Machine Learning Worksheet 6

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Problem 1

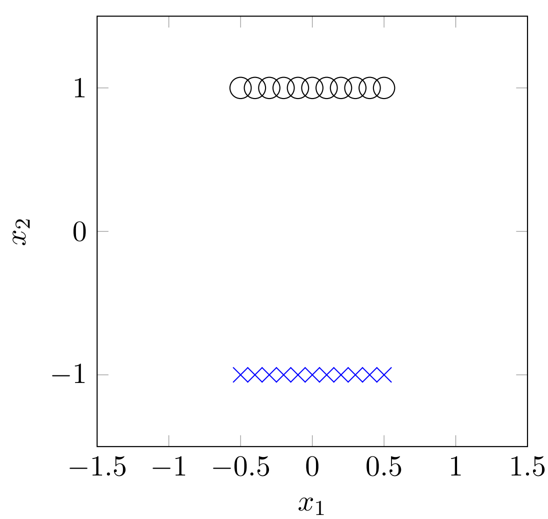
?

Problem 2

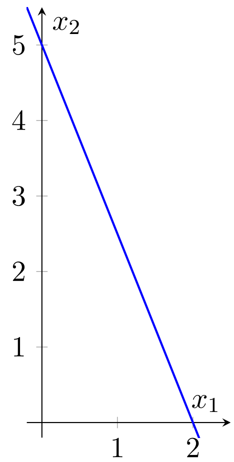
The function

$$\phi(X) : \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \rightarrow \begin{pmatrix} x_1 \cdot x_2 \\ sgn(x_1 \cdot x_2) \end{pmatrix}$$

will transform the data into a space where it will be linearly separable:



Problem 3



General form of this linear classifier:

$$y_k = W^T X + b$$

With W being the normal vector of the two-dimensional hyperplane (line) going through $s_1 = (0, 5)$ and $s_2 = (2, 0)$:

$$\begin{aligned} W^T &= \begin{pmatrix} dx_2 \\ -dx_1 \end{pmatrix}^T \qquad \mathit{with} \quad dx_1 = 0 - 2 = -2 \quad , \; dx_2 = 5 - 0 = 5 \\ &= \begin{pmatrix} 5 \\ 2 \end{pmatrix}^T \end{aligned}$$

The bias can be computed by inserting a point on the plane (e.g. s_2) and setting the classifier to 0, like this:

$$b = 0 - W^T X = 0 - \begin{pmatrix} 5 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix} = -10$$

The classifier with some possible parameters is therefore:

$$y_k = \begin{pmatrix} 5 \\ 2 \end{pmatrix}^T X - 10$$

Problem 4

LinearClassification

December 12, 2016

1 Implementation exercise: Linear Classification

```
In [41]: import numpy as np
         from sklearn import datasets
         import matplotlib.pyplot as plt
         %matplotlib inline
```

1.0.1 Some helper functions for visualisation

```
In [42]: def plot_decision_boundary(X, Z, W=None, b=None):
             fig, ax = plt.subplots(1, 1, figsize=(5, 5))
             ax.scatter(X[:,0], X[:,1], c=Z, cmap=plt.cm.cool)
             ax.set_autoscale_on(False)

             a = - W[0, 0] / W[0, 1]
             xx = np.linspace(-30, 30)
             yy = a * xx - (b[0]) / W[0, 1]

             ax.plot(xx, yy, 'k-', c=plt.cm.cool(1.0/3.0))
```

1.0.2 Dataset Loader

```
In [43]: def loadDataset(split, X=[] , XT=[], Z = [], ZT = []):
             dataset = datasets.load_iris()
             c = list(zip(dataset['data'], dataset['target']))
             np.random.seed(224)
             np.random.shuffle(c)
             x, t = zip(*c)
             sp = int(split*len(c))
             X = x[:sp]
             XT = x[sp:]
             Z = t[:sp]
             ZT = t[sp:]
             names = ['Sepal. length', 'Sepal. width', 'Petal. length', 'Petal. wid
             return np.array(X), np.array(XT), np.array(Z), np.array(ZT), names
```

```
In [44]: # prepare data
         split = 0.67
```

```
X, XT, Z, ZT, names = loadDataset(split)

# combine two of the 3 classes for a 2 class problem
Z[Z==2] = 1
ZT[ZT==2] = 1

# only look at 2 dimensions of the input data for easy visualisation
X = X[:,:2]
XT = XT[:,:2]
```

1.1 Exercise 1: Calculate probability of class 1

Compute the probability of class 1 given the data and the parameters.

arguments: * *X*: data * *W*: weight matrix, part of the parameters * *b*: bias, part of the parameters

returns: * *rate*: probabiliy of the predicted class 1

```
In [45]: def pred(X, W, b):
             x = b + X.dot(W.T)
             p = 1 / (1 + np.exp(-x))

             return p

         W = np.random.randn(1,2) * 0.01
         b = np.random.randn(1) * 0.01
         pred(X, W, b).shape

Out[45]: (100, 1)
```

