Machine Learning Worksheet 7

Tomas Ladek, Michael Kratzer
3602673, 3612903
tom.ladek@tum.de, mkratzer@mytum.de

Problem 1

Let us rewrite the sigmoid activation function as

$$\sigma(x) = \frac{1}{1+e^{-x}} = \frac{e^x}{e^x(1+e^{-x})} = \frac{e^x}{e^x+e^{x-x}} = \frac{e^x}{e^x+1}$$

There exists a network that computes $\sigma(x)$ by scaling and offsetting the hyperbolic tangent function $\tanh(x)$, since

$$\begin{aligned}
\tanh(x) &= \frac{e^x - e^{-x}}{e^x + e^{-x}} \\
&= \frac{e^{-x}(e^{2x}-1)}{e^{-x}(e^{2x}+1)} \\
&= \frac{e^{2x}-1}{e^{2x}+1} \\
&= \frac{2e^{2x} - (1+e^{2x})}{e^{2x}+1} \\
&= \frac{2e^{2x}}{e^{2x}+1} - \frac{1+e^{2x}}{e^{2x}+1} \\
&= 2\frac{e^{2x}}{e^{2x}+1} - 1 \\
&= 2\sigma(2x) - 1
\end{aligned}$$

And therefore

$$\begin{aligned}
\tanh(x) &= 2\sigma(2x) - 1 \\
\tanh(x) + 1 &= 2\sigma(2x) \\
\frac{1}{2}(\tanh(x)+1) &= \sigma(2x) \\
\frac{1}{2}(\tanh(\frac{z}{2})+1) &= \sigma(z) \qquad with\ z = 2x
\end{aligned}$$

Problem 2

$$\begin{aligned}\frac{d}{dx}\,\sigma(x) = \frac{d}{dx}\,\frac{e^x}{e^x+1} &= \frac{(e^x+1)e^x - e^x e^x}{(e^x+1)^2} \\ &= \frac{(e^x+1)e^x}{(e^x+1)^2} - \frac{(e^x)^2}{(e^x+1)^2} \\ &= \frac{e^x}{e^x+1} - \Big(\frac{e^x}{e^x+1}\Big)^2 \\ &= \sigma(x) - \sigma^2(x)\end{aligned}$$

$$\begin{aligned}\frac{d}{dx}\,\tanh(x) = \frac{d}{dx}\,\frac{e^x - e^{-x}}{e^x + e^{-x}} &= \frac{(e^x+e^{-x})(e^x+e^{-x}) - (e^x-e^{-x})(e^x-e^{-x})}{(e^x+e^{-x})^2} \\ = \frac{(e^x+e^{-x})^2 - (e^x-e^{-x})^2}{(e^x+e^{-x})^2} &= 1 - \Big(\frac{e^x-e^{-x}}{e^x+e^{-x}}\Big)^2 = 1 - \tanh^2(x)\end{aligned}$$

Problem 3

Below six plots of the training curves for different learning rates used in `mlp_xor.NeuralNetwork(X,y,l)` and `mlp_sin.NeuralNetwork(X,y,l)` respectively.

mlp_xor.NeuralNetwork(X,y,l):

```
output:
[[  4.56613223e-05]
 [  9.97041292e-01]
 [  9.97041213e-01]
 [  6.24880067e-05]]
```

