

RECURSION, TREE RECURSION AND DATA ABSTRACTION

COMPUTER SCIENCE MENTORS CS 61A

February 18, 2019 to February 20, 2019

1 Recursion

1. Write a function `is_sorted` that takes in an integer `n` and returns true if the digits of that number are nondecreasing from right to left.

```
def is_sorted(n):  
    """  
    >>> is_sorted(2)  
    True  
    >>> is_sorted(22222)  
    True  
    >>> is_sorted(9876543210)  
    True  
    >>> is_sorted(9087654321)  
    False  
    """
```

Solution:

```
    right_digit = n % 10  
    rest = n // 10  
    if rest == 0:  
        return True  
    elif right_digit > rest % 10:  
        return False  
    else:  
        return is_sorted(rest)
```

2. (Spring 2015 MT1 Q3C) Implement the `combine` function, which takes a non-negative integer `n`, a two-argument function `f`, and a number `result`. It applies `f` to the first digit of `n` and the result of combining the rest of the digits of `n` by repeatedly applying `f` (see the doctests). If `n` has no digits (because it is zero), `combine` returns `result`.

```
def combine(n, f, result):
    """
    Combine the digits in non-negative integer n using f.

    >>> combine(3, mul, 2) # mul(3, 2)
    6
    >>> combine(43, mul, 2) # mul(4, mul(3, 2))
    24
    >>> combine(6502, add, 3) # add(6, add(5, add(0, add(2, 3)
    )))
    16
    >>> combine(239, pow, 0) # pow(2, pow(3, pow(9, 0)))
    8
    """
    if n == 0:
        return result
    else:
        return combine(_____, _____,
                       _____)
```

Solution:

```
def combine(n, f, result):
    if n == 0:
        return result
    else:
        return combine(n // 10, f, f(n % 10, result))
```

2 Tree Recursion

1. Mario needs to jump over a series of Piranha plants, represented as a string of 0's and 1's. Mario only moves forward and can either *step* (move forward one space) or *jump* (move forward two spaces) from each position. How many different ways can Mario traverse a level without stepping or jumping into a Piranha plant? Assume that every level begins with a 1 (where Mario starts) and ends with a 1 (where Mario must end up).

Hint: Does it matter whether Mario goes from left to right or right to left? Which one is easier to check?

```
def mario_number(level):
```

```
    """
```

```
    Return the number of ways that Mario can traverse the
    level, where Mario can either hop by one digit or two
    digits each turn. A level is defined as being an integer
    with digits where a 1 is something Mario can step on and
    0 is something Mario cannot step on.
```

```
>>> mario_number(10101)
```

```
1
```

```
>>> mario_number(11101)
```

```
2
```

```
>>> mario_number(100101)
```

```
0
```

```
    """
```

```
    if _____:
```

```
        _____
```

```
    elif _____:
```

```
        _____
```

```
    else:
```

```
        _____
```

Solution:

```
def mario_number(level):  
    """  
    Return the number of ways that mario can traverse the  
    level where mario can either hop by one digit or two  
    digits each turn a level is defined as being an integer  
    where a 1 is something mario can step on and 0 is  
    something mario cannot step on.  
    >>> mario_number(10101)  
    1  
    >>> mario_number(11101)  
    2  
    >>> mario_number(100101)  
    0  
    """  
    if level == 1:  
        return 1  
    elif level % 10 == 0:  
        return 0  
    else:  
        return mario_number(level // 10) + mario_number(  
            level // 10 // 10)
```

2. James wants to print this week's discussion handouts for all the students in CS 61A. However, both printers are broken! The first printer only prints multiples of n pages, and the second printer only prints multiples of m pages. Help James figure out whether or not it's possible to print exactly `total` number of handouts!

```
def has_sum(total, n, m):
    """
    >>> has_sum(1, 3, 5)
    False
    >>> has_sum(5, 3, 5) # 0 * 3 + 1 * 5 = 5
    True
    >>> has_sum(11, 3, 5) # 2 * 3 + 1 * 5 = 11
    True
    """
    if _____:

        return _____

    elif _____:

        return _____

    return _____
```

