

ExteriorSurfaceNode

$$\begin{aligned}
C_{esn} \frac{dT_{esn}}{dt} &= \dot{Q}_{in} - \dot{Q}_{out} \\
C_{esn} \frac{dT_{esn}}{dt} &= \frac{T_{ext} - T_{esn}}{R_{ext,esn}} - \frac{T_{esn} - T_{bmn}}{R_{esn,bmn}} \\
\frac{T_{esn,k-2} - 4T_{esn,k-1} + 3T_{esn}}{2\Delta t} &= \frac{T_{ext} - T_{esn}}{R_{ext,esn} C_{esn}} - \frac{T_{esn} - T_{bmn}}{R_{esn,bmn} C_{esn}} \\
T_{esn,k-2} - 4T_{esn,k-1} + 3T_{esn} &= \frac{2\Delta t}{R_{ext,esn} C_{esn}} T_{ext} - \left(\frac{2\Delta t}{R_{ext,esn} C_{esn}} + \frac{2\Delta t}{R_{esn,bmn} C_{esn}} \right) T_{esn} + \frac{2\Delta t}{R_{esn,bmn} C_{esn}} T_{bmn} \\
-\left(3 + \frac{2\Delta t}{R_{ext,esn} C_{esn}} + \frac{2\Delta t}{R_{esn,bmn} C_{esn}} \right) T_{esn} &+ \frac{2\Delta t}{R_{esn,bmn} C_{esn}} T_{bmn} = T_{esn,k-2} - 4T_{esn,k-1} - \frac{2\Delta t}{R_{ext,esn} C_{esn}} T_{ext}
\end{aligned}$$

BuildingMassNode

$$\begin{aligned}
C_{bmn1} \frac{dT_{bmn1}}{dt} &= \frac{T_{esn} - T_{bmn1}}{R_{esn,bmn1}} - \frac{T_{bmn1} - T_{bmn2}}{R_{bmn1,bmn2}} \\
\frac{T_{bmn1,k-2} - 4T_{bmn1,k-1} + 3T_{bmn1}}{2\Delta t} &= \frac{T_{esn} - T_{bmn1}}{R_{esn,bmn1} C_{bmn1}} - \frac{T_{bmn1} - T_{bmn2}}{R_{bmn1,bmn2} C_{bmn1}} \\
T_{bmn1,k-2} - 4T_{bmn1,k-1} + 3T_{bmn1} &= \frac{2\Delta t}{R_{esn,bmn1} C_{bmn1}} T_{esn} - \left(\frac{2\Delta t}{R_{esn,bmn1} C_{bmn1}} + \frac{2\Delta t}{R_{bmn1,bmn2} C_{bmn1}} \right) T_{bmn1} + \frac{2\Delta t}{R_{bmn1,bmn2} C_{bmn1}} T_{bmn2} \\
\frac{2\Delta t}{R_{esn,bmn1} C_{bmn1}} T_{esn} - \left(3 + \frac{2\Delta t}{R_{esn,bmn1} C_{bmn1}} + \frac{2\Delta t}{R_{bmn1,bmn2} C_{bmn1}} \right) T_{bmn1} &+ \frac{2\Delta t}{R_{bmn1,bmn2} C_{bmn1}} T_{bmn2} = T_{bmn1,k-2} - 4T_{bmn1,k-1}
\end{aligned}$$

InteriorSurfaceNode

$$\frac{T_{bmn} - T_{isn}}{R_{bmn, isn}} - \dot{Q}_{conv} - \dot{Q}_{rad} = 0$$

$$\dot{Q}_{rad} = F_{rad} (\dot{Q}_{conv} + \dot{Q}_{rad})$$

$$\Rightarrow (1 - F_{rad}) \dot{Q}_{rad} = F_{rad} \dot{Q}_{conv}$$

$$\Rightarrow \dot{Q}_{rad} = \frac{F_{rad}}{1 - F_{rad}} \dot{Q}_{conv}$$

$$\frac{T_{bmn} - T_{isn}}{R_{bmn, isn}} - \dot{Q}_{conv} - \frac{F_{rad}}{1 - F_{rad}} \dot{Q}_{conv} = 0$$

$$\frac{T_{bmn} - T_{isn}}{R_{bmn, isn}} - \left(1 + \frac{F_{rad}}{1 - F_{rad}} \right) \dot{Q}_{conv} = 0$$

$$\frac{T_{bmn} - T_{isn}}{R_{bmn, isn}} - \frac{1}{1 - F_{rad}} \cdot \frac{T_{isn} - T_{zan}}{R_{isn, zan}} = 0$$

$$\frac{1}{R_{bmn, isn}} T_{bmn} - \left(\frac{1}{R_{bmn, isn}} + \frac{1}{(1 - F_{rad}) R_{isn, zan}} \right) T_{isn} + \frac{1}{(1 - F_{rad}) R_{isn, zan}} T_{zan} = 0$$