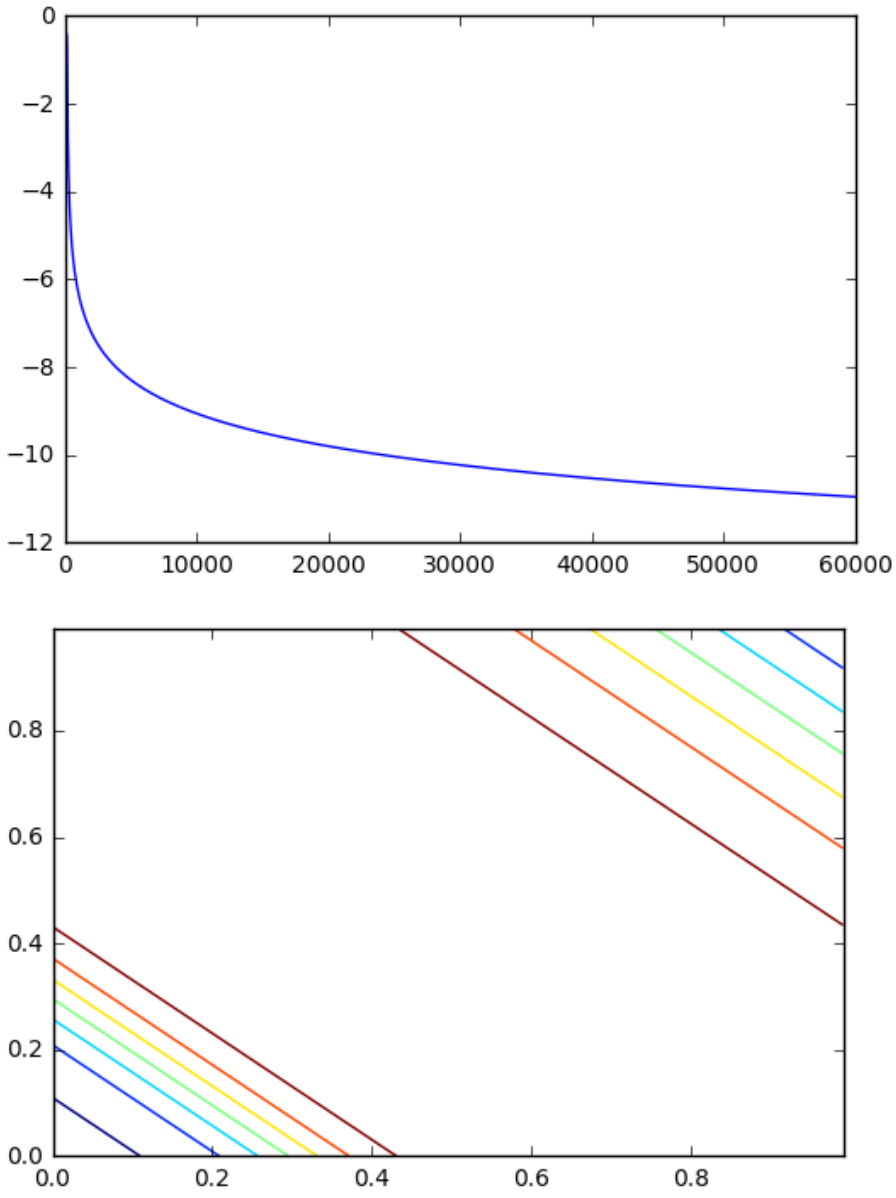
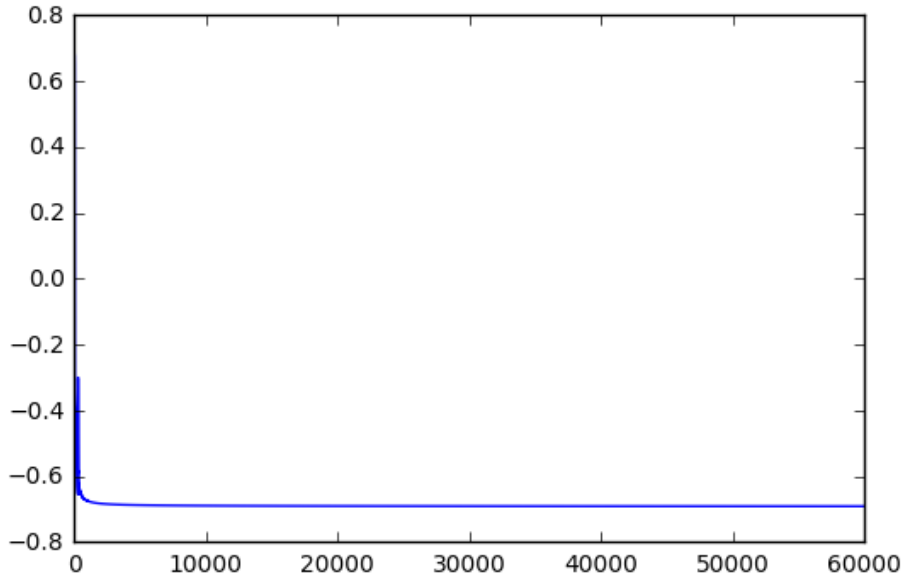


