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Visual rays are parallel

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Abstract. We show that human observers using monocular viewing treat the pencil of 'visual rays' that diverges from the vantage point as experientally parallel. This oddity becomes very noticeable in the case of wide-angle presentations, where the angle subtended by a pair of visual rays may be as large as the angular size of the display. In our presentations such angles subtended over 100 deg. There are various ways to demonstrate the effect; in this study we measure the attitudes of pictorial objects that appear to be situated in mutually parallel attitudes in pictorial space. Our finding is that such objects appear parallel if they are similarly oriented with respect to the local visual rays. This leads to 'errors' in the judgment of mutual orientations of up to 100 deg. Although this appears to be the first quantitative study of the effect, we trace it to qualitative reports by Helmholtz (late 19th century) and Kepler (early 17th century) as well as speculation by early authors (AD 500). The effect has apparently been noticed by visual artists from the late middle ages to the present day.

1 Introduction

Throughout the history of Western art one encounters frequent systematic deviations from true linear perspective. It is debated whether these are due to unfamiliarity (before the 'invention' of linear perspective in the early Renaissance, Alberti 1435 – 1436/1970) or are intentional digressions (as in modern art). In this paper we show that many of such deviations reflect a hitherto hardly undocumented trait of human vision (Kepler 1604/2000; Helmholtz 1896; Koenderink et al 2000; Cuijpers et al 2000, 2001; James et al 2001; Kappers and te Pas 2001; Koenderink et al 2009): although the eye samples the radiance at its entrance pupil from many optical directions converging on that vantage point, the 'visual rays' corresponding to these directions are experienced as a mutually parallel pencil (as predicted by theories of visual space—Koenderink and van Doorn 2008). As a consequence, and perhaps surprisingly, human observers commit characteristic deviations from veridicality of up to 100 deg in the visual estimation of the relative spatial attitudes of objects distributed in a scene. Mutually parallel, congruent objects in the space in front of the observer are perceived as mutually rotated over an angle roughly equal to the angular separation of their respective optical directions. Apparently spatial attitude is referred to the local visual ray, rather than the geometry of the scene, leading to large systematic deviations in wide-angle views (Koenderink et al 2009).

In retrospect, there turn out to exist many suggestive threads relating to this topic, in both the arts and the sciences. Consider figure 1, a 15th century illumination by Jean Fouquet (ca 1460), which shows systematic deviations from true linear perspective. This is especially evident from the rendering of the tiled ground plane. That this is not due to Fouquet's ignorance of technique is evident because he renders the 'pavimento' according to linear perspective in other (including earlier) works. Quite similar deviations are encountered throughout art history, a well known more recent example being Vincent van Gogh's *Bedroom at Arles, Saint-Rémie* of 1889. There exists an earlier drawing

Figure 1. [In colour online, see http://dx.doi.org/10.1068/p6530] Jean Fouquet: *l'Entrée de l'Empereur Charles IV Saint-Denis* (ca 1460). Notice the curvature of the frontoparallels in the tiled floor plane. Jean Fouquet was perfectly able to construct a true perspective 'pavimento' as is evident from many of his works. The curvatures no doubt are used intentionally.

(van Gogh 1888) of the view without the curvatures of the frontoparallels of the floor-boards, again proving that also van Gogh applied such curvatures *intentionally*.

One wonders what goes on here: apparently the artists (at least from the mid-15th to the late-19th centuries) are well aware of the 'correct' way of rendering wide-angle views, yet they chose to apply systematic 'errors'. No doubt the intent is to increase the sense of 'realism' experienced by the intended viewers of the picture. Thus the final cause is apparently the peculiar structure of generic human visual experience.

