Experimental Data Analysis

in ©MATLAB

Lecture 8:
Introduction to models, regression analysis

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Motivation

Association

Question: Can be increased blood pressure associated with stress?
Answer: Correlation analysis.

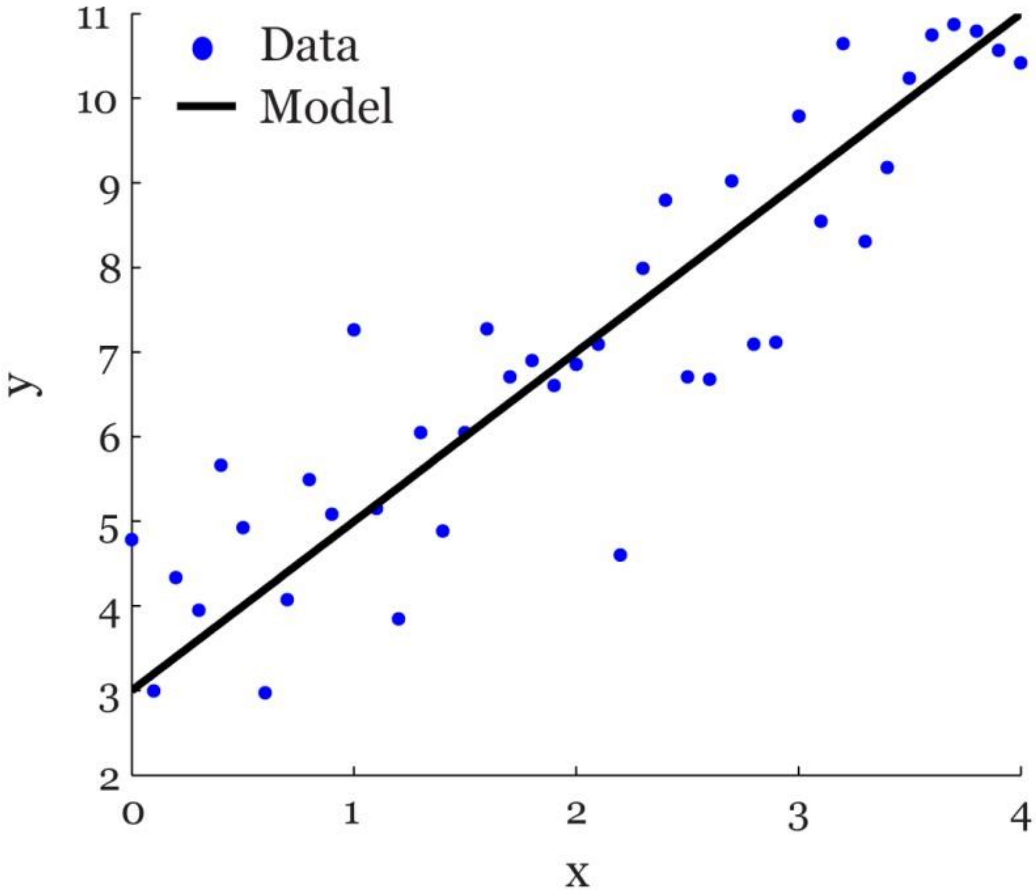
Connection

- Correlation indicates a relationship not causality.
- We need to find a connection to say that relationship is causal (i.e. examine that hormonal response to stress can elevate blood pressure).

Prediction

Question: We interrogate the chief suspect (healthy) and measure his blood pressure. How much was the suspect stressed by our key question?
Answer: Regression analysis.

