First try at LaTeX

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1 First Problem

text text text

$$\nabla \cdot \left(\frac{1}{\rho} \nabla P \right) + \frac{\omega^2}{B} P = 0$$

as ρ is a constant it can be taken out of the gradient and multiplied through the equation.

$$\nabla^2 P + k^2 p = 0$$

$$i.e. \qquad \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + k^2 p = 0$$

If we assume a solution of the form $P(x, y) = X(x)Y(y)$, substituting in to the equation gives:

$$X''Y + XY'' + k^2 XY = 0$$

$$\frac{X''}{X} + \frac{Y''}{Y} + k^2 = 0$$

$$\frac{X''}{X} + k^2 = -\frac{Y''}{Y} = \alpha^2$$

$$\text{so} \qquad Y'' + \alpha^2 Y = 0$$

so Y takes the form $Y = Ae^{icy} \implies Y' = Aice^{icy} \implies Y'' = -Ac^2 e^{icy}$

$$\text{so} \qquad -Ac^2 e^{icy} + \alpha^2 A e^{icy} = 0$$

$$c^2 = \alpha^2$$

$$c = \pm \alpha$$

2 giving it another go ...

text text text

$$\nabla \cdot \left(\frac{1}{\rho} \nabla P \right) + \frac{\omega^2}{B} P \;=\; 0$$

as ρ is a constant it can be taken out of the gradient and multiplied through the equation.

$$\begin{aligned} \nabla^2 P + k^2 p &= 0 \\ \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + k^2 p &= 0 \end{aligned}$$

If we assume a solution of the form $P(x,y) = X(x)Y(y)$, substituting in to the equation gives:

$$\begin{aligned} X''Y + XY'' + k^2XY &= 0 \\ \frac{X''}{X} + \frac{Y''}{Y} + k^2 &= 0 \\ \frac{X''}{X} + k^2 &= -\frac{Y''}{Y} = \alpha^2 \\ \text{so} \qquad Y'' + \alpha^2 Y &= 0 \end{aligned}$$

so Y takes the form $Y = Ae^{icy} \implies Y' = Aice^{icy} \implies Y'' = -Ac^2e^{icy}$

$$\begin{aligned} \text{so} \qquad -Ac^2e^{icy} + \alpha^2 Ae^{icy} &= 0 \\ c^2 &= \alpha^2 \\ c &= \pm\alpha \\ \therefore \qquad Y &= A_1e^{i\alpha y} + A_2e^{-i\alpha y} \end{aligned}$$

With Neumann boundary conditions: $P_x(x,0) = P_x(x,L) = 0 \quad i.e. \quad X'(x)Y(0) = X'(x)Y(L) = 0$
As X'(x) is not necessarily zero for all x this implies $Y(0) = Y(L) = 0$

$$\begin{aligned} \text{so} \qquad Y(0) = A_1e^{i\alpha 0} + A_2e^{-i\alpha 0} &= Y(L) = A_1e^{i\alpha L} + A_2e^{-i\alpha L} = 0 \\ A_1 + A_2 &= 0 \\ \implies \qquad A_1 &= -A_2 \quad (set A_1 = A) \\ \text{then} \qquad Ae^{i\alpha L} - Ae^{-i\alpha L} &= 0 \\ Ae^{i\alpha L} &= Ae^{-i\alpha L} \\ \alpha &= -\alpha \\ \alpha &= 0 \end{aligned}$$

Therefore $Y = A_1 + A_2 = C \quad \forall y \in [0,L]$
Which implies $P(x,y) = P(x)$