There exists at least anecdotal evidence for such an interpretation. For instance, both Kepler and Helmholtz, each of them exceptionally good and analytic observers, remark on the observation that, although the visual field is very wide (about a half-space, corresponding to the field of view of a 'fisheye lens'—see below), it actually appears to be much smaller than a half-space. Everything looks like it being situated 'in front of the observer'. Thus Kepler (1604/2000) spontaneously notes his surprise that:

> "I have often seen, to my amazement, both the sun and my shadow, as if the two were out in front and not opposite each other."

This is evidently the surprise of a genius, for apparently no one in the history of mankind noticed this fact before, yet it is there frequently enough for all to see! Likewise, Helmholtz (1896) notices that

> "... the visual globe, which in the geometrical sense embraces a horizontal angle of about one hundred eighty degrees, seems indeed to be much narrower than this. [But] on looking up at the sky, the bright field in front of us appears to have an angular diameter of about ninety degrees ..."

Thus, Helmholtz estimates the apparent field of view, which he *knows* to subtend about a half-space, as more like a right angle. This corresponds to the width of the 'visual cone' of the ancient authors. For instance, Heliodorus of Larissa (ca 500) holds the visual field to subtend *exactly* 90 deg. However, this is not an *observation*, but a philosophical conclusion: since the visual field must be perfect, it subtends a perfect right circular cone and (being perfect) it can be neither acute nor obtuse, hence *has* to be a right angle. We believe the remarks of Kepler and Helmholtz to result from *actual observations* though, not from merely echoing classical ideas.

2 Experiment

If the pencil of convergent optical directions is indeed experienced as a pencil of more-or-less parallel visual directions, one would expect apparent rotations of objectively parallel objects. This would be a very important effect, a peculiar kind of blindness common to the human condition. However, at present no more than such anecdotal evidence exists. We put this expectation to the empirical test.

2.1 *Methods*

Observers sat in front of a large (about 1 m wide) flat display. The viewing distance was very short (24 cm); thus the angular width of the display was about 130 deg. The viewing was monocular, from a fixed vantage point. Because the observers needed both lateral eye movements and minor head rotations about the vertical, special measures were taken to ascertain the constancy of the vantage point. (Observers pressed the inferior rim of their orbit against a rigid support.) This is important because we need to ensure that the vantage point coincides with the perspective centre. The monocular cues for the distance and slant of the display in the various directions are monocular parallax, accommodation, and texture gradients (due to the pixelation of the screen). Thus the observers were indeed aware of the plane of the display; they could vaguely make out its frame in the semi-darkened room. We have reason to assume that none of these conditions is especially relevant, given previous experiences (Cuijpers et al 2000, 2001).

The display contained renderings of two cubes; one of these could be rotated about the vertical under the observer's control. During the setting, the two cubes were continually visible. The cubes appeared at a number of predetermined positions, equally distributed over the (horizontal extent of the) visual field and including the straight-ahead direction.

The task was to make the cubes appear as being in mutually parallel attitudes in (pictorial) space. Ambiguities were avoided by colouring the six faces of the cubes differently. The cubes were placed in random attitudes so as to prevent the use of (trivial) two-dimensional cues. This is a very important condition. The random attitudes (random rotation about a random axis of rotation) render the task far from trivial. In a canonical attitude, eg with a frontally faced face, the task can be done in a two-dimensional manner: simply make that face a square (or diamond, etc) on the screen. Then a comparison isn't even required—the task can be done with a single cube. (This was the case in the first experiment described by James et al 2001, for example.)

We have good reasons to believe that (granted their random spatial attitudes) the cubes yield results similar to those for any other sufficiently generic objects. For instance, we show below (figure 3) that human figures yield essentially the same results. In the latter case, the apparent rotations are also seen in the scene, not just in the picture of the scene (the illusion reported by James et al 2001).

The stimulus parameter was the attitude of the fixed cube and the response parameter the adjusted attitude of the variable cube. Results were averaged over trials in which the role of the fixed and variable cube was exchanged and over repeats with different random attitudes of the cubes.