Figure 1: Function: XOR, Learning rate 0.3

```
output:
[[ 0.03188002]
 [ 0.9858056 ]
 [ 0.50156382]
 [ 0.50160351]]
```



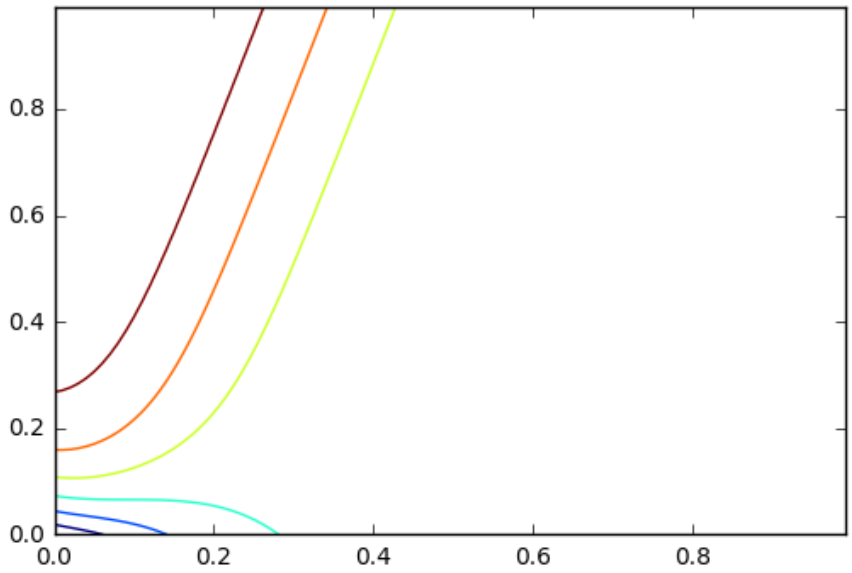


Figure 2: Function: XOR, Learning rate 0.6

```
output:
[[  8.46115472e-05]
 [  4.99961733e-01]
 [  9.90803009e-01]
 [  5.00350863e-01]]
```