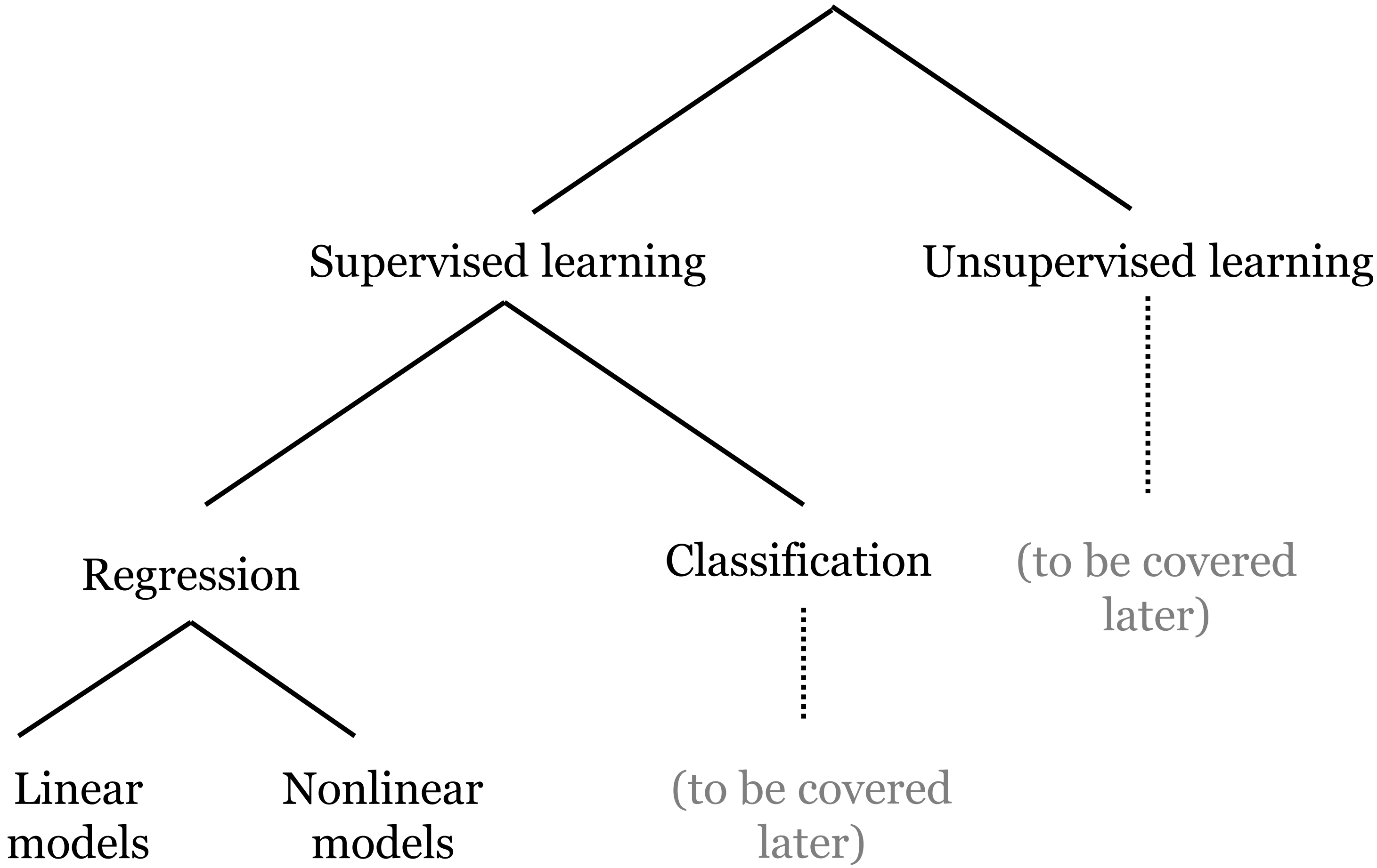
Linear model



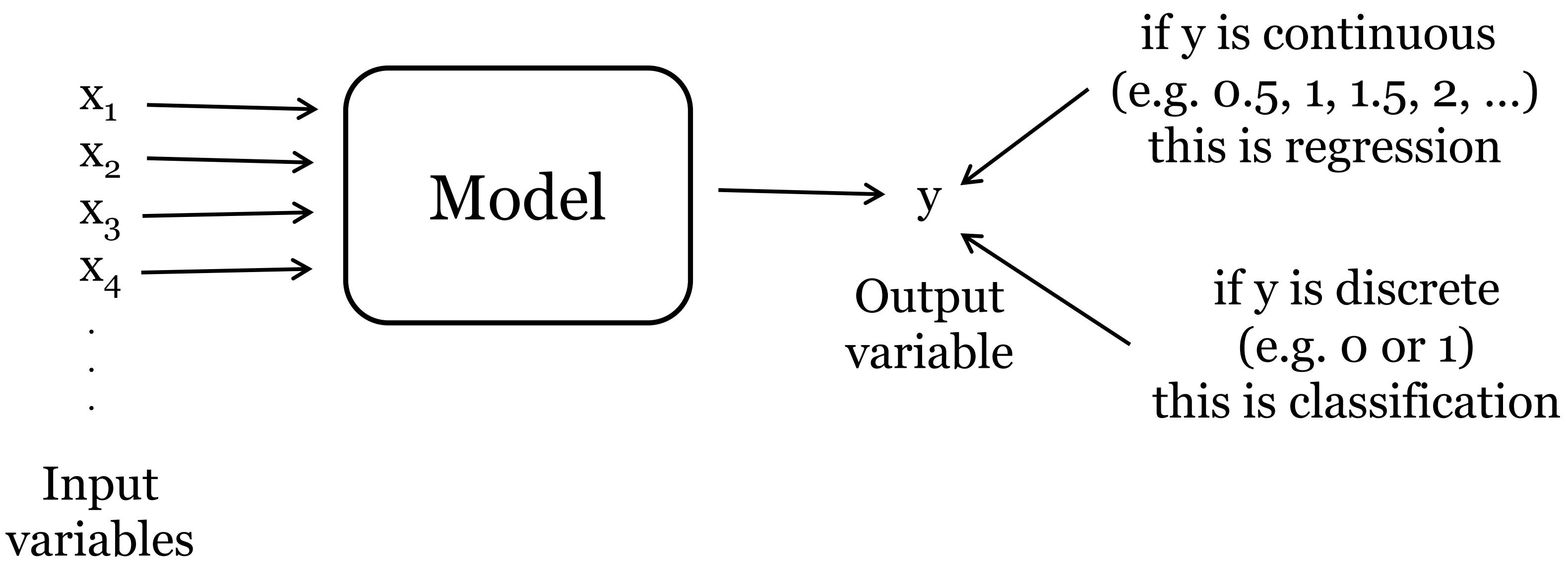
Model specification
y = ax + b

Fitted model
y = 2x + 3

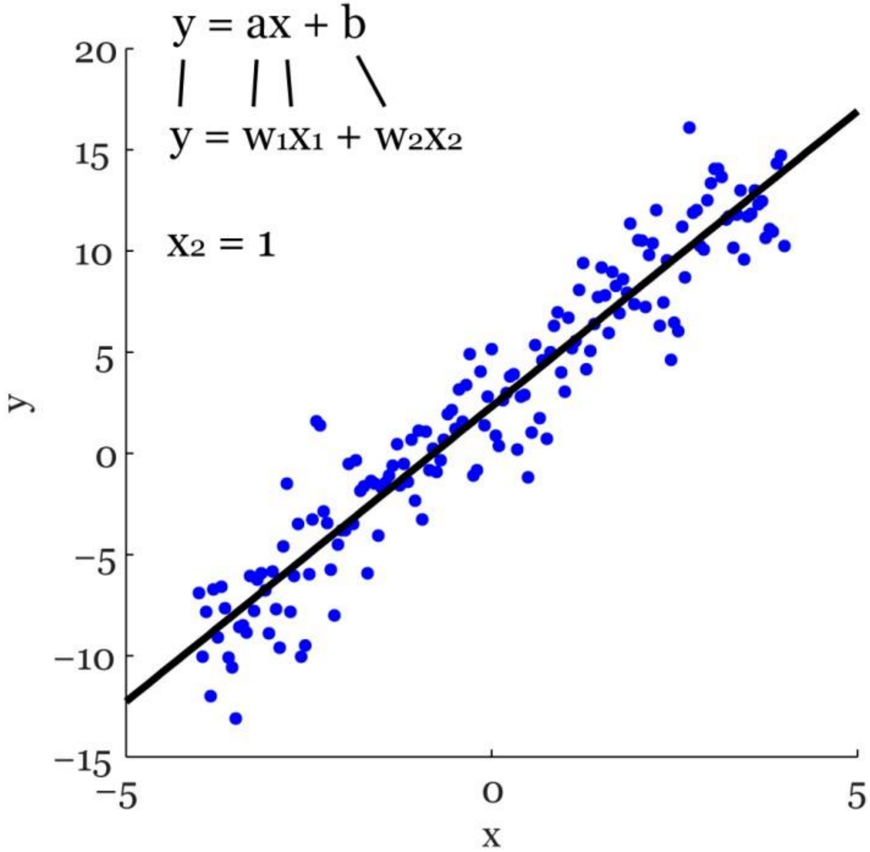
Types of models



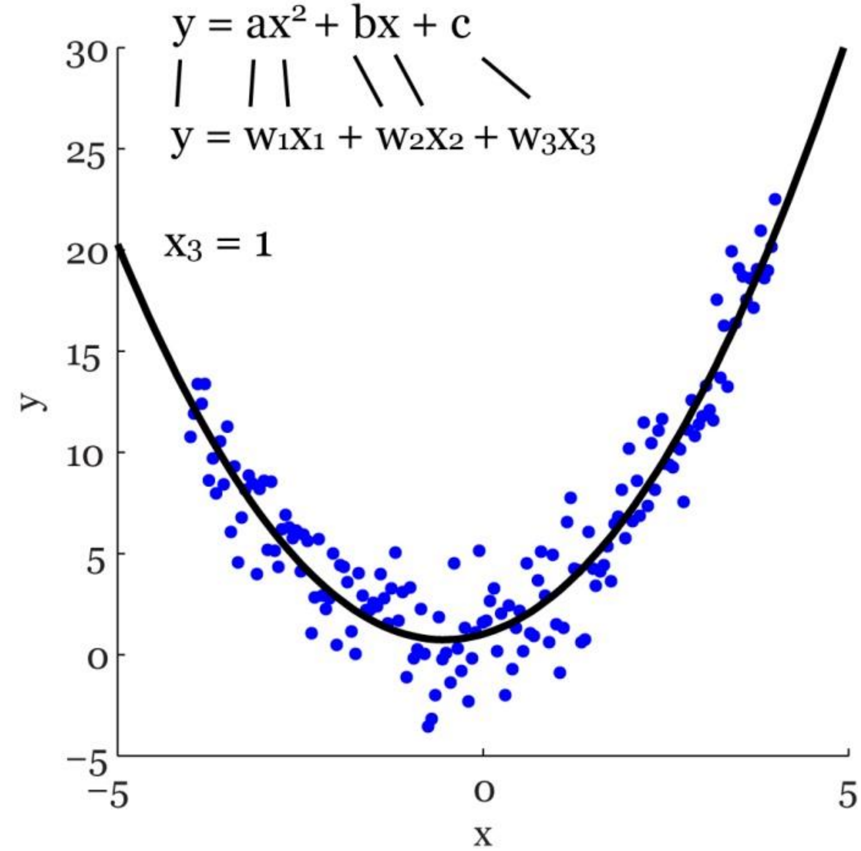
Supervised learning



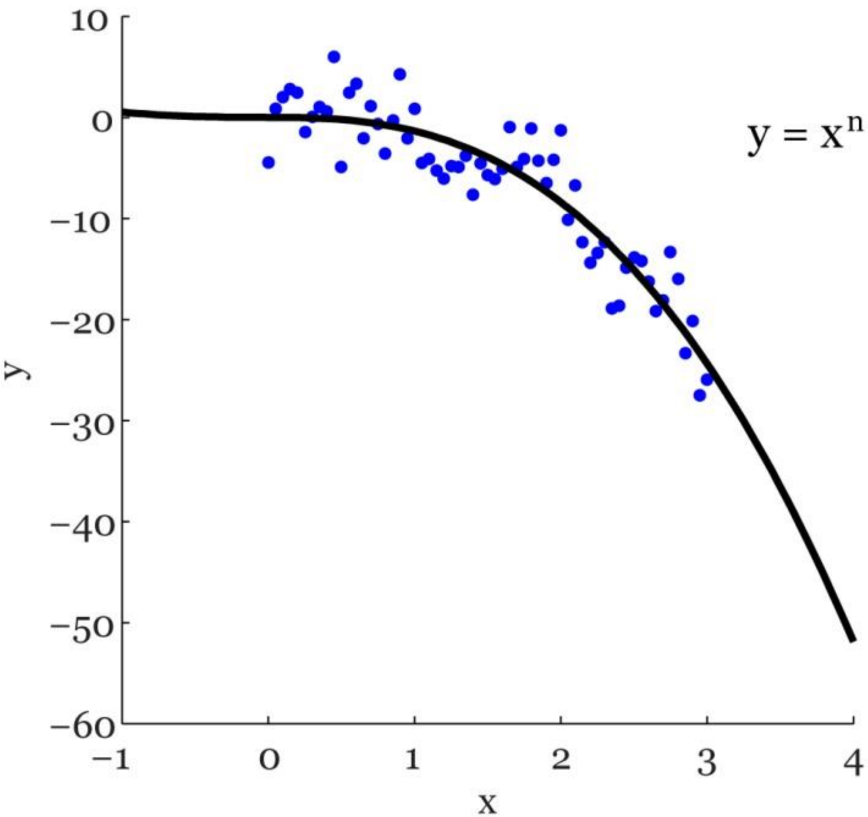
Linear model