2.2 *Results*

We find that signs of apparent rotations are readily apparent if one looks for them because the deviations from veridicality are very large, values of up to 100 deg are easily demonstrated. In the main experiment we compare the mutual spatial attitudes of two mutually congruent objects (cubes in random spatial attitudes) seen in different optical directions. The observers view a display on which the objects are rendered and thus experience them in 'pictorial space'. The rendering is in perfect linear perspective of a (virtual) scene. In this scene the objects are placed on a horizontal, frontoparallel line.

One of the objects can be rotated about the vertical and the task is to set it parallel to the fiducial object *in the scene*. We measure the attitude difference, an angular rotation about the vertical, as a function of the angular separation of the objects with respect to the vantage point.

The result is shown in figure 2. It may be concluded that normal visual observers of all ages and genders effectively ignore optical direction when comparing spatial attitudes of congruent objects in a scene. The slope of the linear regression is 0.84, only slightly shallower than the prediction for the case of perfect neglect of optical directions (the line at unit slope) and very different from the prediction for veridical perception (the horizontal line).

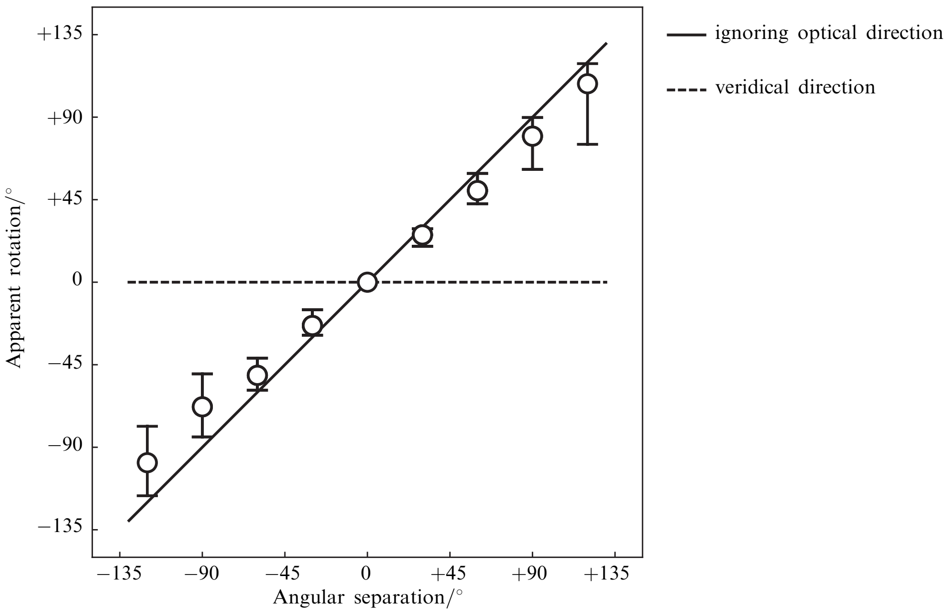

Figure 2. The attitude difference of two congruent objects appearing in parallel attitudes in the scene as a function of their angular separation in the visual field. The circles show the median values, the dashes the 25% and 75% quartiles. Notice that the circle at the origin is obtained 'for free' (not an actual observation). The drawn line shows the prediction for the case of perfect neglect of true optical directions, whereas the dashed line shows the prediction for veridical perception. This result is based on sixteen observers of all ages, matched for gender.

