Exercise session 1: LP problems and sensitivity

1 Giapetto product mix example

Giapetto's Woodcarving, Inc., manufactures three types of wooden toys: soldiers, trains and cars. A soldier sells for $27 and has a production cost of $22.5. A train sells for $21 and has a production cost of $17.5. A car sells for $15 and has a production cost of $13. The manufacture of wooden soldiers, trains and cars requires two types of skilled labor: carpentry and finishing. A soldier requires 1 hour of carpentry labor and 2 hours of finishing labor. A train requires 1 hour of carpentry and 1 hour of finishing labor. A car requires 1 hour of carpentry and 1.5 hour of finishing labor. Each week, Giapetto can obtain only 80 carpentry hours and 100 finishing hours. Demand for trains and cars is unlimited, but at most 40 soldiers are bought each week.

a) Formulate a mathematical model of Giapetto's situation that can be used to maximize Giapetto's weekly profit.
b) Solve the problem with LINGO.
c) Formulate the dual problem by hand and solve it with LINGO. (Check it by generating it with LINGO.) Interpretate the primal variables and constraints. Relate the solution to the dual prices in the original problem and vice versa.

Compute the Range Report. Solve the following problems without solving a new LP and check that you can do this. Afterwards, check the solution by solving a new LP

d) Find the new profit if the profit of one soldier increases to $6.
    - For which values of soldier profit does the current solution basis remain the same?
    - What is the total profit sensitivity to changes in the profit of a soldier?
e) Find the new profit if the number of finishing hours reduces to 90.
    - For which values of finishing hours does the current solution basis remain the same?
    - What is the total profit sensitivity to changes in the number of finishing hours?
f) What is the maximum that Giapetto should be willing to pay for 10 overtime hours extra of carpentry?
g) Find the new profit if one car is produced in three different ways:
    - using the reduced cost,
    - using the dual prices.
h) How much higher must the profit for one car be for cars to be produced in the optimum? Note that this agrees with the range report.
i) Would Giapetto increase his profit by also manufacturing airplanes that sell at $34 , have a production cost of $25, require 2 hours of carpentry and 3 hours of finishing? How much would he win/lose by selling 1 airplane?

For the following problems, you might need to solve additional LP's.

j) Generate a plot of the total profit as a function of the number of available finishing hours (from 0 to 300). How many additional LP's did you solve?
k) Generate a plot of the total profit as a function of the profit of a soldier (from 0 to 10). How many additional LP's did you solve?

2 Farmer Jane

Farmer Jane owns 45 acres of land. She could plant an acre with wheat or corn. Each acre planted with wheat yields \$200 profit; each with corn yields \$300 profit. The labor and fertilizer used for each acre are given in the table below. One hundred workers and 120 tons of fertilizer are available. Maximize Jane's profits.

Solve this problem graphically and check the solution with LINGO.

	Wheat	**Corn**
Labor	3 workers	2 workers
Fertilizer	2 tons	4 tons

3 Special solution cases

a)

An auto company manufactures cars and trucks. Each vehicle must be processed in the paint shop and body assembly shop. If the paint shop were only painting trucks, 40 per day could be painted. If the paint shop were only painting cars, 60 per day could be painted. If the body shop were only producing cars, it could process 50 per day. If the body shop were only producing trucks, it could process 50 per day. Each truck contributes \$300 to profit, and each car contributes \$200 to profit. Use linear programming to determine a daily production schedule that will maximize the company's profits.

b)

Suppose that auto dealers require that the auto company in (a) produces at least 30 trucks and 20 cars. Find the optimal solution to the new LP.

c)

$$\max z = 2x_1 - x_2$$
$$s.t. \quad x_1 - x_2 \leq 1$$
$$2x_1 + x_2 \geq 6$$
$$x_1, x_2 \geq 0$$