ZoneAirNode

$$\frac{T_{isn1} - T_{zan}}{R_{isn1, zan}} + \frac{T_{isn2} - T_{zan}}{R_{isn2, zan}} + \frac{T_{isn} - T_{zan}}{R_{isn, zan}} + \sum \dot{Q}_{conv} = \dot{Q}_{sys}$$

$$\sum \dot{Q}_{conv} = (1 - F_{rad, wnd}) UA_{wnd} (T_{ext} - T_{zan})$$

$$+ (1 - F_{rad, edr}) UA_{edr} (T_{sa} - T_{zan})$$

$$+ (1 - F_{rad, ibe}) UA_{ibe} (T_{adj} - T_{zan})$$

$$+ \dot{Q}_{sol, cv} + \dot{Q}_{hg, cv}$$

$$\sum \dot{Q}_{conv} =$$

$$(1 - F_{rad, wnd}) UA_{wnd} T_{ext}$$

$$+ (1 - F_{rad, edr}) UA_{edr} T_{sa}$$

$$+ (1 - F_{rad, ibe}) UA_{ibe} T_{adj}$$

$$- \left(\begin{aligned} &(1 - F_{rad, wnd}) UA_{wnd} \\ &+ (1 - F_{rad, edr}) UA_{edr} \\ &+ (1 - F_{rad, ibe}) UA_{ibe} \end{aligned} \right) T_{zan}$$

$$+ \dot{Q}_{sol, cv} + \dot{Q}_{hg, cv}$$

$$\frac{1}{R_{isn1, zan}} T_{isn1} + \frac{1}{R_{isn2, zan}} T_{isn2} + \frac{1}{R_{isn, zan}} T_{isn}$$

$$- \left(\frac{1}{R_{isn1, zan}} + \frac{1}{R_{isn2, zan}} + \frac{1}{R_{isn, zan}} + UA_{wnd} + UA_{edr} + UA_{ibe} \right) T_{zan}$$

$$= \dot{Q}_{sys} - UA_{wnd} T_{ext} - UA_{edr} T_{sa} - UA_{ibe} T_{adj} - \dot{Q}_{sol, cv} - \dot{Q}_{hg, cv}$$

ThermalStorageNode

$$C_{tsn} \frac{dT_{tsn}}{dt} = \dot{Q}_{rad, isn1} + \dot{Q}_{rad, isn2} + \sum \dot{Q}_{rad, oth} - \frac{T_{tsn} - T_{zan}}{R_{tsn, zan}}$$

$$T_{tsn, k-2} - 4T_{tsn, k-1} + 3T_{tsn} = \frac{2\Delta t}{C_{tsn}} \left[\dot{Q}_{rad, isn1} + \dot{Q}_{rad, isn2} + \sum \dot{Q}_{rad, oth} - \frac{T_{tsn} - T_{zan}}{R_{tsn, zan}} \right]$$

$$\dot{Q}_{rad, isn1} = \frac{F_{rad, isn1}}{1 - F_{rad, isn1}} \cdot \frac{T_{isn1} - T_{zan}}{R_{isn1, zan}}$$

$$\dot{Q}_{rad, isn2} = \frac{F_{rad, isn2}}{1 - F_{rad, isn2}} \cdot \frac{T_{isn2} - T_{zan}}{R_{isn2, zan}}$$

$$\sum \dot{Q}_{rad, oth} = \dot{Q}_{sol, rd} + \dot{Q}_{ihg, rd} + \dot{Q}_{wnd, rd} + \dot{Q}_{edr, rd} + \dot{Q}_{ibe, rd}$$

$$\sum \dot{Q}_{rad, oth} = \dot{Q}_{sol, rd} + \dot{Q}_{ihg, rd} + F_{rad, wnd} \dot{Q}_{cond, wnd} + F_{rad, edr} \dot{Q}_{cond, edr} + F_{rad, ibe} \dot{Q}_{cond, ibe}$$

$$\sum \dot{Q}_{rad, oth} = \dot{Q}_{sol, rd} + \dot{Q}_{ihg, rd} + F_{rad, wnd} UA_{wnd} (T_{ext} - T_{zan}) + F_{rad, edr} UA_{edr} (T_{sa} - T_{zan}) + F_{rad, ibe} UA_{ibe} (T_{adj} - T_{zan})$$

$$T_{tsn, k-2} - 4T_{tsn, k-1} + 3T_{tsn} = \frac{2\Delta t}{C_{tsn}} \left[\frac{F_{rad, isn1}}{1 - F_{rad, isn1}} \cdot \frac{T_{isn1} - T_{zan}}{R_{isn1, zan}} + \frac{F_{rad, isn2}}{1 - F_{rad, isn2}} \cdot \frac{T_{isn2} - T_{zan}}{R_{isn2, zan}} \right.$$

$$\left. + \dot{Q}_{sol, rd} + \dot{Q}_{ihg, rd} + F_{rad, wnd} UA_{wnd} (T_{ext} - T_{zan}) + F_{rad, edr} UA_{edr} (T_{sa} - T_{zan}) + F_{rad, ibe} UA_{ibe} (T_{adj} - T_{zan}) - \frac{T_{tsn}}{R_{tsn, zan}} + \frac{T_{zan}}{R_{tsn, zan}} \right]$$

$$\begin{aligned}
& \frac{2\Delta t \cdot F_{rad, isn1}}{C_{tsn} (1 - F_{rad, isn1}) R_{isn1, zan}} T_{isn1} - \frac{2\Delta t \cdot F_{rad, isn1}}{C_{tsn} (1 - F_{rad, isn1}) R_{isn1, zan}} T_{zan} \\
& + \frac{2\Delta t \cdot F_{rad, isn2}}{C_{tsn} (1 - F_{rad, isn2}) R_{isn2, zan}} T_{isn2} - \frac{2\Delta t \cdot F_{rad, isn2}}{C_{tsn} (1 - F_{rad, isn2}) R_{isn2, zan}} T_{zan} \\
& + \frac{2\Delta t}{C_{tsn}} F_{rad, wnd} UA_{wnd} T_{ext} - \frac{2\Delta t}{C_{tsn}} F_{rad, wnd} UA_{wnd} T_{zan} \\
& + \frac{2\Delta t}{C_{tsn}} F_{rad, edr} UA_{edr} T_{sa} - \frac{2\Delta t}{C_{tsn}} F_{rad, edr} UA_{edr} T_{zan} \\
& + \frac{2\Delta t}{C_{tsn}} F_{rad, ibe} UA_{ibe} T_{adj} - \frac{2\Delta t}{C_{tsn}} F_{rad, ibe} UA_{ibe} T_{zan} \\
& + \frac{2\Delta t}{C_{tsn}} \dot{Q}_{sol, rd} + \frac{2\Delta t}{C_{tsn}} \dot{Q}_{ihg, rd} \\
& - \frac{2\Delta t T_{tsn}}{R_{tsn, zan} C_{tsn}} + \frac{2\Delta t T_{zan}}{R_{tsn, zan} C_{tsn}} - 3T_{tsn} = T_{tsn, k-2} - 4T_{tsn, k-1}
\end{aligned}$$

$$\begin{aligned}
& \frac{2\Delta t \cdot F_{rad, isn1}}{C_{tsn} (1 - F_{rad, isn1}) R_{isn1, zan}} T_{isn1} + \frac{2\Delta t \cdot F_{rad, isn2}}{C_{tsn} (1 - F_{rad, isn2}) R_{isn2, zan}} T_{isn2} \\
& - \left(\frac{2\Delta t}{R_{tsn, zan} C_{tsn}} + 3 \right) T_{tsn} \\
& - \left(\frac{\frac{2\Delta t \cdot F_{rad, isn1}}{C_{tsn} (1 - F_{rad, isn1}) R_{isn1, zan}} + \frac{2\Delta t \cdot F_{rad, isn2}}{C_{tsn} (1 - F_{rad, isn2}) R_{isn2, zan}}}{\frac{2\Delta t}{R_{tsn, zan} C_{tsn}}} + \frac{\frac{2\Delta t}{C_{tsn}} F_{rad, wnd} UA_{wnd} + \frac{2\Delta t}{C_{tsn}} F_{rad, edr} UA_{edr} + \frac{2\Delta t}{C_{tsn}} F_{rad, ibe} UA_{ibe}}{\frac{2\Delta t}{R_{tsn, zan} C_{tsn}}} \right) T_{zan} \\
& = \left(\begin{aligned} & T_{tsn, k-2} - 4T_{tsn, k-1} \\ & - \frac{2\Delta t}{C_{tsn}} F_{rad, wnd} UA_{wnd} T_{ext} - \frac{2\Delta t}{C_{tsn}} F_{rad, edr} UA_{edr} T_{sa} - \frac{2\Delta t}{C_{tsn}} F_{rad, ibe} UA_{ibe} T_{adj} \\ & - \frac{2\Delta t}{C_{tsn}} \dot{Q}_{sol, rd} - \frac{2\Delta t}{C_{tsn}} \dot{Q}_{ihg, rd} \end{aligned} \right)
\end{aligned}$$