2.3 *Consequences for rendering and display*

These effects are sufficiently large that they give rise to the visually striking effects that are frequently categorised as 'the distortions of wide-angle lenses', despite the fact that such lenses yield perfect linear perspective by design. An example is shown in figure 3 top (made with a well corrected lens, horizontal visual field 104 deg, 'correct' viewing distance 39% of the picture width). In order to obtain a rendering of a row of people in strict military order on a frontoparallel row, one needs to place them on a circular arc about the camera and have each face the camera (thus destroying the *actual* but bringing about the *apparent* military order). This will still look somewhat deformed because the persons at each end will look 'fat' (horizontally dilated), an effect that can be avoided by using the equiangular (Postel, see Barre et al 1967) projection realised in many 'fisheye' lenses (figure 3 bottom). [So-called 'fisheye lenses' derive from the famous Hill (1926) lens. It covers 180 deg yielding a 62 mm diameter image of the whole visible sky. Typical modern designs for 35 mm cameras use equiangular projection.] This projection is very similar to that applied by Jean Fouquet and Vincent van Gogh. Its formal description was possibly pioneered by Leonardo da Vinci

Figure 3. Top: linear perspective of a frontoparallel row of people in strict military order (see figure 5 top). Notice that people at each end are seen in profile, they look 'rotated' with respect to the 'straight ahead' direction. Bottom: equiangular projection of a circular (about the camera) row of people, all facing the camera (see figure 5 bottom). Although the curvature of the rendering of straight lines is apparent, the row of people looks frontoparallel and in strict military order.

(White 1957) and was popularised by Barre et al 1967 (compare Hauck 1879) as a means to render wide architectural scenes. As we have shown here, such a projection perfectly fits the visual experience of generic human observers. [Technically this is not a 'projection' at all, but it refers to 'Riemann (1867) normal coordinates'.]

In a follow-up experiment we asked naive observers which photograph (figure 3 top or bottom) "looks most as a row of people arranged in a frontoparallel line in strict military order". The result—as expected—is that the overwhelming majority (17 out of 20; thus 85%) immediately goes for the lower figure. The remaining three were puzzled by the curvilinear architectural details. The linear perspective renderings (figure 4) look unnatural as viewed from normal reading distance, but the apparent orientations look mutually parallel. This remains the case if the photographs are viewed from the perspective centre, which is about a third of the picture width.

Although rarely (if ever) mentioned, the effect is by no means isolated. A striking example is Ferdinand Hodler's (1892) large painting *Die Lebensmüden* at the Neue Pinakothek, Munich. It shows five persons on a bench in frontoparallel arrangement, much as the arrangement in figure 3 top. The figures have been rotated as in figure 3 bottom, though the bench is rendered in perfect linear perspective. The figures at each end appear severely rotated with respect to the bench, offering the example of a striking local inconsistency. Once this inconsistency has been pointed out (few people appear to notice it spontaneously) it sticks out as a sore thumb. We explore the perspective of this particular painting in the appendix.

A reviewer of this paper suggested that the effect will only occur for pictures, but not for real objects. We have good reasons to doubt this, as closely related effects have been reported to apply in the case of physical space (Koenderink et al 2000; Cuijpers et al 2000, 2001; Kappers and te Pas 2001). Moreover, in taking the photographs (figure 3) the effect was clearly apparent to the photographer. One study that suggests the effect does not occur at all is by James et al (2001) which was done with

Figure 4. Top: linear perspective of a circular (about the camera) row of people, all facing the camera (see figure 5 bottom). Notice the apparent strong size differences. Bottom: linear perspective of a frontoparallel row of people facing the camera (see figure 5 centre). Notice the strong perspective distortions. The equiangular projection yields by far the most convincing impression of a strict military order as viewed from normal reading distance.

real objects. We are at a loss to explain this report because these authors evidently observed the effect described here. In their discussion they remark:

> "We should note that the two-cube set-up leads to a compelling visual illusion. The illusion is best seen when a pair of identically oriented cubes is placed with first cube centred and its front face perpendicular to the line of sight and the second cube in a peripheral position. When one looks at these two cubes, they do not appear to have the same orientation in three-dimensional space. For example, the peripheral cube appears to be rotated too far to the right if it is on one's right. Like other visual illusions, knowing that the two cubes are oriented the same in space does not appear to change the magnitude of the illusion." (page 3453)

This 'illusion' is clearly our 'effect'. However, in their formal results these authors find no sign of the illusion at all, they report 'veridical' results. The striking discrepancy between eye measure and formal data must be due to the precise methodology followed; we don't venture an explanation. Apparently the issue calls for further, detailed, study.

