Literature collection

To go deeper into the area of PINNs (or similar approaches), we collected some literature covering interesting studies, applications and extensions. Additionally, we tried to gather some material concerning your mentioned applications (e.g., fluid flow equations, mechanic problems, etc.) that might help you get started using PI-Deep Learning in your problem.

Surveys

Here, we collect some surveys that give a detailed overview of the field.

- Overview of PINNs: Cuomo et al., *Scientific Machine Learning through Physics-Informed Neural Networks: Where we are and What's next*, 2022, `https://arxiv.org/abs/2201.05624`.
- Overview of multiple approaches: Hao et al., *Physics-Informed Machine Learning: A Survey on Problems, Methods and Applications*, 2023, `https://arxiv.org/abs/2211.08064`.
- General challenges: Krishnapriyan et al., *Characterizing possible failure modes in physics-informed neural networks*, 2021, `https://arxiv.org/abs/2109.01050`.
- Comparison to classical methods:
  - Markidis, *The Old and the New: Can Physics-Informed Deep-Learning Replace Traditional Linear Solvers?*, 2021, `https://arxiv.org/abs/2103.09655`.
  - Grossmann et al., *Can Physics-Informed Neural Networks beat the Finite Element Method?*, 2023, `https://arxiv.org/abs/2302.04107`.

Helpful PINN extensions

Here, we mention different extensions of the PINN approach (some can also be used in other methods) that may be helpful.

- Hard constrains: Liu et al., *A Unified Hard-Constraint Framework for Solving Geometrically Complex PDEs*, 2022, `https://openreview.net/pdf?id=GNt5ntEGjD3`.
- FBPINNs (learn DoF of basis functions): Mosoley et al., *Finite Basis Physics-Informed Neural Networks (FBPINNs): a scalable domain decomposition approach for solving differential equations*, 2021, `https://arxiv.org/abs/2107.07871`.

- VPINNs (use variational loss): Kharazmi et al., *Variational Physics-Informed Neural Networks For Solving Partial Differential Equations*, 2019, `https://arxiv.org/abs/1912.00873`.

- XPINNs (domain decomposition): Jagtap et al., *Extended Physics-Informed Neural Networks (XPINNs): A Generalized Space-Time Domain Decomposition Based Deep Learning Framework for Nonlinear Partial Differential Equations*, 2020, `https://global-sci.org/intro/article_detail/cicp/18403.html`.

- Multiscale/Fourier feature architectures:
  - Tancik et al., *Fourier Features Let Networks Learn High Frequency Functions in Low Dimensional Domains*, 2020, `https://arxiv.org/abs/2006.10739`.
  - Liu et al., *Multiscale DeepONet for Nonlinear Operators in Oscillatory Function Spaces for Building Seismic Wave Responses*, 2021, `https://arxiv.org/abs/2111.04860`.

- Stiff-PINNs: Ji et al., *Stiff-PINN: Physics-Informed Neural Network for Stiff Chemical Kinetics*, 2021, `https://pubs.acs.org/doi/10.1021/acs.jpca.1c05102`.

- Long-Time DeepONets: Wang et al., *Long-time integration of parametric evolution equations with physics-informed DeepONets*, 2023, `https://www.sciencedirect.com/science/article/pii/S0021999122009184`.

Helpful training procedures
Here, we collect some ideas to stabilize the training process and facilitate convergence.

- Study to combine Adam and LBFGS: He et al., *Physics-Informed Neural Networks for Multiphysics Data Assimilation with Application to Subsurface Transport*, 2019, `https://arxiv.org/abs/1912.02968`.

- Dynamic gradient normalization: Deguchi et al., *Dynamic & norm-based weights to normalize imbalance in back-propagated gradients of physics-informed neural networks*, 2023, `https://iopscience.iop.org/article/10.1088/2399-6528/ace416/pdf`.

- Application of dimensionless problem and normalized data: Lin et al., *A seamless multiscale operator neural network for inferring bubble dynamics*, 2021, link-to-paper.

- Normalization of the input of NNs: Rasht-Behesht et al., *Physics-informed Neural Networks (pinns) for wave propagation and full waveform inversions*, 2022, `https://agupubs.onlinelibrary.wiley.com/doi/10.1029/2021JB023120`

Deep Learning for fluid flow problems

Here, we collect some studies that try to solve different kinds of flow equations with (PI)-Deep Learning.

- PINNs for Navier-Stokes: Jin et al., *NSFnets (Navier-Stokes Flow nets): Physics-informed neural networks for the incompressible Navier-Stokes equations*, 2020, `https://arxiv.org/abs/2003.06496`.

