Generative Modeling of Turbulence

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Ergodicity: What Does "Chaotic" Actually Mean?

- Turbulent flow is **chaotic** by nature
- **LES:** Simulation of fine details in vortex flow but associated with **enormous computational costs** $\Rightarrow$ Can we get around this problem?

- Vortices always look different but in the long run their statistical properties are determined **(ergodicty)**:

$$\lim_{T\to\infty} \frac{1}{T} \int_0^T f \circ \varphi_t(x_0)\, \mathrm{d}t = \int_\Omega f(x) \mathrm{d}\,\mu(x)\ \forall x_0 \in \Omega. \tag{1}$$

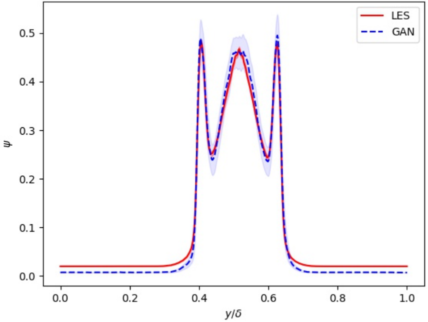
$\Rightarrow$ The time average of a dynamical system equates with the ensemble average of its invariant measure.

- Probability measure μ on the flow configurations x encodes the statistical properties of the chaotic flow/dynamics $\varphi_t(x_0)$
- Goal: Sampling from the **unknown** measure $\mu \Rightarrow$ Can we **learn** μ **from data**?

Turbulence Modeling with GAN

Our contribution[1]:

- Demonstrated that GAN-synthesized images match the output of LES in terms of visual level and physical quantities, while requiring significantly less computation.
- Proof that GAN do converge for ergodic learning problems

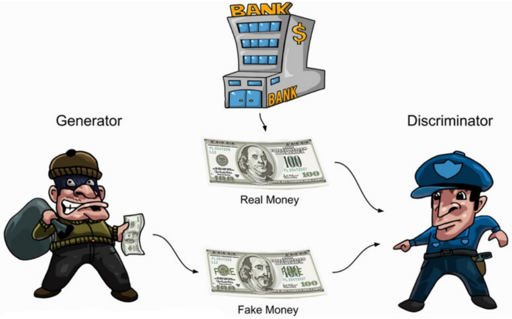


[1] Drygala et. al. (2022). *Generative modeling of turbulence.* Physics of Fluids, 34(3), 035114.

Generative Adversarial Networks (GAN) - Intuition

Generator:
Generates "fake" images, which should look as close as possible to given real-world data.

Discriminator:
Distinguishes real images from generated ones.



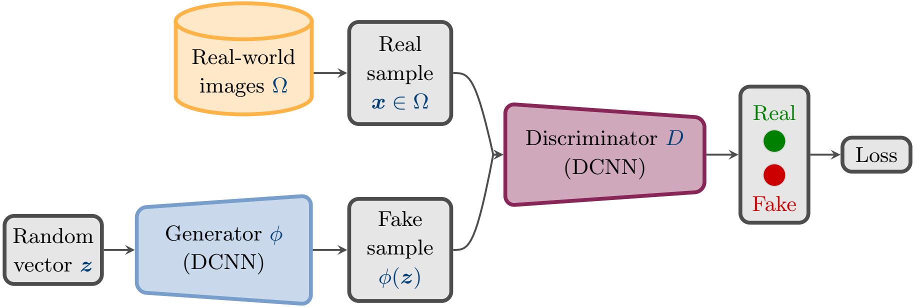
Intuition behind GAN.[2]

[2] Atienza, Rowel. *Advanced Deep Learning with Keras.* Packt Publishing Ltd, 2018.

Generative Adversarial Networks (GAN) - Architecture

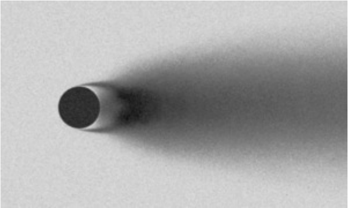
- Two-player Min-Max game:

$$\min_{\phi} \max_{D} \mathbb{E}_{\boldsymbol{x}\sim\mu}[\log(D(\boldsymbol{x}))] + \mathbb{E}_{\boldsymbol{z}\sim\lambda}[\log(1 - D(\phi(\boldsymbol{z})))]$$