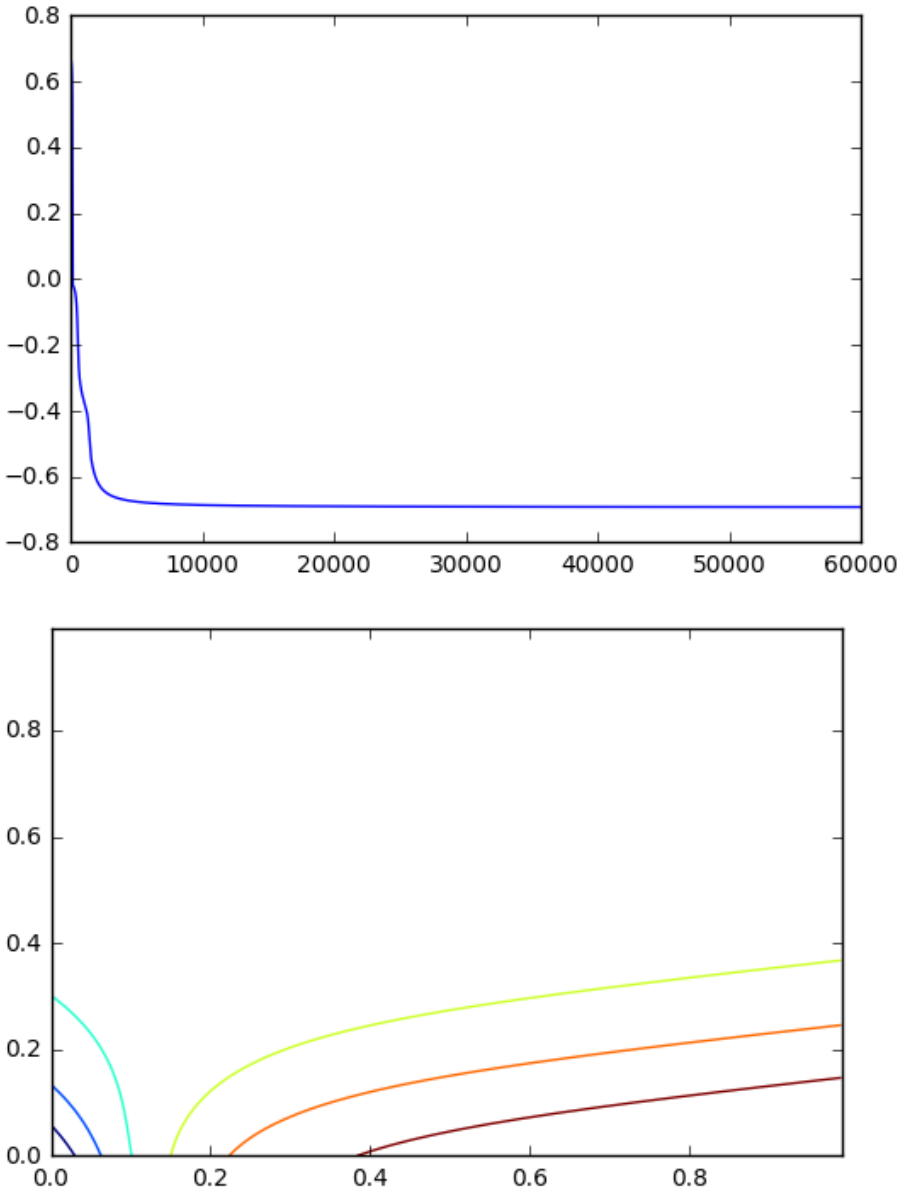


Figure 3: Function: XOR, Learning rate 0.05

mlp_sin.NeuralNetwork(X,y,l):

