

Exercise sessions 2+3:

LP problem formulation and solving using LINGO

For each of the following problems:

- determine the decision variables,
- formulate the appropriate LP,
- solve the problem using LINGO.

Some remarks:

- Some of the problems require an extra constraint, restricting the decision variables to be integer or binary.
- Use extended LINGO syntax if the model is not too trivial.

1 Product mix examples

a) Dorian Auto advertisements

Dorian Auto manufactures luxury cars and trucks. The company believes that its most likely customers are high-income women and men. To reach these groups, Dorian Auto has embarked on an ambitious TV advertising campaign and has decided to purchase 1-minute commercial spots on two types of programs: comedy shows and football games. Each comedy commercial is seen by 7 million high-income women and 2 million high-income men. Each football commercial is seen by 2 million high-income women and 12 million high-income men. A 1-minute comedy ad costs

\$50,000, and a 1-minute football ad costs \$100,000. Dorian would like the commercials to be seen by at least 28 million high-income women and 24 million high-income men. Determine how Dorian Auto can meet its advertising requirements at minimum cost.

b) Aviston Electronics purchases

The Aviston Electronics company manufactures motors for toys and small appliances. The marketing department is predicting sales of 6,100 units of the Dynamonster motor in the next quarter. A Dynamonster is assembled from three components: a shaft, base, and cage. It is clear that some of these components will have to be purchased from outside suppliers because of limited in-house capacity. The variable in-house production cost per unit is compared with the outside purchase cost in the table below:

Component	Outside Cost	Inside Cost
Shaft	1.21	0.81
Base	2.50	2.30
Cage	1.95	1.45

Aviston's plant has three departments. Time requirements in hours of each component in each department if manufactured in-house are summarized in the following table. Hours available for Dynamonster production are listed in the last row. Find out how many units of each component should be purchased outside.

Component	Cutting Department	Shaping Department	Fabrication Department
Shaft	0.04	0.06	0.04
Base	0.08	0.02	0.05
Cage	0.07	0.09	0.06
Capacity	820	820	820

2 Covering/staffing problems:

a) post office staffing

For this post office example, suppose that each full-time employee works 8 hours per day. Thus, Monday's requirement of 17 workers may be viewed as a requirement of $8 \times 17 = 136$ hours. The post office may meet its daily labor requirements by using both full-time and part-time employees. During each week, a full-time employee works 8 hours a day for five consecutive days, and a part-time employee works 4 hours a day for five consecutive days. A full-time employee costs the post office \$15 per hour, whereas a part-time employee (with reduced fringe benefits) costs the post office only \$10 per hour. Union requirements limit the total part-time labor to 25 % of the weekly labor requirements. Formulate an LP to minimize the post office's weekly labor costs.

Day of Week	Number of employees required
Sunday	11
Monday	17
Tuesday	13
Wednesday	15
Thursday	19
Friday	14
Saturday	16

b) political mass mailing

A political candidate wants to make a mass mailing of some literature to counteract some nasty remarks that his opponent has recently made. Our candidate has identified five mailing lists that contain names and addresses of voters that our candidate might like to reach. Each list may be purchased for a price. A partial list cannot be purchased. The number of names that each list contains in each of four professions are listed below.

Names on each list (in 1000s) by profession					
Mailing List	Law	Health	Business Executives	Craft Professionals	Cost of List
1	28	4	7	2	\$41,000
2	9	29	11	3	\$52,000
3	6	3	34	18	\$61,000
4	2	4	6	20	\$32,000
5	8	9	12	14	\$43,000
Desired Coverage:	20	18	22	20	

Our candidate has estimated how many voters he wants to reach in each profession. This is listed in the row "Desired Coverage". Our candidate has a more limited budget than his opponent. Therefore, our candidate does not want to spend any more than he has to in order to "do the job".

3 Blending problems:

a) Whiskey blending

Hackensack Blended Whiskey Company imports three grades of whiskey: Prime, Choice, and Premium. These unblended grades can be used to make up the following two brands of whiskey associated characteristics:

Brand	Specifications	Selling price per liter
Scottish Club	Not less than 60 % Prime Not more than 20 % Premium	\$6.80
Johnny Gold	Not more than 60% Premium Not less than 15 % Prime	\$5.70

The costs and availabilities of the three raw whiskeys are:

Whiskey	Available this week (in liter)	Cost per liter
Prime	2,000	\$7.00
Choice	2,500	\$5.00
Premium	1,200	\$4.00

Hackensack wishes to maximize this week's profit contribution. How much should be made of each of the two brands? How should the three raw whiskeys be blended into each of the two brands?

b) Student school assignment

Recent federal regulations strongly encourage the assignment of students to schools in a city so that the racial composition of any school approximates the racial composition of the entire city. Consider the case of the Greenville city schools. The city can be considered as composed of five areas with the following characteristics.

Area	Fraction minority	Number of students
1	.20	1,200
2	.10	900
3	.85	1,700
4	.60	2,000
5	.90	2,500

The ruling handed down for Greenville is that a school can have neither more than 75 percent nor less than 30 percent minority enrollment. There are three schools in Greenville with the following capacities:

School	Capacity
Bond	3,900
Pocahontas	3,100
Pierron	2,100

The objective is to design an assignment of students to schools so as to stay within the capacity of each school and satisfy the composition constraints while minimizing the distance traveled by students. The distances in kilometers between areas and schools are:

Area					
School	1	2	3	4	5
Bond	2.7	1.4	2.4	1.1	0.5
Pocahontas	0.5	0.7	2.9	0.8	1.9
Pierron	1.6	2.0	0.1	1.3	2.2

There is an additional condition that no student can be transported more than 2.6 kilometers. Find the number of students that should be assigned to each school from each area. Assume that any group of students from an area has the same ethnic mix as the whole area.

4 Multiperiod planning problems

a) Pilot planning

An airline predicts the following pilot requirements for the next five quarters: 80, 90, 110, 120, 110. Current staff is 90 pilots. The question of major concern is the number of pilots to hire in each of the next five quarters. A new pilot must spend the quarter in which he is hired in training. The line's training facilities limit the number of pilots in training to at most 15. Further, the training of pilots requires the services of experienced pilots at the ratio of 5 to 1; that is, 5 pilots in training require 1 experienced pilot. An experienced pilot so assigned cannot be used to satisfy regular requirements. However, he can be used part-time for training and part-time for flying during the same period. The cost of hiring and training a pilot is estimated at \$20,000 exclusive of the experienced pilot time required. Experienced pilots cost \$25,000 per quarter. The company policy does not include firing pilots. Determine how many pilots should be hired in each period.

b) Financial planning

Finco Investment Corporation must determine an investment strategy for the firm during the next three years. At present time (time 0), \$100,000 is available for investment. Investments A, B, C, D, and E are available. The cash flow associated with investing \$1 in each investment is given in the following table.

Cash flow at time *				
	0	1	2	3
A	-\$1	+\$0.50	+\$1	\$0
B	\$0	-\$1	+\$0.50	+\$1
C	-\$1	+\$1.2	\$0	\$0
D	-\$1	\$0	\$0	+\$1.9
E	\$0	\$0	-\$1	+\$1.5
* Time 0 = present time; time 1 = 1 year from now; time 2 = 2 years from now; time 3 = 3 years from now.				

For example, \$1 invested in investment B requires a \$1 cash outflow at time 1 and returns 50¢ at time 2 and \$1 at time 3. To ensure that the company's portfolio is diversified, Finco requires that at most \$75,000 be placed in any single investment. In addition to investments A-E, Finco can earn interest at 8 % per year by keeping uninvested cash in money market funds. Returns from investments may be immediately reinvested. For example, the positive cash flow received from investment C at time 1 may immediately be reinvested in investment B. Finco cannot borrow funds, so the cash available for investment at any time is limited to cash on hand. Note that the investment table is absolute, not relative. I.e. investments A, C and D can be made a time '0' only, and are closed thereafter. Formulate an LP that will maximize cash on hand at time 3.