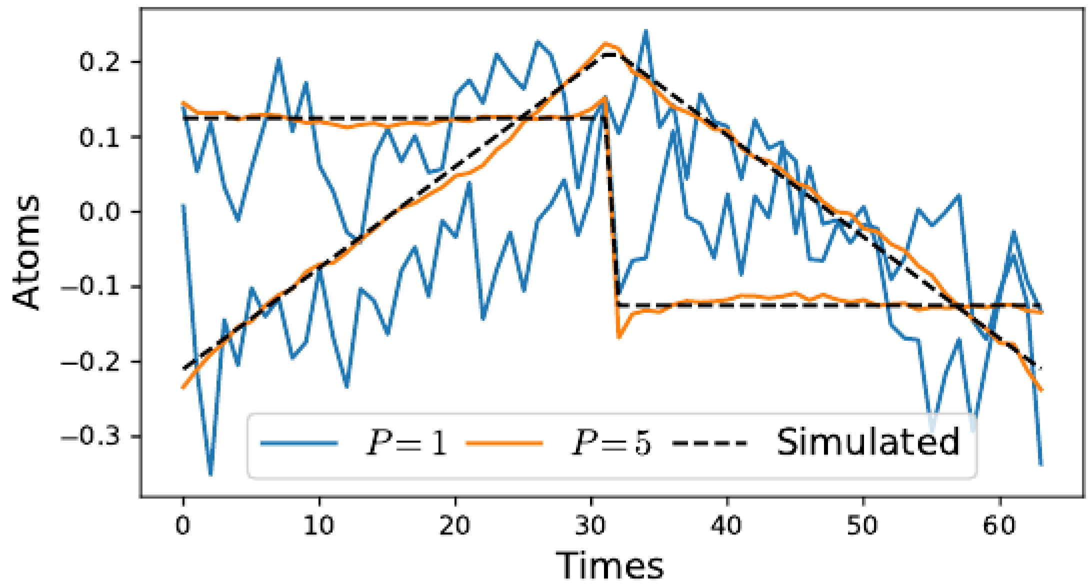
Expectation-Maximization algorithm

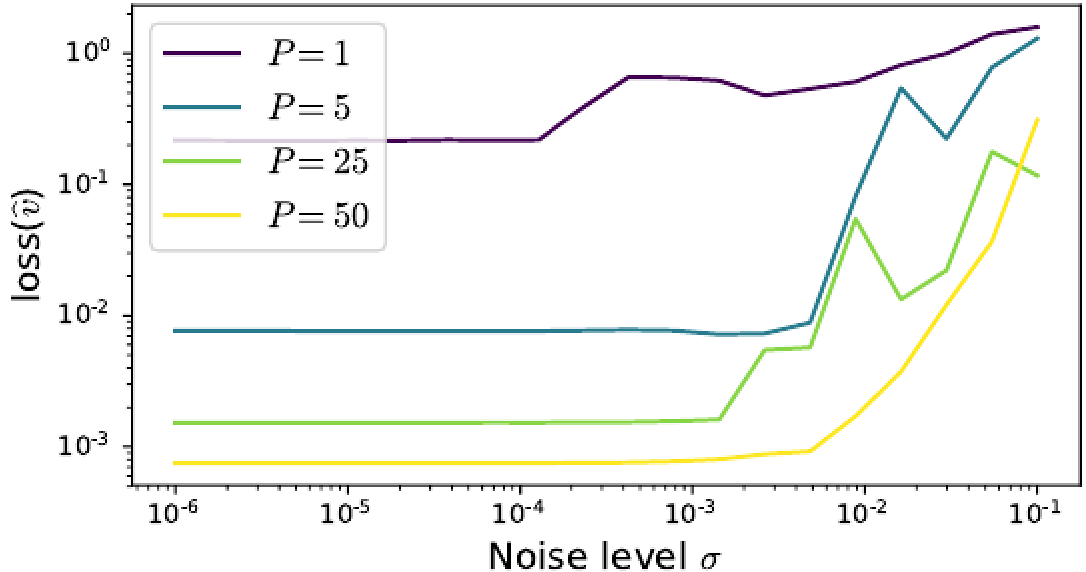
E-step

$$w^n[t]^{(i)} \triangleq \mathbb{E}\left[1/\phi^n[t]\right]_{p(\phi|x,z^{(i)},d^{(i)})}$$