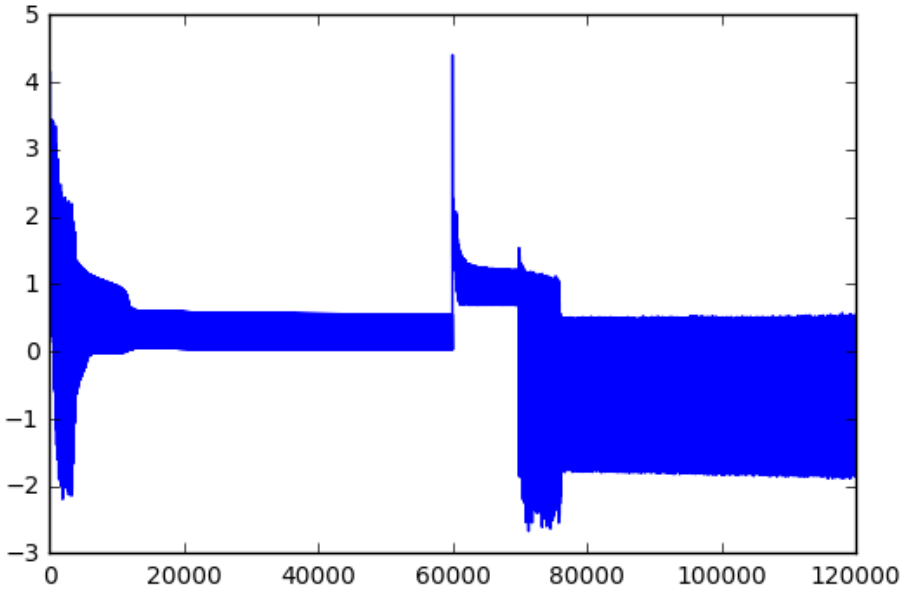


Figure 4: Function: Sin, Learning rate 0.3

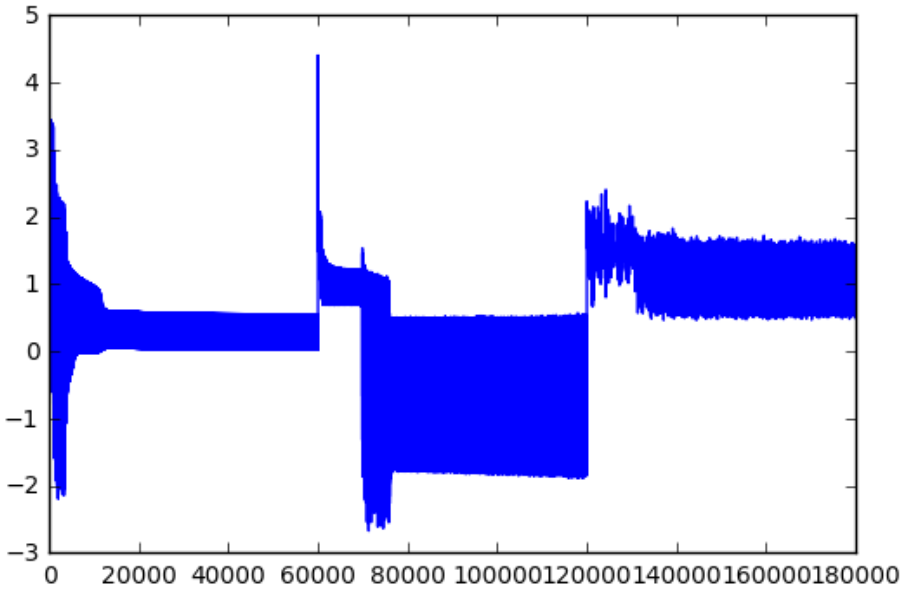


Figure 5: Function: Sin, Learning rate 0.7

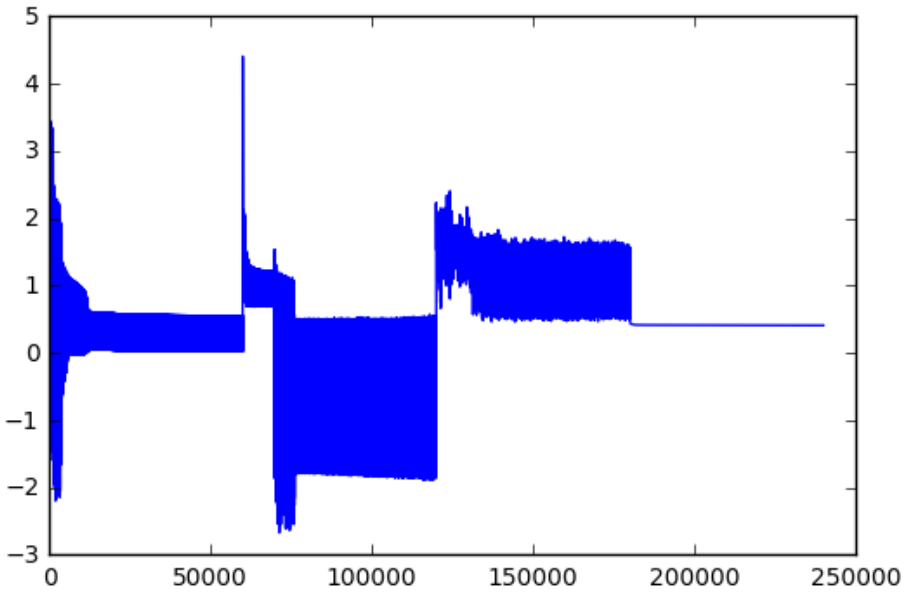


Figure 6: Function: Sin, Learning rate 0.05