Machine Learning Worksheet 8

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Problem 1

$$\nabla E(w) = \begin{pmatrix} \frac{\partial E(w)}{\partial w_1} \\ \vdots \\ \frac{\partial E(w)}{\partial w_d} \end{pmatrix}$$

with

$$\frac{\partial E(w)}{\partial w_i} = \frac{1}{m} \sum_{i=1}^{m} f'(z_i - w x_i) x_i + \gamma w_i$$

$$f'(x) = \begin{cases} x & |x| < 1 \\ sgn(x) & otherwise \end{cases}$$

Problem 2

To minimize the error or loss we have to find better values for the weights w. This is normaly not a convex problem so we have to use an incremental approach like steepest decent to propagate changes back through the network and adjusting the weights.

$$w_{i+1} = w_i - \alpha \nabla E(w) \tag{1}$$

α is the learing rate.

Problem 3

Only implementation in the perceptron

neuralnetworks1

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1 Neural network learning

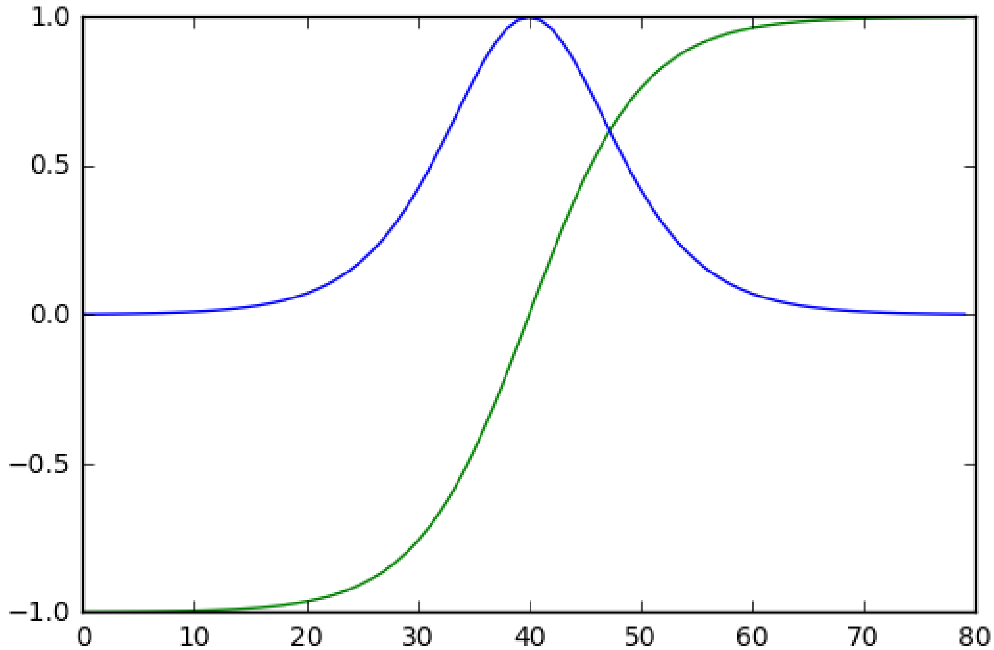
notebook written by Patrick van der Smagt

```
In [40]: import numpy as np
         import matplotlib.pyplot as plt
         %matplotlib inline
```

We start off with defining transfer functions. These are the (nonlinear) functions ϕ in the neurons.

The implementation is a bit weird: it returns $y = \phi(x, \text{False})$ or $s = \phi(y, \text{True})$ where $y = \phi(x)$. So if the second parameter is True, the first argument is interpreted to be $\phi(x)$ and is used to compute the derivative. This is done for computational efficiency only.

```
In [41]: # sigmoid function.  returns y=sigmoid(x) or y' = sigmoid(y, True)
         def sigmoid(x,deriv=False):
             if(deriv==True):
                 return x*(1-x)
             return 1/(1+np.exp(-x))
         # tanh function.
         def tanh(x,deriv=False):
             if(deriv==True):
                 return 1-x*x
             return np.tanh(x)
         # tanh function.
         def linear(x,deriv=False):
             if(deriv==True):
                 return 1
             return x
         def rectifier(x, deriv=False):
             if (deriv==True):
                 return (x > 0.)
             return x * (x > 0.)

         actfunc = tanh                  # transfer function
         y = actfunc(np.arange(-4,4,.1))
         plt.plot(y, "green");
         s = actfunc(y,True)        # derivative
         plt.plot(s, "blue");
```



1.1 Let's start off with a simple neural network with no hidden layer

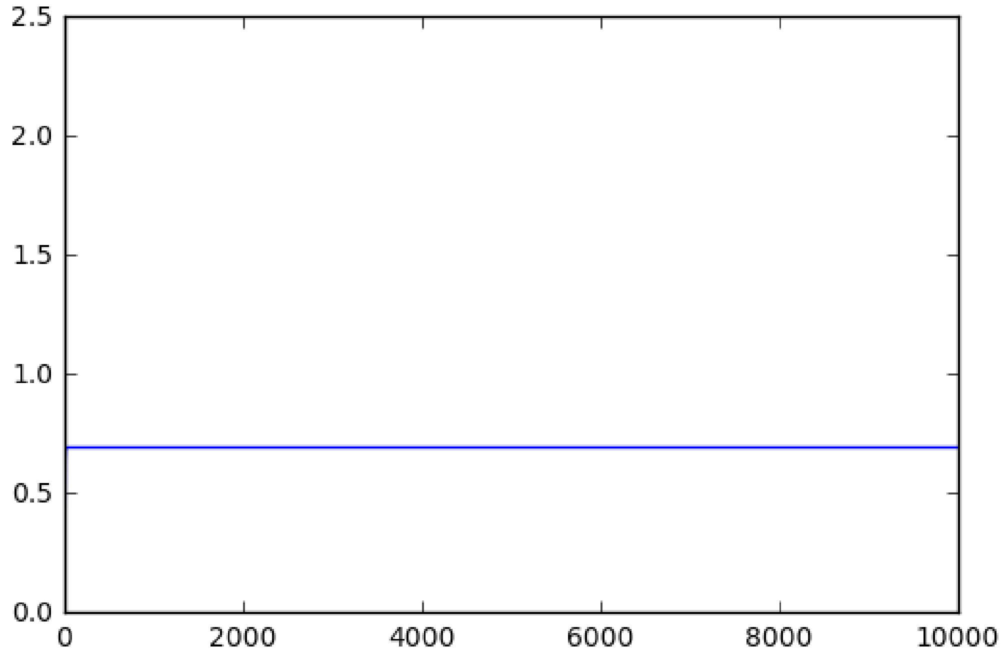
This simple "neural network"—in this form also known as perceptron—does a simple forward pass, then computes the output errors, and uses those to adapt the weights.

Note that this thing does something like linear regression—and the error is a convex function in its parameters w.

```
In [42]: # input dataset.  Why is the third input always 1?
         X = np.array([  [0,0,1],
                         [0,1,1],
                         [1,0,1],
                         [1,1,1] ])

         # output dataset
         z = np.array([[0,0,1,1]]).T

         # seed random numbers
         np.random.seed()

         # initialize weights randomly with mean 0
         # What happens if I start with w0 = 0?
         w0 = 2*np.random.random((X.shape[1],1)) - 1

         loss = []
```

```python
# the transfer function
transfer = tanh

# the learning rate
learn_rate = 0.1
# the momentum
momentum = 0.1
u = 0

# make so many steps
for iter in range(10000):
    # forward propagation: layer 0 is the input
    l0 = X
    # layer one is dot product of l0 with w0,
    # then the transfer function.  l1 is the output, i.e., y
    l1 = transfer(np.dot(l0,w0))

    # What is the residual?  Note that the error
    # equals the absolute residual.
    residual = z - l1
    # In the MLE interpretation we assume Gaussian errors in
    # the data.  So the loss is the sum of squared errors:
    loss.append(sum(i**2 for i in residual))

    # To find the residual at the inputs of the output unit, we
    # "propagate it through the unit", i.e., we multiply the
    # residual with the derivative of the transfer function.
    # We call the result "delta".  You can easily show that
    # this is mathematically correct.
    l1_delta = residual * transfer(l1,True)
    if iter == 1:
        u = l1_delta
    else:
        u += momentum * l1_delta
    # Update weights.  How can this be extended with a momentum term?
    w0 += learn_rate * np.dot(l0.T,u)

print ("perceptron output:")
print (l1)
plt.plot(np.log(loss));
```

```
perceptron output:
[[ 1.]
 [ 1.]
 [ 1.]
 [ 1.]]
```



1.2 Neural Network Implementation

A neural network with one hidden layer. Biases are not added by this implementation (why is that no problem?). Also, momentum is not implemented yet—please implement that to get faster learning, as soon as you know what is meant by "momentum".

```
In [43]: class MyMLP:
             def __init__(self, n_inpt, n_hidden, n_output, hid_transfer = tanh, ou
                 self.w0 = 2*np.random.random((n_inpt, n_hidden))
                 self.w1 = 2*np.random.random((n_hidden, n_output))
                 self.hid_transfer = hid_transfer
                 self.out_transfer = out_transfer
                 self.loss = []

             def ffnn_forward(self, X):
                 l0 = X
                 l1 = self.hid_transfer(np.dot(l0,self.w0))
                 l2 = self.out_transfer(np.dot(l1,self.w1))
                 return (l0,l1,l2)

             def NeuralNetwork(self, X, z, learn_rate = 0.1, momentum = 0.9):
                 # search direction for the weights in layers 1 and 0
                 optim_step1 = 0
                 optim_step0 = 0
```

```
            for j in xrange(60000):

                # Feed forward through layers 0, 1, and 2
                # l0 is the input layer
                # l1 is the hidden layer
                # l2 is the output layer
                (l0,l1,l2) = self.ffnn_forward(X)

                # (signed) error at the output
                residual = z - l2
                # the loss is the sum of squared errors
                self.loss.append(sum(i**2 for i in residual))

                # compute the delta.
                l2_delta = residual*self.out_transfer(l2,deriv=True)

                # back-propagate the output delta to the hidden units
                l1_error = l2_delta.dot(self.w1.T)

                # compute the delta at the hidden units
                l1_delta = l1_error * self.hid_transfer(l1,deriv=True)

                optim_step1 = learn_rate*l1.T.dot(l2_delta)
                optim_step0 = learn_rate*l0.T.dot(l1_delta)

                self.w1 += optim_step1
                self.w0 += optim_step0

            plt.figure()
            plt.plot(np.log(self.loss));
```

Print the contour plot of this NN with a two-dimensional input

```
In [44]: def nncontour (start, end, step, mlp):
             xx, yy = np.meshgrid( np.arange(start, end, step),  np.arange(start, e
             x = np.array([[0,0,1]])
             z = np.zeros(( len(xx[:,0]), len(yy[:,0])) )
             for i in range(len(xx[:,0])):
                 for j in range(len(yy[:,0])):
                     x = [[xx[i,j], yy[i,j], 1]]
                     (l0,l1,l2) = mlp.ffnn_forward(x)
                     z[i, j] = l2.item(0)
             plt.figure()
             plt.contour(xx, yy, z)
```

Now do a typical silly example: the XOR function. Please play around with the parameters of the NN: number of hiddens, the learning rate, the momentum…

```
In [45]: X = np.array([[0,0,1],
                       [0,1,1],
```

```
              [1,0,1],
              [1,1,1]])

y = np.array([[0],
                          [1],
                          [1],
                          [0]])

mlp_xor = MyMLP(X.shape[1], 2, y.shape[1])
mlp_xor.NeuralNetwork(X,y,0.5)
(l0, l1, l2) = mlp_xor.ffnn_forward(X)
print "output:"
print l2
nncontour(0,1, 0.01, mlp_xor)
```

```
  File "<ipython-input-45-f12424a15e72>", line 14
    print "output:"
                  ^
SyntaxError: Missing parentheses in call to 'print'
```

1.3 Draw Hinton diagrams

Hinton diagrams can be used to represent weights in a NN. Each box represents a single weight. The colour of the box indicates the sign of the weight (black: negative; white: positive); the size of the box the size of the weight.

```
In [ ]: def hinton(matrix, max_weight=None, ax=None):
            """Draw Hinton diagram for visualizing a weight matrix."""
            ax = ax if ax is not None else plt.gca()

            if not max_weight:
                #max_weight = 0.5 * 2**np.ceil(np.log(np.abs(matrix).max())/np.log
                max_weight = np.max(np.abs(matrix))

            ax.patch.set_facecolor('gray')
            ax.set_aspect('equal', 'box')
            ax.xaxis.set_major_locator(plt.NullLocator())
            ax.yaxis.set_major_locator(plt.NullLocator())

            for (x, y), w in np.ndenumerate(matrix):
                color = 'white' if w > 0 else 'black'
                size = np.sqrt(np.abs(w)) / max_weight
                rect = plt.Rectangle([x - size / 2, y - size / 2], size, size,
                                     facecolor=color, edgecolor=color)
                ax.add_patch(rect)
```

```
        ax.autoscale_view()
        ax.invert_yaxis()
```

```
In [ ]: hinton(mlp_xor.w0)
```

Now try to fit a sine wave. Experiment with, e.g., very many and very few hidden units. Try it with and without momentum in the learning.

```
In [ ]: data = np.arange(-3,3,0.1) # is it enough data??
        X = np.array([[x, 1] for x in data])
        y = np.array([[np.sin(x)+np.random.randn()/100 for x in data]]).T
        mlp_sin = MyMLP(X.shape[1],10,y.shape[1], hid_transfer=sigmoid)
```

```
In [ ]: mlp_sin.NeuralNetwork(X,y,0.1) # note that momentum is not implemented yet
```

Plot the output of the neural network over the training data.

```
In [ ]: (l0,l1,l2) = mlp_sin.ffnn_forward(X)
        plt.plot(l2, "green")
        plt.plot(y, "blue")
```