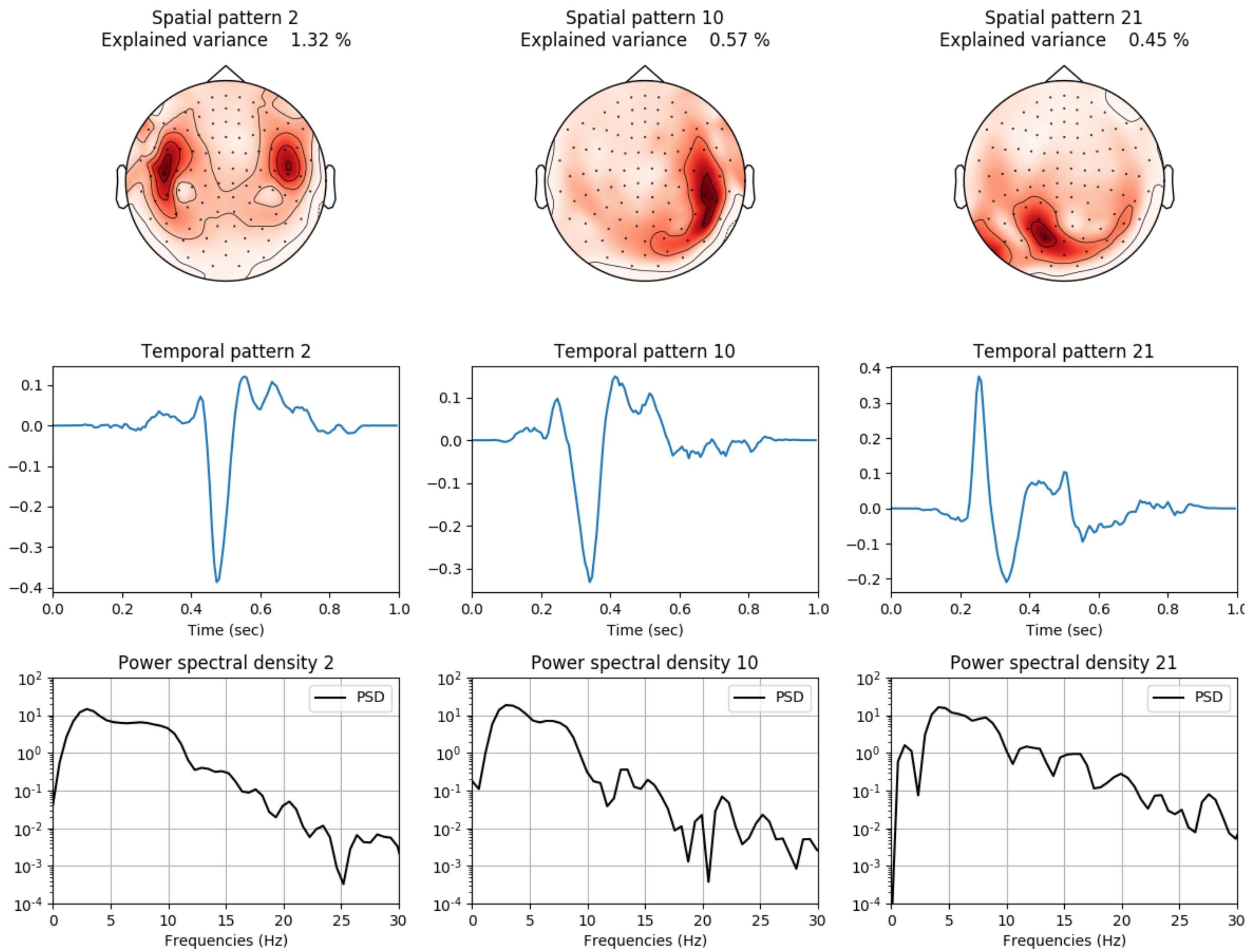


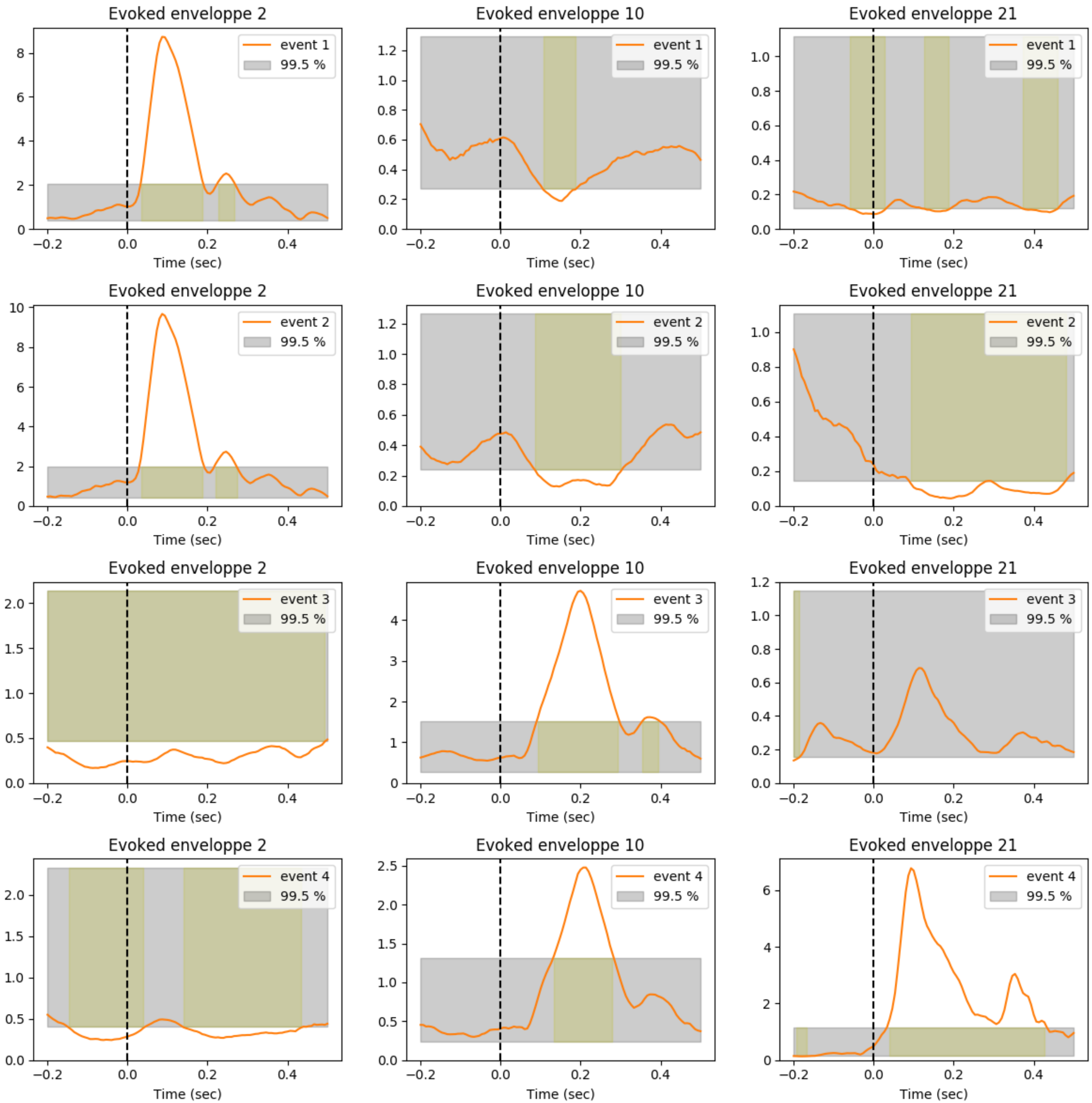
Simulations on multivariate CSC





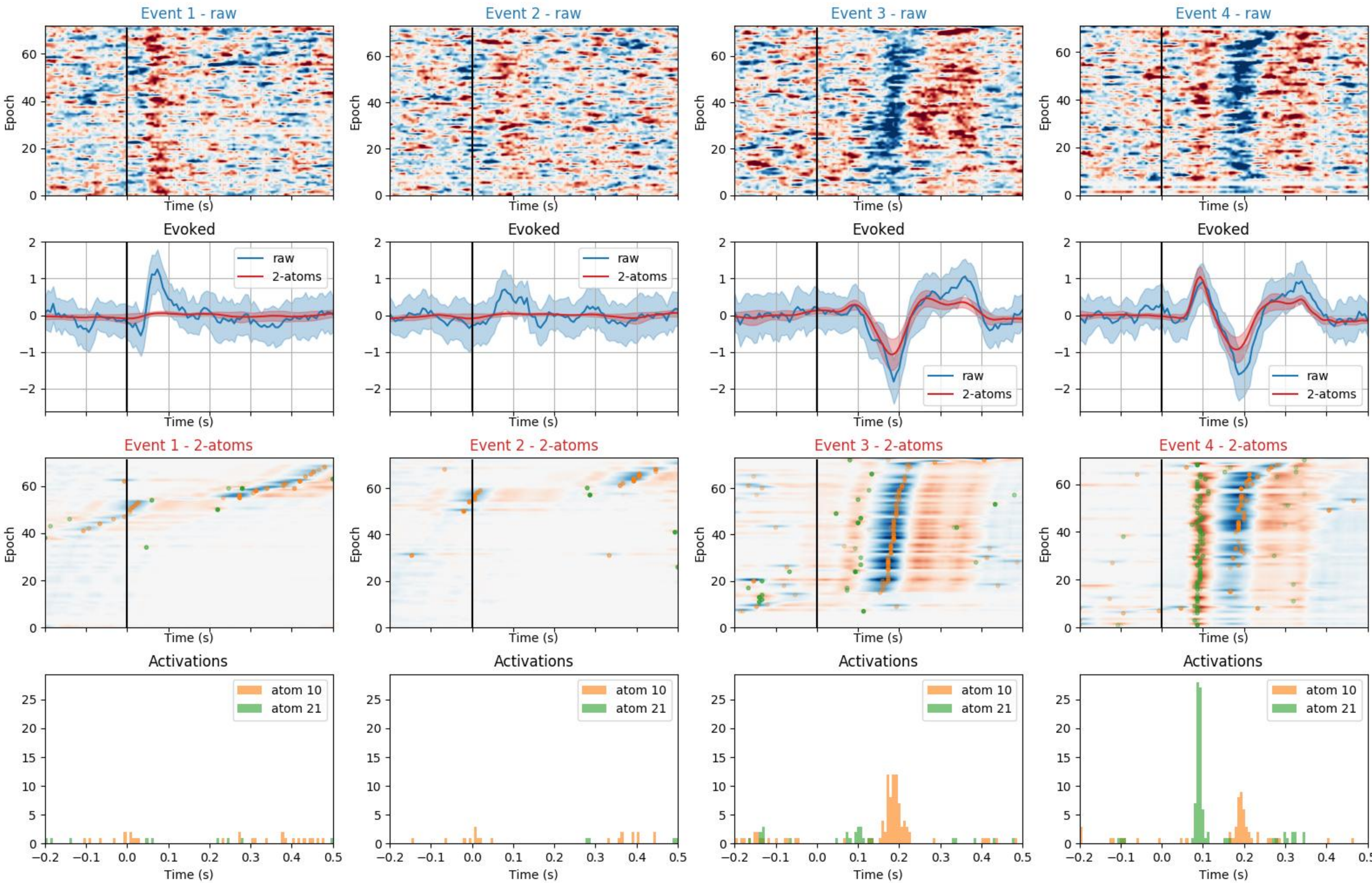
Event-related atoms



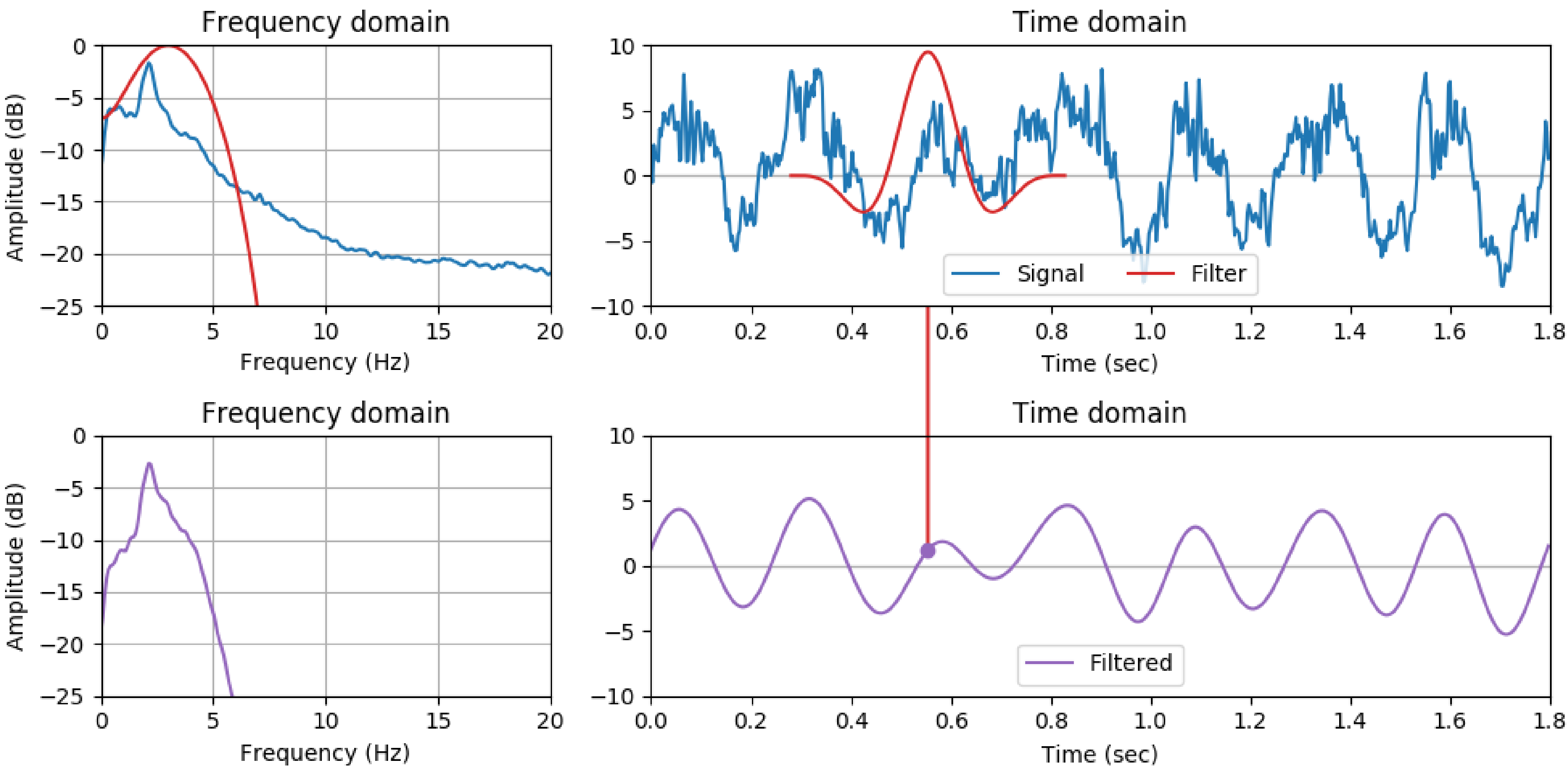


Evoked enveloppe 2
event 1
99.5 %
Time (sec)
Evoked enveloppe 10
event 1
99.5 %
Time (sec)
Evoked enveloppe 21
event 1
99.5 %
Time (sec)
Evoked enveloppe 2
event 2
99.5 %
Time (sec)
Evoked enveloppe 10
event 2
99.5 %
Time (sec)
Evoked enveloppe 21
event 2
99.5 %
Time (sec)
Evoked enveloppe 2
event 3
99.5 %
Time (sec)
Evoked enveloppe 10
event 3
99.5 %
Time (sec)
Evoked enveloppe 21
event 3
99.5 %
Time (sec)
Evoked enveloppe 2
event 4
99.5 %
Time (sec)
Evoked enveloppe 10
event 4
99.5 %
Time (sec)
Evoked enveloppe 21
event 4
99.5 %
Time (sec)

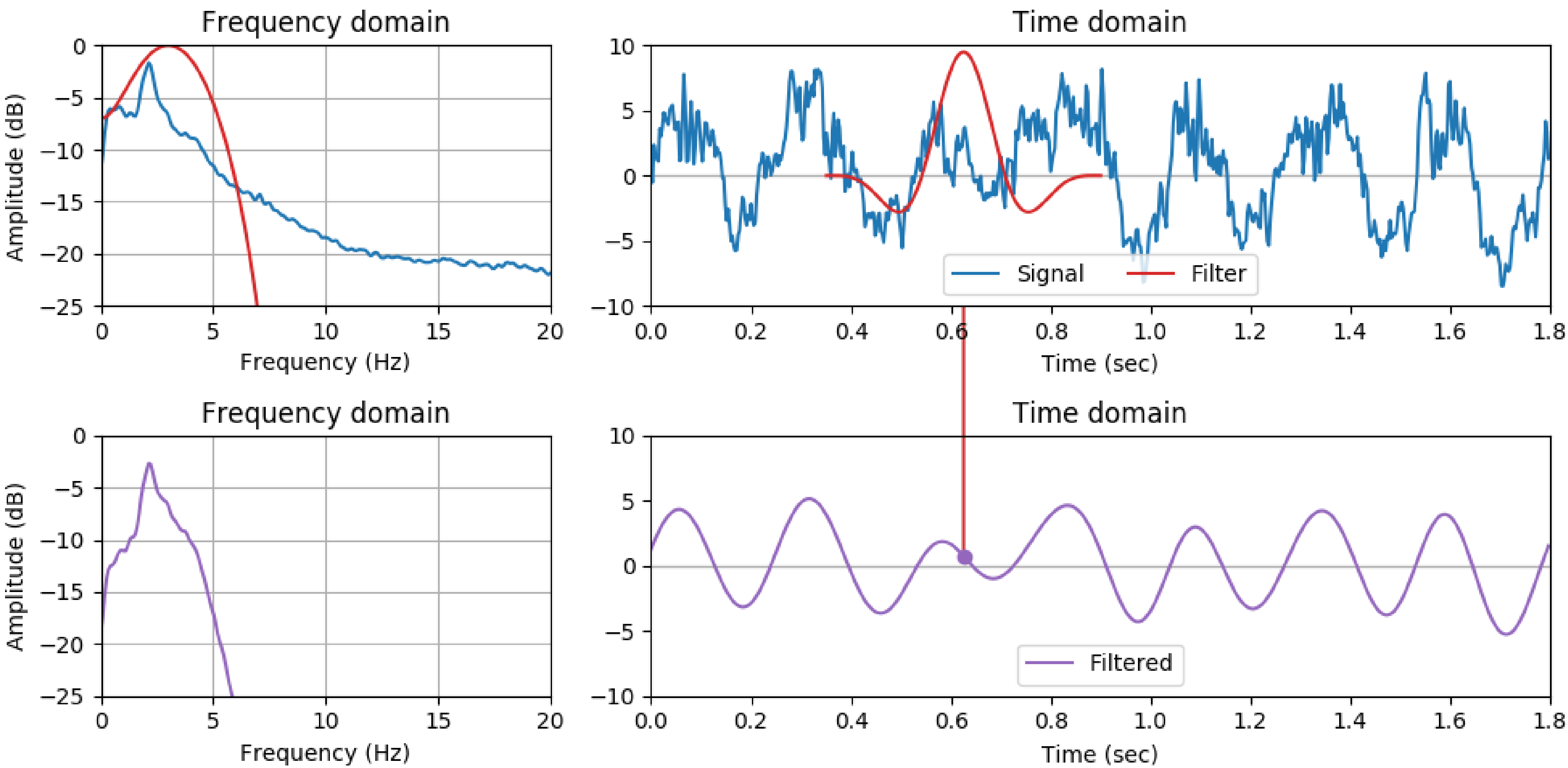
Event-related activations



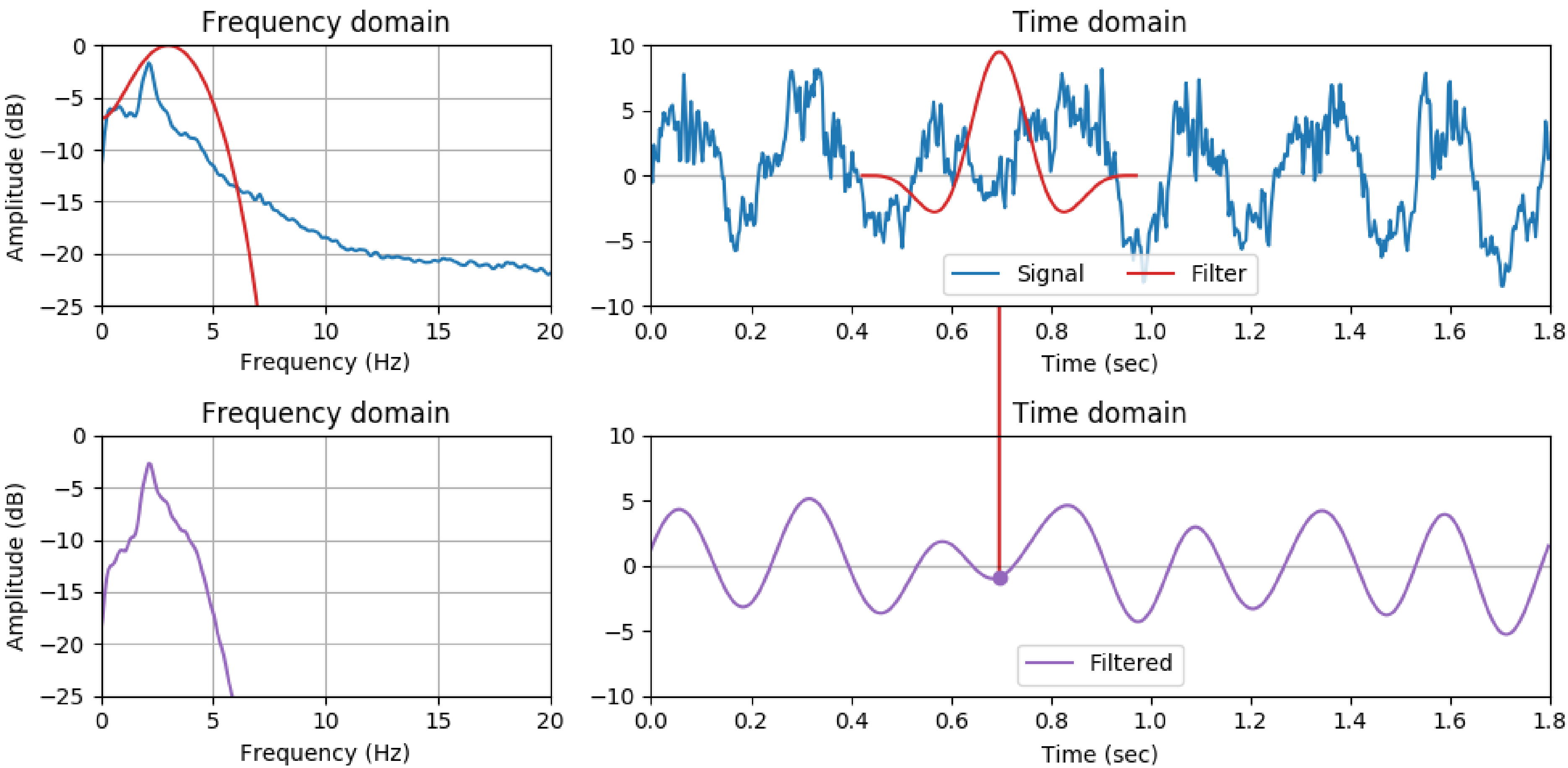
Narrow-band linear filtering