This striking result at first blush might seem very odd; one wonders how people might negotiate the world successfully? A likely answer is that vision is just an interface, not necessarily 'veridical' in the naive sense (Hoffman 2009). Moreover, spatial attitudes can often be referred to the local context (eg a car being parked to the curb). Perhaps surprisingly, local inconsistencies can often be found in the visual arts albeit apparently rarely noticed by either the artist or the viewers.

This seemingly odd finding is not specific to the visual faculty, for there exists a close parallel in haptic perception (Kappers 1999), where (blindfolded) observers ignore the rotation of their arm in the shoulder joint and judge the relative spatial attitudes of two mutually congruent objects presented at a laterally diverse location in their reach space according to their hand frames. The deviations from veridicality in the haptic case are as large as those in the optical case. Until quite recently such huge haptic effects—as in the optical case—went undocumented.

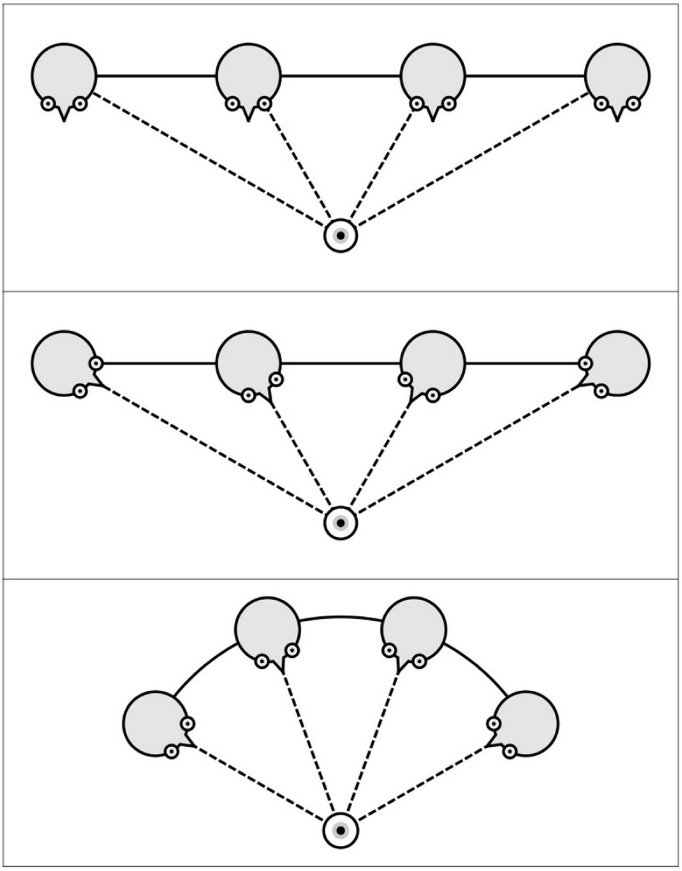

Figure 5. The actual setups, for the photographs shown in figures 3 and 4. At top the actors are lined up in perfect military order; this yields the photograph shown in figure 3 top. The actors are mutually parallel though they don't look it. At bottom the actors are arranged on a circular arc concentric with the camera. Moreover, they all face the camera, thus are in no way arranged parallel to each other in space. This yields the photograph shown at figure 3 bottom and figure 4 top. The actors are far from being mutually parallel though they do look it. In the picture at centre the actors are lined up in frontoparallel, but face the camera. This is the case of the photograph in figure 4 bottom.

