**RO47001 ROBOT DYNAMICS & CONTROL**

Cognitive Robotics

# LECTURE 7: Kinematics & Dynamics of Mobile Robot / Automated Vehicle

**Barys Shyrokau**

**Cognitive Robotics, 3mE, Delft University of Technology, The Netherlands**

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TU Delft

# LEARNING GOALS

After this lecture, the student should be able to:

1. Describe the basic wheel types and wheel configurations.
2. Derive the kinematic model of the mobile robot.
3. Describe the concept of vehicle handling.
4. Perform steady-state analysis of steering characteristics.
5. Derive the equations of motion of a linear vehicle model.

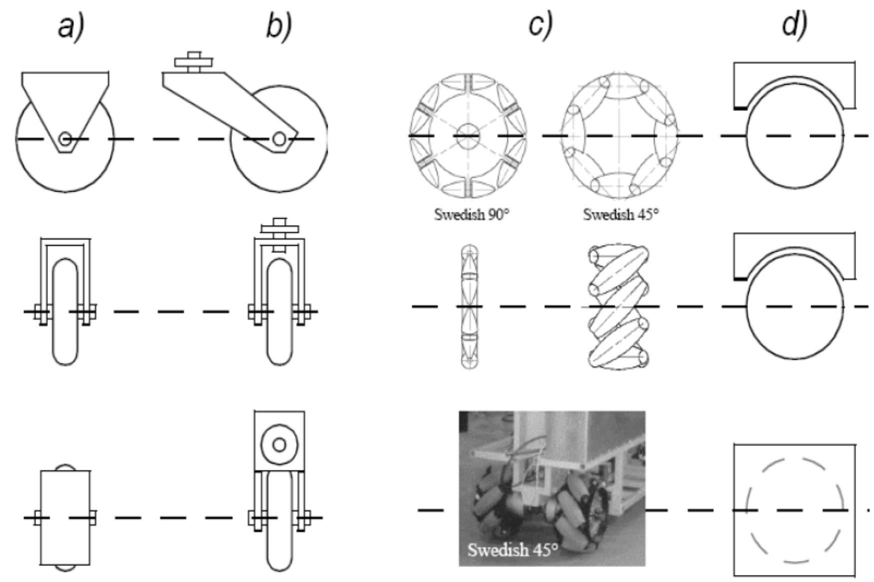
# WHEELED MOBILE ROBOTS


- Wheels are the most common solution for the majority of applications.
- Three wheels are sufficient to guarantee stability and improved with four.
- There are many possibilities of wheel configurations when considering possible techniques for mobile robot locomotion.
- The selection of wheels depends on the application.

# BASIC WHEELS TYPES

The choice of wheel type has a large effect on the overall kinematics of the mobile robot:

- a) Standard wheel: Two degrees of freedom; rotation around the (motorized) wheel axle and the contact point.
- b) Castor wheel: Three degrees of freedom; rotation around the wheel axle, the contact point and the castor axle.
- c) Swedish wheel: Three degrees of freedom; rotation around the (motorized) wheel axle, around the rollers and the contact point.
- d) Ball or spherical wheel.

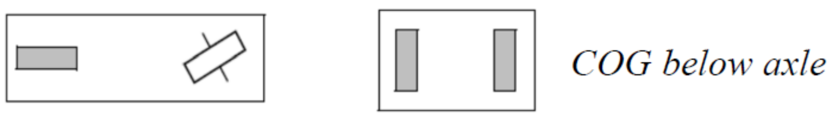


# CHARACTERISTICS OF WHEELED ROBOTS

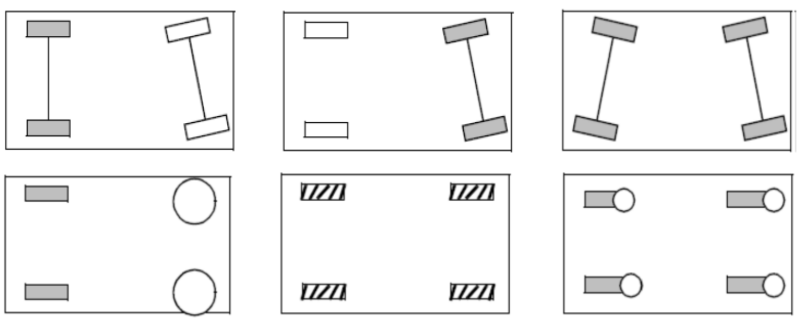
- The stability of a vehicle is guaranteed with 3 wheels. If center of gravity is within the triangle formed by the ground contact point of the wheels.
- Stability is improved by 4 and more wheels. However, these arrangements are hyper static and require a flexible suspension system.
- Bigger wheels allow to overcome higher obstacles, but they require higher torque or reductions in the gearbox.
- Most arrangements are non-holonomic requiring high control effort.
- Trade-off between Maneuverability and Controllability.
- Combining actuation and steering on one wheel increases complexity.

# WHEEL CONFIGURATIONS

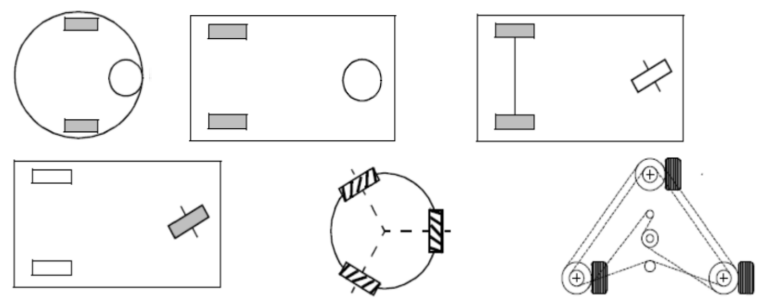
*Two-wheel configurations*
One steering wheel and one traction wheel; Two-wheel differential drive (independent actuators for each wheel)




*Four-wheel configurations*



*Three-wheel configurations*:
Two-wheel differential drive and one unpowered omnidirectional wheel; Two connected traction wheels and one steering wheel; Two free wheels and one steered traction wheel; Three Swedish wheels: omnidirectional movement; Three synchronously driven and steered wheels: orientation not controllable



Omnidirectional Drive Synchro Drive

# KINEMATIC MODEL AND CONSTRAINTS

- Each wheel contributes to the robot's motion and, at the same time, imposes constraints on robot motion.
- Wheels are tied together based on robot chassis geometry.

Main assumptions:

- The robot is built from rigid mechanisms
- No slip occurs in the orthogonal direction of rolling (non-slipping).
- No translational slip occurs between the wheel and the floor (pure rolling)
- All steering axes are perpendicular to the floor

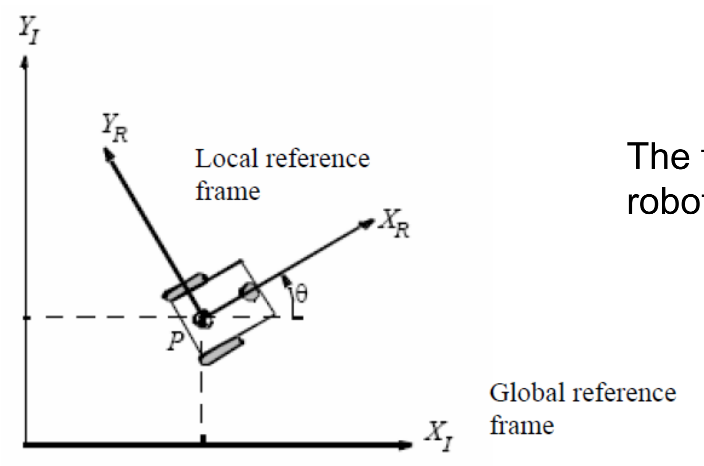
# KINEMATIC MODEL

The position of *P* in the global reference frame is specified by coordinates *x* and *y*, and the angular difference between the global and local reference frames is given by *θ*.

$$\xi_I = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

We need to map motion along the axes of the global reference frame to motion along the axes of the robot's local reference frame.

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

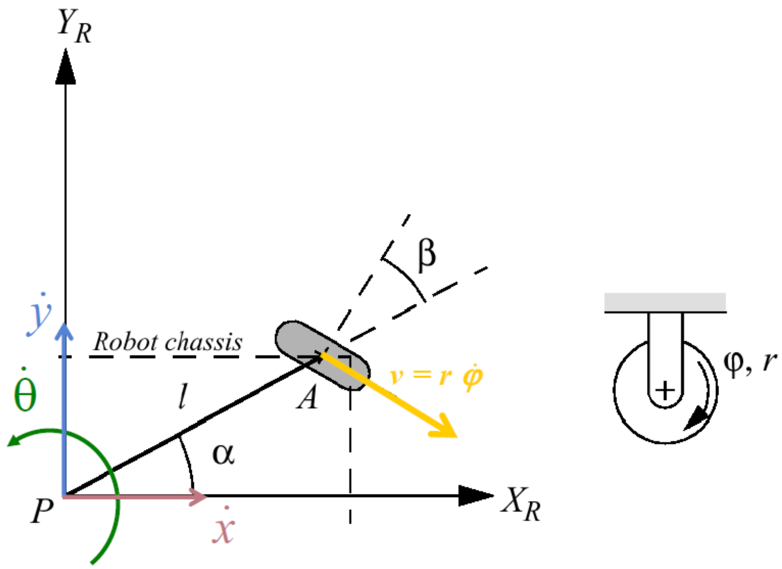



The forward kinematic model describes the motion of a mobile robot in the global reference frame as a function of wheel velocities

$$\dot{\xi}_R = R(\theta)\dot{\xi}_I = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$

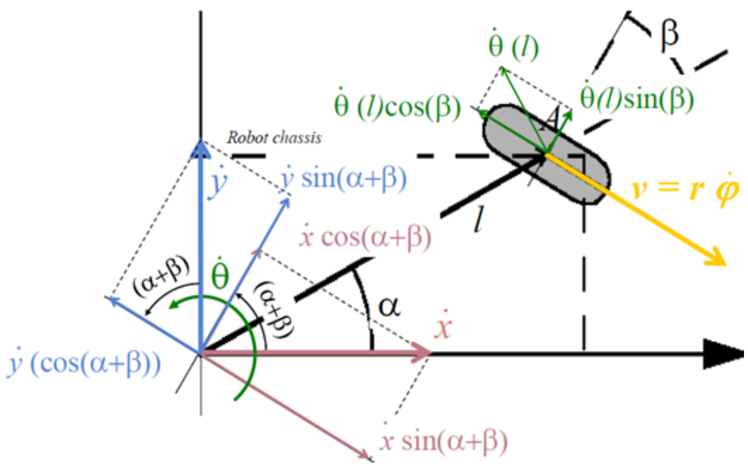
# WHEEL CONSTRAINTS: FIXED STANDARD WHEEL

- The first constraint enforces the concept of rolling contact — that the wheel must roll when motion takes place in the appropriate direction.
- The second constraint enforces the concept of no lateral slippage — that the wheel must not slide orthogonal to the wheel plane.