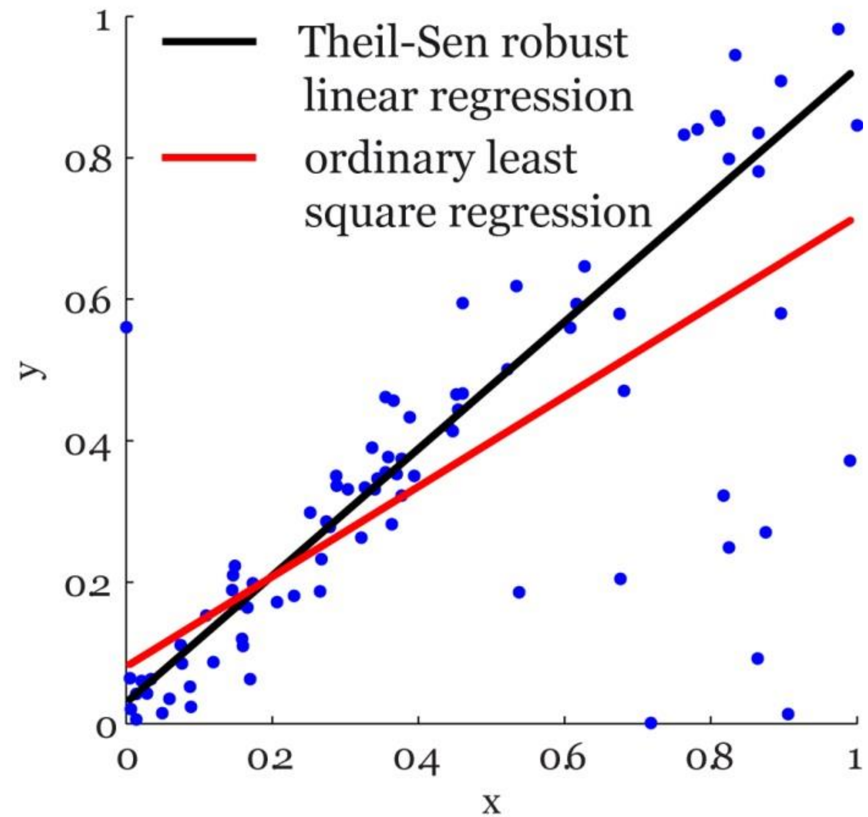
Linearized model



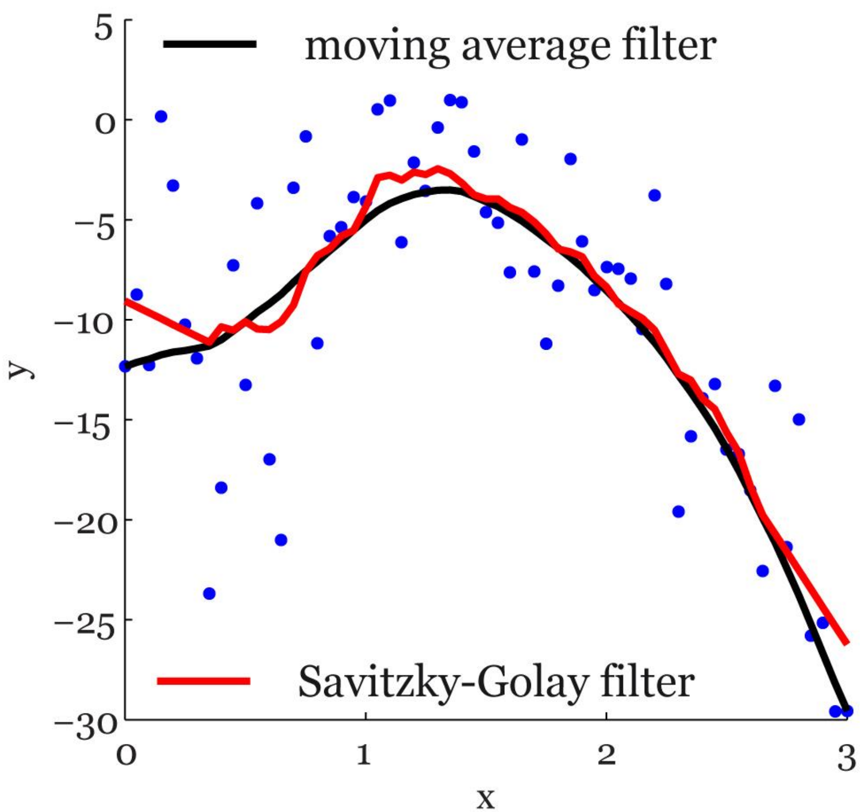
Parametric nonlinear model

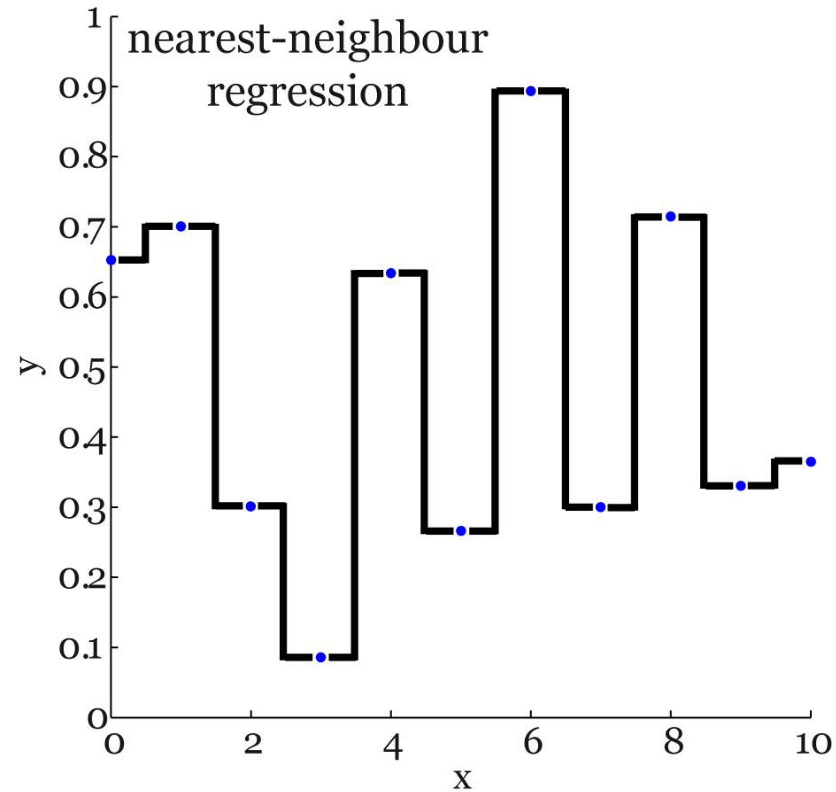


Nonparametric linear model



Nonparametric nonlinear model

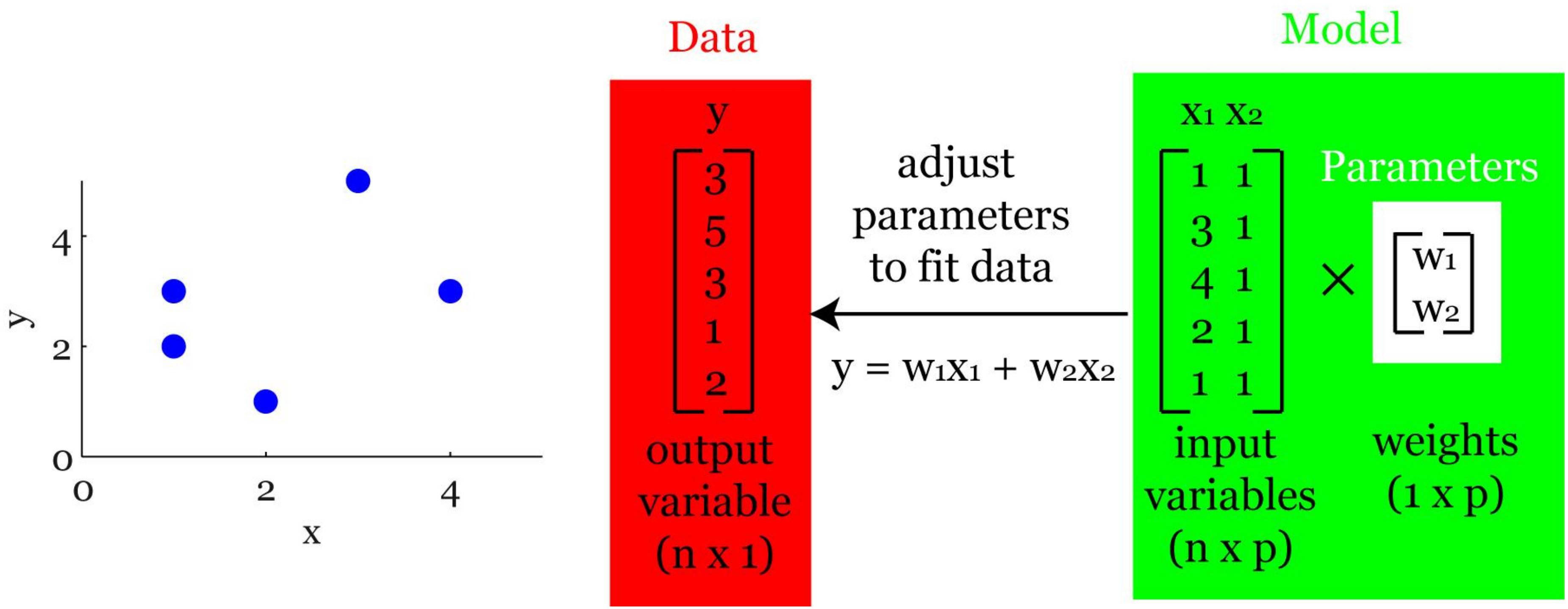




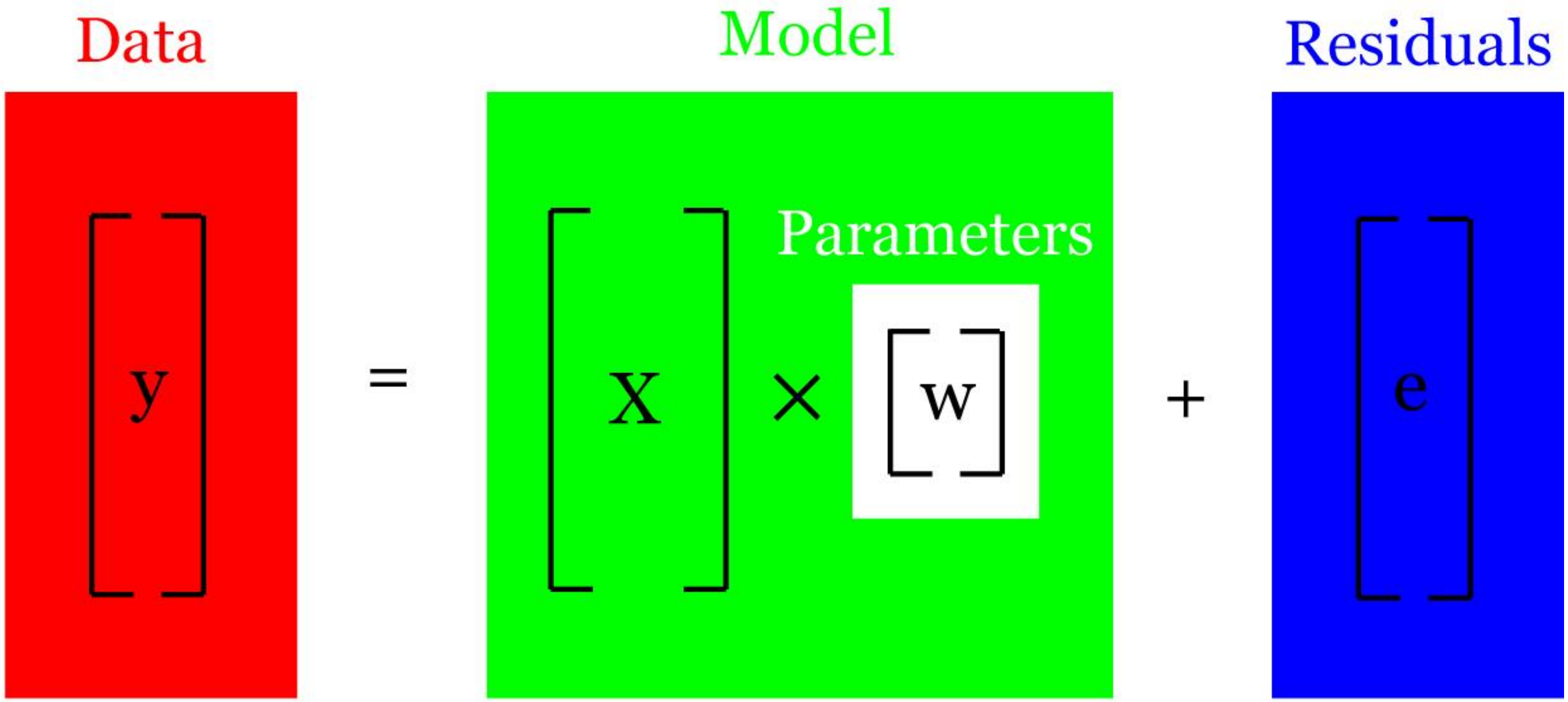
Model characteristics

	Linear	**Parametric**	**Linear in parameters**
