Multiple Measurement Vectors

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1 Introduction

Given a set of (jointly) sparse vectors B, and a sensing matrix A, we wish to solve the following problem:

$$Y = AX + W \tag{1.0.1}$$

where $A \in \mathbb{R}^{m \times n}$, $Y \in \mathbb{R}m \times L$, $X \in \mathbb{R}^{n \times L}$, and $W \in \mathbb{R}^{m \times L}$ i additive Gaussian white noise. This is a generalisation of the Single measurement vector problem:

$$y = Ax + w \tag{1.0.2}$$

which can be inverted by solving the following optimisation problem:

$$\hat{x} = \arg\min_x ||y - Ax||_2^2 + \lambda \, ||x||_1 \tag{1.0.3}$$

The problem (1.0.1) can also be solved via optimisation:

$$\hat{X} = \arg\min_X ||Y - AX||_F^2 + \lambda \, ||X||_{2,1} \tag{1.0.4}$$

where

$$||Z||_F^2 = \text{trace}\left(Z^T Z\right) \tag{1.0.5}$$

and

$$||Z||_{2,1} = \sum_{i=1}^{m} ||z_j||_2 \tag{1.0.6}$$

Problem (1.0.3) has a solution given by the iterative ADMM algorithm:

$$x^{k+1} := \left(A^T A + \rho I\right)^{-1} \left(A^T b + \rho \left(z^k - y^k\right)\right) \tag{1.0.7}$$
$$z^{k+1} := S_{\lambda/\rho}\left(x^{k+1} + y^k/\rho\right) \tag{1.0.8}$$
$$y^{k+1} := y^k + \rho\left(x^{k+1} - z^{k+1}\right) \tag{1.0.9}$$

To derive the iterations in (1.0.9), we introduce a dummy variable z to separate the ℓ_2 and ℓ_1 norms in the objective function, and constrain them to be equal:

$$\hat{x} = \arg\min_{x,z} ||y - Ax||_2^2 + \lambda \, ||z||_1 \text{ s.t. } x - z = 0 \tag{1.0.10}$$

the iterations (1.0.9) are then straightforwardly derived by differentiating the Lagrangian:

$$L_p\left(x, z, \gamma\right) = ||y - Ax||_2^2 + ||z||_1 + \gamma^T\left(x - z\right) + \frac{\rho}{2}\|x - z\|_2^2 \tag{1.0.11}$$

Similarly to (1.0.2), the solution to (1.0.1) can be cast as an iterative algorithm via ADMM. Cast the problem as a constrained optimisation:

$$\hat{X} = \arg\min_{X,Z} ||Y - AX||_F^2 + \lambda ||Z||_{2,1} \text{ s.t. } X - Z = 0 \tag{1.0.12}$$

Then, form the augmented Lagrangian:

$$L_p(X, Z, \gamma) = ||Y - AX||_F^2 + ||Z||_{2,1} + \gamma^T (X - Z) + \frac{\rho}{2} ||X - Z||_F^2 \tag{1.0.13}$$

2 DADMM Formulation

3 Results

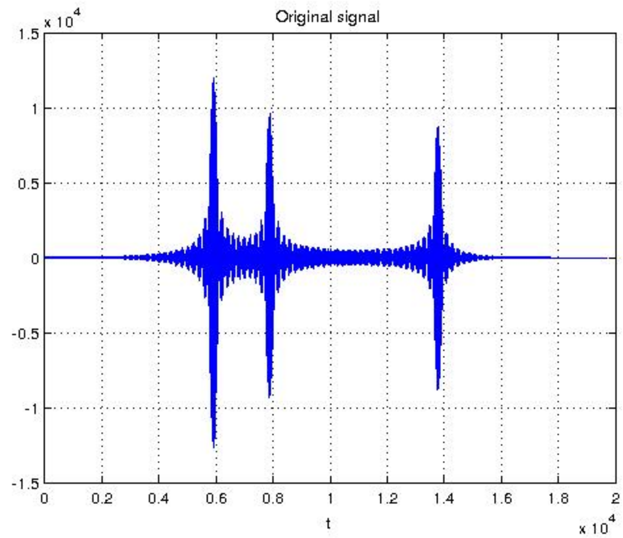

Figure 3.1: Original signal

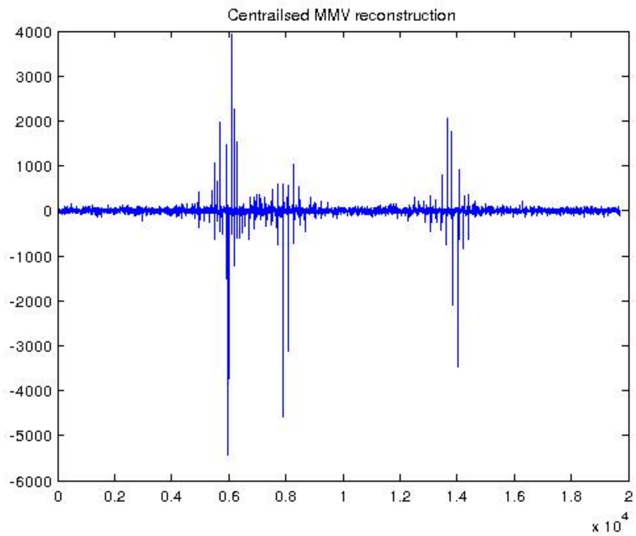

Figure 3.2:

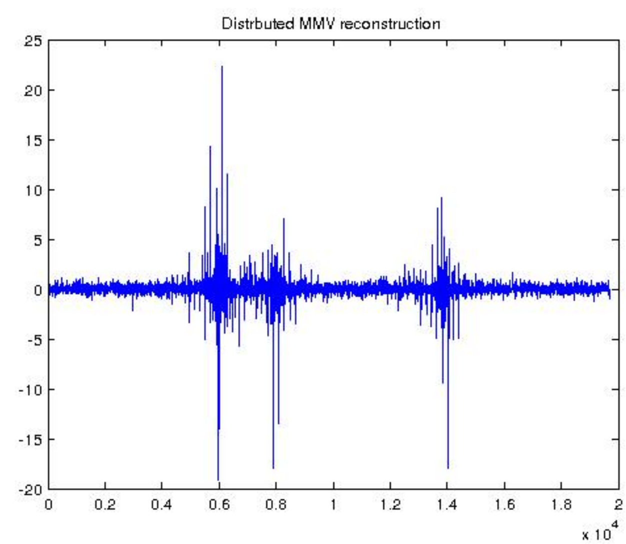

Figure 3.3: