Machine Learning Worksheet 10

Tomas Ladek, Michael Kratzer
3602673, 3612903
tom.ladek@tum.de, mkratzer@mytum.de

Problem 1

We don't know.

Problem 2

$$p(y) = \mathcal{N}(A\mu, A(WW^T + \psi)A^T)$$

i) With A and ψ being diagonal, then $A(WW^T + \psi)A^T$ is also diagonal.

$$\begin{aligned} A(WW^T + \psi)A^T &= AWW^TA^T + A\psi A^T \\ &= (AW)(AW)^T + A\psi A^T \\ &= \widetilde{W}\widetilde{W}^T + A\psi A^T \\ &= \widetilde{W}\widetilde{W}^T + \widetilde{\psi} \end{aligned}$$

With $\widetilde{\psi}$ being diagonal.

ii)

$$\begin{aligned} A(WW^T + \sigma^2 I)A^T &= AWW^TA^T + A\sigma^2 IA^T \\ &= (AW)(AW)^T + \sigma^2 AA^T \\ &= (AW)(AW)^T + \sigma^2 I \\ &= \widetilde{W}\widetilde{W}^T + \sigma^2 I \end{aligned}$$

Problem 3

Given the solution for the posterior of the FA model:

$$\begin{aligned} \Sigma &= (I + W^T\psi^{-1}W)^{-1} \\ m_i &= \Sigma(W^T\psi^{-1}(x_i - \mu)) \end{aligned}$$

then

$$\begin{aligned} \Sigma &= (\Sigma_0^{-1} + W^T\psi^{-1}W)^{-1} \\ m_i &= \Sigma(W^T\psi^{-1}(x_i - \mu) + \Sigma_0^{-1}\mu_0) \end{aligned}$$

with $\psi = \sigma^2 I$ and $\Sigma_0^{-1} = I$.

$$\begin{aligned}
\Sigma &= (I + W^T \frac{1}{\sigma^2} I W)^{-1} \\
&= (I + \frac{1}{\sigma^2} W^T W)^{-1} \\
&= (i + \frac{1}{\sigma^2} I)^{-1} \\
&= (diag(\frac{1}{\sigma^2}))^{-1} \\
&= \sigma^2 I \\
m &= \sigma^2 I (W^T \frac{1}{\sigma^2} I (x_i - \mu) + I \mu_0) \\
&= \sigma^2 I (W^T \frac{1}{\sigma^2} I (x_i - \mu) + \sigma^2 I \mu_0) \\
&= W^T (x_i - \mu)
\end{aligned}$$

With $\sigma^2 \to 0$ it is an orthogonal projection.