# Frames

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Given a set of observations described by a linear model:

$$y = Ax + n \tag{0.0.1}$$

where $y \in \mathbb{R}^m, x \in \mathbb{R}^n, A \in \mathbb{R}^{m \times n}, andn\ N\left(0,1\right) \in \mathbb{R}^m$, we wish to recover $x$ from these measurements.

One approach is to form a basis, or an over-complete basis, from $y$ as:

$$Q = yy^t \tag{0.0.2}$$

if we let $v = Ax$, we see that $Q$ is approximately $vv^t$. One way to form a solution is to find an $\hat{x}$ which is maximally correlated with the $k$ largest eigenvectors of $Q$.

I.e we can express $\hat{x}$ as:

$$\hat{x} = \sum_{i=1}^{k} \lambda_i yy^t \tag{0.0.3}$$

in particular $v$ is a $||v||^2$ eigen-vector of $vv^t$.

One way to find such an $\hat{x}$ would be to form the matrix $\mathbf{U}$ by taking the $k$ largest eigen-vectors of $Q$ normalised by their respective eigen-values, and then correlating these eigen-vectors against the matrix $A$.

### OMP

**Algorithm 1:** Orthogonal Matching Pursuit

**input** : Matrix $\mathbf{U}$, sensing matrix $\mathbf{A}$, stopping criterion $s$
initialization
$k \leftarrow 0$
$\Lambda_0 = \emptyset$
$\mathbf{r}_k \leftarrow \mathbf{U}$ ;
**while** $k \neq s$ *or* $\boldsymbol{r}_k \neq 0$ **do**
  $k \leftarrow k+1$;
  `match`$(h_k = A^T \boldsymbol{r}^{k-1})$;
  $\Lambda_k = \Lambda_{k-1} \cup \arg\min_j h_k\left(j\right)$;
  `update`:;
  $\hat{\mathbf{x}} \leftarrow \mathbf{A}^{\dagger}_{\Lambda_k} \mathbf{U}$;
  $\mathbf{r}_k \leftarrow \mathbf{U} - \mathbf{A}^{\dagger}_{\Lambda_k} \mathbf{U}$
**end**
**output**: $\hat{\mathbf{x}} = \arg\min_{x:\mathrm{supp}(x) \subseteq \Lambda_k} \|U - Ax\|_2$

OMP iteratively detects the support set of the signal - proceeding by extending the estimate by a single index per iteration. The algorithm uses a matched filter to find the column which is maximally correlated with the current estimate, and solves a least-squares problem to update the residual vector.