Narrow-band linear filtering



Narrow-band linear filtering



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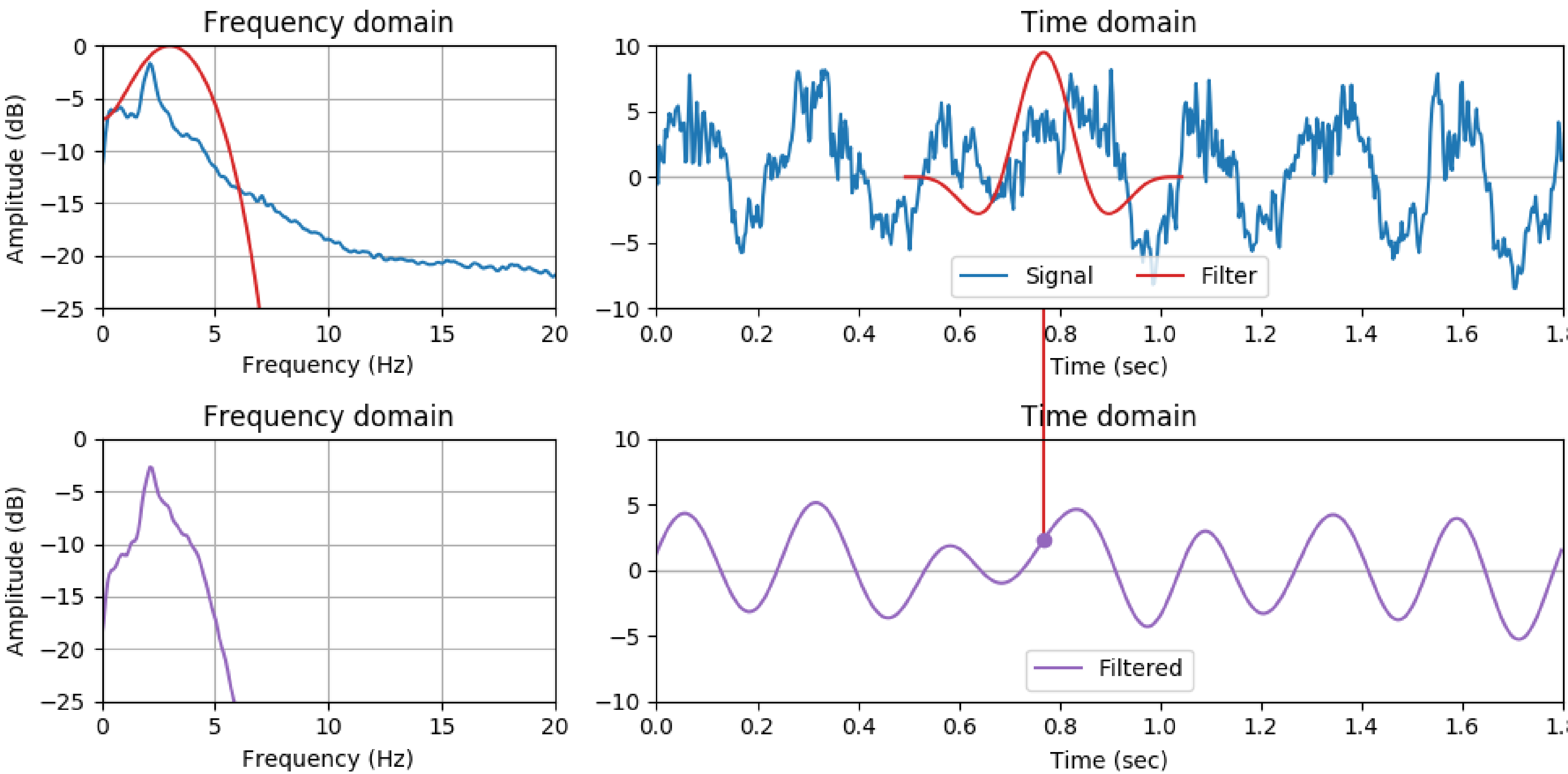
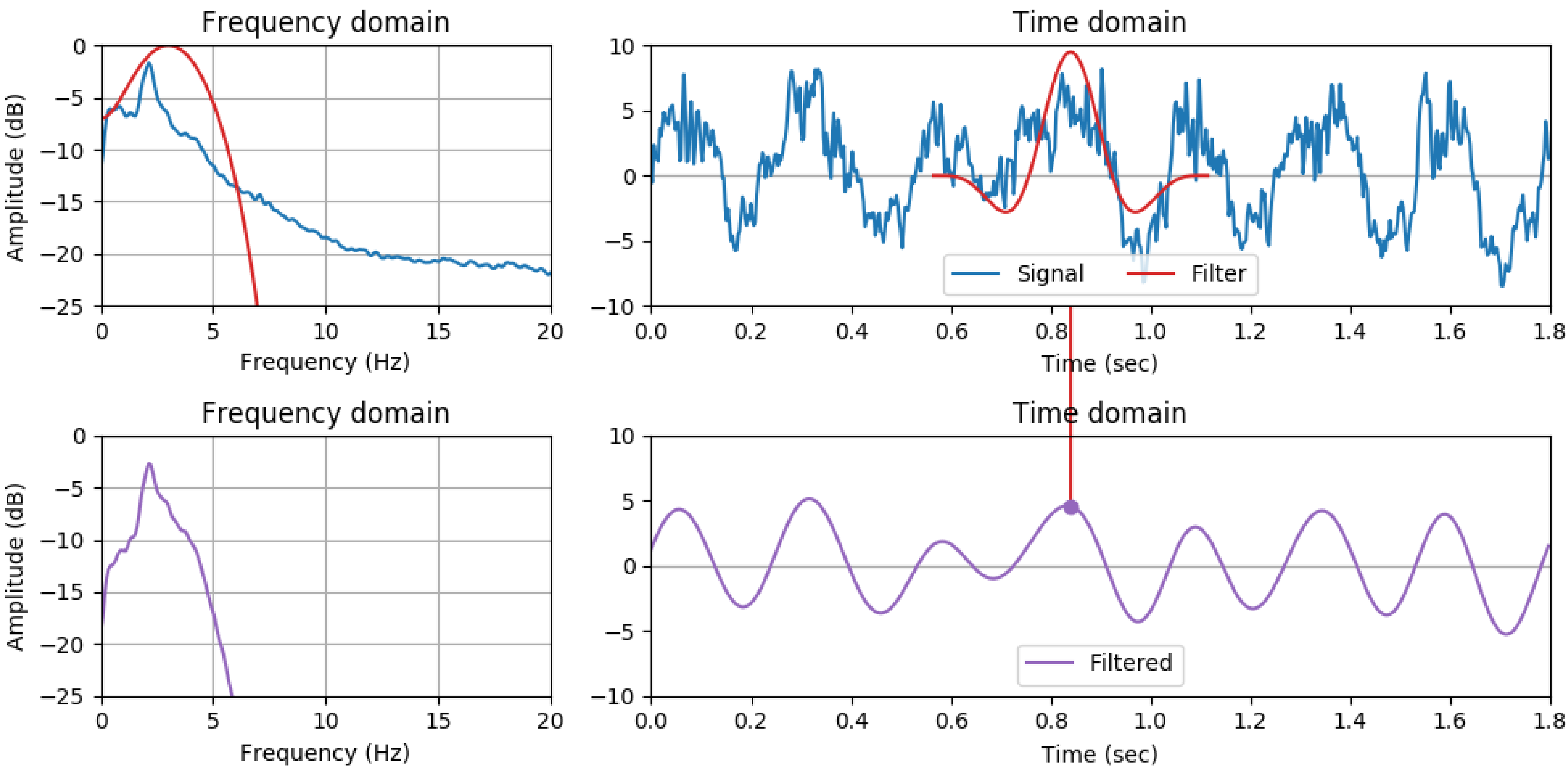
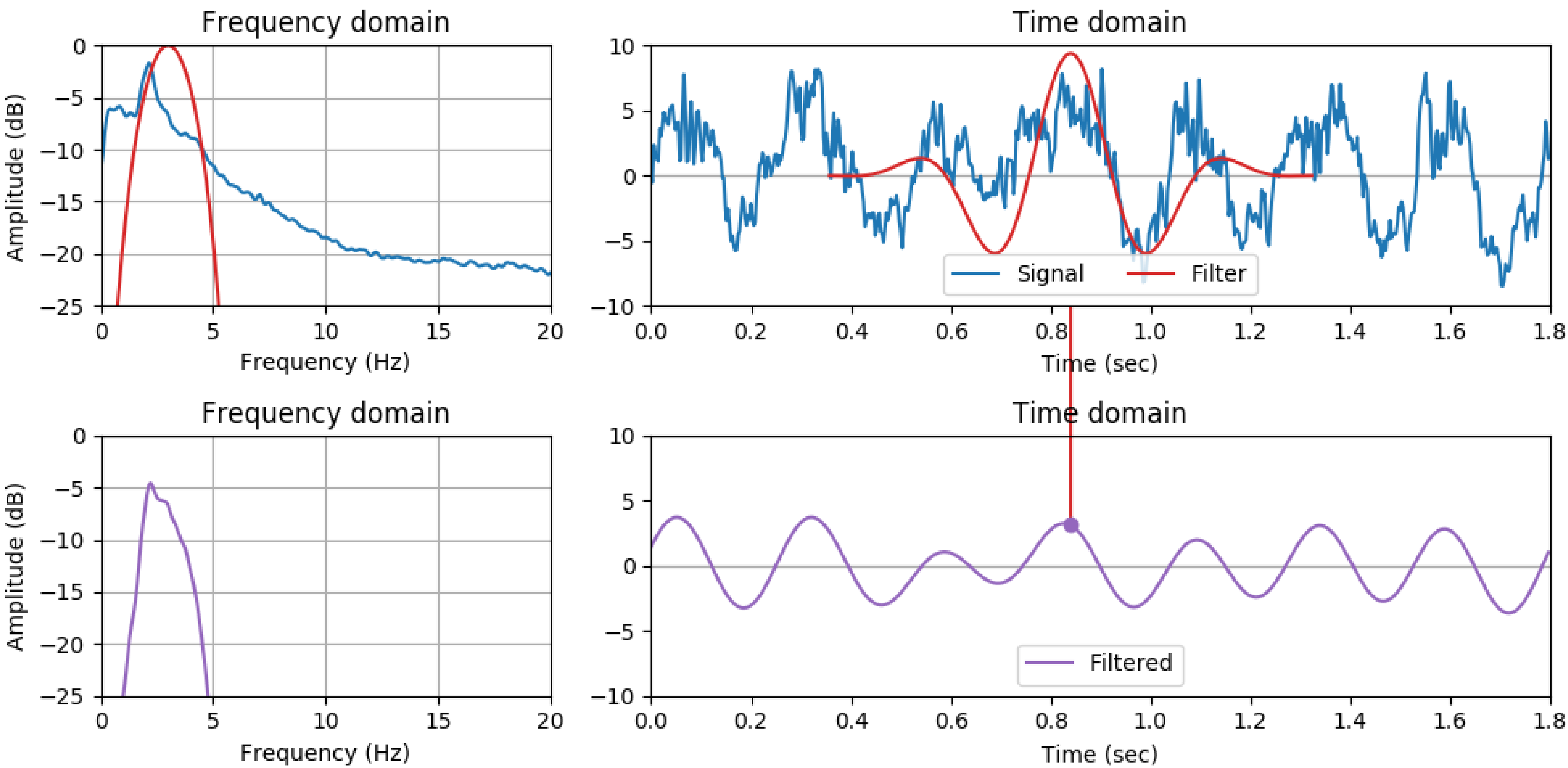