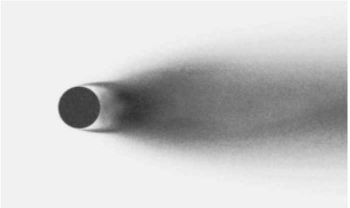


DCGAN vs. Vanilla GAN & WGAN

- DCNN expecially successful and applicable in field of **image processing**
- Guidelines for stable training at higher resolution and deeper architectures for GAN with CNN



Vanilla GAN



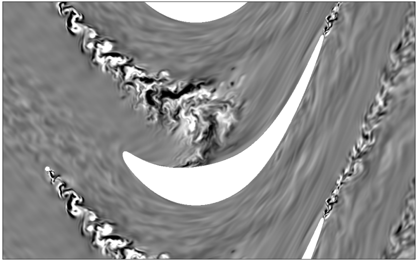
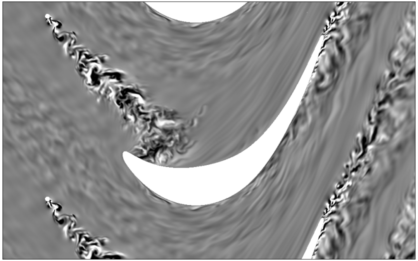
Wasserstein GAN

DCGAN

Generalization Capabilities under Changes of Geometry

Our contribution:[3]

- Demonstration of generalization capabilities of GAN-synthesized turbulence generators when geometric changes occur in the flow configuration
- Data: Flow around a low-pressure turbine (LPT) stator with periodic wake impact obtained from highly resolved LES
- Turbulence Modeling by conditional deep convolutional GAN `pix2pixHD`

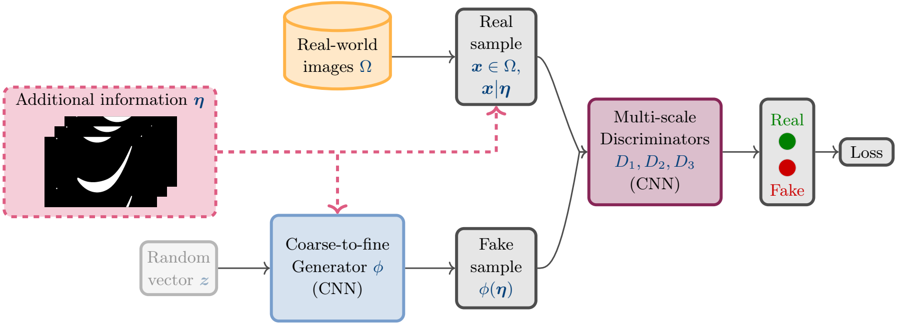
 

[3] Drygala et. al. (2023). *Generalization capabilities of conditional GAN for turbulent flow under changes of geometry*. EUROGEN 2023.

Conditional Deep Convolutional GAN (cDCGAN)

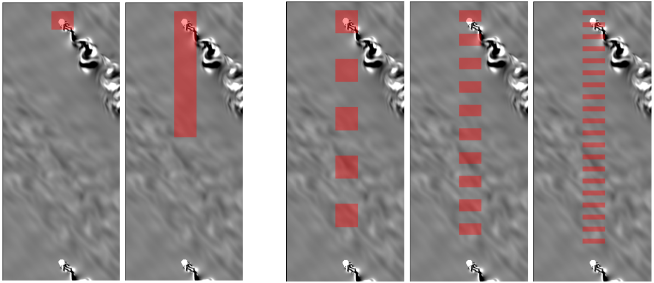
- pix2pixHD: Generation of **high-resolution photo-realistic** images by **conditioning** the input of the adversarial network on the corresponding **semantic label maps**

$$\mathcal{L}_{cond}(D,\phi) = \mathbb{E}_{\substack{\boldsymbol{x}\sim\mu \\ \boldsymbol{\eta}\sim\nu}}[\log(D(\boldsymbol{x}|\boldsymbol{\eta}))] + \mathbb{E}_{\substack{\boldsymbol{z}\sim\lambda \\ \boldsymbol{\eta}\sim\nu}}[\log(1 - D(\phi(\boldsymbol{z}|\boldsymbol{\eta})))] \tag{2}$$



