Greedy Pursuit Algorithms

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OMP

Algorithm 1: Orthogonal Matching Pursuit

input : Observation vector $\mathbf{y}$, sensing matrix $\mathbf{A}$, stopping criterion s
initialization
$k \leftarrow 0$
$\Lambda_0 = \emptyset$
$\mathbf{r}_k \leftarrow \mathbf{y}$;
while $k \neq s$ *or* $\boldsymbol{r}_k \neq 0$ **do**
    $k \leftarrow k + 1$;
    `match(`$h_k = A^T \boldsymbol{r}^{k-1}$`)`;
    $\Lambda_k = \Lambda_{k-1} \cup \arg\min_j h_k(j)$;
    `update`:;
    $\hat{\mathbf{x}} \leftarrow \mathbf{A}^{\dagger}_{\Lambda_k}\mathbf{y}$;
    $\mathbf{r}_k \leftarrow \mathbf{y} - \mathbf{A}^{\dagger}_{\Lambda_k}\mathbf{y}$
end
output: $\hat{\mathbf{x}} = \arg\min_{x:\text{supp}(x)\subseteq\Lambda_k} \|y - Ax\|_2$

OMP iteratively detects the support set of the signal - proceeding by extending the estimate by a single index per iteration. The algorithm uses a matched filter to find the column which is maximally correlated with the current estimate, and solves a least-squares problem to update the residual vector.

Remarks:

- The residual vector at step k, $\mathbf{r}_k$, can be viewed as the orthoganalisation of the observation vector against the previously chosen columns of $\mathbf{A}$.