Machine Learning Worksheet 4

Tomas Ladek, Michael Kratzer
3602673, 3612903
tom.ladek@tum.de, mkratzer@mytum.de

Problem 1

The Inverse of a diagonal atrix is just a diagonal matrix with each of the elements inverted.

$$\Lambda^{-1} = \begin{pmatrix} \frac{1}{\lambda_1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \frac{1}{\lambda_d} \end{pmatrix}, U = \begin{pmatrix} u_{1,1} & \dots & u_{1,d} \\ \vdots & \ddots & \vdots \\ u_{d,1} & \dots & u_{d,d} \end{pmatrix} \tag{1}$$

Just multiplying $U\Lambda U^T$ results in what should be proven.

$$U\Lambda U^T = \begin{pmatrix} u_{1,1} & \dots & u_{1,d} \\ \vdots & \ddots & \vdots \\ u_{d,1} & \dots & u_{d,d} \end{pmatrix} \begin{pmatrix} \frac{1}{\lambda_1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \frac{1}{\lambda_d} \end{pmatrix} \begin{pmatrix} u_{1,1} & \dots & u_{d,1} \\ \vdots & \ddots & \vdots \\ u_{1,d} & \dots & u_{d,d} \end{pmatrix} \tag{2}$$

$$= \begin{pmatrix} \frac{u_{1,1}}{\lambda_1} & \dots & \frac{u_{1,d}}{\lambda_d} \\ \vdots & \ddots & \vdots \\ \frac{u_{d,1}}{\lambda_1} & \dots & \frac{u_{d,d}}{\lambda_d} \end{pmatrix} \begin{pmatrix} u_{1,1} & \dots & u_{d,1} \\ \vdots & \ddots & \vdots \\ u_{1,d} & \dots & u_{d,d} \end{pmatrix} \tag{3}$$

$$= \begin{pmatrix} \frac{u_{1,1}u_{1,1}}{\lambda_1} + \dots + \frac{u_{1,d}u_{1,d}}{\lambda_d} & \dots & \frac{u_{1,1}u_{d,1}}{\lambda_1} + \dots + \frac{u_{1,d}u_{d,d}}{\lambda_d} \\ \vdots & \ddots & \vdots \\ \frac{u_{d,1}u_{1,1}}{\lambda_1} + \dots + \frac{u_{1,d}u_{d,d}}{\lambda_d} & \dots & \frac{u_{d,1}u_{d,1}}{\lambda_1} + \dots + \frac{u_{d,d}u_{d,d}}{\lambda_d} \end{pmatrix} \tag{4}$$

$$= \frac{1}{\lambda_1} \begin{pmatrix} u_{1,1}u_{1,1} & \dots & u_{1,1}u_{d,1} \\ \vdots & \ddots & \vdots \\ u_{d,1}u_{1,1} & \dots & u_{d,1}u_{d,1} \end{pmatrix} + \dots + \frac{1}{\lambda_d} \begin{pmatrix} u_{1,d}u_{1,d} & \dots & u_{1,d}u_{d,d} \\ \vdots & \ddots & \vdots \\ u_{1,d}u_{d,d} & \dots & u_{d,d}u_{d,d} \end{pmatrix} \tag{5}$$

$$= \frac{1}{\lambda_1} u_1 u_1^T + \dots + \frac{1}{\lambda_d} u_d u_d^T \tag{6}$$

$$= \sum_{i=1}^{d} \frac{1}{\lambda_i} u_i u_i^T \tag{7}$$

Problem 2

HW2

Problem 3

HW3

Problem 4

HW4