# WHEEL CONSTRAINTS: FIXED STANDARD WHEEL

The rolling constraint enforces that the motion along the direction of the wheel plane must be accompanied by the appropriate amount of wheel spin so that there is pure rolling at the contact point:




$$v = \dot{\varphi} r = \dot{x}_R \sin(\alpha+\beta) - \dot{y}_R \cos(\alpha+\beta) - \dot{\theta} l \cos\beta =$$

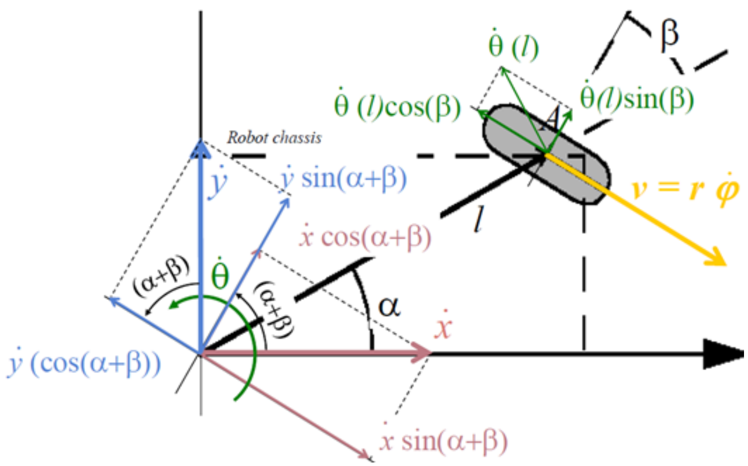
$$\begin{bmatrix} \sin(\alpha+\beta) & -\cos(\alpha+\beta) & -l\cos\beta \end{bmatrix} \begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\theta} \end{bmatrix}$$

The *rolling constraint* can be written as:

$$\begin{bmatrix} \sin(\alpha+\beta) & -\cos(\alpha+\beta) & -l\cos\beta \end{bmatrix} R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

# WHEEL CONSTRAINTS: FIXED STANDARD WHEEL

The sliding constraint for this wheel enforces that the component of the wheel's motion orthogonal to the wheel plane must be zero:




$$\dot{x}_R \cos(\alpha+\beta) + \dot{y}_R \sin(\alpha+\beta) + \dot{\theta} l \sin\beta = 0$$

The *sliding constraint* can be written as:

$$\left[\cos(\alpha+\beta) \quad \sin(\alpha+\beta) \quad l\sin\beta\right] R(\theta)\dot{\xi}_I = 0$$

# WHEEL CONSTRAINTS: COMPLETE ROBOT

Given a robot with N wheels

- each wheel imposes zero or more constraints on the robot's motion
- **only fixed (f) and steerable (s) standard wheels impose constraints**

Suppose we have a total of $N = Nf + Ns$ standard wheels

- We can develop the equations for the constraints in matrix forms.
- Rolling ($J_1$ - matrix with projections for all wheels to their motions along their individual wheel planes, $J_2$ - constant diagonal matrix)

$$J_1(\beta_s)R(\theta)\dot{\xi}_I + J_2\dot{\varphi} = 0$$

$$\varphi(t) = \underbrace{\begin{bmatrix} \varphi_f(t) \\ \varphi_s(t) \end{bmatrix}}_{(N_f+N_s)\times 1} \qquad J_1(\beta_s) = \underbrace{\begin{bmatrix} J_{1f} \\ J_{1s}(\beta_s) \end{bmatrix}}_{(N_f+N_s)\times 3} \qquad J_2 = diag(r_1 \cdots r_N)$$

$$\boxed{\begin{bmatrix} \sin(\alpha+\beta) & -\cos(\alpha+\beta) & (-l)\cos\beta \end{bmatrix}} R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

- Lateral movement

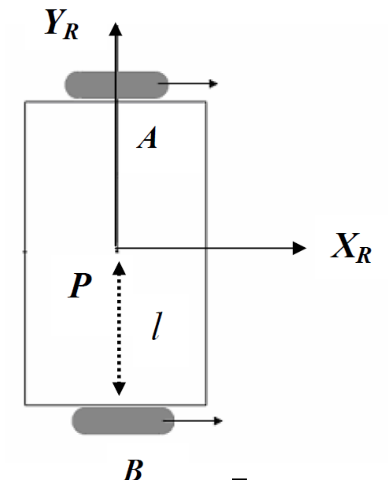
$$C(\beta_s)R(\theta)\dot{\xi}_I = 0$$

$$C_1(\beta_s) = \underbrace{\begin{bmatrix} C_{1f} \\ C_{1s}(\beta_s) \end{bmatrix}}_{(N_f+N_s)\times 3} \longrightarrow \boxed{\begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) & l\sin\beta \end{bmatrix}} R(\theta)\dot{\xi}_I = 0$$

# EXAMPLE

Wheel A: α=π/2; β = 0;

Wheel B: α=-π/2; β = π;




The rolling constraints are:

$$\begin{bmatrix}1 & 0 & -l\end{bmatrix} R(\theta)\dot{\xi}_I - r\dot{\varphi}_A = 0$$

$$\begin{bmatrix}1 & 0 & l\end{bmatrix} R(\theta)\dot{\xi}_I - r\dot{\varphi}_B = 0$$

$$\underbrace{\begin{bmatrix}1 & 0 & -l \\ 1 & 0 & l\end{bmatrix}}_{J_1} R(\theta)\dot{\xi}_I - \underbrace{\begin{bmatrix}r & 0 \\ 0 & r\end{bmatrix}}_{J_2}\begin{bmatrix}\dot{\varphi}_A \\ \dot{\varphi}_B\end{bmatrix} = 0$$

The sliding constraints are:

$$\underbrace{\begin{bmatrix}0 & 1 & 0 \\ 0 & 1 & 0\end{bmatrix}}_{C} R(\theta)\dot{\xi}_I = 0$$

Fusing these two equations yields:

$$\begin{bmatrix}J_1 \\ C_1\end{bmatrix} R(\theta)\dot{\xi}_I = \begin{bmatrix}J_2\dot{\varphi} \\ 0\end{bmatrix}$$

Hence:

$$\begin{bmatrix}1 & 0 & -l \\ 1 & 0 & l \\ 0 & 1 & 0\end{bmatrix} R(\theta)\dot{\xi}_I = \begin{bmatrix}r\dot{\varphi}_A \\ r\dot{\varphi}_B \\ 0\end{bmatrix}$$

The final kinematic equation of the mobile robot is:

$$\dot{\xi}_I = R^T(\theta)\begin{bmatrix}1 & 0 & -l \\ 1 & 0 & l \\ 0 & 1 & 0\end{bmatrix}^{-1}\begin{bmatrix}r\dot{\varphi}_A \\ r\dot{\varphi}_B \\ 0\end{bmatrix}$$

# MOBILE ROBOT MANEUVERABILITY

The maneuverability of a mobile robot is the combination

- of the mobility available based on the sliding constraints
- plus additional freedom contributed by the steering

To avoid any lateral slip the motion vector has to satisfy:

$$C(\beta_s) = \begin{bmatrix} C_f \\ C_s(\beta_s) \end{bmatrix} R(\theta)\dot{\xi}_I = 0$$

The rank of $C(\beta_s)$ is the number of independent constraints.

The **degree of mobility** is defined by the dimensionality of the null space of $C(\beta_s)$

Restricted by no-sliding constraints

The greater the rank, the more constrained the mobility

$$\delta_m = \dim N\left[C(\beta_s)\right] = 3 - rank\left[C(\beta_s)\right]$$

No standard wheels $rank\left[C(\beta_s)\right] = 0$

All motion directions constrained $rank\left[C(\beta_s)\right] = 3$

# MOBILE ROBOT MANEUVERABILITY

**Degree of steerability** $\delta_s = rank\left[C_s(\beta_s)\right]$

Increase in the rank implies more degrees of steering freedom and greater eventual maneuverability

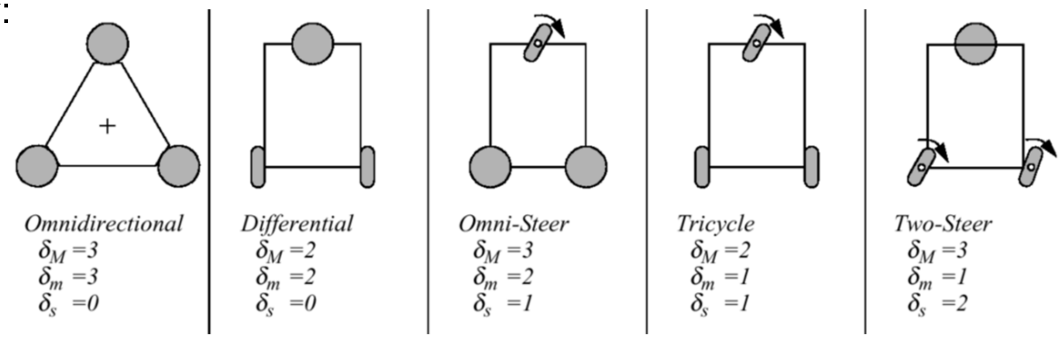
Since $C(\beta_s)$ includes $C_s(\beta_s)$, this means that a steered standard wheel can both decrease mobility and increase steerability: its particular orientation at any instant imposes a kinematic constraint, but its ability to change that orientation can lead to additional trajectories.

The range of $\delta_s$ can be specified: $0 \leq \delta_s \leq 2$.

The overall degrees of freedom that a robot can manipulate, called the **degree of maneuverability** can be defined in terms of mobility and steerability:

$$\delta_M = \delta_m + \delta_s$$

*Two robots with the same value of degree of maneuverability are not maneuverable in the same way*



# Outdoor Mobile Robot / Automated Vehicle

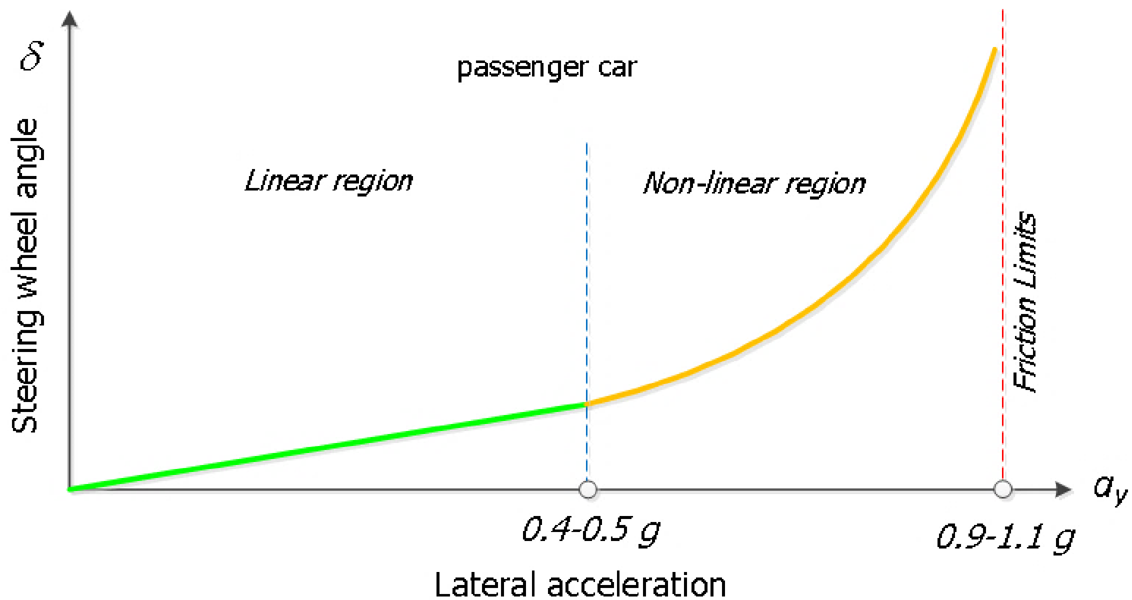
# AUTOMATED DELIVERY: Vilnius City Centre, Lithuania


# VEHICLE HANDLING

- lateral response of a vehicle to driver input
- steady-state lateral motion is a core part of handling

Motion regions:

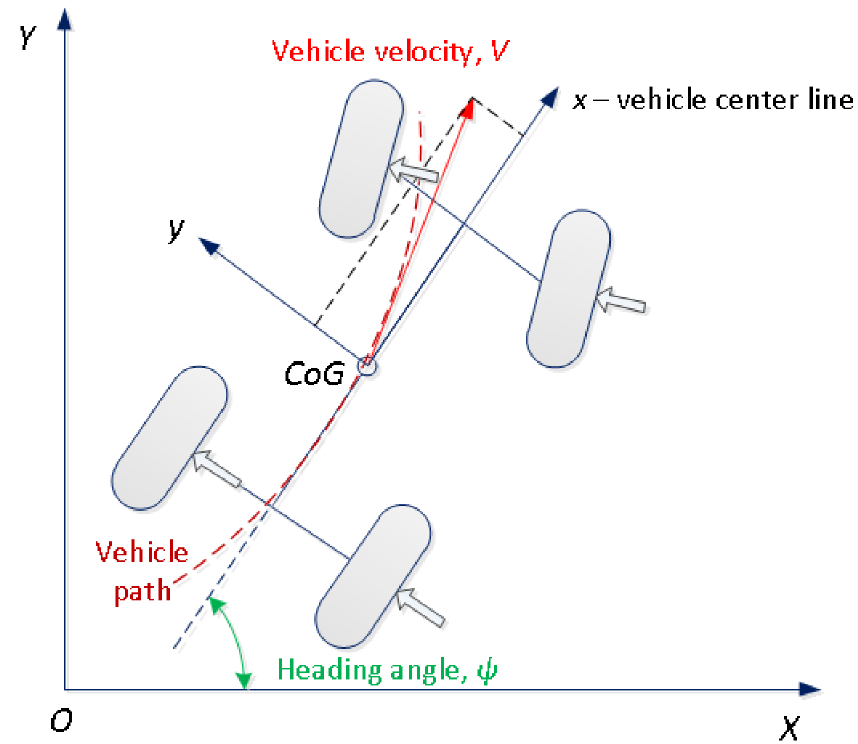
- kinematic (low speed)
- linear
- nonlinear
- near friction limits



# TERMINOLOGY

<u>Heading angle</u> $\psi$ is the angle between the trace on the X-Y plane of the vehicle x-axis and the X-axis of the earth fixed system.

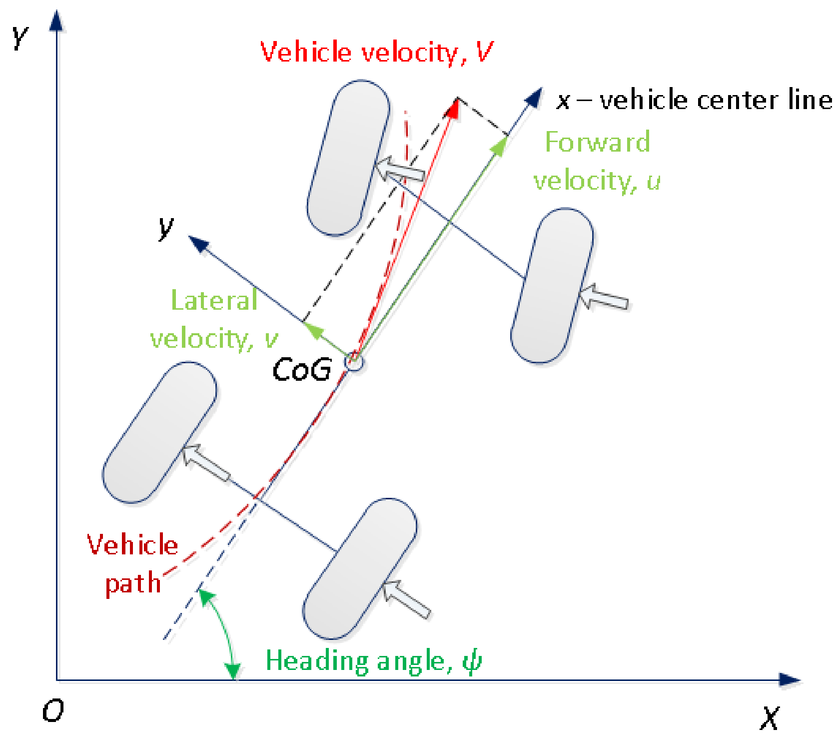
<u>Velocity vector</u> $V$ is vehicle velocity in earth-fixed coordinates, it is always tangent to the path of the vehicle CoG.



# TERMINOLOGY

Forward Velocity $u$ is the component of the vehicle velocity in the x-direction.

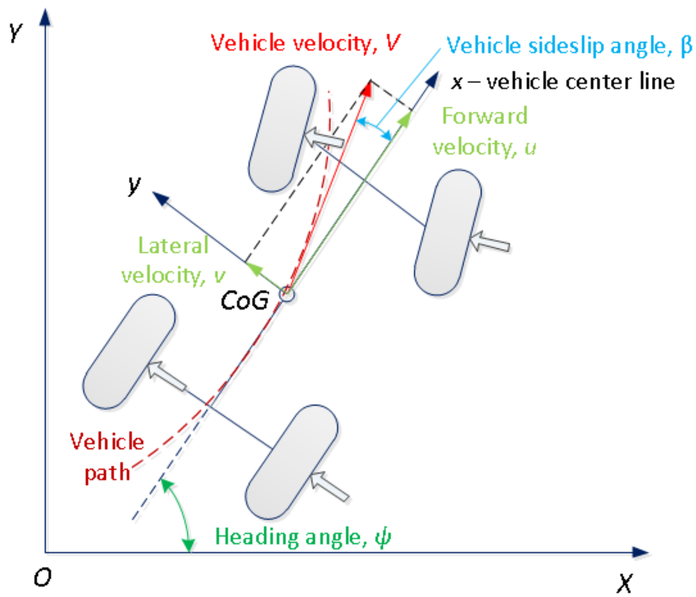
Lateral or side velocity $v$ is the component of the vehicle velocity in the y-direction.



# TERMINOLOGY

Sideslip angle $\beta$ is the angle between the vehicle $x$-axis (vehicle center line) and the vehicle velocity vector

In stable driving situations the sideslip angle is small (< 5 deg)

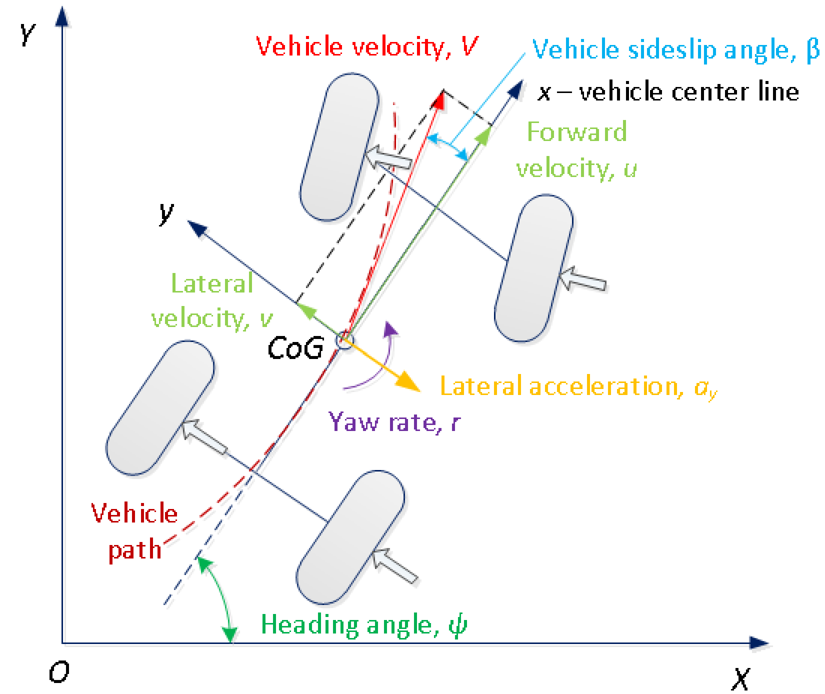



$$\beta = \arctan\left(\frac{v}{u}\right) \approx \frac{v}{u}$$

# TERMINOLOGY

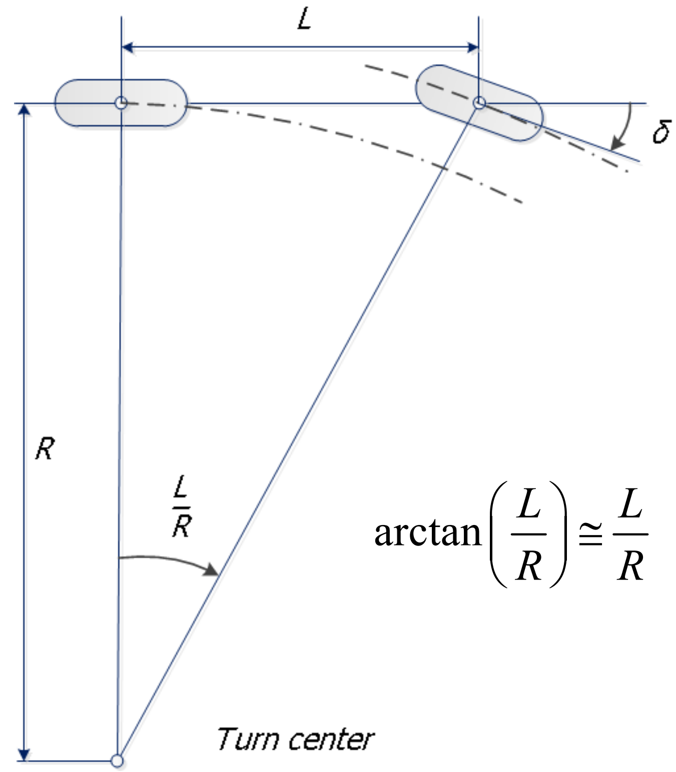
Yaw rate $r$ is the angular rate about the vertical axis

Lateral acceleration $a_y$ is the component of the vector acceleration of a point in the vehicle perpendicular to the vehicle's $x$-axis



# TURNING AT VERY LOW SPEED

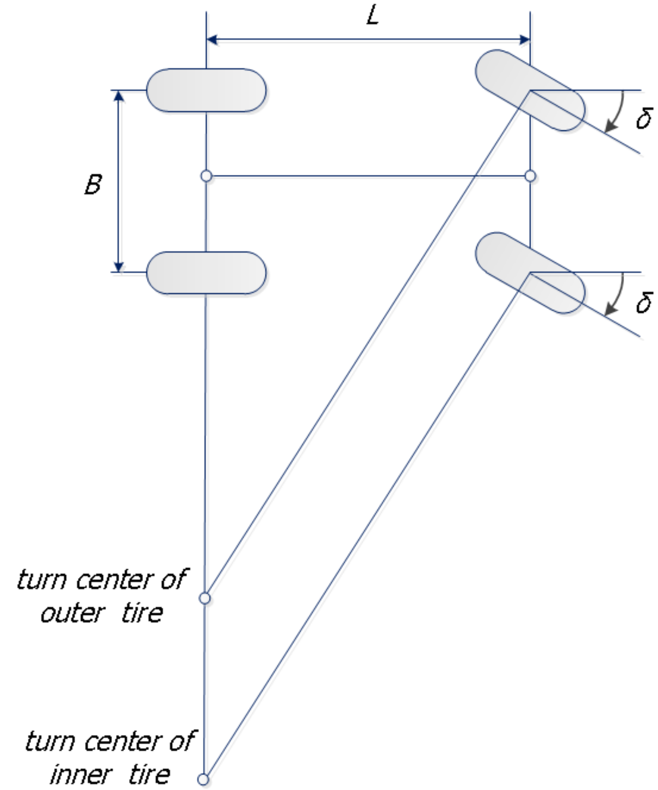
- bicycle vehicle
- inertial forces and centrifugal acceleration are negligible
- the problem transforms into pure geometrical
- instantaneous center of rotation (ICR)
- longer wheelbase *L* results in wider turn radius *R* or more steer input
- this geometric steering angle is known as the **Ackermann angle**




$$\arctan\left(\frac{L}{R}\right) \cong \frac{L}{R}$$

# TURNING AT VERY LOW SPEED

- four-wheel vehicle
- same steer angle for inner and outer tires
- creates **tire scrub** as the wheels tend to fight each other's motion
- necessary to allow different steer angles on inner and outer wheels



