Literature collection PINNs

Surveys

Here, we collect some surveys that give a detailed overview of the field.

- Overview of PINNs: Cuomo et al., *Scientific Machine Learning through Physics-Informed Neural Networks: Where we are and What's next*, 2022, `https://arxiv.org/abs/2201.05624`.

- Overview of multiple approaches: Hao et al., *Physics-Informed Machine Learning: A Survey on Problems, Methods and Applications*, 2023, `https://arxiv.org/abs/2211.08064`.

- General challenges: Krishnapriyan et al., *Characterizing possible failure modes in physics-informed neural networks*, 2021, `https://arxiv.org/abs/2109.01050`.

- Comparison to classical methods:
  - Markidis, *The Old and the New: Can Physics-Informed Deep-Learning Replace Traditional Linear Solvers?*, 2021, `https://arxiv.org/abs/2103.09655`.
  - Grossmann et al., *Can Physics-Informed Neural Networks beat the Finite Element Method?*, 2023, `https://arxiv.org/abs/2302.04107`.

- Toscano et al., *From PINNs to PIKANs: Recent Advances in Physics-Informed Machine Learning*, 2024, `https://arxiv.org/abs/2410.13228`

Helpful PINN extensions

Here, we mention different extensions of the PINN approach (some can also be used in other methods) that may be helpful.

- Hard constrains: Liu et al., *A Unified Hard-Constraint Framework for Solving Geometrically Complex PDEs*, 2022, `https://openreview.net/pdf?id=GNt5ntEGjD3`.

- FBPINNs (learn DoF of basis functions): Mosoley et al., *Finite Basis Physics-Informed Neural Networks (FBPINNs): a scalable domain decomposition approach for solving differential equations*, 2021, `https://arxiv.org/abs/2107.07871`.

- VPINNs (use variational loss): Kharazmi et al., *Variational Physics-Informed Neural Networks For Solving Partial Differential Equations*, 2019, `https://arxiv.org/abs/1912.00873`.

- XPINNs (domain decomposition): Jagtap et al., *Extended Physics-Informed Neural Networks (XPINNs): A Generalized Space-Time Domain Decomposition Based Deep Learning Framework for Nonlinear Partial Differential Equations*, 2020, `https://ceur-ws.org/Vol-2964/article_60.pdf`.

- Multiscale/Fourier feature architectures:
  - Tancik et al., *Fourier Features Let Networks Learn High Frequency Functions in Low Dimensional Domains*, 2020, `https://arxiv.org/abs/2006.10739`.
  - Liu et al., *Multiscale DeepONet for Nonlinear Operators in Oscillatory Function Spaces for Building Seismic Wave Responses*, 2021, `https://arxiv.org/abs/2111.04860`.

- Stiff-PINNs: Ji et al., *Stiff-PINN: Physics-Informed Neural Network for Stiff Chemical Kinetics*, 2021, `https://pubs.acs.org/doi/10.1021/acs.jpca.1c05102`.

Helpful training procedures
Here, we collect some ideas to stabilize the training process and facilitate convergence.

- Study to combine Adam and LBFGS: He et al., *Physics-Informed Neural Networks for Multiphysics Data Assimilation with Application to Subsurface Transport*, 2019, `https://arxiv.org/abs/1912.02968`.

- Dynamic gradient normalization: Deguchi et al., *Dynamic & norm-based weights to normalize imbalance in back-propagated gradients of physics-informed neural networks*, 2023, `https://iopscience.iop.org/article/10.1088/2399-6528/ace416/pdf`.

- Application of dimensionless problem and normalized data: Lin et al., *A seamless multiscale operator neural network for inferring bubble dynamics*, 2021, link-to-paper.

- Normalization of the input of NNs: Rasht-Behesht et al., *Physics-informed Neural Networks (pinns) for wave propagation and full waveform inversions*, 2022, `https://agupubs.onlinelibrary.wiley.com/doi/10.1029/2021JB023120`

- Wang et al., *When and why PINNs fail to train: A neural tangent kernel perspective*, 2020, `https://arxiv.org/abs/2007.14527`

Literature collection Operator Learning

DeepONet

The Deep Operator Networks (DeepONet) are one other frequently used approach for operator learning which we skipped in the workshop. Here some introduction and applications of the approach:

- Lu et al., *DeepONet: Learning nonlinear operators for identifying differential equations based on the universal approximation theorem of operators*, 2020, `https://arxiv.org/abs/1910.03193`

- Goswami et al. *A physics-informed variational DeepONet for predicting crack path in quasi-brittle materials*, 2022, `https://www.sciencedirect.com/science/article/pii/S004578252200010X`

- Li et al., *An architectural analysis of DeepOnet and a general extension of the physics-informed DeepOnet model on solving nonlinear parametric partial differential equations*, 2025, `https://www.sciencedirect.com/science/article/pii/S0925231224014462`

- Zhu et al., *Fourier-DeepONet: Fourier-enhanced deep operator networks for full waveform inversion with improved accuracy, generalizability, and robustness*, 2023, `https://arxiv.org/abs/2305.17289`

PCA-Nets and FNOs

While both PCA-Nets and FNOs were presented in the workshop, many details where left out for an easier understanding, here we collect some papers for both approaches:

- Li et al., *Fourier Neural Operator for Parametric Partial Differential Equations*, 2021, `https://arxiv.org/abs/2010.08895`

- Bhattacharya et al., *Model Reduction And Neural Networks For Parametric PDEs*, 2021, `https://smai-jcm.centre-mersenne.org/item/10.5802/smai-jcm.74.pdf`

- Li et al., *Fourier Neural Operator with Learned Deformations for PDEs on General Geometries*, 2023, `https://www.jmlr.org/papers/volume24/23-0064/23-0064.pdf`

- Tripura et al., *Wavelet Neural Operator for solving parametric partial differential equations in computational mechanics problems*, 2023, `https://www.sciencedirect.com/science/article/pii/S0045782522007393`
- Li et al., *Physics-Informed Neural Operator for Learning Partial Differential Equations*, 2022, `https://arxiv.org/abs/2111.03794`

Surveys/Comparison studies
Again multiple surveys as well as studies that compare different operator learning approaches.

- Kovachki et al., *Operator learning Algorithms and analysis*, 2024, link
- Tanyu et al., *Deep Learning Methods for Partial Differential Equations and Related Parameter Identification Problems*, 2023, `https://arxiv.org/abs/2212.03130`
- Lu et al., *A comprehensive and fair comparison of two neural operators (with practical extensions) based on FAIR data*, 2022, `https://arxiv.org/abs/2111.05512`