

## Aperiodic History: Annotated Bibliography

### Analyses of the Aperiodic Exponent

*This annotated bibliography include empirical analyses and methods for spectral analyses of aperiodic (1/f or 1/f-like) activity in neural field data (M/EEG, iEEG, LFP). Reports are listed chronologically.*

Motokawa, K. (1949). Energy of Brain Waves and Energetics of the Brain. *The Tohoku Journal of Experimental Medicine*, 51(1–2), 119–129. <https://doi.org/10.1620/tjem.51.119>

This paper examines patterns of energy in brain activity (motivated by a discussion of metabolism), and in doing so appears to be the first to demonstrate something akin to a 1/f distribution of power in EEG recordings, reporting an “exponential law of energy distribution in eeg”.

Kingma, Y. J., Pronk, C. N. A., & Sparreboom, D. (1976). Parameter estimation of power spectra using Gaussian functions. *Computers and Biomedical Research*, 9(6), 591–599. [https://doi.org/10.1016/0010-4809\(76\)90018-5](https://doi.org/10.1016/0010-4809(76)90018-5)

One of the earliest approaches for fitting the power spectrum, the proposed method is focuses on fitting Gaussians (as oscillations), but also includes the fitting of an exponential function to capture the overall pattern of decreasing power over increasing frequencies.

Dumermuth, G., Gasser, T., Germann, P., Hecker, A., Herdan, M., & Lange, B. (1977). Studies on EEG Activities in the Beta Band. *European Neurology*, 16(1–6), 197–202. <https://doi.org/10.1159/000114900>

With the goal of studying beta activity, introduces an analysis approach that measures the spectral baseline (effectively, the aperiodic exponent), and measuring peak activity above this.

Note – a similar / overlapping project is reported in the following report (in French):

Dumermuth, G., Lange, B., & Herdan, M. (1983). Analyse spectrale de l'activite EEG rapide (beta). *Revue d'électroencéphalographie et de neurophysiologie clinique*, 13(2), 122–127. [https://doi.org/10.1016/S0370-4475\(83\)80070-7](https://doi.org/10.1016/S0370-4475(83)80070-7)

Matthis, P., Scheffner, D., & Benninger, C. (1981). Spectral Analysis of the EEG: Comparison of Various Spectral Parameters. *Electroencephalography and Clinical Neurophysiology*, 52(2), 218–221. [https://doi.org/10.1016/0013-4694\(81\)90171-1](https://doi.org/10.1016/0013-4694(81)90171-1)

A brief overview and comparison of different spectral parameters, with an example application to clinical EEG (brain tumor), including the discussion and application of the aperiodic exponent.

Feinberg, I., March, J. D., Floyd, T. C., Fein, G., & Aminoff, M. J. (1984). Log amplitude is a linear function of log frequency in NREM sleep EEG of young and elderly normal subjects. *Electroencephalography and Clinical Neurophysiology*, 58(2), 158–160. [https://doi.org/10.1016/0013-4694\(84\)90029-4](https://doi.org/10.1016/0013-4694(84)90029-4)

One of the earliest studies of the pattern of power across all frequencies, this paper analyzed the log(frequency) vs. log(power) relationship in sleep data, noting also that there is a difference across age groups (perhaps the earliest reported age-related finding).

Dumermuth, G., & Molinari, L. (1987). Spectral Analysis of the EEG. *Neuropsychobiology*, 17, 85–99. <https://doi.org/10.1159/000118345>

Reports on a tutorial on spectral analysis of EEG data, including discussing spectral parameters – including a ‘pink noise’ (1/f) component, and peaks. Discusses measuring the aperiodic exponent.

Freeman, W. J., & van Dijk, B. W. (1987). Spatial patterns of visual cortical fast EEG during conditioned reflex in a rhesus monkey. *Brain Research*, 422(2), 267–276. [https://doi.org/10.1016/0006-8993\(87\)90933-4](https://doi.org/10.1016/0006-8993(87)90933-4)

Examining patterns of visual cortical activity in Rhesus monkeys, and note that the power spectra resemble a 1/f pattern with peaks in the 20-40 Hz range.

Pascual-Marqui, R. D., Valdes-Sosa, P. A., & Alvarez-Amador, A. (1988). A Parametric Model for Multichannel EEG Spectra. *International Journal of Neuroscience*, 40(1–2), 89–99. <https://doi.org/10.3109/00207458808985730>

This paper reflects early methodological work on measuring different components in the spectral domain, proposing the Xi-Alpha model, both fit with t-distributions whereby Xi reflects a pattern of power across all frequencies and Alpha reflects the dominant alpha oscillation.

Note: some follow up papers apply this method, and discuss the Xi parameter, e.g.:

Amador, A. A., Valdés Sosa, P. A., Pascual Marqui, R. D., Garcia, L. G., Lirio, R. B., & Bayard, J. B. (1989). On the structure of EEG development. *Electroencephalography and Clinical Neurophysiology*, 73(1), 10–19. [https://doi.org/10.1016/0013-4694\(89\)90015-1](https://doi.org/10.1016/0013-4694(89)90015-1)

Nakata, M., & Mukawa, K. (1989). Fourier analysis of broad spectral EEG from a fluctuation point of view. *The Pavlovian Journal of Biological Science*, 24(3), 90–97. <https://doi.org/10.1007/BF02701923>

This paper examines 1/f and 1/f-like patterns of activity in LFP, examining the frequency range across which such properties hold. They report different patterns of aperiodic activity when comparing cortical and sub-cortical regions, that go to very high frequency ranges (~3 kHz).

Pritchard, W. S. (1992). The Brain in Fractal Time: 1/F-Like Power Spectrum Scaling of the Human Electroencephalogram. *International Journal of Neuroscience*, 66(1–2), 119–129. <https://doi.org/10.3109/00207459208999796>

This paper explicitly examines 1/f scaling of EEG power spectra, reporting it in eyes-open and eyes-closed conditions, as well as differences across channels.

Nakata, M., Mukawa, J., & Fromm, G. H. (1993). Evaluation of human consciousness level by means of "automated fluctuation analysis" of high frequency electroencephalogram fitted by double Lorentzians. *Integrative Physiological and Behavioral Science*, 28(4), 343–352. <https://doi.org/10.1007/BF02690931>

Analyze EEG recordings across a broad frequency range, fitting Lorentzians (1/k with a knee) and note differences in model form and fit parameters between wake and sleep.

Inouye, T., Matsumoto, Y., Shinosaki, K., Iyama, A., & Toi, S. (1994). Increases in the power spectral slope of background electroencephalogram just prior to asymmetric spike and wave complexes in epileptic patients. *Neuroscience Letters*, 173, 197–200. [https://doi.org/10.1016/0304-3940\(94\)90182-1](https://doi.org/10.1016/0304-3940(94)90182-1)

This paper is one of the earliest analyses of aperiodic neural activity in a clinical context, showing that aperiodic activity systematically related to epileptiform activity.

Barrie, J. M., Freeman, W. J., & Lenhart, M. D. (1996). Spatiotemporal analysis of prepyriform, visual, auditory, and somesthetic surface EEGs in trained rabbits. *Journal of Neurophysiology*, 76(1), 520–539. <https://doi.org/10.1152/jn.1996.76.1.520>

Examine ECoG grids across the cortex of rabbits, fit 1/f parameters, and compare pre / post stimulus. Report a stimulus-related change in aperiodic parameters.

Novikov, E., Novikov, A., Shannahoff-Khalsa, D., Schwartz, B., & Wright, J. (1997). Scale-similar activity in the brain. *Physical Review E*, 56(3), R2387–R2389. <https://doi.org/10.1103/PhysRevE.56.R2387>

Fits aperiodic exponent to power spectra from 2 MEG subjects. Motivates and discusses the approach and findings in relation to scale-similar activity.

Chang, H.-J., Freeman, W. J., & Burke, B. C. (1998). Optimization of olfactory model in software to give 1/f power spectra reveals numerical instabilities in solutions governed by aperiodic (chaotic) attractors. *Neural Networks*, 11(3), 449–466. [https://doi.org/10.1016/S0893-6080\(97\)00116-0](https://doi.org/10.1016/S0893-6080(97)00116-0)

Describes a model of the olfactory system, which is optimized to match the 1/f property of empirical recordings, and does so. Suggest the model settles into a strange attractor.

Chen, Z., Tretyakov, A., Takayasu, H., & Nakasato, N. (1998). Spectral Analysis of Multichannel Meg Data. *Fractals*, 06(04), 395–400. <https://doi.org/10.1142/S0218348X98000432>

Fits aperiodic exponent to power spectra from 1 MEG subject. Motivates the analysis in relation to fractal properties. Notes that alpha activity appears to be independent of 1/f activity.

Pereda, E., Gamundi, A., Rial, R., & González, J. (1998). Non-linear behaviour of human EEG: Fractal exponent versus correlation dimension in awake and sleep stages. *Neuroscience Letters*, 250(2), 91–94. [https://doi.org/10.1016/S0304-3940\(98\)00435-2](https://doi.org/10.1016/S0304-3940(98)00435-2)

Compute aperiodic exponent (and other measures) in EEG data, reporting a fractal structure to EEG data (exponent values from 1-3), and changes in the exponent between wake & sleep.

González, J., Gamundi, A., Rial, R., Nicolau, M. C., De Vera, L., & Pereda, E. (1999). Nonlinear, fractal, and spectral analysis of the EEG of lizard, *Gallotia galloti*. *American Journal of Physiology-Regulatory, Integrative and Comparative Physiology*, 277(1), R86–R93.  
<https://doi.org/10.1152/ajpregu.1999.277.1.R86>

Examine EEG from lizard, to test for the presence of fractal / nonlinear structure. Report an aperiodic exponent of ~2 in the lizard brain, that flattens with increased temperature.

Freeman, W. J., Rogers, L. J., Holmes, M. D., & Silbergeld, D. L. (2000). Spatial spectral analysis of human electrocorticograms including the alpha and gamma bands. *Journal of Neuroscience Methods*, 95(2), 111–121. [https://doi.org/10.1016/S0165-0270\(99\)00160-0](https://doi.org/10.1016/S0165-0270(99)00160-0)

Examines scaling properties in intracranial human recordings, including reporting on the aperiodic exponent, including in both awake and anesthetized patients.

Freeman, W. J., & Barrie, J. M. (2000). Analysis of Spatial Patterns of Phase in Neocortical Gamma EEGs in Rabbit. *Journal of Neurophysiology*, 84(3), 1266–1278.  
<https://doi.org/10.1152/jn.2000.84.3.1266>

Examining scaling properties in animal model recordings, including reporting on the aperiodic exponent, noting 1/f properties across the cortex.

Caplan, J. B., Madsen, J. R., Raghavachari, S., & Kahana, M. J. (2001). Distinct patterns of brain oscillations underlie two basic parameters of human maze learning. *Journal of Neurophysiology*, 86(1), 368–380. <https://doi.org/10.1152/jn.2001.86.1.368>

With the goal of measure oscillations, this paper includes a method to fit a 1/f function to neural power spectrum to learn a threshold for peak detection.

Note – later reports further expand upon the method (BOSC - better oscillation detector):

Whitten, T. A., Hughes, A. M., Dickson, C. T., & Caplan, J. B. (2011). A better oscillation detection method robustly extracts EEG rhythms across brain state changes: The human alpha rhythm as a test case. *NeuroImage*, 54(2), 860–874.  
<https://doi.org/10.1016/j.neuroimage.2010.08.064>

Hughes, A. M., Whitten, T. A., Caplan, J. B., & Dickson, C. T. (2012). BOSC: A better oscillation detection method, extracts both sustained and transient rhythms from rat hippocampal recordings. *Hippocampus*, 22(6), 1417–1428.  
<https://doi.org/10.1002/hipo.20979>

Freeman, W. J., Holmes, M. D., Burke, B. C., & Vanhatalo, S. (2003). Spatial spectra of scalp EEG and EMG from awake humans. *Clinical Neurophysiology*, 114(6), 1053–1068.  
[https://doi.org/10.1016/S1388-2457\(03\)00045-2](https://doi.org/10.1016/S1388-2457(03)00045-2)

Examine scaling properties in EEG and EMG, including reporting aperiodic exponent values for EEG, which is reported to vary across scalp location (steeper frontal than occipital).

