PARAMETRIC ESTIMATION OF SPECTRUM DRIVEN BY AN EXOGENOUS SIGNAL

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Abstract

- In this paper, we introduce new parametric and generative *driven auto-regressive* (DAR) models. DAR models provide a non-linear and non-stationary spectral estimation of a signal, conditionally to another *exogenous* signal.
- We detail how inference can be done efficiently while guaranteeing model stability. We show how model comparison and hyper-parameter selection can be done using likelihood estimates. We also point out the limits of DAR models when the exogenous signal contains too high frequencies.
- Finally, we illustrate how DAR models can be applied on neuro-physiologic signals to characterize *phase-amplitude coupling*.

Model selection

- Model likelihood

$$L = \prod_{t=p+1}^{T} \frac{1}{\sqrt{2\pi\sigma(t)^2}} \exp\left(-\frac{\varepsilon(t)^2}{2\sigma(t)^2}\right)$$

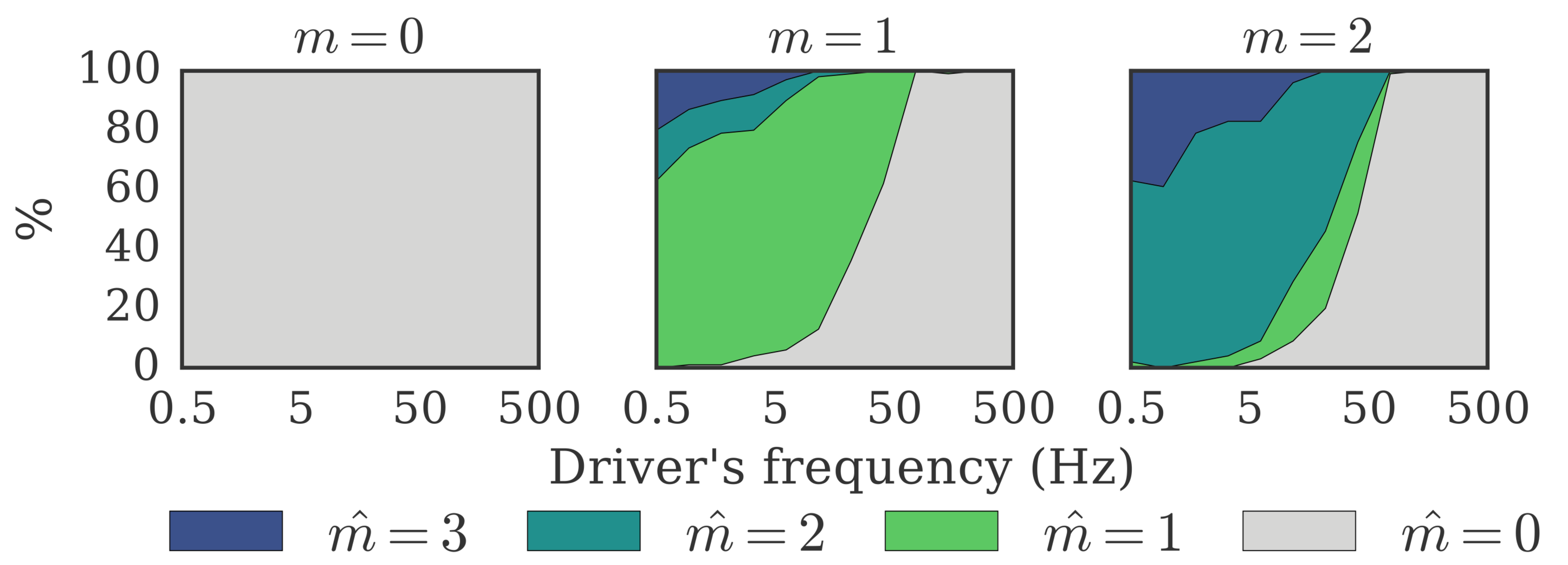
- Bayesian information criterion (BIC)

$$BIC = -2\log(L) + d\log(T)$$

- Degrees of freedom

$$d = (p+1)(m+1)$$

- Simulations: we create DAR model, synthetize a driver and a signal, and try to recover the model order with BIC selection.