Linear models	✓	✓	✓
Linearized models	✗	✓	✓
Parametric nonlinear models	✗	✓	✗
Nonparametric linear models	✓	✗	✗ ✓
Nonparametric nonlinear models	✗	✗	✗ ✓

Matrix representation of linear model

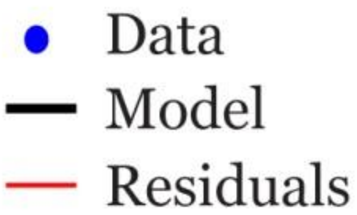


Matrix representation of linear model

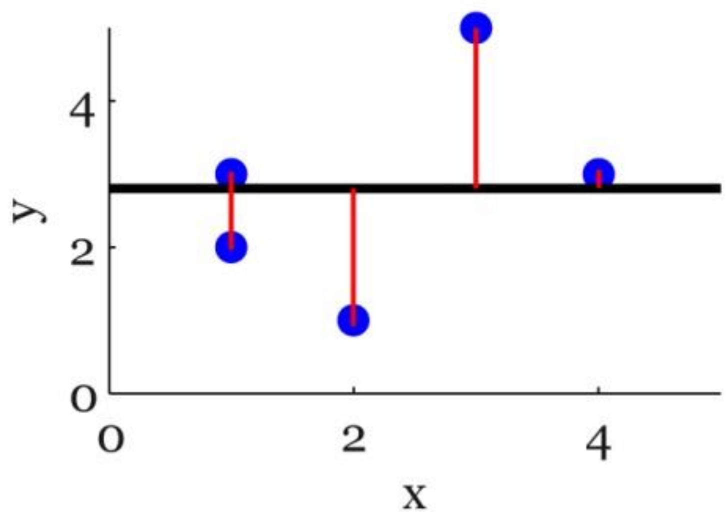


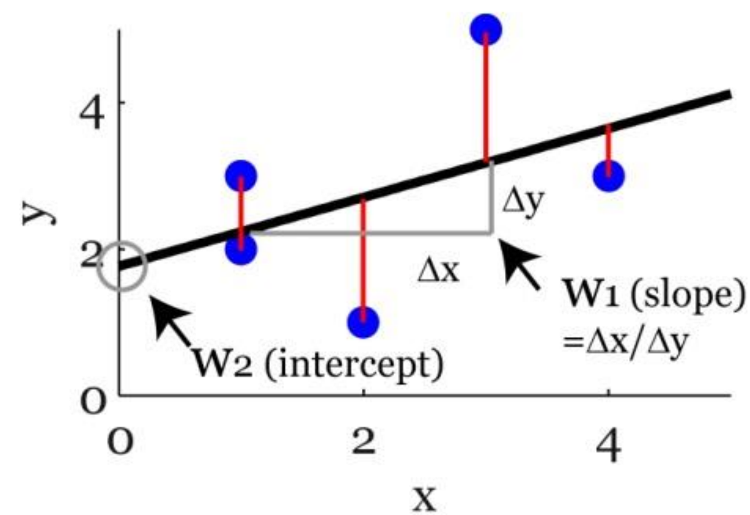
$$\mathbf{y} = \mathbf{X}\mathbf{w} + \mathbf{e}$$

Squared error



$$squared\ error = \sum_{i=1}^{n}(d_i - m_i)^2$$



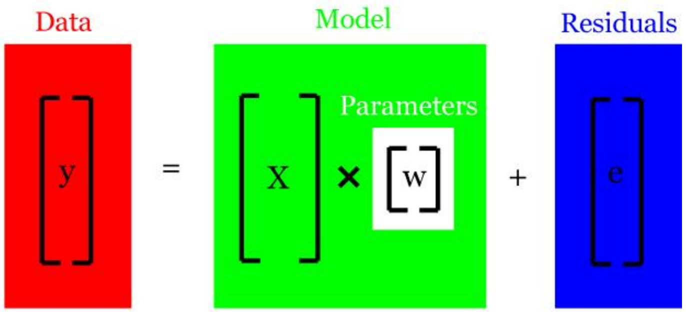


$y = w_1x + w_2$
$w_1 = 0$
$w_2 = mean(x) = 2.8$
$y = 2.8$
squared error = 8.8

$y = w_1x + w_2$
$w_1 = 0.47$
$w_2 = 1.76$
$y = 0.47x + 1.76$
squared error = 7.29

