Spatial Model

Tom Kealy

December 1, 2014

1 Model

We are trying to sense and reconstruct a wideband signal, divided into L channels. We have a (connected) network of M $(= 50)$ nodes placed uniformly at random within the square $[0, 1] \times [0, 1]$.

We write the power spectral density (psd) of the sth transmitter as:

$$\phi_s = \beta_{bs} \psi_b (f) \tag{1.0.1}$$

with the convention that repeated indices are summed over.

This model expresses in psd of the transmitter in a suitable basis - for example $\psi_b (f)$ could be zero everywhere except for the set of frequencies where $f = b$ i.e. ψ is a rectangular function with height β_{bs} and support f. Other candidates for ψ include splines (e.g. raised cosines), and complex exponentials.

Given this, the psd at the rth receiver is:

$$\phi_r = g_{sr} \phi_s = g_{sr} \beta_{bs} \psi_b (f) \tag{1.0.2}$$

where

$$g_{sr} = \exp \left(-||x_r - x_s||_2^\alpha \right) \tag{1.0.3}$$

is the channel response between the sth transmitter and the rth reciver.

We can write the psd at the rth receiver, over all k frequencies as:

$$\phi_{rk} = g_{sr} \phi_s \tag{1.0.4}$$

This model can be summarised using Kronecker products as follows:

$$\phi_{rk} = \left(g_{sr} \bigotimes \psi_{bk} \right) \left(\psi_{bk} \bigotimes \beta_{bs} \right) = g_{sr} \psi_{kb} \bigotimes \psi_{kb} \beta_b s \tag{1.0.5}$$

$\beta_{bs} \in \mathbb{R}^{1 \times n_s}$, $g_{sr} \in \mathbb{R}^{n_r \times n_s}$ and $\psi_{kb} \in 1 \times n_k n_b$ where n_k is the number of frequency bands (in this example $n_k = n_b$.

In the absence of knowledge of the location of the transmitters we introduce a grid of *candidate* locations, to make the above model linear. s now runs over the set of these candidate locations.

The problem of estimating the coefficients, β, from noisy observations $y = \phi_r + N(0, 1)$ is now one that can be tackled by linear regression/convex optimisation.

2 Thoughts

In the noiseless case we would like to calculate:

$$\mathbb{P}\left(\text{obs at r}|\text{transmission from s}\right) = \mathbb{P}(Obs|Tx) \tag{2.0.6}$$

Applying Bayes formula:

$$\mathbb{P}(Obs|Tx) = \frac{\mathbb{P}(Tx|Obs)\mathbb{P}(Obs)}{\mathbb{P}(Tx)} \tag{2.0.7}$$

From 1.0.1, $\mathbb{P}(Tx|Obs)$ is g_{rs}, and $\mathbb{P}(Tx)$ is $|Tx|/8$ as reciever r can possible mistake transmitter s for any of it's 8 neighbours. $\mathbb{P}(Obs)$ is a Rayleigh random variable normalised by the number of receivers $|Rx|$.

$$\mathbb{P}(Obs|Tx) = \exp\left(-||x_r - x_s||_2^{\alpha}\right)\exp(-\gamma||x_r - x_s||_2^2)\frac{|Tx|}{8|Rx|} \tag{2.0.8}$$