BIC selection works well if the driver is not too fast

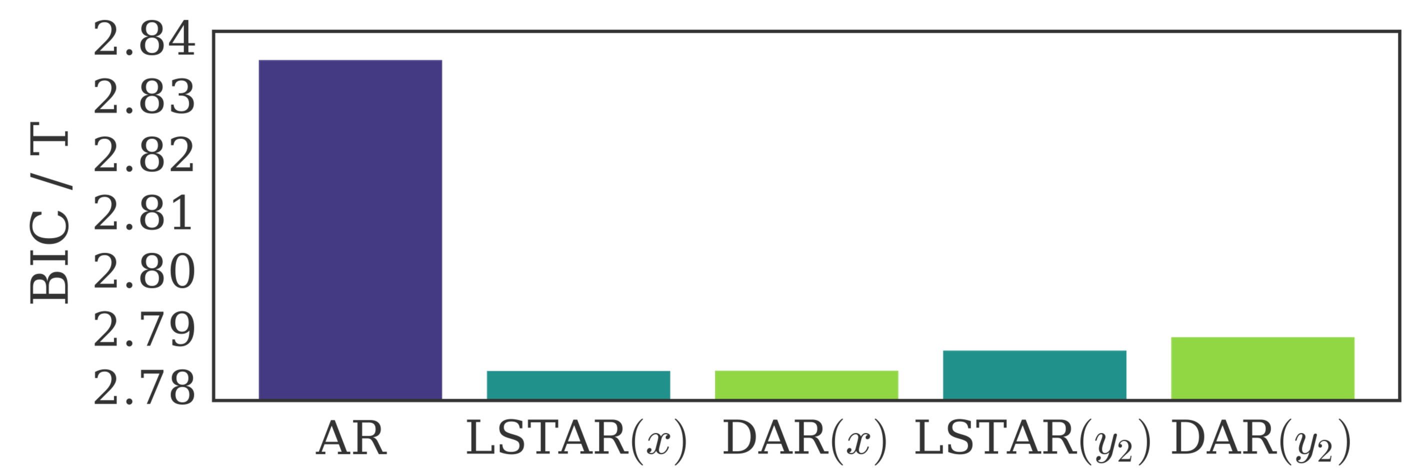
Comparison with LSTAR

- Logistic smooth-transition AR

$$F_j(x(t)) = (1 + e^{-\gamma_j(x(t)-c_j)})^{-1} \qquad a_i(t) = \sum_{j=0}^{m} a_{ij} F_j(x(t))$$

- For fair comparison we added

$$\log(\sigma(t)) = \sum_{j=0}^{m} b_j F_j(x(t))$$



BIC comparison on an electro-physiologic signal

Driven Auto-Regressive (DAR) models

- AR model

$$y(t) + \sum_{i=1}^{p} a_i y(t-i) = \varepsilon(t)$$

- Driven AR model

$$\boxed{a_i(t) = \sum_{j=0}^{m} a_{ij} x(t)^j} \qquad \boxed{\log(\sigma(t)) = \sum_{j=0}^{m} b_j x(t)^j}$$

To guarantee local stability, we use :

- Lattice parameterization

$$a_p^{(p)} = k_p; \qquad \forall i \in [1, p-1],\ a_i^{(p)} = a_i^{(p-1)} + k_p a_{p-i}^{(p-1)}$$

- Local stability criterion

$$-1 < k_i < 1$$

- Log area ratio

$$\gamma_i = \log\left(\frac{1+k_i}{1-k_i}\right) \iff k_i = \frac{e^{\gamma_i}-1}{e^{\gamma_i}+1}$$

- Locally stable Driven AR model

$$\gamma_i(t) = \sum_{j=0}^{m} \gamma_{ij} x(t)^j$$

$$\boxed{\gamma_{ij}} \rightarrow \boxed{\gamma_i(t)} \rightarrow \boxed{k_i(t)} \rightarrow \boxed{a_i(t)}$$

$$\boxed{x(t)}$$

$$\boxed{b_j} \rightarrow \boxed{\sigma(t)}$$

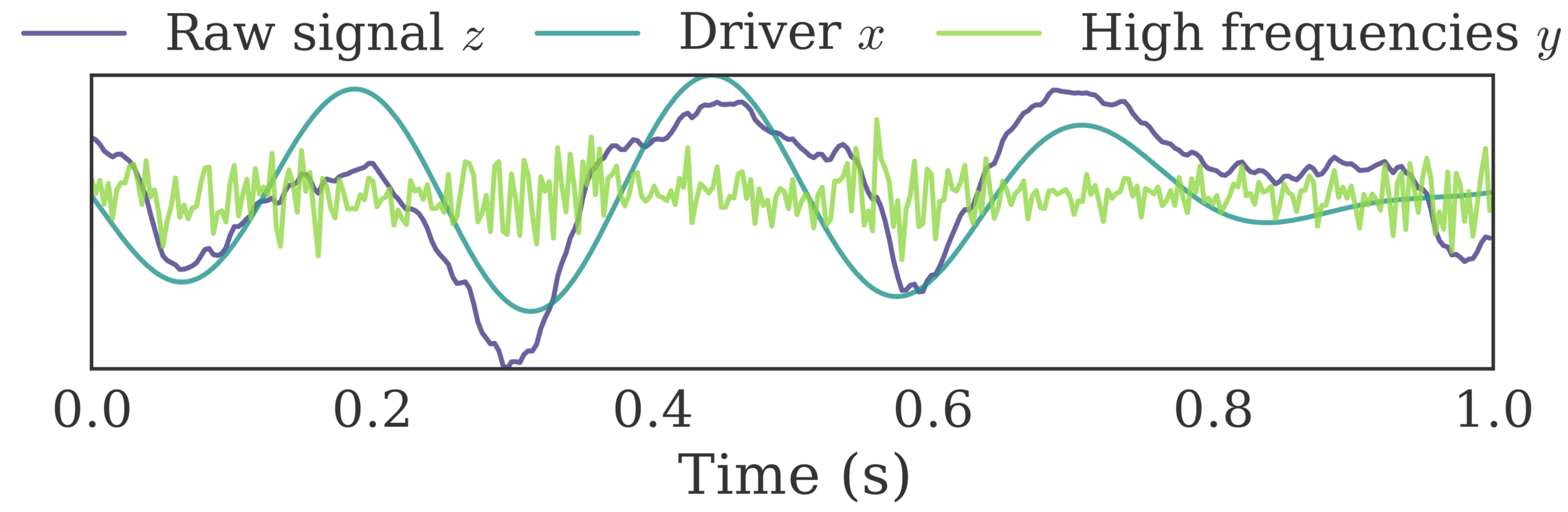
- Power spectral density (PSD) conditionally to the driver x