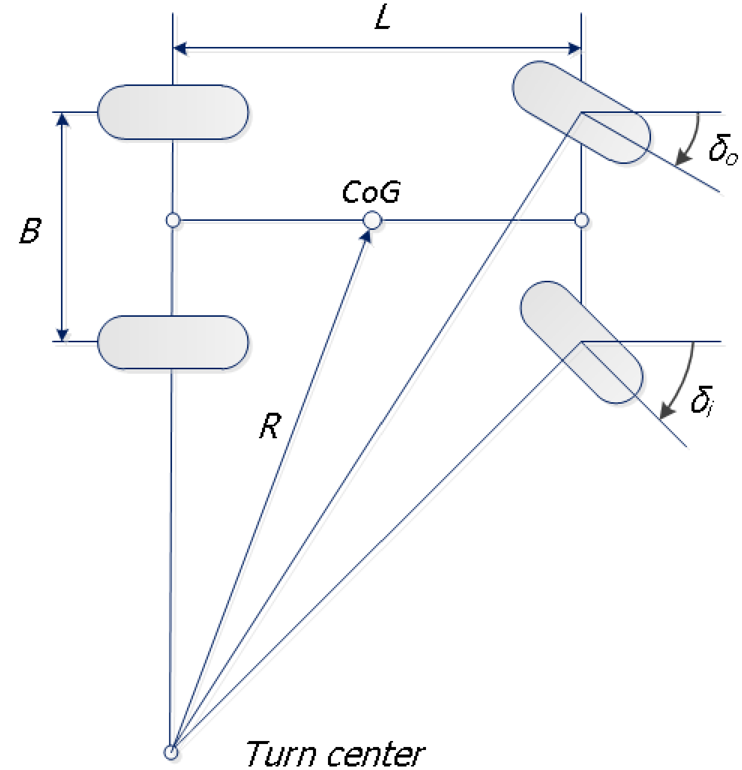
# TURNING AT VERY LOW SPEED

- wheels should follow curved paths with radii originating from a <u>common center ICR</u>
- for proper steering, the steer angles are given by:

$$\delta_o \cong \frac{L}{R+0.5B} < \delta_i \cong \frac{L}{R-0.5B}$$

- **Ackermann geometry** ignoring kingpin offsets is defined as:

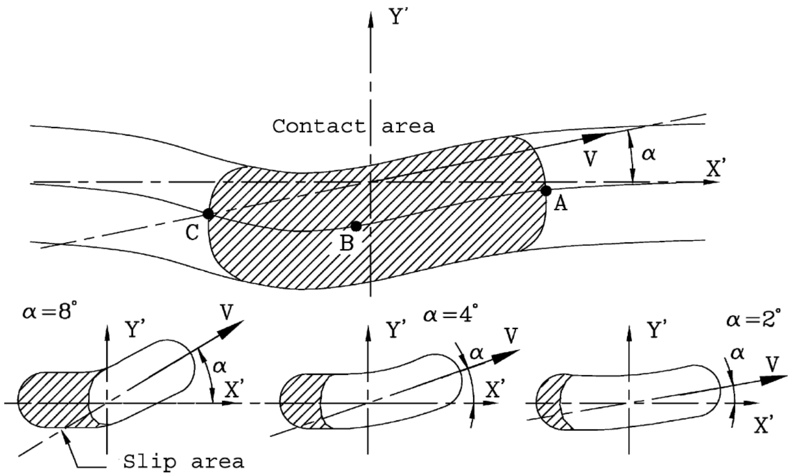
$$\cot\delta_o - \cot\delta_i = \frac{B}{L}$$



# TIRE SLIP ANGLE

<u>Slip angle</u> $\alpha$ is the angle between the expected/desired direction and actual direction of travel of the center of tire contact

$$\alpha = \tan^{-1}\left(\frac{V_{yw}}{V_{xw}}\right)$$



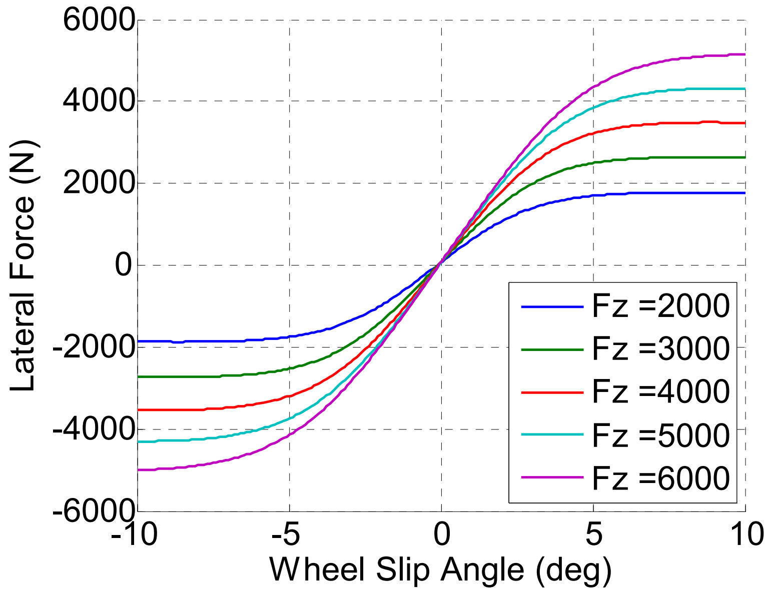

# LATERAL FORCE

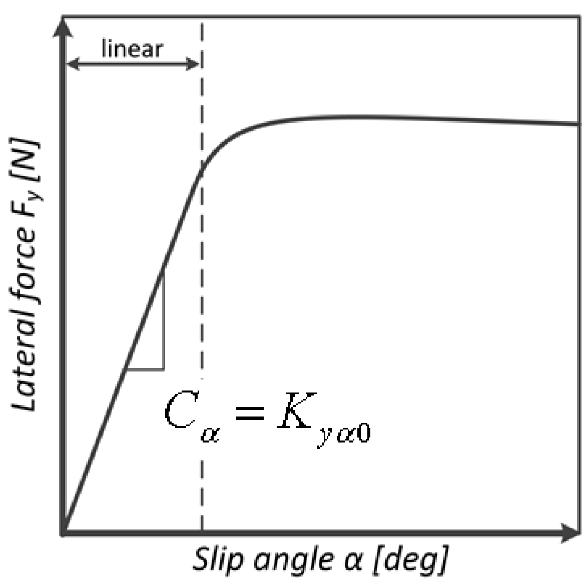
Assuming linear relationship between lateral forces and slip angle

$C_{\alpha f}$ and $C_{\alpha r}$ is cornering stiffness of front and rear axles respectively

$$F_{yf} = C_{\alpha f}\alpha_f$$

$$F_{yr} = C_{\alpha r}\alpha_r$$

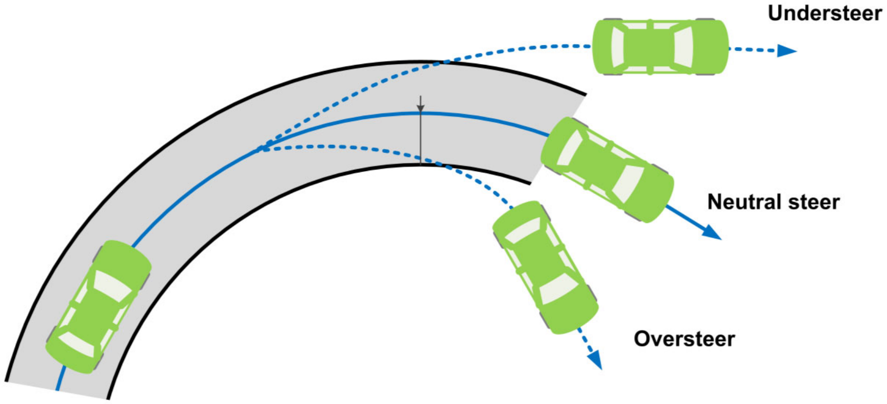



$$F_z = \text{const}$$

# STEERING CHARACTERISTICS

vehicle speeds up in a constant radius turn

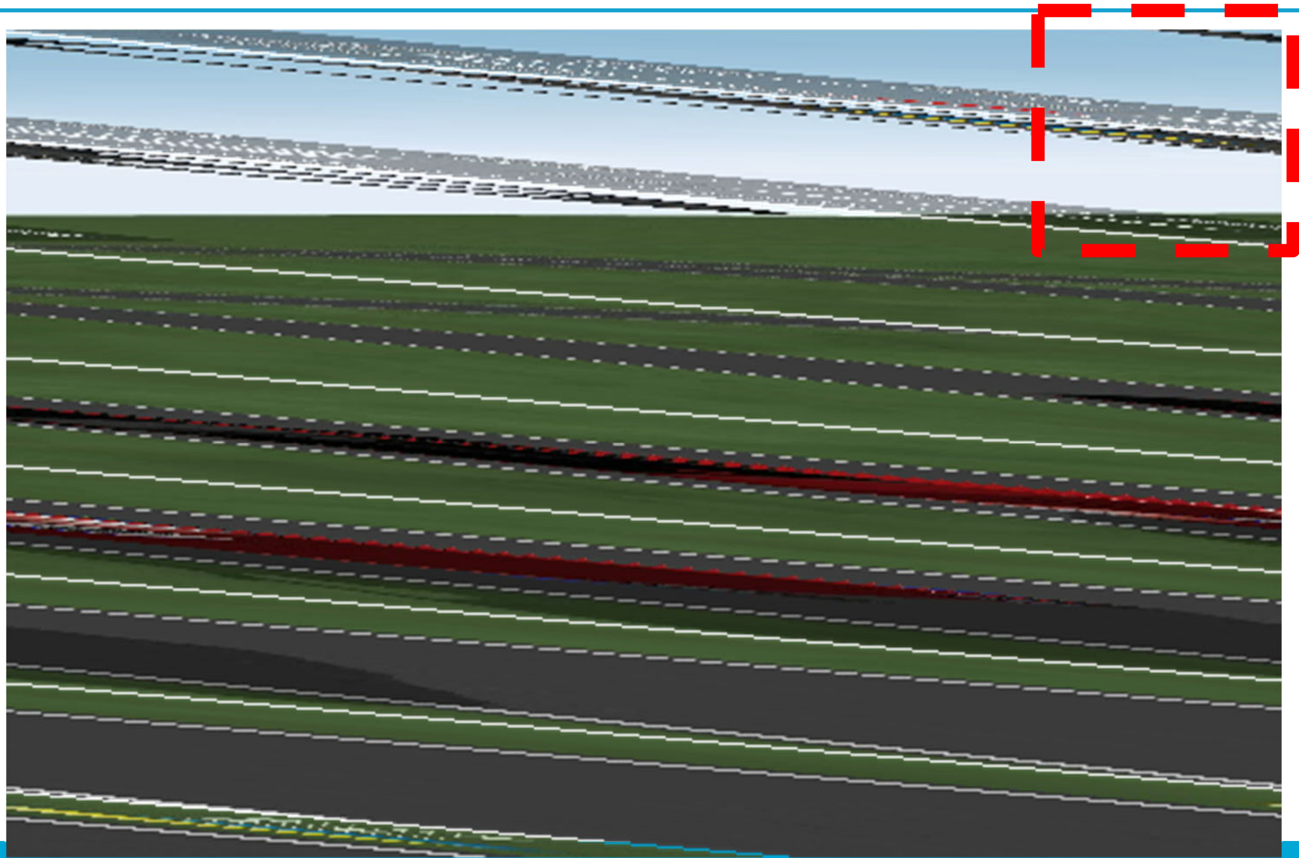



Neutral steer the driver should maintain the same steering wheel position. That is, when accelerated with the steering wheel fixed, the turning radius remains the same.

