Universität Bremen

Center for Industrial Mathematics (ZeTeM)

Faculty 03

Mathematics / Computer science

Introduction to TorchPhysics

Parameter studies

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Heidelberg, 08.11.2023

Yesterday

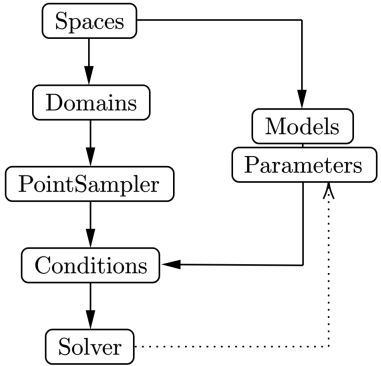
- PINNs:

$$\min \sum_{x_i} ||\partial_t u(x_i) - \Delta u(x_i)||^2$$

- TorchPhysics
  - Domain construction

```
1 bar    = tp.domains.Parallelogram(X, ...)
2 circle = tp.domains.Circle(X, ...)
3 omega  = circle - bar
```

  - Point sampling
  - Conditions



Parameter Studies and Parameter Identification

Realization with PINN and TorchPhysics

Parameter-dependent wave equation:

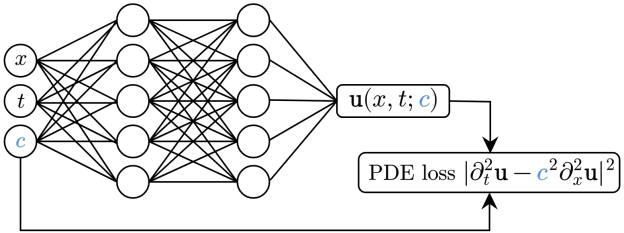
$$\begin{cases} \partial_t^2 u = c\,\partial_x^2 u, & \text{in } I_x \times I_t, \\ u = 0 & \text{in } \partial I_x \times I_t, \\ \partial_t u(\cdot, 0) = 0 & \text{in } I_x, \\ u(\cdot, 0) = \sin(x) & \text{in } I_x, \end{cases}$$

many industrial applications involve:

- **Parameter studies:** Solving the same PDE for many different choices of c
- **Parameter identification:** Finding the c that leads to given solution data $\{\hat{u}_i\}$ (later)

Parameter Studies with PINNs

Solving the same PDE for many different choices of c

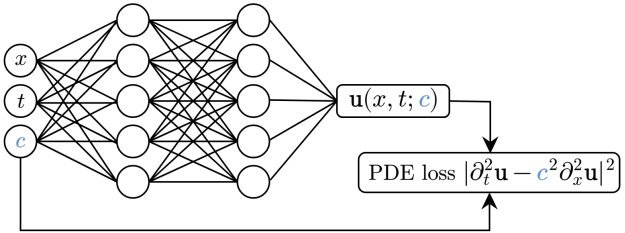


Method:

- Include parameter(s) as additional input(s) to the PINN
- Training: Sample parameter range together with function domain

Parameter Studies with PINNs

Solving the same PDE for many different choices of c



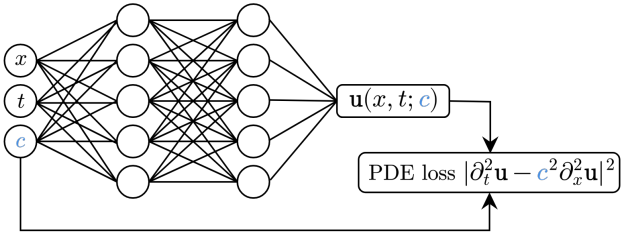

```
model = tp.models.FCN(input_space=X*T*C, output_space=U)

def pde_residual(u, t, x, c):
    return ...
```

Method:

- Include parameter(s) as additional input(s) to the PINN
- Training: Sample parameter range together with function domain

Parameter Studies with PINNs

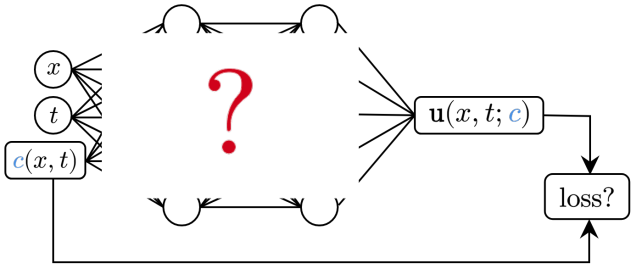
Solving the same PDE for many different choices of c



Result:

- Inference of solution for new parameter by a forward pass to the trained network
- Very little additional effort in evaluation of the network
- Increased amount of training points necessary

Function-Valued Parameter Studies



How to use a function-valued parameter as an input to the NN?

$\rightsquigarrow$ Later: **DeepONets**

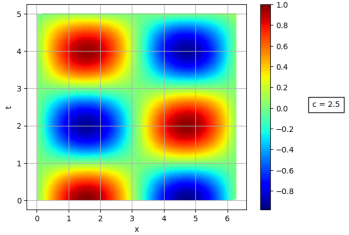
Exercise

$$\begin{cases} \partial_t^2 u = c\,\partial_x^2 u & \text{in } I_x \times I_t, \\ u = 0 & \text{in } \partial I_x \times I_t, \\ \partial_t u(\cdot, 0) = 0 & \text{in } I_x, \\ u(\cdot, 0) = \sin(x) & \text{in } I_x, \end{cases}$$

where $I_x = [0, 2\pi]$ and $I_t = [0, 5]$.

Parameter Study (`Exercise_6.ipynb`)

- Learn the solution for multiple c



Exercise: Stokes

- Learn flow profile for different inflow angles α

$$\begin{cases} -\frac{\mu}{2}\mathrm{div}(\nabla u + \nabla u^T) + \nabla p = 0 & \text{in } \Omega, \quad \text{(Conservation of momentum)} \\ \mathrm{div}(u) = 0 & \text{in } \Omega, \quad \text{(Conservation of mass)} \\ u = u_{in}(\alpha, x) & \text{on } \Gamma_D, \quad \text{(Dirichlet condition)} \\ -\frac{\mu}{2}(\nabla u + \nabla u^T) \cdot n + pn = 0, & \text{on } \Gamma_N, \quad \text{(Outflow condition).} \end{cases}$$

- Flow field $u : \Omega \to \mathbb{R}^2$ and pressure $p : \Omega \to \mathbb{R}$
- Notebook: `Exercise_7.ipynb`