

## Machine Learning Worksheet 9

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### Problem 1

Noise-free:  $m(x_*) = 0 = \mu_*$

$$\begin{aligned}
 K &= \exp\left(-\frac{1}{2}(x-x')^T(x-x')\right) \\
 &= \exp\left(-\frac{1}{2}\begin{pmatrix} x_1-x_1 & x_1-x_2 \\ x_2-x_1 & x_2-x_2 \end{pmatrix}^T \begin{pmatrix} x_1-x_1 & x_1-x_2 \\ x_2-x_1 & x_2-x_2 \end{pmatrix}\right) \\
 &= \exp\left(-\frac{1}{2}\begin{pmatrix} 0 & 0 & -0.5 & -1 \\ 0.5 & 1 & 0 & 0 \end{pmatrix}^T \begin{pmatrix} 0 & 0 & -0.5 & -1 \\ 0.5 & 1 & 0 & 0 \end{pmatrix}\right) \\
 &= \exp\left(-\frac{1}{2}\begin{pmatrix} 0.25 & 0.5 & & 0 \\ 0.5 & 1 & & \\ & & 0.25 & 0.5 \\ & & 0.5 & 1 \end{pmatrix}\right) \\
 &= \begin{pmatrix} 0.88 & 0.78 & & \\ 0.78 & 0.61 & & 1 \\ & & 0.88 & 0.78 \\ 1 & & 0.78 & 0.61 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 K_* &= \exp\left(-\frac{1}{2}\begin{pmatrix} x_1-x_* \\ x_2-x_* \end{pmatrix}^T \begin{pmatrix} x_1-x_* \\ x_2-x_* \end{pmatrix}\right) \\
 &= \exp\left(-\frac{1}{2}\begin{pmatrix} -0.5 & 0 \\ 0 & 1 \end{pmatrix}^T \begin{pmatrix} -0.5 & 0 \\ 0 & 1 \end{pmatrix}\right) \\
 &= \exp\left(-\frac{1}{2}\begin{pmatrix} 0.25 & 0 \\ 0 & 1 \end{pmatrix}\right) \\
 &= \begin{pmatrix} 0.88 & 1 \\ 1 & 0.61 \end{pmatrix} = K_*^T
 \end{aligned}$$

$$\begin{aligned}
 K_{**} &= \exp\left(-\frac{1}{2}(x_*-x_*)^T(x_*-x_*)\right) \\
 &= \exp(0) = 1
 \end{aligned}$$

Therefore

$$f_{jt} = \begin{pmatrix} y_1 \\ y_2 \\ f(x_*) \end{pmatrix} \sim \mathcal{N}\left(0, \begin{bmatrix} K & K_* \\ K_*^T & K_{**} \end{bmatrix}\right) = \mathcal{N}\left(0, \begin{pmatrix} 0.88 & 0.78 & 1 & 1 & 0.88 & 1 \\ 0.78 & 0.61 & 1 & 1 & 1 & 0.61 \\ 1 & 1 & 0.88 & 0.78 & 0.88 & 1 \\ 1 & 1 & 0.78 & 0.61 & 1 & 0.61 \\ 0.88 & 1 & 0.88 & 1 & 1 & 1 \\ 1 & 0.61 & 1 & 0.61 & 1 & 1 \end{pmatrix}\right)$$