1.2 Exercise 2: Calculate the log-likelihood given the target

Compute the logarithm of the likelihood for logistic regression. The negative log-likelihood is our loss function.

arguments: * *X*: data * *Z*: target * *W*: weight matrix, part of the parameters * *b*: bias, part of the parameters

returns: * *log likelihood*: logarithm of the likelihood

```
In [46]: def loglikelihood(X, Z, W, b):
             y = pred(X, W, b)
             Zr = np.reshape(Z, (Z.shape[0], 1))
             like = Zr * np.log(y) + (1 - Zr) * np.log(1 - y)

             return like

         W = np.random.randn(1,2) * 0.01
         b = np.random.randn(1) * 0.01
         loglikelihood(X, Z, W, b).shape

Out[46]: (100, 1)
```

1.3 Exercise 3: Implement the gradient of the loss/log-likelihood

Compute the gradient of the loss with respect to the parameters
arguments: * X: data * Z: target * W: weight matrix, part of the parameters * b: bias, part of the parameters
returns: * $dLdW$: gradient of loss wrt to W * $dLdb$: gradient of loss wrt to b

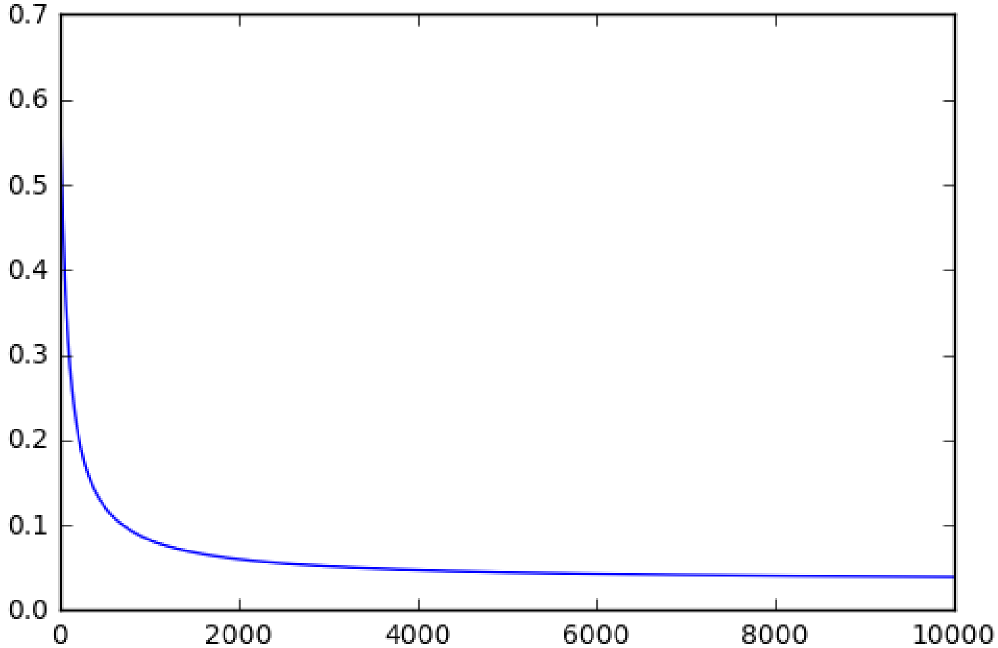
```
In [47]: def grad(X, Z, W, b):
             y = pred(X, W, b)
             dy = y * (1 - y)
             Zr = np.reshape(Z, (Z.shape[0], 1))

             dw = (y - Zr) * X
             dw = dw.sum(axis=0)

             db = np.log(y) + (Zr * dy)/y - ((1 - Zr) * dy)/(1 - y)
             db = -1 * db.sum(axis=0)

             return dw, db

         W = np.random.randn(1,2) * 0.01
         b = np.random.randn(1) * 0.01

         grad(X, Z, W, b)
```

```
Out[47]: (array([-115.53753093,  -32.42879411]), array([ 55.95783241]))
```