Polonnikov, R. I., Wasserman, E. L., & Kartashev, N. K. (2003). Regular Developmental Changes in EEG Multifractal Characteristics. *International Journal of Neuroscience*, 113(11), 1615–1639.  
<https://doi.org/10.1080/00207450390240086>

Look at age-related changes of the aperiodic exponent in EEG data, and find that the aperiodic exponent decreases, in both healthy and clinical (cerebral palsy) patients.

Popivanov, D., Janyan, A., Andonova, E., & Stamenov, M. (2003). *Common Dynamic Properties of Biosignals During Cognition: Self-Similarity and Chaotic Dynamics of Both Response Times and EEG During Movement Imagery*.

Examine nonlinear properties of behavioral and EEG measures during imagined movement, measuring aperiodic exponent. Discuss results in relation to chaos theory.

Bédard, C., Kröger, H., & Destexhe, A. (2006). Does the  $1/f$  Frequency Scaling of Brain Signals Reflect Self-Organized Critical States? *Physical Review Letters*, 97(11).  
<https://doi.org/10.1103/PhysRevLett.97.118102>

Examine  $1/f$  activity in neural recordings in relation to self-organized criticality. Report  $1/f$  properties in intracortical recordings, but argue they are not consistent with critical states.

Freeman, W. J., Holmes, M. D., West, G. A., & Vanhatalo, S. (2006). Fine spatiotemporal structure of phase in human intracranial EEG. *Clinical Neurophysiology*, 117(6), 1228–1243.  
<https://doi.org/10.1016/j.clinph.2006.03.012>

Report about ‘phase cones’ (spatial phase patterns). Also discusses  $1/f$  power spectra and reports a difference in aperiodic exponent between wake and sleep in human ECoG recordings.

Phothisonothai, M., & Nakagawa, M. (2007). Fractal-Based EEG Data Analysis of Body Parts Movement Imagery Tasks. *The Journal of Physiological Sciences*, 57(4), 217–226.  
<https://doi.org/10.2170/physiolsci.RP006307>

Analyze multiple fractal measures, including the aperiodic exponent, in EEG data during imagined movement, and evaluate that these methods characterize the data.

Shimono, M., Owaki, T., Amano, K., Kitajo, K., & Takeda, T. (2007). Functional modulation of power-law distribution in visual perception. *Physical Review E*, 75(5), 051902.  
<https://doi.org/10.1103/PhysRevE.75.051902>

Investigate aperiodic exponent in MEG data during a visual detection task, and report that aperiodic exponent differs between correct and incorrect trials.

Freeman, W. J., O’Nuallain, S., & Rodriguez, J. (2008). Simulating cortical background activity at rest with filtered noise. *Journal of Integrative Neuroscience*, 7(03), 337–344.  
<https://doi.org/10.1142/S0219635208001885>

Explore simulations in which the cumulative summation of random events gives rise to 1/f power spectra and explores this framework as it compares to empirical human ECoG data.

Krakovská, A., & Štolc, S. (2008). Spectral decay vs. Correlation dimension of EEG. *Neurocomputing*, 71(13–15), 2978–2985. <https://doi.org/10.1016/j.neucom.2007.06.007>

Examine the relationship between aperiodic exponent and correlation dimension in awake EEG data, finding that the two measures are highly correlated.

Šušmáková, K., & Krakovská, A. (2008). Discrimination ability of individual measures used in sleep stages classification. *Artificial Intelligence in Medicine*, 44(3), 261–277.  
<https://doi.org/10.1016/j.artmed.2008.07.005>

Apply multiple methods, including aperiodic exponent, to sleep EEG data, and compare between different sleep stages. Report that the aperiodic exponent differentiates between wake and sleep.

Aoki, R., Wake, H., Sasaki, H., & Agata, K. (2009). Recording and spectrum analysis of the planarian electroencephalogram. *Neuroscience*, 159(2), 908–914.  
<https://doi.org/10.1016/j.neuroscience.2008.11.011>

Examining neural recordings in planarian (flatworms) to investigate EEG patterns in a ‘primitive’ brain. Report that the power spectrum follows a 1/f patterns after 1 Hz.

Freeman, W. J., & Zhai, J. (2009). Simulated power spectral density (PSD) of background electrocorticogram (ECoG). *Cognitive Neurodynamics*, 3(1), 97–103.  
<https://doi.org/10.1007/s11571-008-9064-y>

Describe a model of the source of ‘background activity’ in LFP recordings, that generates 1/f data, and compare this to empirical human ECoG recordings. Suggest an approach examining non-oscillatory asynchronous, ‘scale-free’ activity that relates to changes in population firing.

Miller, K. J., Sorensen, L. B., Ojemann, J. G., & den Nijs, M. (2009). Power-Law Scaling in the Brain Surface Electric Potential. *PLoS Computational Biology*, 5(12), e1000609.  
<https://doi.org/10.1371/journal.pcbi.1000609>

Report powerlaw scaling in human ECoG recordings, consistent across subjects in the analyzed range of 80-500 Hz. Examines neural models that may explain these patterns.

Miller, K. J., Zanos, S., Fetz, E. E., den Nijs, M., & Ojemann, J. G. (2009). Decoupling the Cortical Power Spectrum Reveals Real-Time Representation of Individual Finger Movements in Humans. *Journal of Neuroscience*, 29(10), 3132–3137. <https://doi.org/10.1523/JNEUROSCI.5506-08.2009>

Introduce and apply a principal component analysis (PCA) based method to separate low-frequency rhythms from broadband activity, and apply this method to human ECoG data with a motor task. Applying the method find movement-specific broadband spectral motifs.

Dehghani, N., Bédard, C., Cash, S. S., Halgren, E., & Destexhe, A. (2010). Comparative power spectral analysis of simultaneous electroencephalographic and magnetoencephalographic recordings in humans suggests non-resistive extracellular media. *Journal of Computational Neuroscience*, 29(3), 405–421. <https://doi.org/10.1007/s10827-010-0263-2>

Compare the frequency scaling of simultaneous EEG & MEG recordings, to evaluate if results are consistent with a resistive medium. Find spatial variation of the aperiodic exponent (in both) and differences between modalities, which is interpreted as relating to a non-resistive media.

He, B. J., Zempel, J. M., Snyder, A. Z., & Raichle, M. E. (2010). The Temporal Structures and Functional Significance of Scale-free Brain Activity. *Neuron*, 66(3), 353–369. <https://doi.org/10.1016/j.neuron.2010.04.020>

Examine powerlaw properties in neural power spectra (human ECoG), and analyzes discusses nested frequencies, and variations of powerlaw properties with task modulation.

Valencia, M., López-Azcárate, J., Nicolás, M. J., Alegre, M., & Artieda, J. (2012). Dopaminergic modulation of the spectral characteristics in the rat brain oscillatory activity. *Chaos, Solitons & Fractals*, 45(5), 619–628. <https://doi.org/10.1016/j.chaos.2011.12.019>

Looking at the influence of dopaminergic drugs on the power law component of neural power spectra, in rats. Report that dopamine antagonists and agonists cause rotations of the aperiodic exponent around a fulcrum point of 20 Hz. Discuss results in relation to self-organized criticality.

Fransson, P., Metsäranta, M., Blennow, M., Åden, U., Lagercrantz, H., & Vanhatalo, S. (2013). Early Development of Spatial Patterns of Power-Law Frequency Scaling in fMRI Resting-State and EEG Data in the Newborn Brain. *Cerebral Cortex*, 23(3), 638–646. <https://doi.org/10.1093/cercor/bhs047>

Examine powerlaw scaling in both EEG and fMRI in newborns and adults, with a goal of analyzing spatial patterns. Report variation across spatial locations in EEG (and fMRI) powerlaw properties.

Janjarsjitt, S., & Loparo, K. A. (2013). Comparison of complexity measures using two complex system analysis methods applied to the epileptic ECoG. *Journal of the Korean Physical Society*, 63(8), 1659–1665. <https://doi.org/10.3938/jkps.63.1659>

Examines aperiodic exponent (and other measures) in human ECoG data from epilepsy patients, and reports changes in aperiodic exponent relates to seizure activity.

Note: there are additional related reports from Janjarsjitt & colleagues, including conference papers prior to this report, and subsequent further work on aperiodic activity in epilepsy.

Mareš, J., Vyšata, O., Procházka, A., & Vališ, M. (2013). Age-dependent complex noise fluctuations in the brain. *Physiological Measurement*, 34(10), 1269–1279. <https://doi.org/10.1088/0967-3334/34/10/1269>

Examines a large EEG dataset (n=17722) and reports a systematic decrease in exponent with increasing age. Discusses results in relation to self-organized criticality.