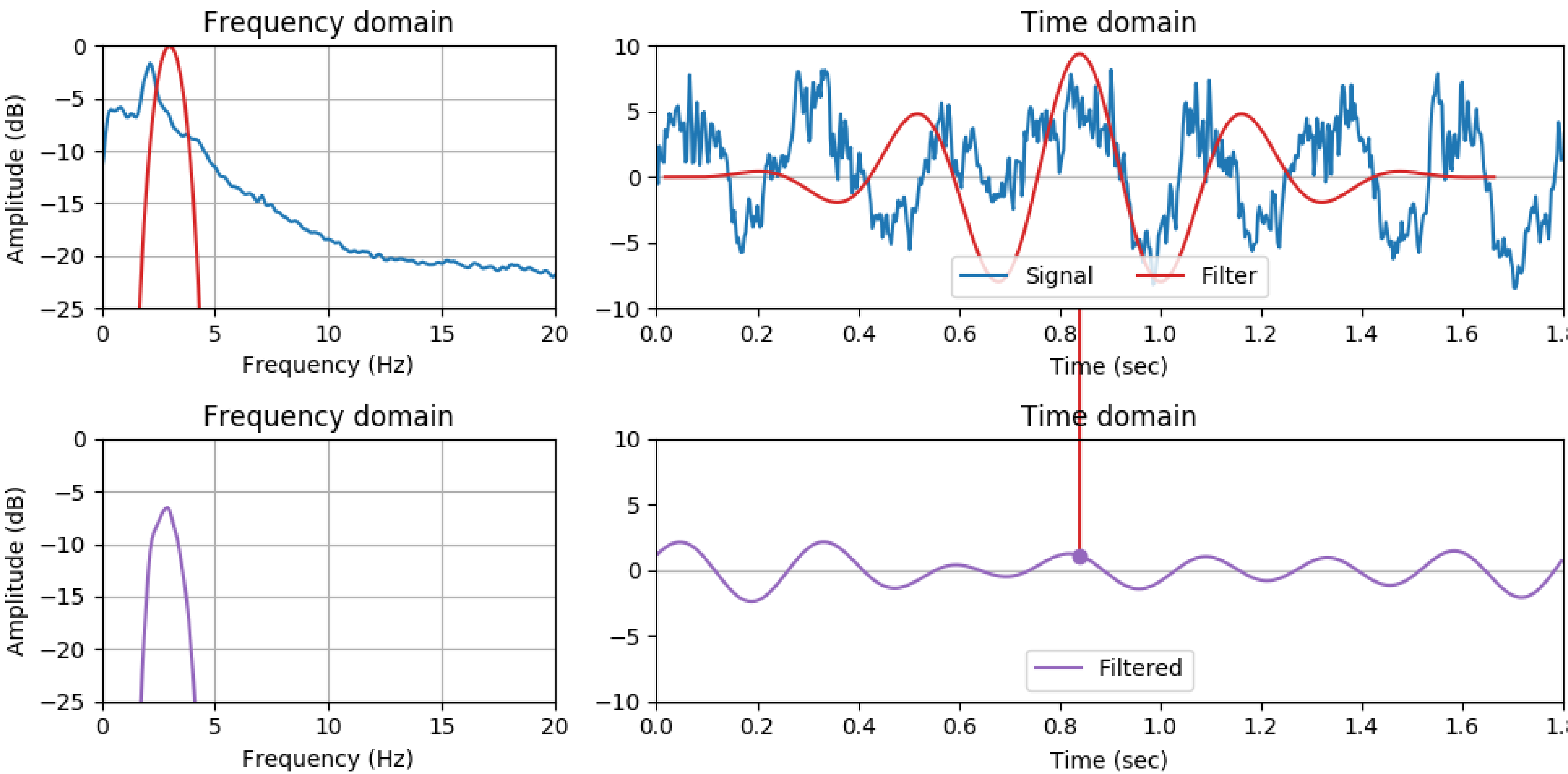
Narrow-band linear filtering



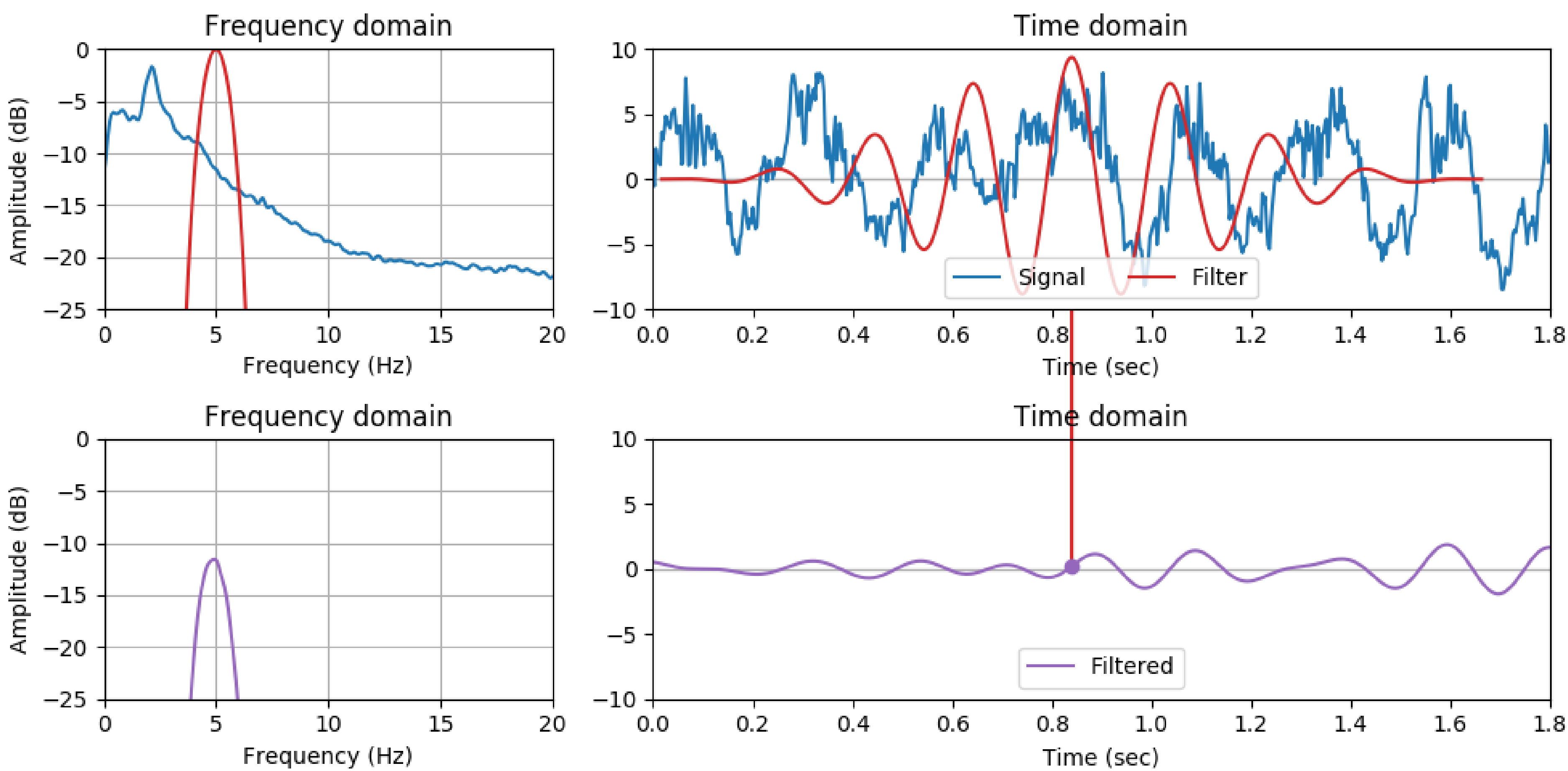
Narrow-band linear filtering



Narrow-band linear filtering



Narrow-band linear filtering



Narrow-band linear filtering