1.4 Exercise 4: Test everything

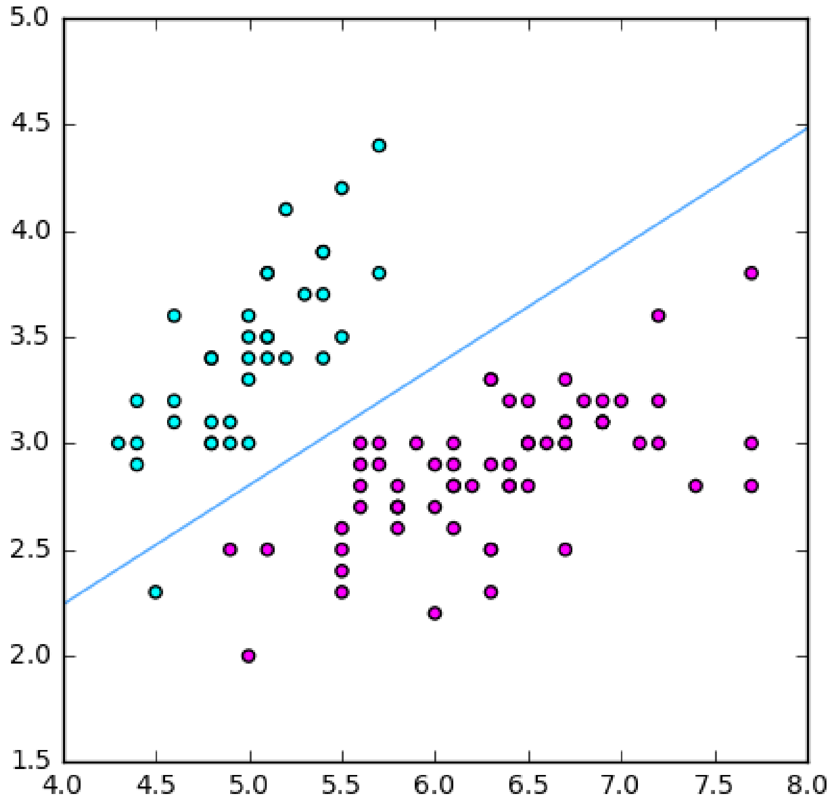
Run the provied simple gradient descent algorithm to optimize the model parameters and plot the resuling decision boundary.

```
In [48]: W = np.random.randn(1,2) * 0.01
         b = np.random.randn(1) * 0.01

         learning_rate = 0.001
         train_loss = []
         validation_loss = []

         for i in range(10000):
             dLdW, dLdb = grad(X, Z, W, b)

             W -= learning_rate * dLdW
             # The gradient gets too large. There is something wrong with the calcu
             # of the gradient ...
             #b -= learning_rate * dLdb

             train_loss.append( - loglikelihood(X, Z, W, b).mean())

         _ = plt.plot(train_loss)
```



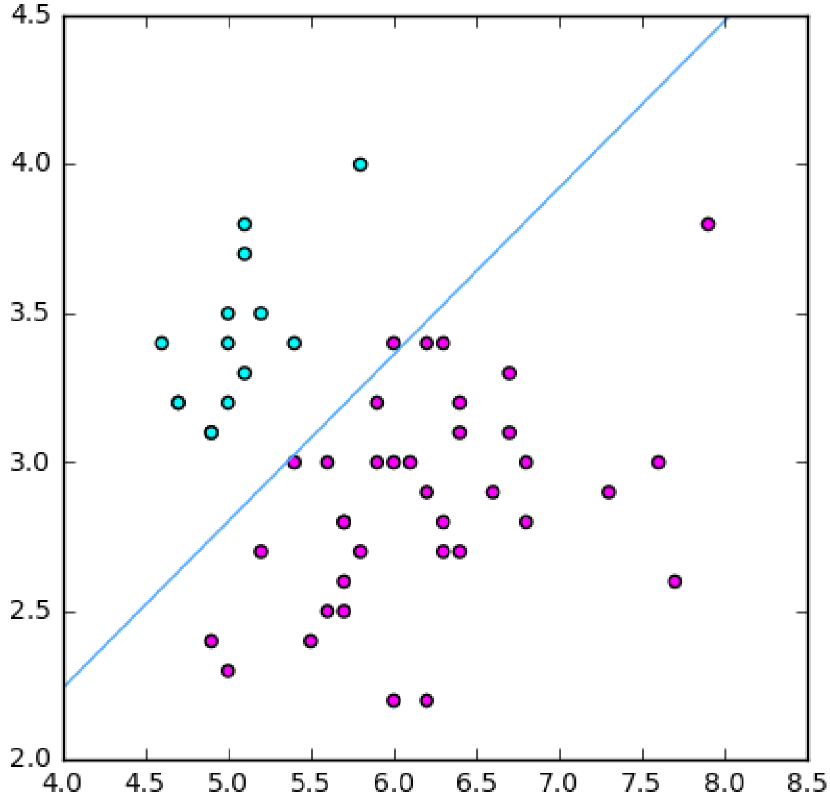
1.4.1 Decision boundary on the training set

```
In [49]: plot_decision_boundary(X, Z, W=W, b=b)
```



1.4.2 Decision boundary on the test set

```
In [50]: plot_decision_boundary(XT, ZT, W=W, b=b)
```



In [ ]: