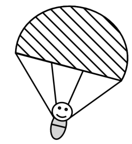
2.1 Solving a ODE with TorchPhysics



Use TorchPhysics to solve the ODE for falling with a parachute

$$\begin{aligned}\partial_t^2 u(t) &= D(\partial_t u(t))^2 - g, \\ u(0) &= H, \\ \partial_t u(0) &= 0,\end{aligned} \tag{1}$$

which we also considered yesterday. To open the prepared code template:

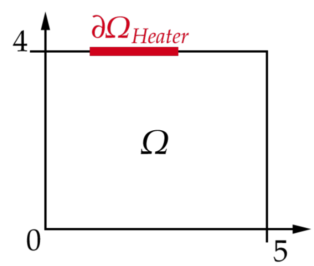
1. Open Google Colab
2. Select *open Notebook* and then the tab *GitHub*
3. Search: TomF98/torchphysics
4. Select the branch: *Workshop* and then Exercise2_1.ipynb

As a guideline, the example of the morning lecture can be found here.

⋆) **Bonus**: Extend your implementation, to learn the solution for multiple values of $D \in [0.01, 1.]$ and then also for different $H \in [50, 100]$ and $g \in [5, 10]$. Similar to the exercises of yesterday. **Hint:** Create seperate samplers for the respective parameters. Then multiply ("$*$") the time sampler with the parameter sampler in order to obtain a sampler which samples tuples (t, D).

2.2 Solving a PDE with TorchPhysics

Use TorchPhysics to solve the following heat equation:



$$\begin{aligned}\partial_t u(x,t) &= D\Delta_x u(x,t), && \text{on } \Omega \times I, && (2)\\ u(x,0) &= u_0, && \text{on } \Omega, && (3)\\ u(x,t) &= h(t), && \text{at } \partial\Omega_{Heater} \times I, && (4)\\ \nabla_x u(x,t) \cdot \vec{n}(x) &= 0, && \text{at } (\partial\Omega \setminus \partial\Omega_{Heater}) \times I. && (5)\end{aligned}$$

The above system describes an isolated room Ω, with a heater at the wall $\partial\Omega_{Heater} = \{(x,y) | 1 \leq x \leq 3, y = 4\}$. We set $I = [0, 20]$, $D = 1$, the initial temperature to $u_0 = 16\,^\circ\text{C}$ and the temperature of the heater to:

$$h(t) = \begin{cases} (16 + 24\frac{t}{5})\,^\circ\text{C}, & \text{if } t \leq 5, \\ 40\,^\circ\text{C}, & \text{if } t > 5. \end{cases}$$

a) **A PDE in TorchPhysics**: Implement the above equation with TorchPhysics. A template for this problem can be found under: Exercise2_2.ipynb.

b) **Domain Operations**: Next, we assume that the room contains a pillar (a circle) at position $(2, 2)$ with radius 0.5, remove this part from your domain. Here, also the boundary condition (5) holds.
Hint: Inside TorchPhysics, the difference of two domains `A` and `B` can be computed with `A - B`.

$\star$) **Bonus**: Add a window at $\partial\Omega_{\text{Window}} = \{(x, y) | 2 \leq x \leq 4, y = 0\}$ with fixed temperature of 16 °C.

$\star$) **Bonus**: Let the network learn all solutions for $D \in [0.1, 5]$, like in the problem before.

2.3 Solving an inverse Problem with TorchPhysics

We are given a noisy dataset $\{(u_i, x_i, t_i)\}_{i=1}^N$ which corresponds to the solution of the wave equation

$$\begin{aligned} \partial_t^2 u &= c\,\partial_x^2 u, && \text{in } I_x \times I_t, \\ u &= 0, && \text{on } \partial I_x \times I_t, \\ \partial_t u &= 0, && \text{on } \partial I_x \times I_t, \\ u(x, 0) &= \sin(x), && \text{in } I_x, \end{aligned}$$

with $I_x = [0, 2\pi]$ and $I_t = [0, 20]$. Here, we aim to determine the unknown parameter c with the PINN approach. Follow the template given in Exercise2_3.ipynb to solve this exercise.

$\star$) **Bonus**: In the notebook we added 1 % noise to the data and picked only half for the training. First, try out what results can be achieved with 5 % and 10 % noise. Second, if you keep 1 % noise but only use 10 % of the available data. Lastly, combine the case of 10 % noise with only 10 % of the available data and check the accuracy of the learned c and u.