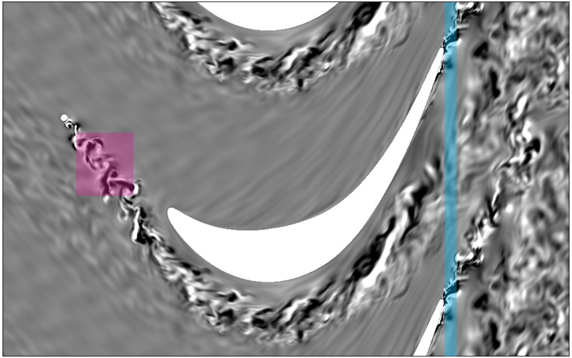
Generalization - Setup Data

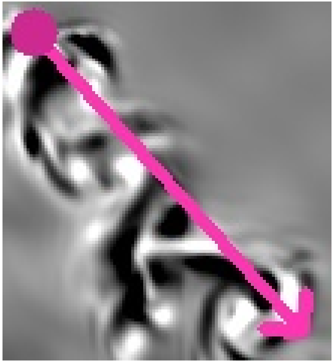
- **Data set:** $2,250$ images with resolution $1,000 \times 625$ ($\hat{=}10$ bar passing periods)
- **Data set splits:** Exclusion of a certain percentage from each period from the training that represents the test data
  - Strategy 1: Extraction of sequential frames
  - Strategy 2: Reduction of the frame rate at regular intervals
- **Generalization capabilities**: Iterative increase of percentage of excluded images
  $\Rightarrow$ Gradual reduction of variation in training data



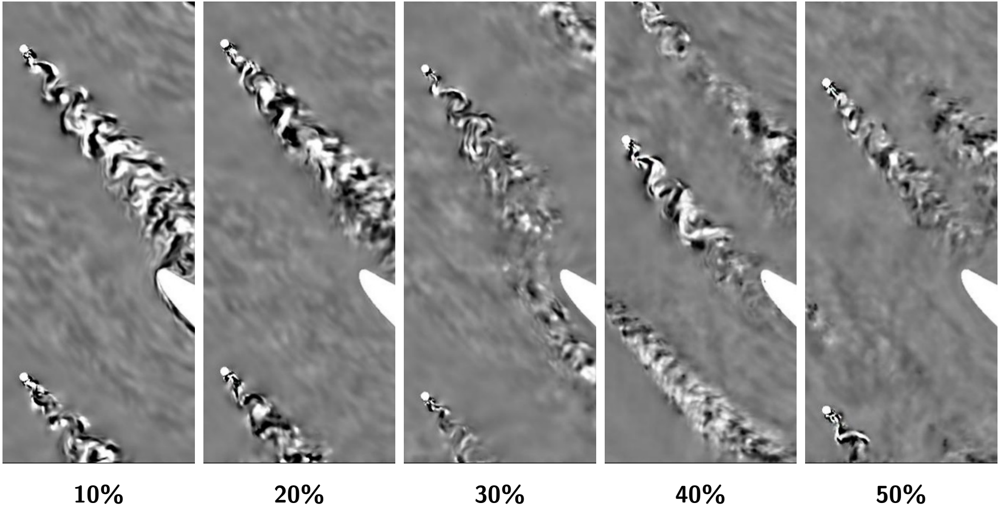
Generalization - Physics-based Metrics

- Turbulent flow field **correlations** in front of the rotor blade
- **Moving average** for the turbulence pattern in the rear region of the LPT stator suction side