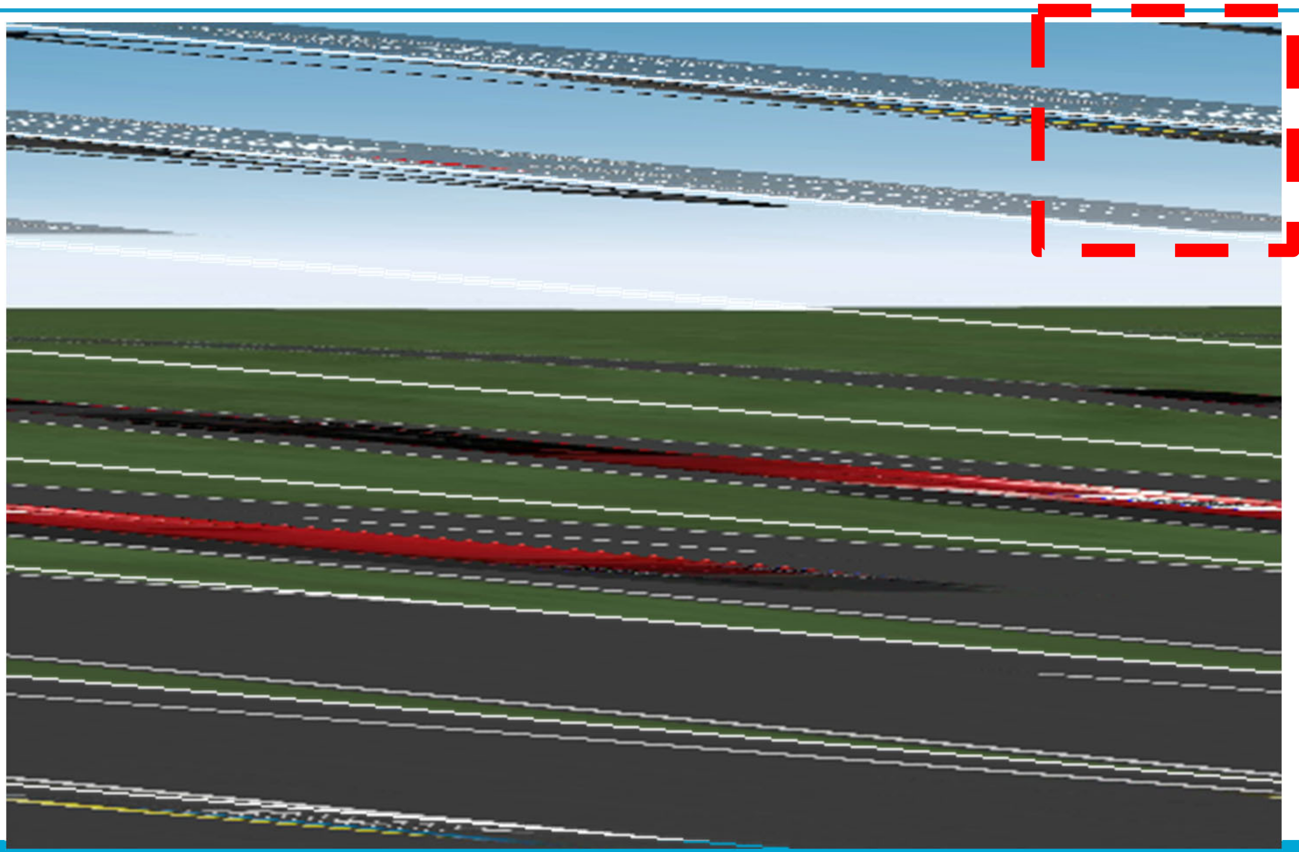
Understeer the driver must increase the steering angle. That is when accelerated with the steering wheel fixed, the turning radius increases.

Oversteer the driver must decrease the steering angle. That is when accelerated with the steering wheel fixed, the turning radius decreases.

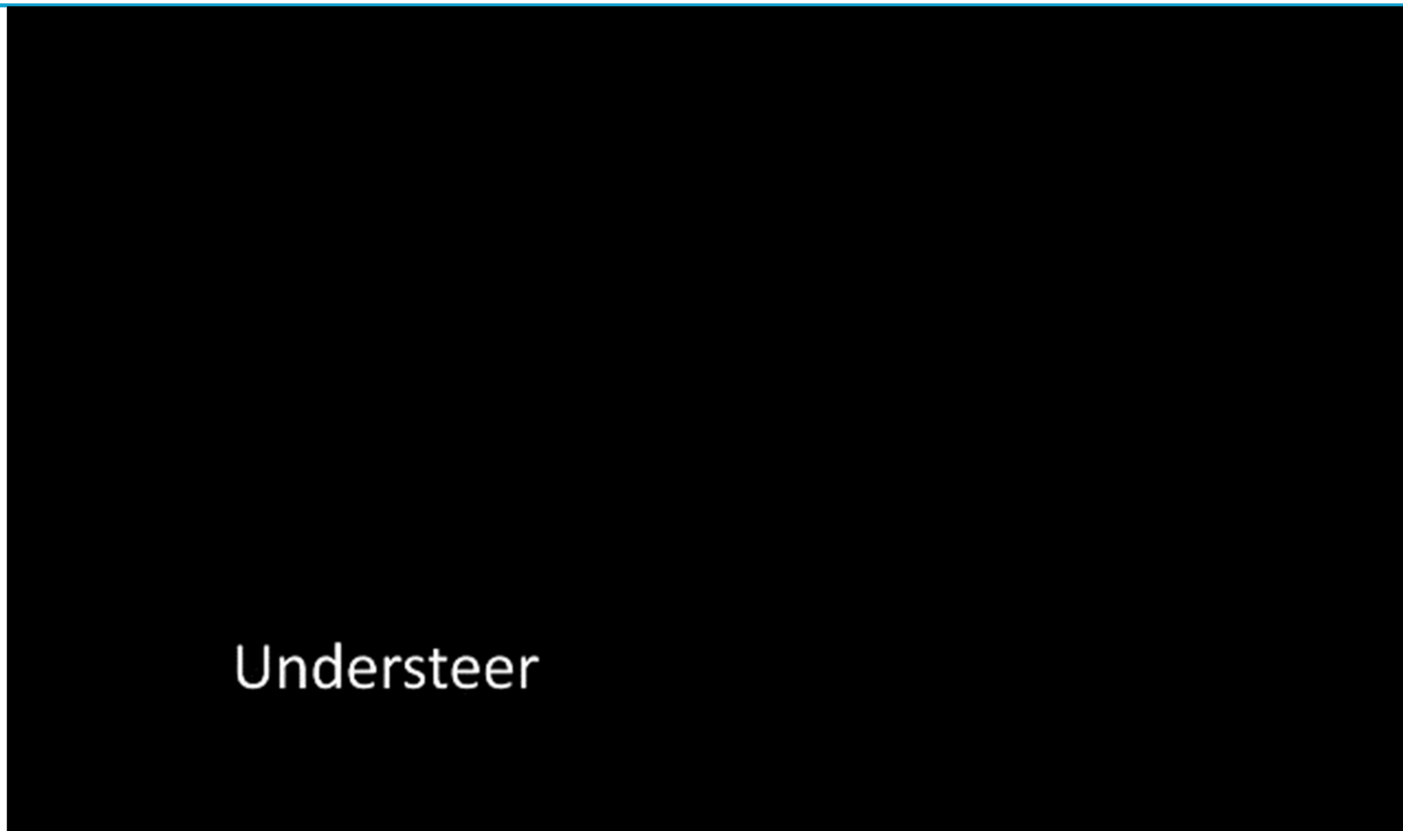
# VEHICLE UNDERSTEER



# VEHICLE OVERSTEER



## BEHAVIOR FOR BOTH CASES



## STEADY-STATE CORNERING

Assume front and rear wheels turn about the same turn center.

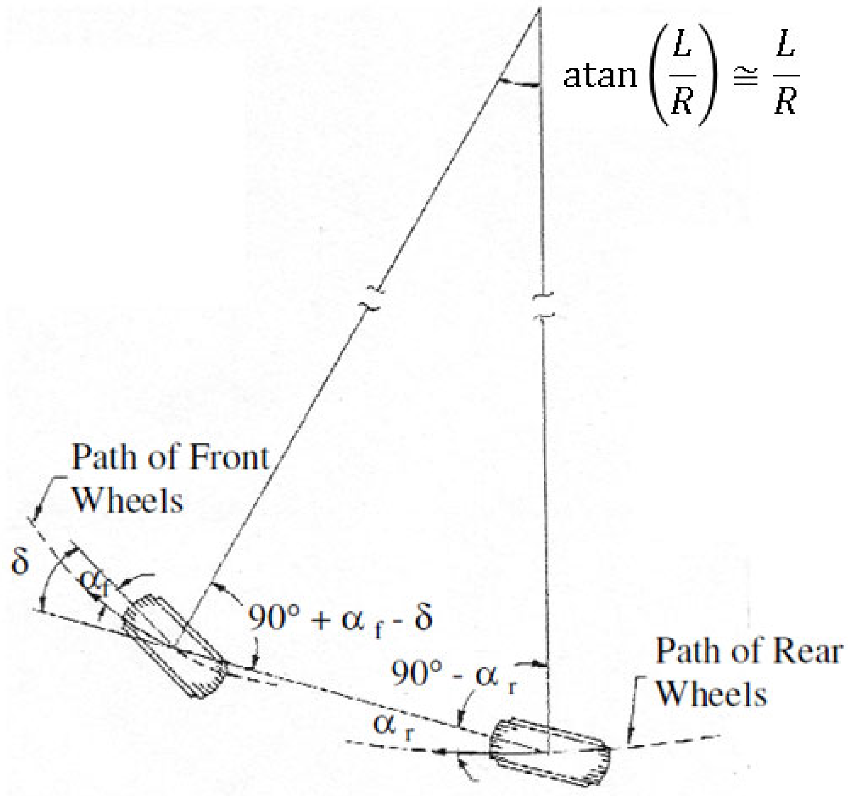
The sum of inner angles inside the triangle gives:

$$\frac{\pi}{2} + \alpha_f - \delta + \frac{\pi}{2} - \alpha_r + \frac{L}{R} = \pi$$

Now substitute for slip angles:

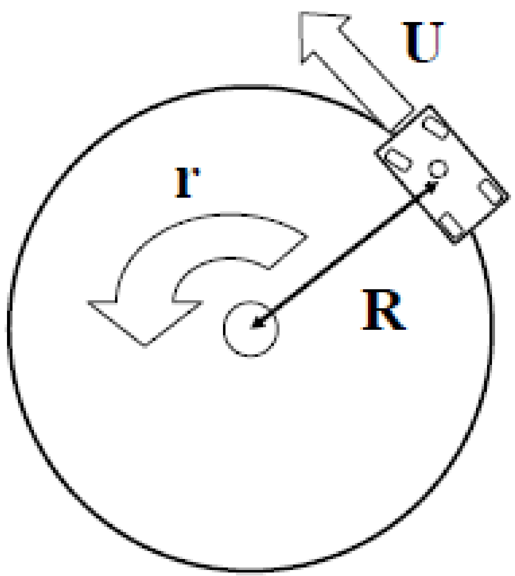
$$\delta = \frac{L}{R} + \alpha_f - \alpha_r$$

$$\delta = \frac{L}{R} + \frac{F_{yf}}{C_{\alpha f}} - \frac{F_{yr}}{C_{\alpha r}}$$



# STEADY-STATE CORNERING

- lateral acceleration $a_y = \frac{u^2}{R}$
- yaw rate $r = \frac{u}{R}$
- path curvature $\rho = \frac{1}{R}$



# STEADY-STATE CORNERING

Performing a force balance maintaining steady state motion:

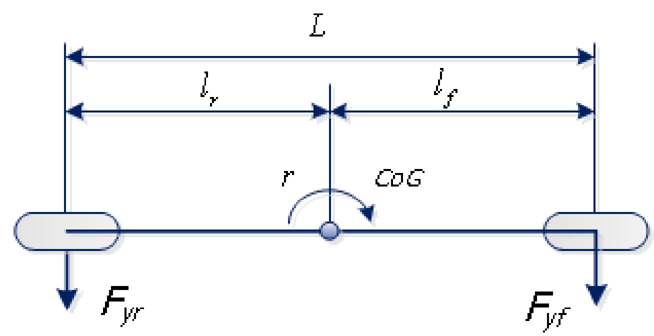
$$\sum F_y = F_{yf} + F_{yr} = ma_y = \frac{mu^2}{R}$$

Also the moment balance yields:

*In steady state there is no yaw acceleration*

$$\sum M_z = l_f F_{yf} - l_r F_{yr} = 0$$

$$\Rightarrow F_{yf} = \frac{l_r}{l_f} F_{yr}$$




Substituting into the first equation results:

$$F_{yr} = \frac{l_f}{L}\frac{mu^2}{R} \qquad F_{yf} = \frac{l_r}{L}\frac{mu^2}{R}$$

# UNDERSTEER GRADIENT

Finally lets put the equations together:

$$\delta = \frac{L}{R} + \frac{ml_r u^2}{LRC_{\alpha f}} - \frac{ml_f u^2}{LRC_{\alpha r}} = \frac{L}{R} + \underbrace{\frac{mg}{L}\left(\frac{l_r}{C_{\alpha f}} - \frac{l_f}{C_{\alpha r}}\right)}_{K_{us}}\frac{a_y}{g} = \frac{L}{R} + K_{us}\frac{a_y}{g}$$

- $K_{us}$ is understeer gradient

$$K_{us} = \frac{mg}{L}\left(\frac{l_r}{C_{\alpha f}} - \frac{l_f}{C_{\alpha r}}\right)$$

- two main contributors:
    - Kinematic contribution (Ackermann angle).
    - Speed (or acceleration) dependent part.
- determine the magnitude and direction of the steering inputs required to achieve certain vehicle response.

## STEERING CHARACTERISTICS

**neutral steer**

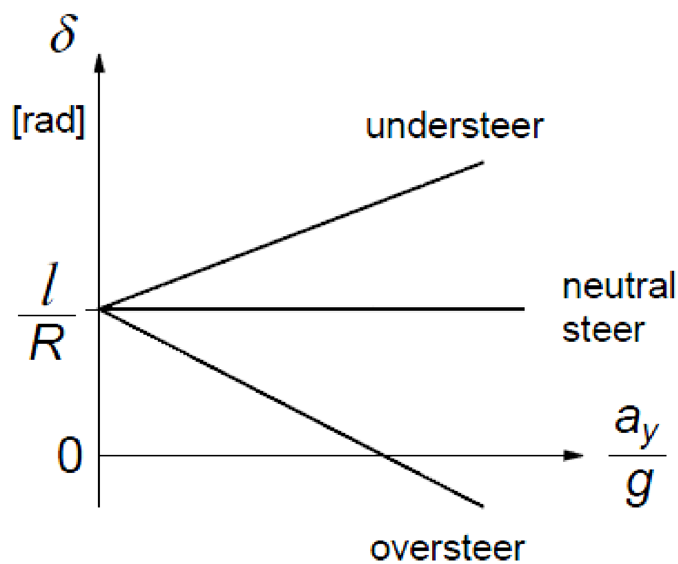
- $K_{us} = 0$ and $\alpha_f = \alpha_r$
- no change in steer angle will be required to follow the path.

**understeer**

- $K_{us} > 0$ and $\alpha_f > \alpha_r$
- front slip angle is greater than the rear one, to keep trajectory the steering angle should be increased.

**oversteer**

- $K_{us} < 0$ and $\alpha_f < \alpha_r$
- rear slip angle is greater than the front one, to compensate trajectory the steering angle should be decreased.



# STEADY-STATE YAW RATE RESPONSE

Considering steady-state vehicle motion: $r = \dfrac{u}{R}; \quad a_y = \dfrac{u^2}{R}$

$$\delta = \frac{Lr}{u} + K_{us}\frac{u^2 r}{gu} = r\left(\frac{L}{u} + K_{us}\frac{u}{g}\right) = \frac{r}{u}\left(L + K_{us}\frac{u^2}{g}\right)$$

Yaw velocity response gain can be formulated as: $G_{yaw}^{ss} = \left.\dfrac{r}{\delta}\right|_{ss} = \dfrac{u}{L + K_{us}u^2 / g}$

## CALCULATION EXAMPLE

- baseline, $K_{us}$ = 5.54e-4 rad = 0.032 deg
  - distance $l_f$ / $l_r$ from front / rear axle to CoG 1.47 / 1.41 m
  - full mass $m$ 1900 kg
  - inertia moment $I_z$ 3500 kgm$^2$
  - front cornering stiffness $C_{af}$ 2*92000 N/rad
  - rear cornering stiffness $C_{ar}$ 2*97000 N/rad

- understeer, $K_{us}$ = 0.0169 rad = 0.968 deg
  - rear cornering stiffness 50% stiffer

- neutral, $K_{us}$ = 0.0
  - rear cornering stiffness $\dfrac{C_{\alpha f} l_f}{l_r}$

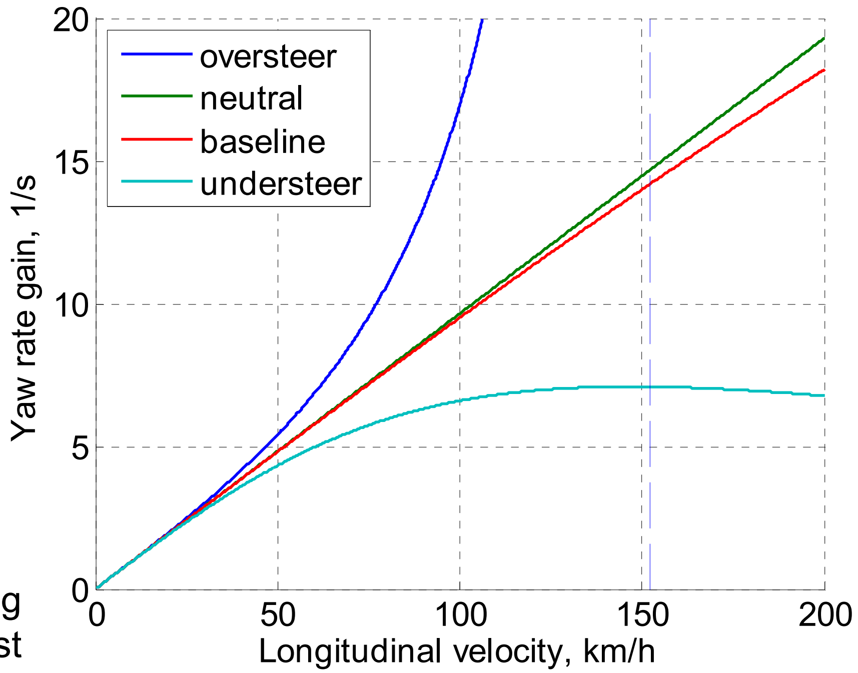
- oversteer, $K_{us}$ = -0.0158 rad = -0.905 deg
  - rear cornering stiffness 25% softer

# YAW VELOCITY RESPONSE GAIN

Defined as the ratio of the steady state yaw velocity to the steer angle

- **neutral**
  gain increases linearly with speed
- **understeer**
  gain increases to a maximum and then drops
- **oversteer**
  gain increases with speed

*Oversteer* car has the most sensitive handling characteristics, while the *understeer* vehicle is least responsive.



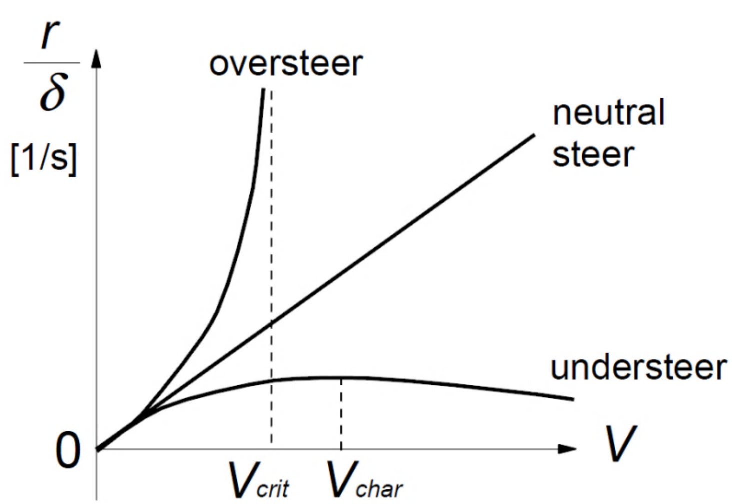
# CHARACTERISTIC SPEED

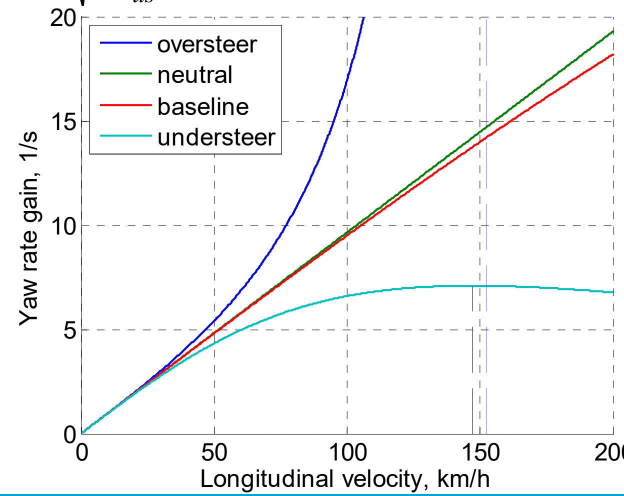
*Characteristic speed* is the speed at which the steering input required to negotiate the turn is twice the angle needed at speeds approaching zero

- only for understeer vehicle
- maximum steady-state steering sensitivity

$$\frac{2L}{R} = \frac{L}{R} + K_{us}\frac{V_{char}^2}{gR} \Rightarrow$$

$$V_{char} = \sqrt{\frac{gL}{K_{us}}} = \sqrt{\frac{2.88 \cdot 9.81}{0.968 / 180 \cdot \pi}} = 40.9 m / s$$



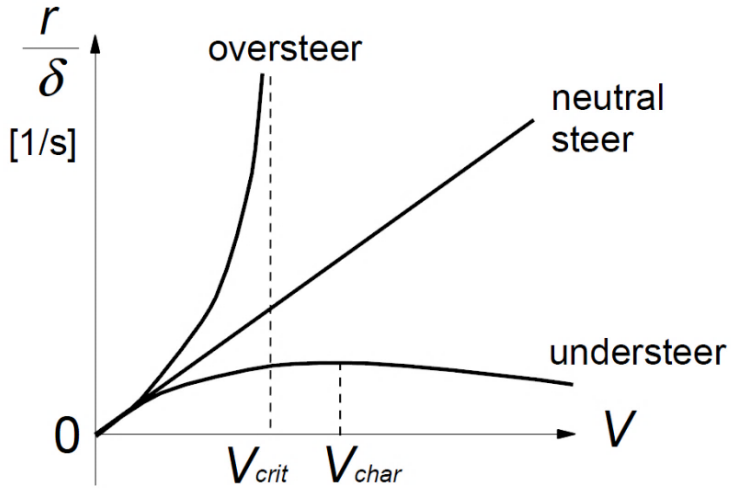



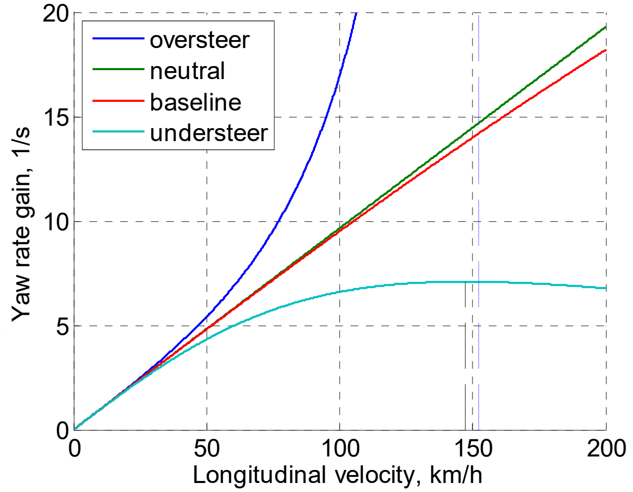
# CRITICAL SPEED

*Critical speed* is the speed at which the steering input required to negotiate the turn is zero

- only for oversteer vehicle
- vehicle is unstable beyond it

$$0 = \frac{L}{R} + K_{us}\frac{V_{crit}^2}{gR} \quad \Rightarrow \quad V_{crit} = \sqrt{\frac{gL}{-K_{us}}} = \sqrt{\frac{2.88 \cdot 9.81}{0.905/180 \cdot \pi}} = 42.29 m/s$$



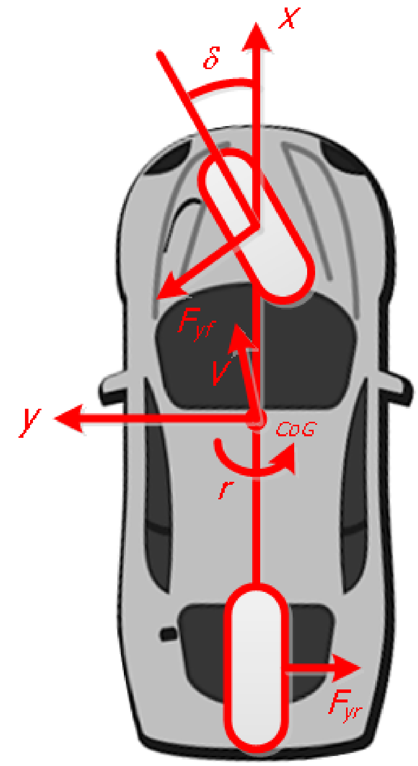



# BICYCLE MODEL

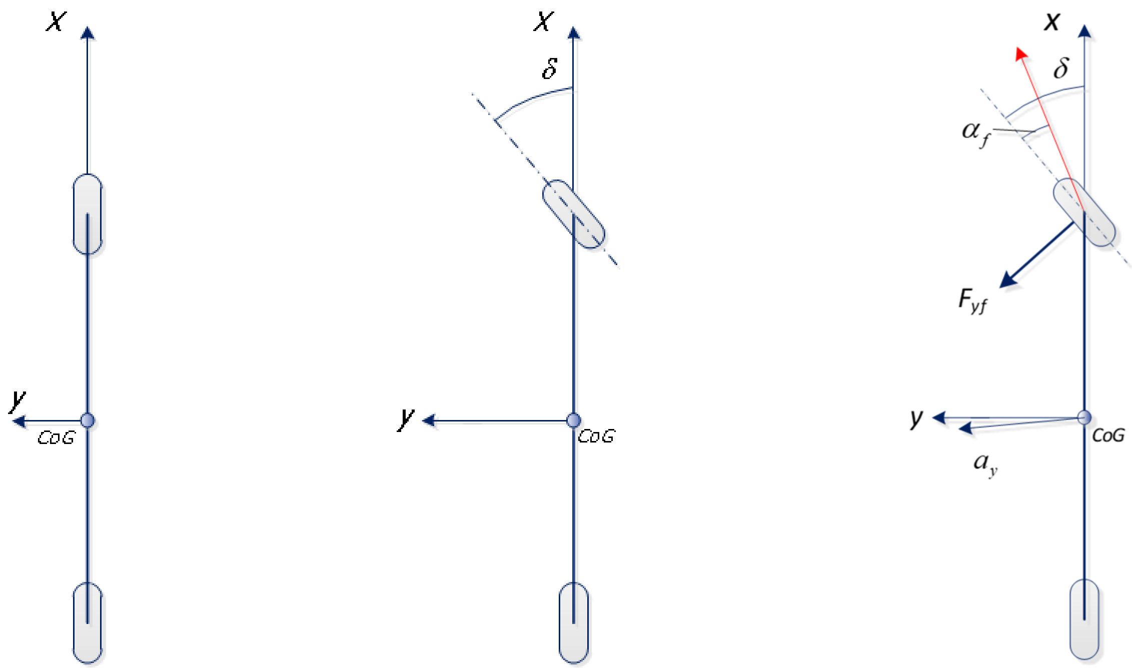
<u>2DoF "bicycle" model</u> is the simplest form to analyze vehicle handling.

<u>Assumptions and limitations:</u>

- constant forward velocity
- no longitudinal / lateral load transfer
- linear range tires
- no roll, pitch or vertical motion
- "ideal" steering dynamics
- no suspension
- no compliance effects



# SEQUENCE OF CORNERING

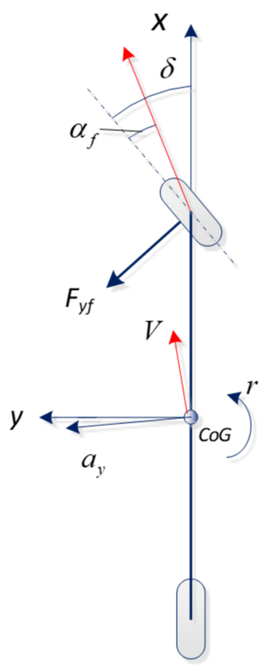



straight motion

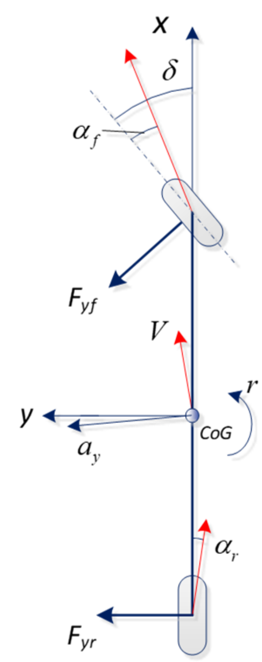
front wheel steering

front slip angles ->
front lateral force ->
lateral acceleration

## SEQUENCE OF CORNERING




yaw moment -> yaw velocity




rear slip angle -> rear lateral force

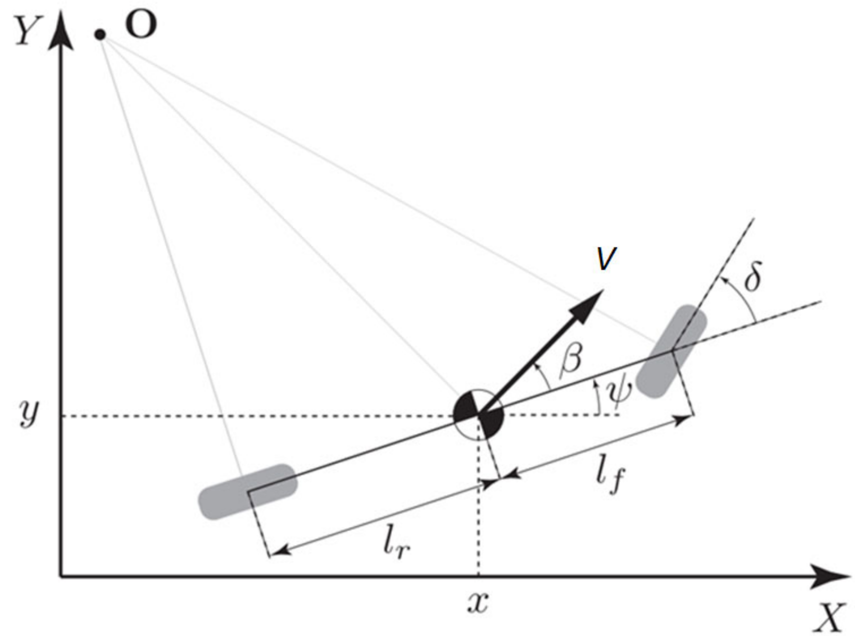
more lateral acceleration,
more yaw velocity
but less yaw moment

# KINEMATIC BICYCLE MODEL

- Model provides a mathematical description of the motion without considering the forces.
- The equations of motion are based purely on geometric relationships governing the system.

$$\dot{X} = V\cos(\psi + \beta)$$
$$\dot{Y} = V\sin(\psi + \beta)$$
$$\dot{\psi} = \frac{V}{L}\cos\beta\tan\delta = \frac{V}{l_r}\sin\beta$$
$$\beta = \tan^{-1}\left(\frac{l_r}{L}\tan\delta\right)$$

*V* – vehicle speed




Note: full derivation in Chapter 2.2, Rajamani, R., 2011. Vehicle dynamics and control. Springer Science & Business Media.