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Appendix: Analysis of a painting by Ferdinand Hodler

The painting *Die Lebensmüden* (see figure A1) is apparently in perfect one-point perspective. It may be assumed that the seat of the bench is rectangular, from the left and right ends one immediately finds the vantage point VP. It lies slightly above the top of the painting (about a head's height) and a little to the right of the middle, above the left (hanging) hand of the central person. This allows one to draw the horizon. In order to do actual measurements one additionally needs to calibrate the depth (eg establish the distance points).

Apart from the central figure all figures are sitting on the bench with the upper parts of their legs in the sagittal plane of their bodies, which again are aligned with the bench. This accentuates the bilateral symmetry of the figures that helps greatly to make this such a striking picture. It allows us to infer the fore – aft axes of the figures. In the perspective these are vertical lines that do not converge on the main vanishing point on the horizon (as they should if the figures were arranged in perfect military order) but in a common point at a very large, effectively infinite, distance above the horizon. This indicates that the figures are not at all lined up in parallel fashion, but it does not allow us to read off the angle. However, it does show that the mutual angle of rotation of any two figures is equal to their angular separation, which is the main message of the paper, even independently of any calibration. Such a calibration is perhaps desirable because it would serve to put a numerical magnitude on the deviations.

In order to achieve such a calibration one needs to establish a fiducial length as the unit in the depth dimension. One possibility is to use the fact that the depth of a bench tends to equal the width of the average person. For example, seats of kitchen-chairs

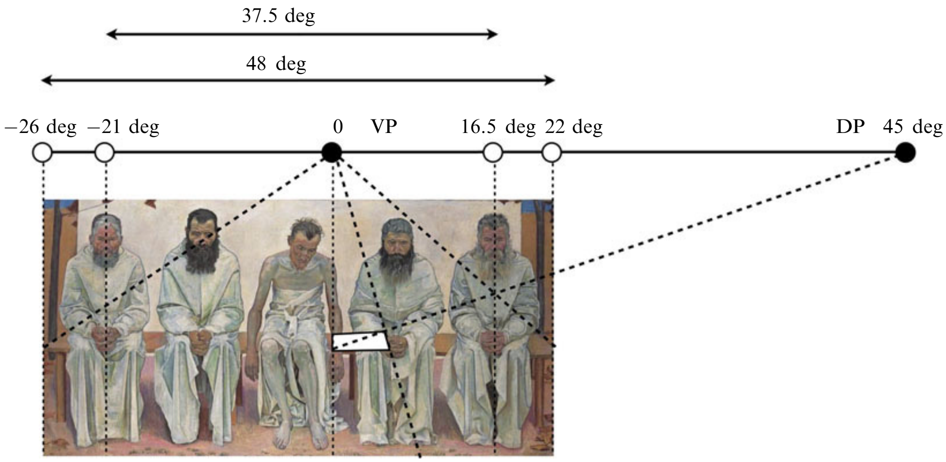

Figure A1. [In colour online] The painting *Die Lebensmüden*, by Ferdinand Hodler (1892, oil-tempera on canvas, 149.7 cm × 294.0 cm, Neue Pinakothek, Munich), overlaid with a perspective calibration grid. The vanishing point of the one-point perspective of the bank is VP, the right distance point is estimated on the horizon at DP. The planes of bilateral symmetry are indicated by the vertical lines.

tend to be square. This immediately allows one to draw a diagonal and find a distance point DP on the horizon (figure A1), the point at top far right. This again allows one to attach a nonlinear azimuth scale to the horizon. The horizontal angular width of the painting, that is the extent of the field of view, then can be read off. It is about 48 deg of visual angle, roughly the width of a photograph taken with a 'normal' or 'standard' lens.

From the fore – aft body axes of the two outermost figures we find that they mutually subtend an angle of about 38 deg.

Here we have perhaps slightly underestimated the various angles, just to stay on the safe side. The 'error'—perhaps an unfortunate term because no doubt the arrangement was intentional by the painter—is thus very appreciable, about an angle of 40 deg. The painter did something very similar to what we did in the photographs, thus greatly augmenting the impact of the composition.

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