$$\boxed{S_y(x_0)(f) = \left|\sum_{i=0}^{p} \frac{a_i(x_0)}{\sigma(x_0)} e^{-j2\pi f i}\right|^{-2}}$$

Application to neuroscience

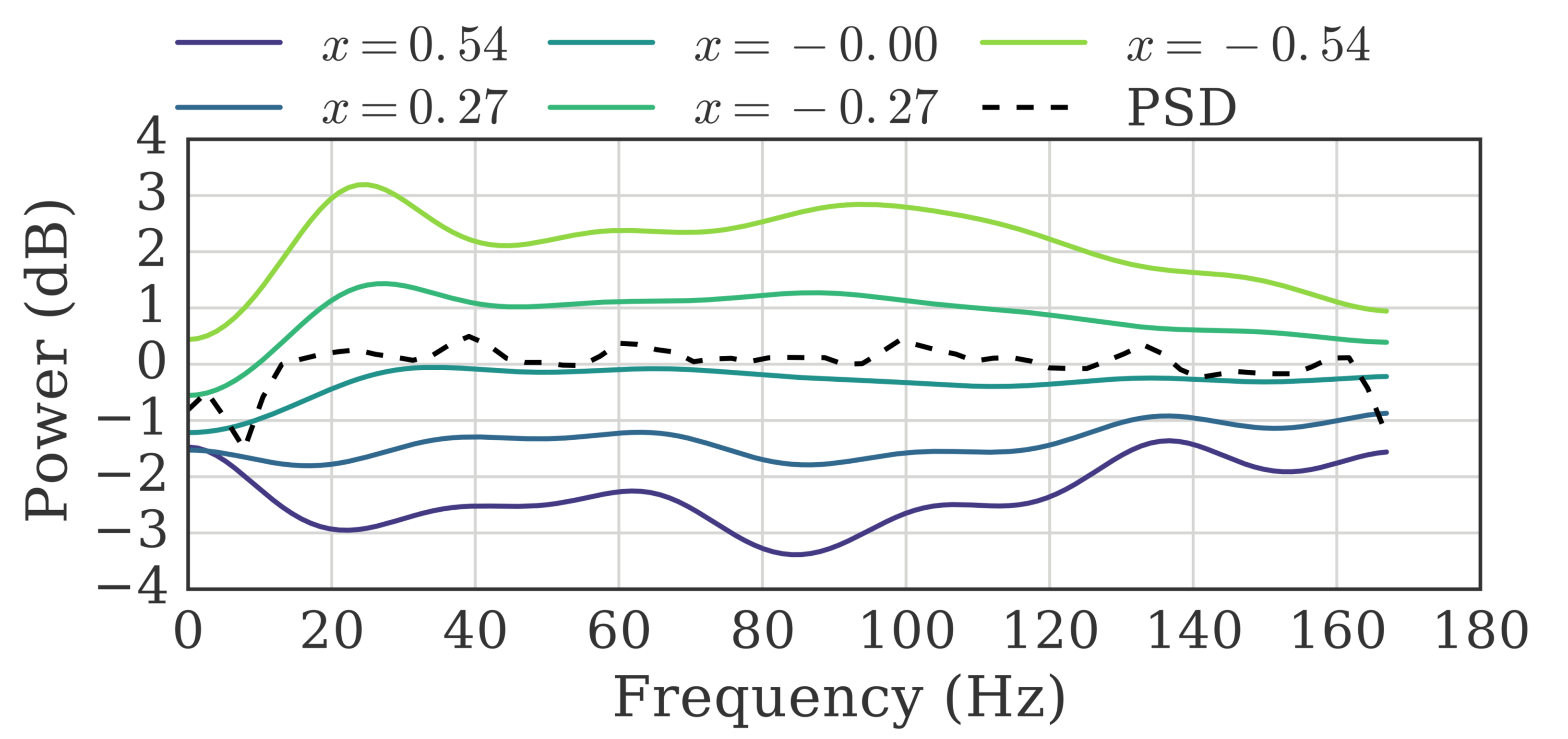
In neuroscience, *phase-amplitude coupling* (PAC) refers to the interaction between:

- The phase of a slow neural oscillation x
- The amplitude of high frequencies y



Portion of a human electro-corticogram (ECoG) channel

We band-pass filter the driver x from the signal, and apply DAR models on the high frequencies y, to estimate the PAC.



Power spectral density (PSD) conditionally to the driver x

References

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