Solution:

```
def has_sum(total, n, m):
    if total == 0: # (total == n or total == m) works too
                    except when total equals 0
        return True
    elif total < 0: # (total < min(n1, n2)) works given
                    alternate base case
        return False
    return has_sum(total - n, n, m) or has_sum(total - m, n
        , m)
```

3. The next day, the printers break down even more! Each time they are used, the first printer prints a random x copies $50 \leq x \leq 60$, and the second printer prints a random y copies $130 \leq y \leq 140$. James also relaxes his expectations: he's satisfied as long as there's at least `lower` copies so there are enough for everyone, but no more than `upper` copies to prevent waste.

```
def sum_range(lower, upper):
    """
    >>> sum_range(45, 60) # Printer 1 prints within this range
    True
    >>> sum_range(40, 55) # Printer 1 can print a number 56-60
    False
    >>> sum_range(170, 201) # Printer 1 + 2 will print between
    180 and 200 copies total
    True
    """
    def copies(pmin, pmax):
        if _____:

            return _____

        elif _____:

            return _____

        return _____

    return copies(0, 0)
```

Solution:

```
def sum_range(lower, upper):
    def copies(pmin, pmax):
        if lower <= pmin and pmax <= upper:
            return True
        elif upper < pmin:
            return False
        return copies(pmin + 50, pmax + 60) or copies(pmin
            + 130, pmax + 140)
    return copies(0, 0)
```

3 Data Abstraction

1. The following is an **Abstract Data Type (ADT)** for elephants. Each elephant keeps track of its name, age, and whether or not it can fly. Given our provided constructor, fill out the selectors:

```
def elephant(name, age, can_fly):  
    """  
    Takes in a string name, an int age, and a boolean can_fly.  
    Constructs an elephant with these attributes.  
    >>> dumbo = elephant("Dumbo", 10, True)  
    >>> elephant_name(dumbo)  
    "Dumbo"  
    >>> elephant_age(dumbo)  
    10  
    >>> elephant_can_fly(dumbo)  
    True  
    """  
    return [name, age, can_fly]  
def elephant_name(e):
```

Solution:

```
    return e[0]
```

```
def elephant_age(e):
```

Solution:

```
    return e[1]
```

```
def elephant_can_fly(e):
```

Solution:

```
    return e[2]
```

2. This function returns the correct result, but there's something wrong about its implementation. How do we fix it?

```
def elephant_roster(elephants):  
    """  
    Takes in a list of elephants and returns a list of their  
    names.  
    """  
    return [elephant[0] for elephant in elephants]
```

Solution: `elephant[0]` is a Data Abstraction Violation (DAV). We should use a selector instead. The corrected function looks like:

```
def elephant_roster(elephants):  
    return [elephant_name(elephant) for elephant in  
            elephants]
```

3. Fill out the following constructor for the given selectors.

```
def elephant(name, age, can_fly):
```

Solution:

```
    return [[name, age], can_fly]
```

```
def elephant_name(e):  
    return e[0][0]  
def elephant_age(e):  
    return e[0][1]  
def elephant_can_fly(e):  
    return e[1]
```

4. How can we write the fixed `elephant_roster` function for the constructors and selectors in the previous question?

Solution: No change is necessary to fix `elephant_roster` since using the `elephant` selectors “protects” the roster from constructor definition changes.

5. (Optional) Fill out the following constructor for the given selectors.

```
def elephant(name, age, can_fly):  
    """  
    >>> chris = elephant("Chris Martin", 38, False)  
    >>> elephant_name(chris)  
        "Chris Martin"  
    >>> elephant_age(chris)  
        38  
    >>> elephant_can_fly(chris)  
        False  
    """  
    def select(command)
```

Solution:

```
        if command == "name":  
            return name  
        elif command == "age":  
            return age  
        elif command == "can_fly":  
            return can_fly  
        return "Breaking abstraction barrier!"  
  
    return select  
def elephant_name(e):  
    return e("name")  
def elephant_age(e):  
    return e("age")  
def elephant_can_fly(e):  
    return e("can_fly")
```