Miller, K. J., Honey, C. J., Hermes, D., Rao, R. P., den Nijs, M., & Ojemann, J. G. (2014). Broadband changes in the cortical surface potential track activation of functionally diverse neuronal populations. *NeuroImage*, 85, 711–720. <https://doi.org/10.1016/j.neuroimage.2013.08.070>

Examines broadband changes in human ECoG data shows / discusses broadband changes (offset) during task activity. Also discusses the aperiodic exponent.

Vyšata, O., Procházka, A., Mareš, J., Rusina, R., Pazdera, L., Vališ, M., & Kukal, J. (2014). Change in the Characteristics of EEG Color Noise in Alzheimer's Disease. *Clinical EEG and Neuroscience*, 45(3), 147–151. <https://doi.org/10.1177/1550059413491558>

Compare aperiodic exponent in EEG recordings comparing between control and Alzheimer's patients, and report differences in the prefrontal areas.

Podvalny, E., Noy, N., Harel, M., Bickel, S., Chechik, G., Schroeder, C. E., Mehta, A. D., Tsodyks, M., & Malach, R. (2015). A unifying principle underlying the extracellular field potential spectral responses in the human cortex. *Journal of Neurophysiology*, 114(1), 505–519. <https://doi.org/10.1152/jn.00943.2014>

Examine 1/f in human ECoG data during a visual task, and report that the aperiodic exponent changes with stimulus presentation and relate this to neuronal activation.



## Aperiodic History: Annotated Bibliography

### Additional Related Papers on Methodological Aspects

*This annotated bibliography lists additional papers related to the analysis of aperiodic activity in neural field recordings, specifically those that do not explicitly employ spectral measures of aperiodic activity.*

Jasper, H. H. (1948). Charting the Sea of Brain Waves. *Science*, 108(2805), 343–347.  
<https://doi.org/10.1126/science.108.2805.343>

A discussion of contemporary work on analyzing oscillations in EEG recordings, and notes the 'Fourier fallacy' – the fallacy of assuming that measured frequencies in a frequency domain representation necessarily actually occur in the analyzed data.

Imahori, E., & Suhara, K. (1949). On the Statistical Method in the Brain-Wave Study. Part I. *Psychiatry and Clinical Neurosciences*, 3(2), 137–155. <https://doi.org/10.1111/j.1440-1819.1949.tb02697.x>

Propose and apply a method for analyzing EEG, based on auto-correlation. Describe a rhythmic component and an 'aperiodic motion or heavily damped undulation'. Suggest overall that 'brain rhythms' are not sinusoidal, and may be damped oscillations and/or 'non-periodic motions'.

Brazier, M. A. B., & Barlow, J. S. (1956). Some applications of correlation analysis to clinical problems in electroencephalography. *Electroencephalography and Clinical Neurophysiology*, 8(2), 325–331. [https://doi.org/10.1016/0013-4694\(56\)90124-9](https://doi.org/10.1016/0013-4694(56)90124-9)

Discusses applications of autocorrelation related analyses of EEG data, including in relation to what such measures look like for signals reflecting both periodic and aperiodic components (using this terminology) and how to analyze and interpret such measures.

Wennberg, A., & Zetterberg, L. H. (1971). Application of a Computer-Based Model for EEG Analysis. *Electroencephalography and Clinical Neurophysiology*, 31(5), 457–468.  
[https://doi.org/10.1016/0013-4694\(71\)90167-2](https://doi.org/10.1016/0013-4694(71)90167-2)

Early example of quantifying EEG features - applies an auto-regressive model to EEG data and use to examine and parameterize features of different 'types' of activity, including those that contributes power across all frequencies ('Type I') and rhythmic components ('Type II').

Bullock, T. H., McClune, M. C., & Enright, J. T. (2003). Are the electroencephalograms mainly rhythmic? Assessment of periodicity in wide-band time series. *Neuroscience*, 121(1), 233–252.  
[https://doi.org/10.1016/S0306-4522\(03\)00208-2](https://doi.org/10.1016/S0306-4522(03)00208-2)

Examine and test the idea that EEG is largely made up of oscillations with a new method, and argue against this idea, suggesting that most of the power spectrum is non-rhythmic.