Asynchronous Network Model: Notes

Tom Kealy

October 21, 2014

1 Model

We are given a connected graph $G = (V, E)$ with vertex set V, $|V| = n$ i.e. there are n nodes. E is the edge set, which is finite: $|E| = m$, for some $m \in \mathbb{Z}$.

We are given some measurements, according to some (possibly underdetermined) linear system, and we wish to find a solution. For example, we wish to compute the average of a set of numbers. We write $x(0) = [x_1(0) \dots x_n(0)]^T$ for the initial data at each node, and $x_{av} = \frac{1}{n}\sum_i x_i(0)$ for the average of the entries. We wish to compute x_{av} in a distributed manner.

1.1 Asynchronous time model

At node i, place a Poisson clock, with rate λ_i (for the sake of simplicity let $\lambda_i = 1\ \forall i$ in what follows). The inter-tick times at each node are rate 1 exponentials, independent across nodes and over time. This is equivalent to having a single clock ticking according to a rate n Poisson process at times Z_k, $k \geq 1$, where $\{Z_{k+1} - Z_k\}$ are i.i.d exponentials of rate n. We denote the node whose clock ticked by time Z_k as $I_k \in \{1 \dots n\}$. The I_k are i.i.d distributed across $\{1 \dots n\}$.

Time is discretised according to clock ticks (these are the only times that $x(\circ)$ changes). Thus, the interval $[Z_k, Z_{k+1})$ denotes the k^{th} time slot, and (on average) there are n clock ticks per unit of absolute time. The following lemma makes this precise:

Lemma 1.1. *For any $k \geq 1$, $\mathbb{E}[Z_k] = k/n$, and for any $\delta > 0$:*

$$\mathbb{P}\left(|Z_k - \frac{k}{n}| \geq \frac{\delta k}{n}\right) \geq 2\exp\left(-\frac{\delta^2 k}{2}\right) \tag{1.1.1}$$

Proof. By definition:

$$\mathbb{E}[Z_k] = \sum_{j=1}^{k} \mathbb{E}[Z_j - Z_{j-1}] = \sum_{j=1}^{k} \frac{1}{n} = \frac{k}{n} \tag{1.1.2}$$

The concentration result follows directly from Cramer's theorem. □

As a consequence, for $k \geq n$:

$$Z_k = \frac{k}{n}\left(1 \pm \sqrt{\frac{2\log n}{n}}\right) \tag{1.1.3}$$

with probability at least $1 - 1/n^2$. I.e. dividing the quantities measured in terms of clock ticks by n gives the corresponding quantities measured in absolute time.