Matlab example 1

Ordinary least squares solution



$\mathbf{y} = \mathbf{Xw} + \mathbf{e}$

regressors · residuals = 0

$X^T e = 0$

regressors · (data – modelfit) = 0

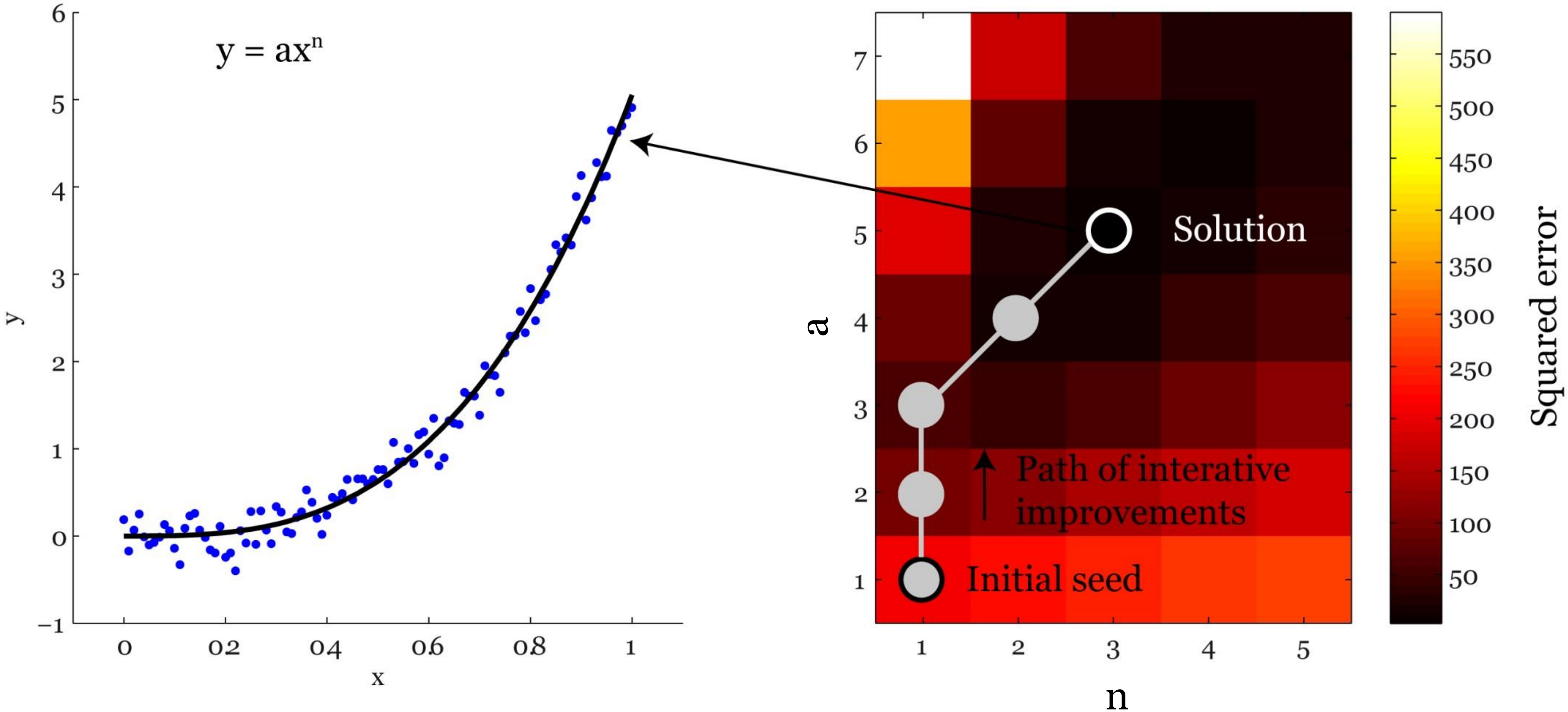
$X^T(y - Xw) = 0$

$X^T y - X^T X w = 0$

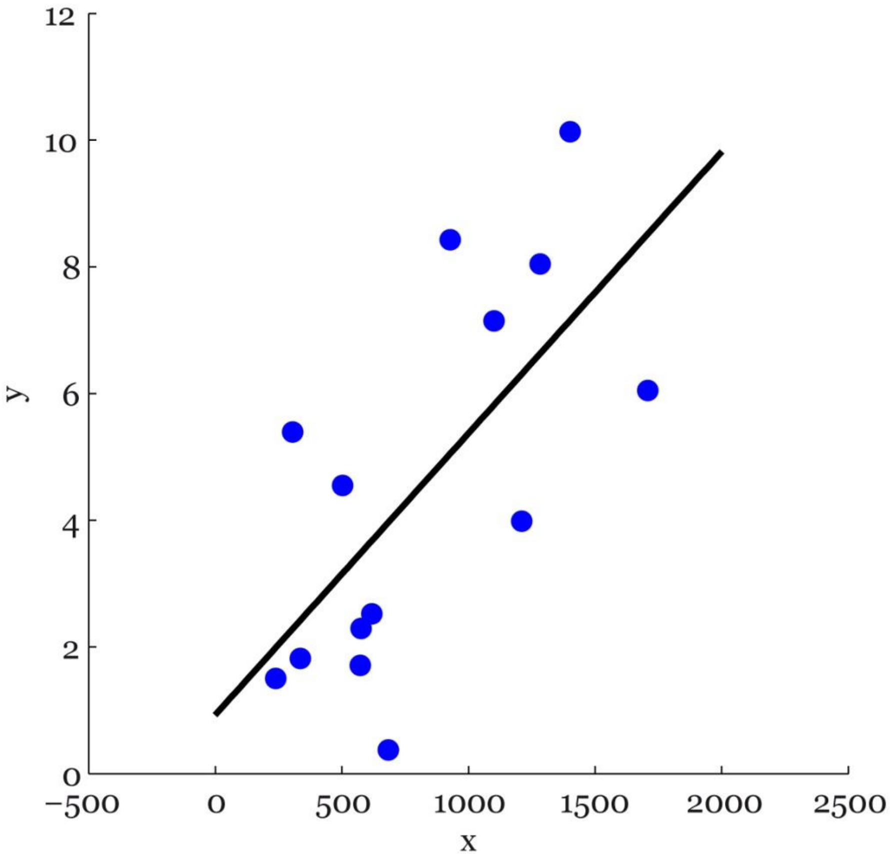
$w = (X^T X)^{-1} X^T y$

Matlab example **2**

Fitting nonlinear model based on local, iterative optimization



Quantifying model accuracy

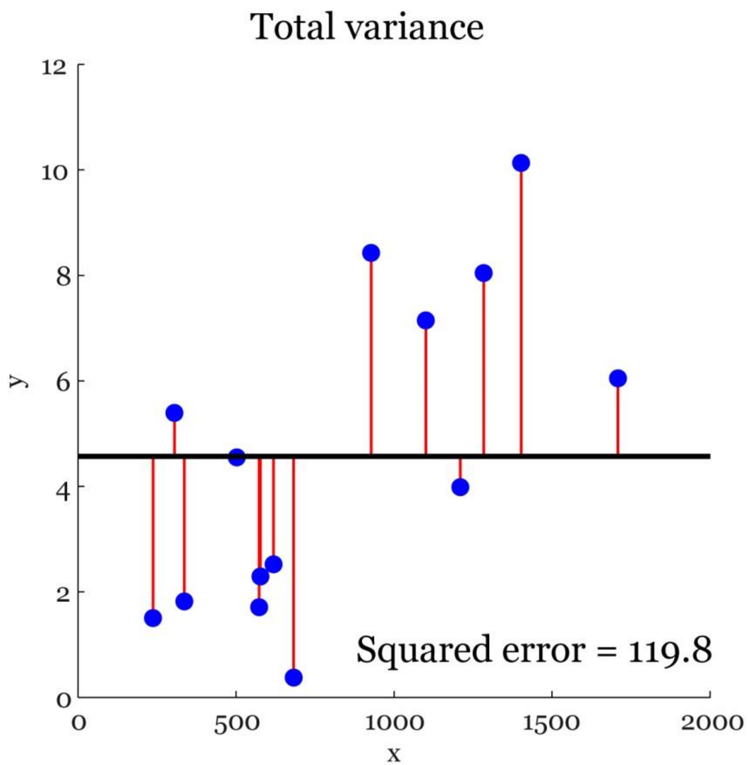


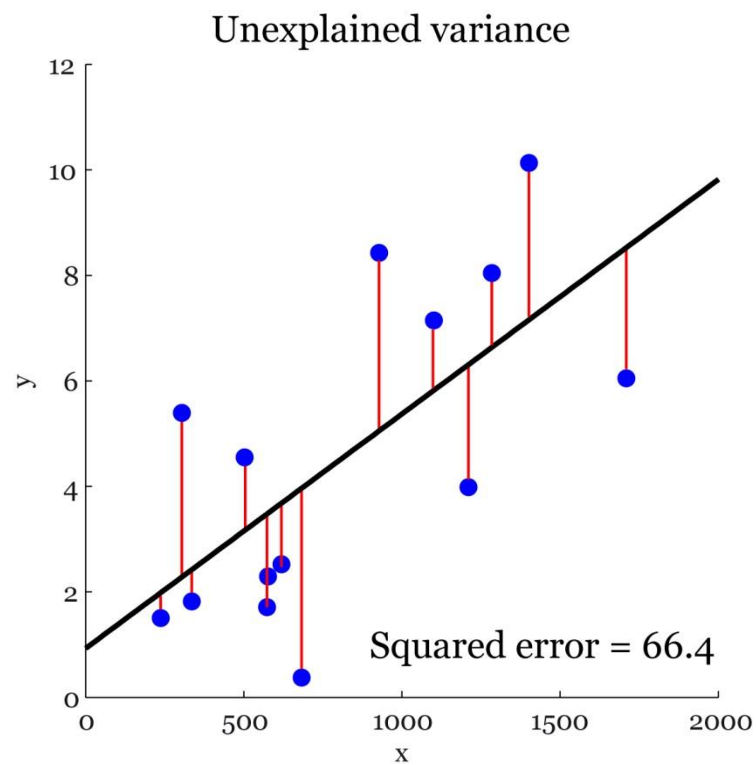
Squared error = 66.4
(dependent on units, hard to interpret)

R^2 = 44.6%
(independent on units, easy to interpret)

$$variance = \frac{\sum_{i=1}^{n}(d_i - \bar{d})^2}{n-1}$$

Coefficient of determination (R^2)





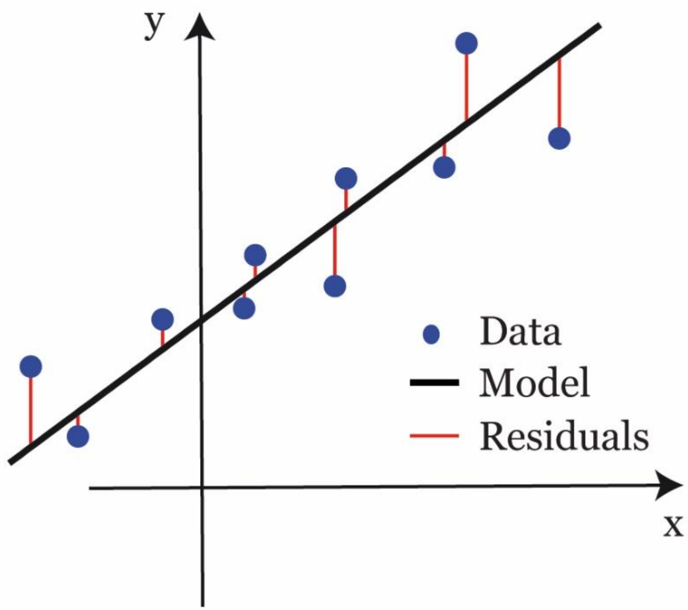
$$R^2 = 100 \times \left(1 - \frac{unexplained\ variance}{total\ variance}\right)$$

$$R^2 = 100 \times \left(1 - \frac{SE\ model\ fit}{SE\ model\ mean}\right)$$

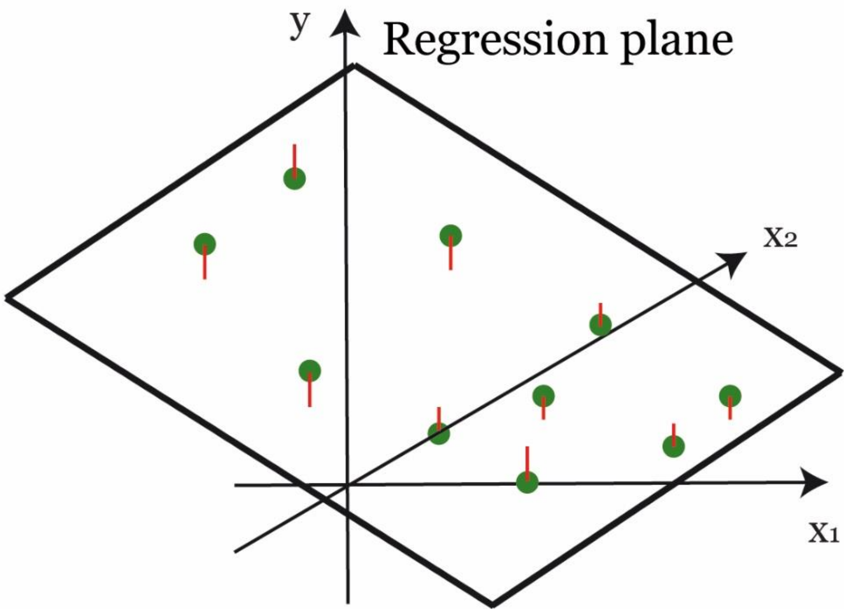
$$R^2 = 100 \times \left(1 - \frac{\frac{\sum_{i=1}^{n}(d_i - m_i)^2}{n-1}}{\frac{\sum_{i=1}^{n}(d_i - \bar{d})^2}{n-1}}\right)$$

$$R^2 = 100 \times \left(1 - \frac{\sum_{i=1}^{n}(d_i - m_i)^2}{\sum_{i=1}^{n}(d_i - \bar{d})^2}\right)$$

