

MACROS, TAIL RECURSION AND INTERPRETERS

COMPUTER SCIENCE MENTORS CS 61A

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1 Let in Scheme

1. **let** is a special form in Scheme which allows you to create local bindings. Consider the example

```
(let ((x 1)) (+ x 1))
```

Here, we assign `x` to 1, and then evaluate the expression `(+ x 1)` using that binding, returning 2. However, outside of this expression, `x` would not be bound to anything.

Each `let` special form has a corresponding lambda equivalent. The equivalent lambda expression for the above example is

```
((lambda (x) (+ x 1)) 1)
```

The following line of code does not work. Why? Write the lambda equivalent of the `let` expressions.

```
(let ((foo 3)
      (bar (+ foo 2)))
  (+ foo bar))
```

$((\text{lambda}(\text{foo bar}) (+ \text{foo bar})) 3 (+ \text{foo } 2))$

2 Macros

1. What will Scheme output?

```

scm> (define x 6)
      X
scm> (define y 1)
      y
scm> '(x y a)
      (x y a)
scm> '(,x ,y a)
      (6 1 a)
scm> '(,x y a)
      (6 y a)
scm> '(, (if (- 1 2) '+ '-') 1 2)
      (+ 1 2)
scm> (eval '(, (if (- 1 2) '+ '-') 1 2))
      3
scm> (define (add-expr a1 a2)
      (list '+ a1 a2))
      add-expr
scm> (add-expr 3 4)
      (+ 3 4)
scm> (eval (add-expr 3 4))
      7
scm> (define-macro (add-macro a1 a2)
      (list '+ a1 a2))
      add-macro
scm> (add-macro 3 4)
      7

```

2. Implement `if-macro`, which behaves similarly to the `if` special form in Scheme but has some additional properties. Here's how the `if-macro` is called:

```
if <cond1> <expr1> elif <cond2> <expr2> else <expr3>
```

If `cond1` evaluates to a truth-y value, `expr1` is evaluated and returned. Otherwise, if `cond2` evaluates to a truth-y value, `expr2` is evaluated and returned. If neither condition is true, `expr3` is evaluated and returned.

```
; Doctests
```

```
scm> (if-macro (= 1 0) 1 elif (= 1 1) 2 else 3)
```

```
2
```

```
scm> (if-macro (= 1 1) 1 elif (= 2 2) 2 else 3)
```

```
1
```

```
scm> (if-macro (= 1 0) (/ 1 0) elif (= 2 0) (/ 1 0) else 3)
```

```
3
```

```
(define-macro (if-macro cond1 expr1 elif cond2 expr2 else
  expr3)
```

Regular
Scheme
code

```
(cond (cond1 expr1)
      (cond2 expr2)
      (else expr3)))
```

Macro
List

```
(list 'cond (list 'cond1 expr1)
      (list 'cond2 expr2)
      (list 'else expr3)))
```

```
)
```

Macro
Pseudoquote

```
(cond (,cond1 ,expr1)
      (,cond2 ,expr2)
      (else ,expr3)))
```

3. Could we have implemented `if-macro` using a function instead of a macro? Why or why not?

No, using a function, we would always be evaluating each argument once before passing in to function call.

4. Implement `apply-twice`, which is a macro that takes in a call expression with a single argument. It should return the result of applying the operator to the operand twice.

;Doctests

```
scm> (define add-one (lambda (x) (+ x 1)))
```

```
add-one
```

```
scm> (apply-twice (add-one 1))
```

```
3
```

```
scm> (apply-twice (print 'hi))
```

```
hi
```

```
undefined
```

```
(define-macro (apply-twice call-expr)
```

Handwritten implementation of the macro:

```
(let ((operator (car call-expr))
      (operand (cdr call-expr)))
  (operator (operator operand)))
```

Diagram illustrating the evaluation of the macro call:

```
call-expr → (print) → 'hi'
```

The macro expands to:

```
(print (print 'hi))
```

Tail context:

- last body subexp in λ

- subexp 2, 3 in if

- all non-predicate subexp in cond

- last subexp in and or begin let

3 Tail Recursion

1. What is a tail context? What is a tail call? What is a tail recursive function?

func call in tail context

2. Why are tail calls useful for recursive functions?

$O(1)$ space

3. Consider the following function:

```

(define (count-instance lst x)
  (cond ((null? lst) 0)
        ((equal? (car lst) x) (+ 1 (count-instance
                                   (cdr lst) x)))
        (else (count-instance (cdr lst) x))))

```

What is the purpose of `count-instance`? Is it tail recursive? Why or why not?

Optional: draw out the environment diagram of this sum-list with `lst = (1 2 1)` and `x = 1`.

No, $(+ 1 (\text{count-instance } (\text{cdr } \text{lst}) x))$

recursive call is one of the arguments of the addition

Doesn't span entire space of tail context
 \Rightarrow not a tail call

4. Rewrite count-instance to be tail recursive.

```
(define (count-tail lst x)
```

```
  (define (count-tail-tr lst x count)
```

```
    (cond ((null? lst) count)
```

```
          ((= x (car lst)) (count-tail-tr (cdr lst) x (+ count 1)))
```

```
          (else (count-tail-tr (cdr lst) x count)))
```

```
    )
```

```
  ) (count-tail-tr lst x 0)
```

5. Implement `filter`, which takes in a one-argument function `f` and a list `lst`, and returns a new list containing only the elements in `lst` for which `f` returns true. Your function must be tail recursive.

You may wish to use the built-in `append` function, which takes in two lists and returns a new list containing the elements of the first list followed by the elements of the second.

```
; Doctests
```

```
scm> (filter (lambda (x) (> x 2)) '(1 2 3 4 5))
```

```
(3 4 5)
```

```
(define (filter f lst)
```

```
  (define (filter-tr f lst retlst)
```

```
    (cond ((null? lst) retlst)
```

```
          ((f (car lst)) (filter-tr f (cdr lst) (append retlst
```

```
            (else (filter-tr f (cdr lst) retlst)) (cons (car lst) nil))))
```

```
    )
```

```
  ) (filter-tr f lst nil)
```

```
,
```

4 Interpreters

1. Circle the number of calls to `scheme_eval` and `scheme_apply` for the code below.

```

(+ 1 2)
-----
scheme_eval  1 3 4 6
scheme_apply 1 2 3 4

```

2. Write the number of calls to `scheme_eval` and `scheme_apply` for the code below.

```

(if 1 (+ 2 3) (/ 1 0))
-----
scheme_eval  1 3 4 6
scheme_apply 1 2 3 4

```

if : special form

```

(or #f (and (+ 1 2) 'apple) (- 5 2))
-----
scheme_eval  6 8 9 10
scheme_apply 1 2 3 4

```

or, and, quote: special form

```

(define (square x) (* x x))
-----
(+ (square 3) (- 3 2))
-----
scheme_eval  2 5 14 24
scheme_apply 1 2 3 4

```

define = special form

```

(define (add x y) (+ x y))
-----

```

```

(add 5 3) (or 0 2)
-----

```

or = special form

13 eval
3 apply