# MOTION WITH FIXED COORDINATES

Vehicle motion by fixed coordinates on the vehicle

Velocity vector $\dot{\mathbf{R}} = u\mathbf{i} + v\mathbf{j}$

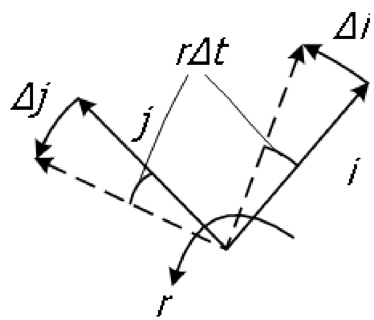
Acceleration vector $\ddot{\mathbf{R}} = \dot{u}\mathbf{i} + u\dot{\mathbf{i}} + \dot{v}\mathbf{j} + v\dot{\mathbf{j}}$

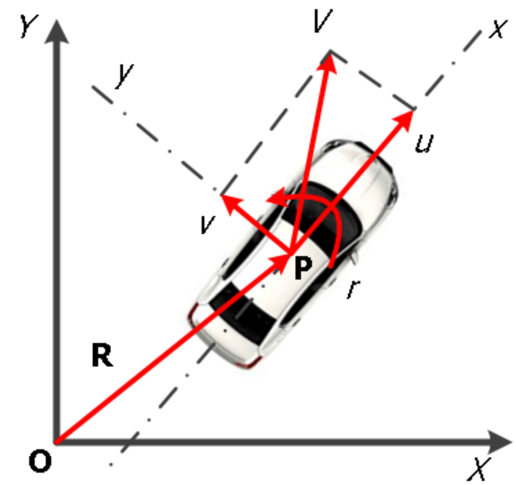
Changes in unit vectors **i** and **j** with time Δt:

$$\Delta\mathbf{i} = r\Delta t\mathbf{j} \qquad \Delta\mathbf{j} = -r\Delta t\mathbf{i}$$

Thus: $\dot{\mathbf{i}} = \lim\limits_{\Delta t \to 0} \frac{\Delta\mathbf{i}}{\Delta t} = r\mathbf{j} \qquad \dot{\mathbf{j}} = \lim\limits_{\Delta t \to 0} \frac{\Delta\mathbf{j}}{\Delta t} = -r\mathbf{i}$

$$\ddot{\mathbf{R}} = \underbrace{(\dot{u} - vr)}_{a_x}\mathbf{i} + \underbrace{(\dot{v} + ur)}_{a_y}\mathbf{j}$$




**Acceleration consists of two components!**

# LINEAR BICYCLE MODEL

Newton-Euler equations for the vehicle motion:

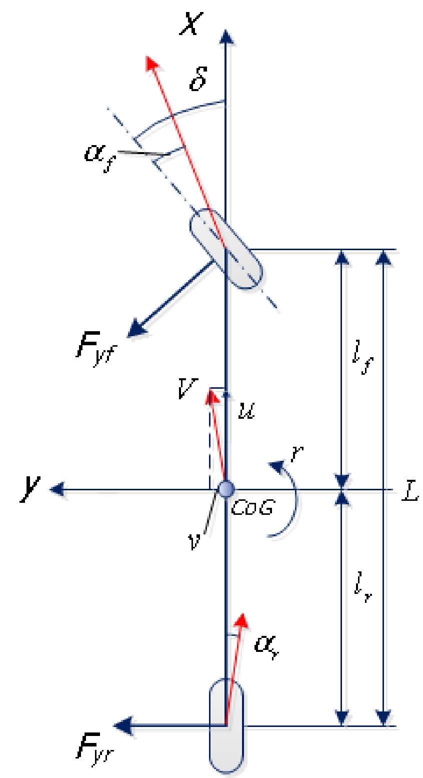
$$ma_x = 0$$
$$ma_y = m(\dot{v} + ur) = \sum F_y$$
$$I_z \dot{r} = \sum M_z$$

The trajectory in an inertial frame:

$$\dot{X} = u\cos\psi - v\sin\psi$$
$$\dot{Y} = u\sin\psi + v\cos\psi$$

Assuming small steering angle $\delta$:

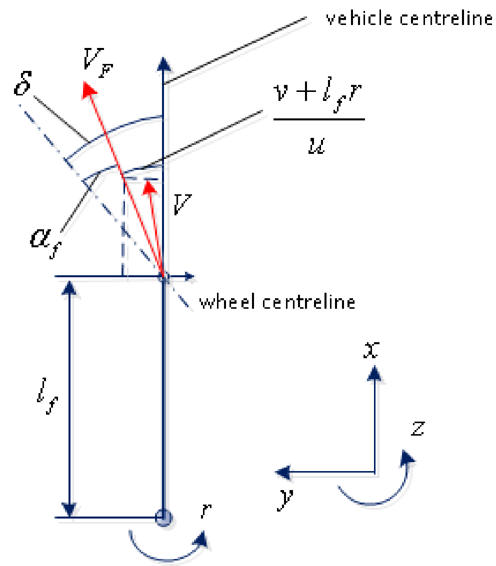
$$\sum F_y \approx F_{yf} + F_{yr}$$
$$\sum M_z = l_f F_{yf} - l_r F_{yr}$$

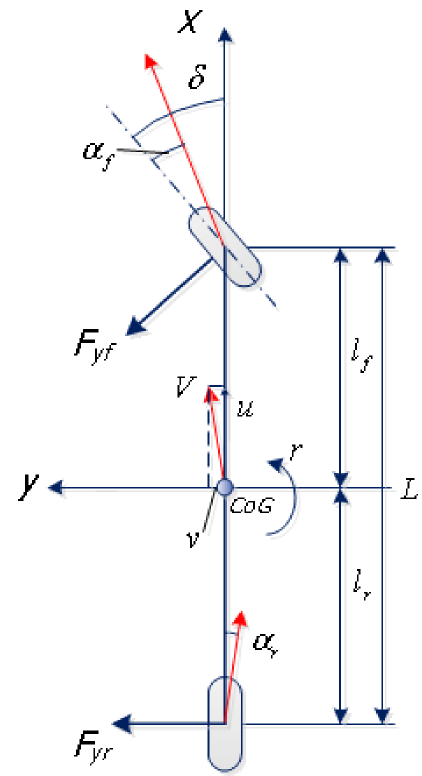


# FRONT SLIP ANGLE

Front slip angle turns the vehicle out of the turn

$$\alpha_f = \delta - \frac{v + l_f r}{u} = -\beta + \delta - \frac{l_f r}{u}$$

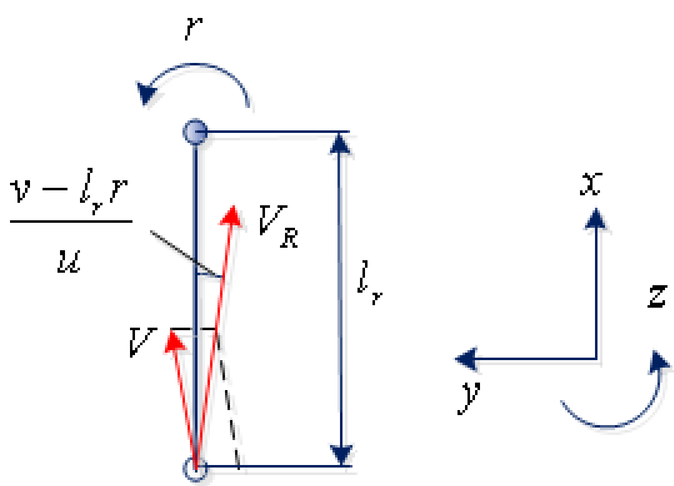


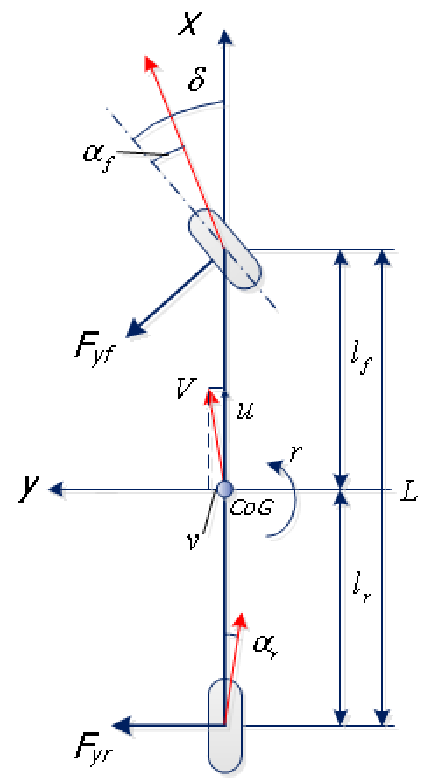



# REAR SLIP ANGLE

Rear slip angle turns the vehicle into the turn

$$\alpha_r = -\frac{v - l_r r}{u} = -\beta + \frac{l_r r}{u}$$



# BICYCLE MODEL

Considering derived equations of wheel slip angle and lateral forces

$$m(\dot{v}+ur)=F_{yf}+F_{yr}=C_{\alpha f}\left(\delta-\frac{v+l_f r}{u}\right)+C_{\alpha r}\left(-\frac{v-l_r r}{u}\right)$$

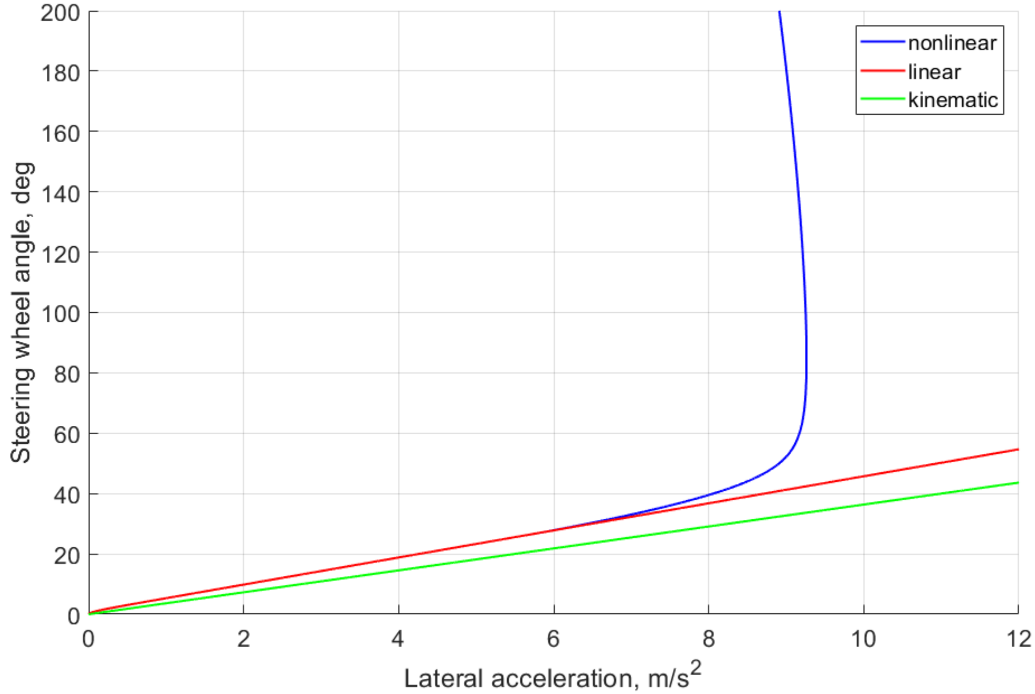
$$I_z\dot{r}=l_f F_{yf}-l_r F_{yr}=l_f C_{\alpha f}\left(\delta-\frac{v+l_f r}{u}\right)-l_r C_{\alpha r}\left(-\frac{v-l_r r}{u}\right)$$

Finally, lets put the equations together:

$$\dot{v}=-\left(\frac{C_{\alpha f}+C_{\alpha r}}{mu}\right)v+\left(-u+\frac{l_r C_{\alpha r}-l_f C_{\alpha f}}{mu}\right)r+\frac{C_{\alpha f}}{m}\delta$$

$$\dot{r}=\left(\frac{l_r C_{\alpha r}-l_f C_{\alpha f}}{I_z u}\right)v+\left(-\frac{l_f^2 C_{\alpha f}+l_r^2 C_{\alpha r}}{I_z u}\right)r+\left(\frac{l_f C_{\alpha f}}{I_z}\right)\delta$$

# VEHICLE HANDLING: SIMULATION RESULTS




Due to no constraints in the linear tire model, acceleration is linearly proportional to steering input resulting in unrealistic values.

RO47001 ROBOT DYNAMICS & CONTROL

Cognitive Robotics

# LECTURE 7:
# Kinematics & Dynamics of Mobile Robot / Automated Vehicle


**Barys Shyrokau**

Cognitive Robotics, 3mE, Delft University of Technology,
The Netherlands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