

Machine Learning Worksheet 7

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Problem 1

Let us rewrite the sigmoid activation function as

$$\sigma(x) = \frac{1}{1 + e^{-x}} = \frac{e^x}{e^x(1 + e^{-x})} = \frac{e^x}{e^x + e^{x-x}} = \frac{e^x}{e^x + 1}$$

There exists a network that computes $\sigma(x)$ by scaling and offsetting the hyperbolic tangent function $\tanh(x)$, since

$$\begin{aligned}\tanh(x) &= \frac{e^x - e^{-x}}{e^x + e^{-x}} \\ &= \frac{e^{-x}(e^{2x} - 1)}{e^{-x}(e^{2x} + 1)} \\ &= \frac{e^{2x} - 1}{e^{2x} + 1} \\ &= \frac{2e^{2x} - (1 + e^{2x})}{e^{2x} + 1} \\ &= \frac{2e^{2x}}{e^{2x} + 1} - \frac{1 + e^{2x}}{e^{2x} + 1} \\ &= 2\frac{e^{2x}}{e^{2x} + 1} - 1 \\ &= 2\sigma(2x) - 1\end{aligned}$$

And therefore

$$\begin{aligned}\tanh(x) &= 2\sigma(2x) - 1 \\ \tanh(x) + 1 &= 2\sigma(2x) \\ \frac{1}{2}(\tanh(x) + 1) &= \sigma(2x) \\ \frac{1}{2}(\tanh(\frac{z}{2}) + 1) &= \sigma(z) \quad \text{with } z = 2x\end{aligned}$$
