Soft Connection

Tom Kealy

December 18, 2013

This is a short note to quickly justify some bounds on the power constant β, needed to connect a rgg with soft connection function parametrised by β.

A result of Gupta and Kumar (Critical Power for connectivity in Wireless Networks) is that if all nodes radiate power to fill a disk of radius r, then the network is asymptotically connected iff:

$$\pi r^2 = \frac{logn + c\left(n\right)}{n} \tag{1}$$

This implies that there is a radius r_{conn} above which the network is connected:

$$r_{conn} = \sqrt{\frac{logn}{\pi n}} \tag{2}$$

and the diameter of the connected component is

$$d = \Theta\left(\sqrt{\frac{\pi n}{logn}}\right) \tag{3}$$

In the following we compare the rgg with soft connections to the equivalent rgg with hard connections: the minimal β we should choose, so that the (soft) graph is connected is the beta that puts a $1-\varepsilon$ ammount of mass in the (hard) circle around the nodes

In the case of a Rayleigh distributed variable:

$$F\left(r\right) = \int_0^{r_\beta} 1 - e^{-\beta r^2} dr = r_\beta - \frac{\sqrt{\pi}\,\mathrm{erf}\left(r_\beta\sqrt{\beta}\right)}{2\sqrt{\beta}} = 1 - \epsilon \tag{4}$$

choose $r_\beta \geq r_{conn}$ and the graph should be connected w.h.p. I.e $r_\beta = r_{conn} + \delta$ (choose a circle so that most of the mass of the distribution is in the circle, but the radius ensures connection).

$$\sqrt{\frac{logn}{\pi n}} - \frac{\sqrt{\pi}\,\mathrm{erf}\, r_\beta \sqrt{\beta}}{2\sqrt{\beta}} = 1 - \epsilon \tag{5}$$

This should allow you see how varying β changes the diameter of the connected component.