Simple linear regression



Multiple linear regression



Research project: Parkinson's disease (PD), stuttering & L-dopa

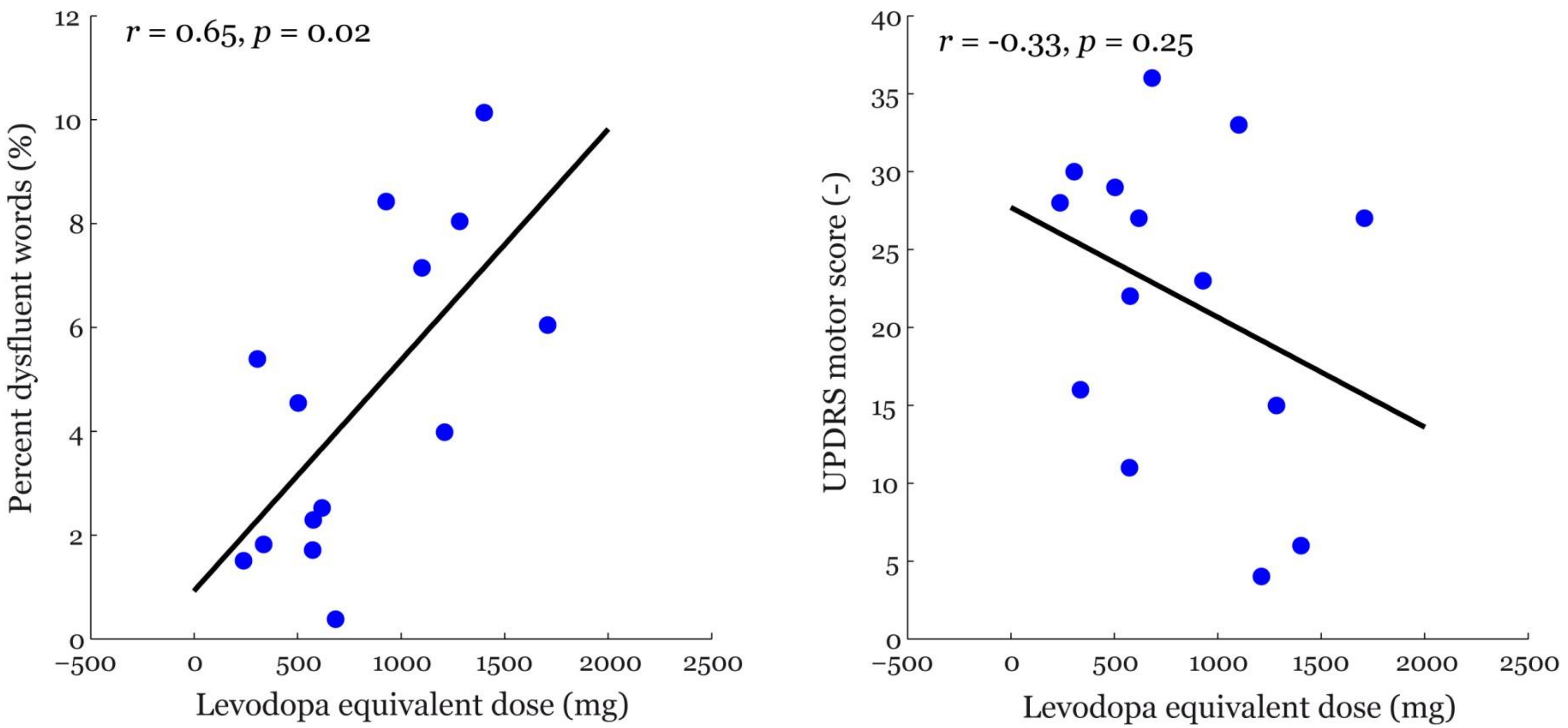
Background:

- Excess dopamine theory of stuttering suggests that stuttering may be related to an excess amount of dopamine within the brain
- Some patients with PD develop stuttering in the course of their illness
- Levodopa is precursor of dopamine used to treat motor manifestations of patients with PD

Hypothesis:

- Stuttering is related to extent of L-dopa doses
- Stuttering is not related to motor speech manifestations in PD

Research project: Parkinson's disease (PD), stuttering & L-dopa



Matlab example 3

Research project: Parkinson's disease (PD), stuttering & L-dopa

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Linear regression model:
    y ~ 1 + x1 + x2

Estimated Coefficients:
                   Estimate        SE          tStat       pValue
                  __________    _________    ________    ________

    (Intercept)       1.1987       2.4258     0.49413     0.63093
    x1             0.0043749    0.0015763      2.7755    0.018049
    x2            -0.0097572     0.071821    -0.13586     0.89439

Number of observations: 14, Error degrees of freedom: 11
Root Mean Squared Error: 2.46
R-squared: 0.447,  Adjusted R-Squared 0.346
F-statistic vs. constant model: 4.44, p-value = 0.0386
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stuttering: (Intercept); L-dopa: x1; UPDRS: x2

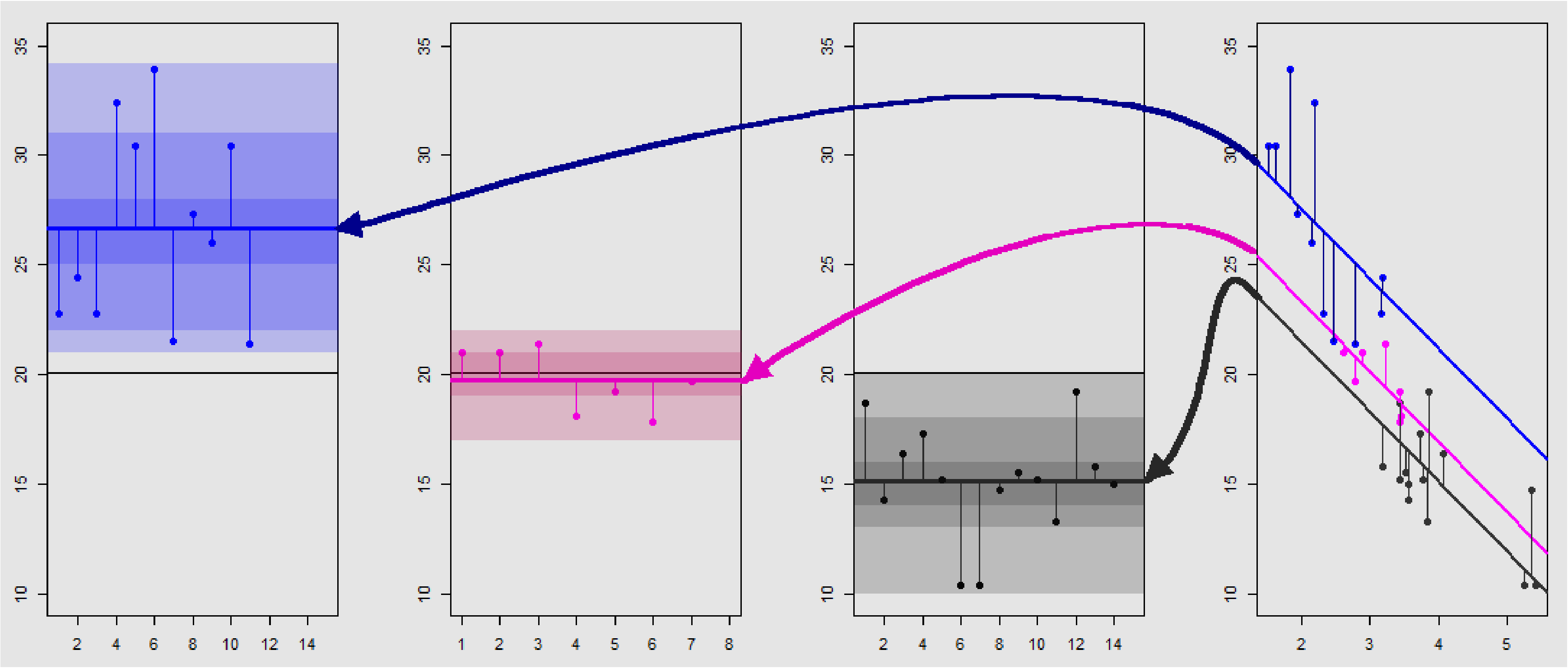
How to report results of regression?

Our case: [$F(2,11) = 4.4$, $p = 0.04$, $R^2 = 0.45$]

Some conclusions? :-)

Matlab example **4**

- ANOVA and linear regression analysis are the "same thing"



- *Intercept is the mean of the reference group*
- *The coefficients for the other two groups are the differences in the mean between the reference group and the other groups*