- PINNs for Cahn-Hillard: Wight et al., *Solving Allen-Cahn and Cahn-Hilliard Equations using the Adaptive Physics Informed Neural Networks*, 2020, `https://arxiv.org/abs/2007.04542`.

- DeepONet for Navier-Stokes: Meng et al., *A comprehensive and fair comparison of two neural operators (with practical extensions) based on FAIR data*, 2021, `https://arxiv.org/abs/2111.05512`.

- DeepONet for Cahn-Hillard: Xu et al., *Transfer Learning Enhanced DeepONet for Long-Time Prediction of Evolution Equations*, 2022, `https://arxiv.org/abs/2212.04663`.

- DeepONet for Euler equation: Witman et al., *Neural Basis Functions for Accelerating Solutions to High Mach Euler Equations*, 2022, `https://arxiv.org/abs/2208.01687`.

- FNO for Burgers, Darcy and Navier-Stokes: Li et al., *Fourier Neural Operator for Parametric Partial Differential Equations*, 2020, `https://arxiv.org/abs/2010.08895`.

- PI-DeepONet and PINO for Navier-Stokes: Li et al., *Physics-Informed Neural Operator for Learning Partial Differential Equations*, 2021, `https://arxiv.org/abs/2111.03794`.

Mechanic problems

Here, we collect a few papers regarding (elastic) mechanic problems.

- PINNs for linear elasticity: Haghighat et al., *A deep learning framework for solution and discovery in solid mechanics*, 2020, `https://arxiv.org/abs/2003.02751`.

- Energy based PINNs: Li et al., *A Physics-Guided Neural Network Framework for Elastic Plates: Comparison of Governing Equations-Based and Energy-Based Approaches*, 2020, `https://arxiv.org/abs/2010.06050`.

- Accuracy studies: Guo et al., *Energy-based error bound of physics-informed neural network solutions in elasticity*, 2020, `https://arxiv.org/abs/2010.09088`.

- Stress prediction with DeepONet: He et al., *Novel DeepONet architecture to predict stresses in elastoplastic structures with variable complex geometries and loads*, 2023, `https://arxiv.org/abs/2306.03645`.

Wave and Helmholtz equation

Here, we collect studies for solving wave or Helmholtz equations with deep learning.

- PINNs for wave equations: Alkhadhr et al., *Wave Equation Modeling via Physics-Informed Neural Networks: Models of Soft and Hard Constraints for Initial and Boundary Conditions*, 2023, `https://www.mdpi.com/1424-8220/23/5/2792`.

- PINNs for Helmholtz equation: Song et al., *A versatile framework to solve the Helmholtz equation using physics-informed neural networks*, 2021, link-to-paper.

- DeepONet for Helmholtz eq. : Zhang et al., *A Hybrid Iterative Numerical Transferable Solver (HINTS) for PDEs Based on Deep Operator Network and Relaxation Methods*, 2022, `https://arxiv.org/abs/2208.13273`.

- FNO for photo-acoustic wave equation: Guan et al., *Fourier Neural Operator Networks: A Fast and General Solver for the Photoacoustic Wave Equation*, 2021, `https://arxiv.org/abs/2108.09374`.

Reaction-(Diffusion)-Equations

Here, we collect a few papers that handle diffusion and reaction problems with deep learning.

- PINNs for chemical kinetics: Gusmão et al., *Kinetics-informed neural networks*, 2023, `https://www.sciencedirect.com/science/article/pii/S0920586122001195`.

- PINNs for reaction-advection-diffusion equation: Gomes et al., *Physics-Aware Neural Networks for Boundary Layer Linear Problems*, 2022, link-to-paper.

- PINNs for biological application: Rebai et al., *Unsupervised physics-informed neural network in reaction-diffusion biology models (Ulcerative colitis and Crohn's disease cases) A preliminary study*, 2023, `https://arxiv.org/abs/2302.07405`.

- DeepONet for reaction diffusion: Deng et al., *Convergence rate of DeepONets for learning operators arising from advection-diffusion equations*, 2021, `https://arxiv.org/abs/2102.10621`.

Electrochemical Impedance Spectroscopy (EIS)
Here, we collect a few papers that handle EIS. To our knowledge no PI approaches are currently studied. Therefore, we only collected some classical deep learning for EIS.

- Deconvolution approach: Quattrocchi et al., *Deconvolution of electrochemical impedance spectroscopy data using the deep-neural-network-enhanced distribution of relaxation times*, 2023, link-to-paper.

- Gaussian process machine learning for EIS in lithium ion batteries: Zhang et al., *Identifying degradation patterns of lithium ion batteries from impedance spectroscopy using machine learning*, 2020, `https://www.nature.com/articles/s41467-020-15235-7`.