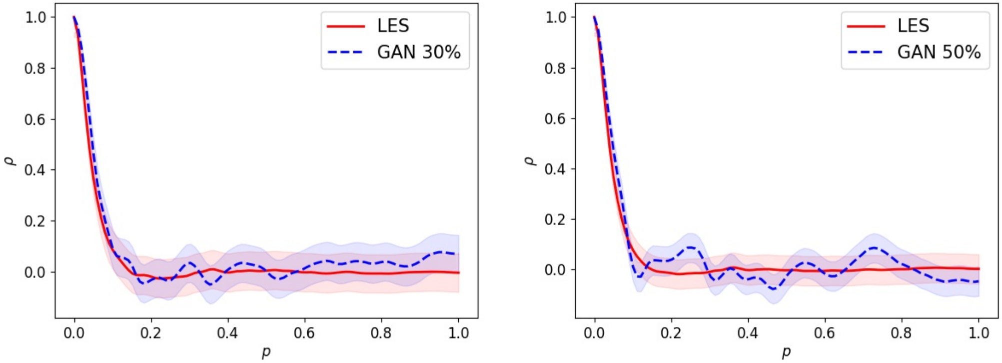




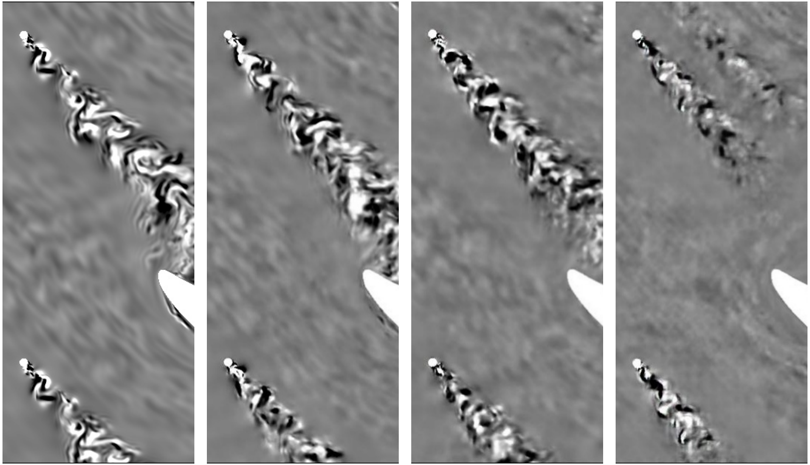
Inference Results - Sequential Frame Exclusion



Sequential Frame Exclusion - Correlations

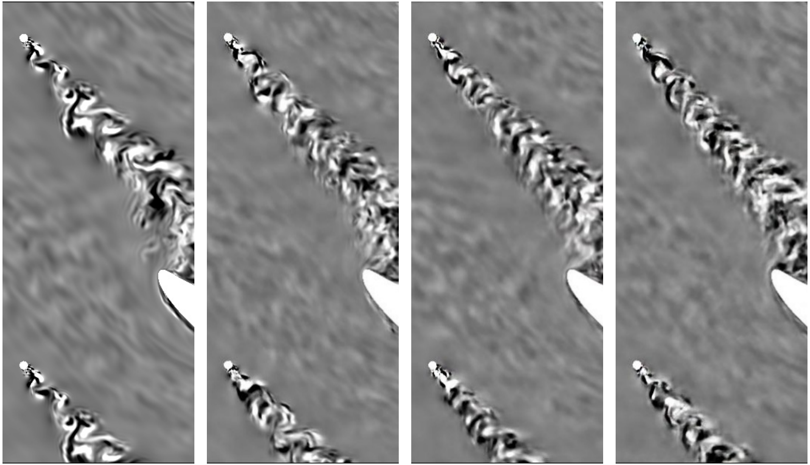


Inference Results - Frame Rate Reduction at 5 Intervals



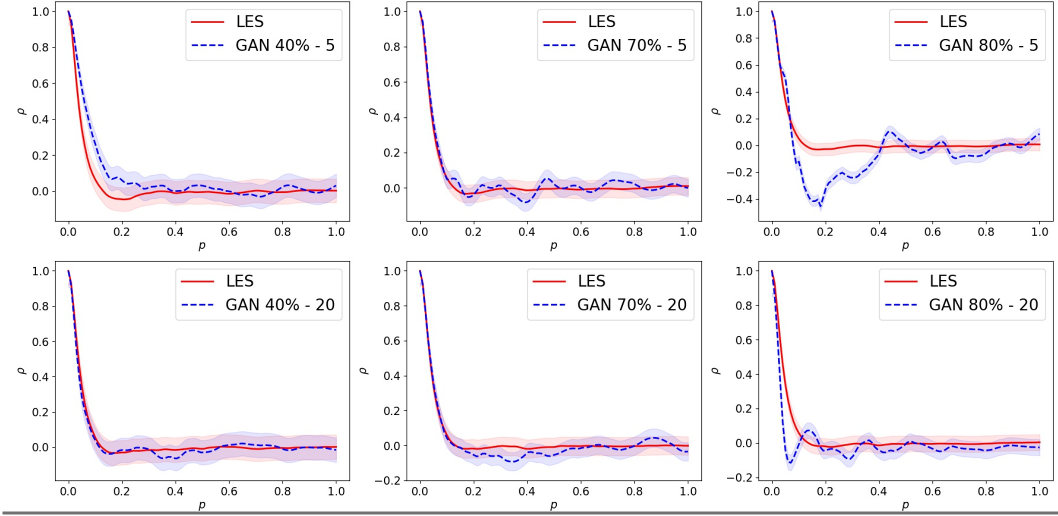
LES GAN 40% GAN 60% GAN 80%

Inference Results - Frame Rate Reduction at 20 Intervals



LES | GAN 40% | GAN 60% | GAN 80%

Frame Rate Reduction at Regular Intervals - Correlations



Computational Cost

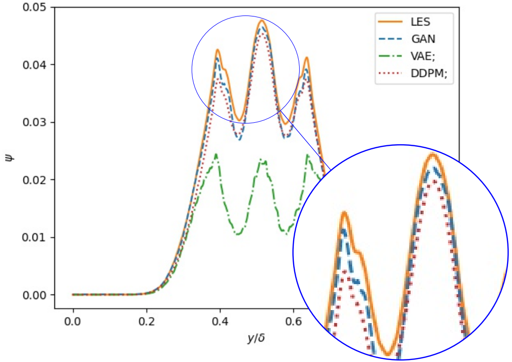
Flow	LES[a]	GAN-Training[b]	GAN-Inference[b]
Cylinder (DCGAN)	72 core weeks $\hat{=}$ 1 day (for $5,000$ images)	1.5 min/epoch ($\approx$ 2 days for $2,000$ epochs)	0.001 sec/image ($\approx$ 5 sec for $5,000$ images)
LPT stator (`pix2pixHD`)	640 core weeks $\hat{=}$ 8 days (for $2,250$ images)	Full dataset: 15 min/epoch ($\approx$ 2 days for 200 epochs); 30% of dataset: 6 min/epoch ($\approx$ 20 hours for 200 epochs)	0.01 sec/image ($\approx$ 22.5 sec for $2,250$ images)

Machines: a: 560 CPU cores, Intel Xeon SSkylake"Gold 6132 @2.6 GHz, b: GPU Quadro RTX 8000, 48 GB

Comparison of Generative Learning Methods

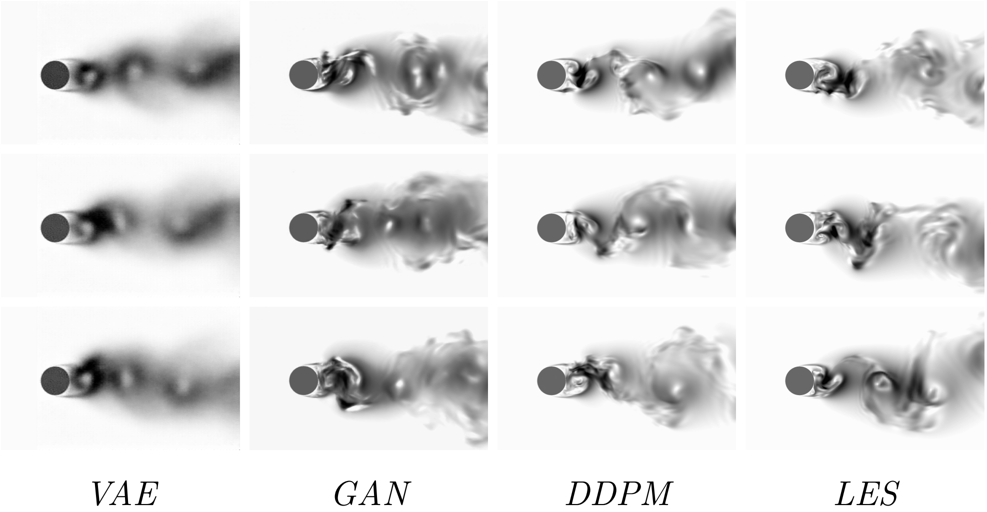
Our contribution:[4]

- Comparison of three generative learning methods for turbulence modeling: Variational Autoencoder (VAE), GAN and Denoising Diffusion Probabilistic Models (DDPM)
- Demonstrated that DDPM can generate adequate turbulent flow like DCGAN.
- In comparison to DDPM, DCGAN offers three key advantages: Significantly faster training and inference times, and a much lower data requirement.



[4] Drygala, Ross et. al. (2024). *Comparison of Generative Learning Methods for Turbulence Modeling*. arXiv: tba.

Comparison of Generative Learning Methods - Results



Outlook

Let's watch some videos!

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Thanks for your attention!

Reference:

(1) Drygala et. al. (2022). "Generative modeling of turbulence." Physics of Fluids, 34(3), 035114.

(2) Drygala et al. "Generalization capabilities of conditional GAN for turbulent flow under changes of geometry." EUROGEN 2023.

(3) Drygala, Ross et al. "Comparison of Generative Learning Methods for Turbulence Modeling." arXiv: tba.

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