Quarkonium beyond the E/T expansion: Non-Lindblad master equations

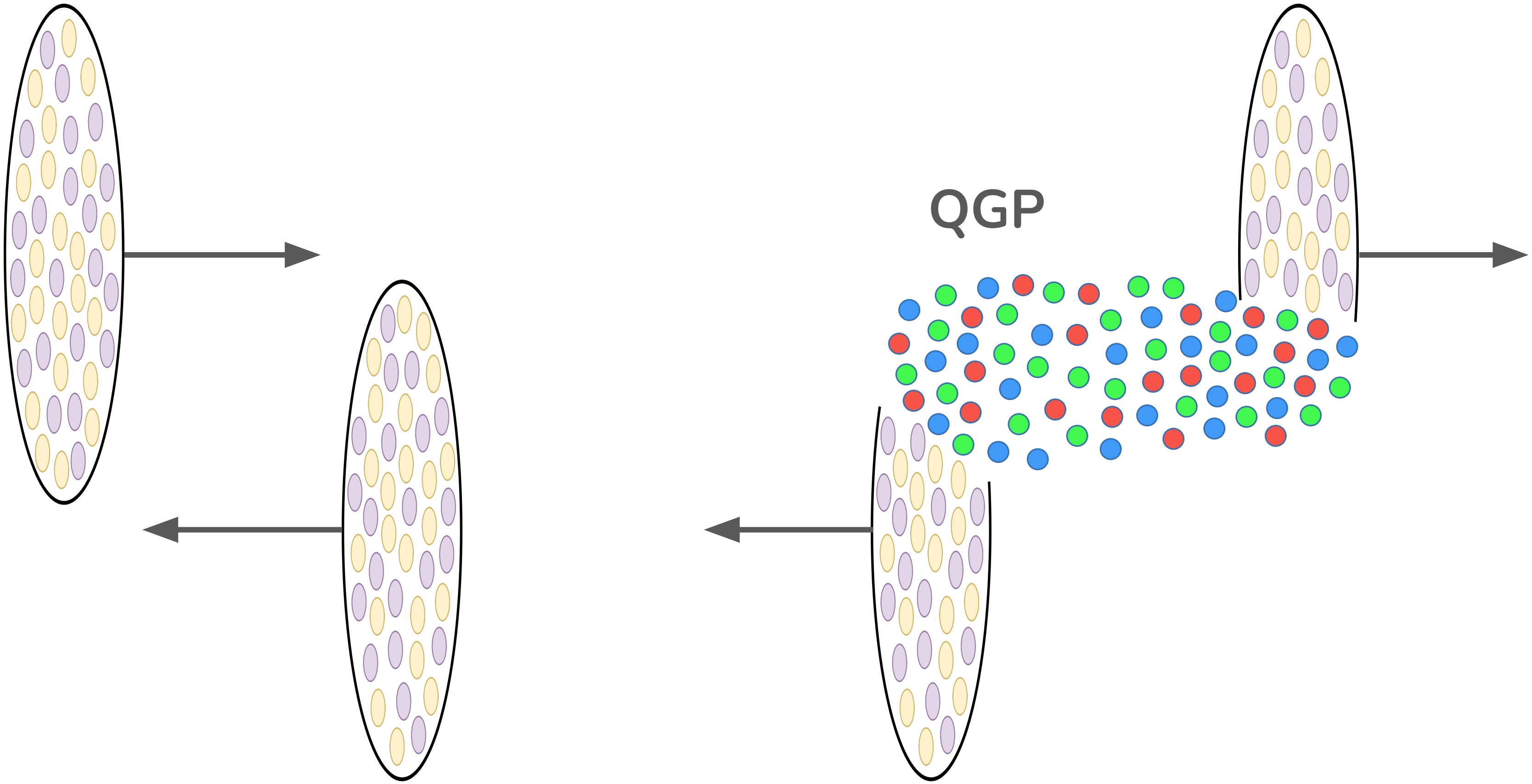
Tom Magorsch

in collaboration with
Nora Brambilla, Arthur Lin and Antonio Vairo

Open Quantum Systems:
Dissipative Dynamics from Quarks to the Cosmos

03.12.25

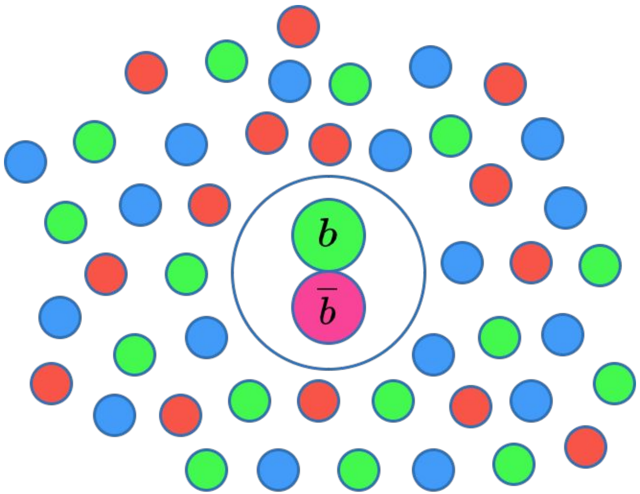
Quarkonium suppression



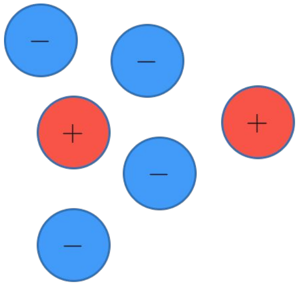
Quarkonium suppression

Veres, Gábor I. "Heavy ion physics at CMS and ATLAS: hard probes." *arXiv preprint arXiv:1905.10461* (2019).

Propagation through QGP



T. Matsui, H. Satz, Phys. Lett. B 178 (1986) 416

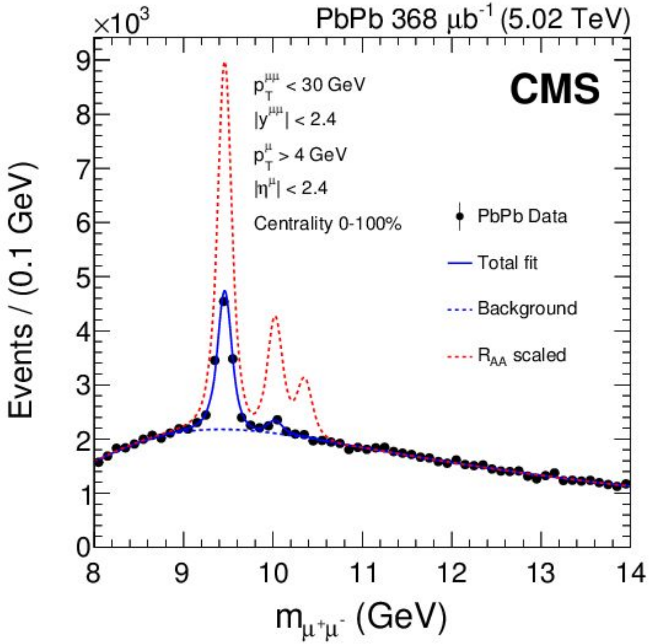


$$V(r) = -\frac{\alpha}{r}$$

$$\downarrow$$

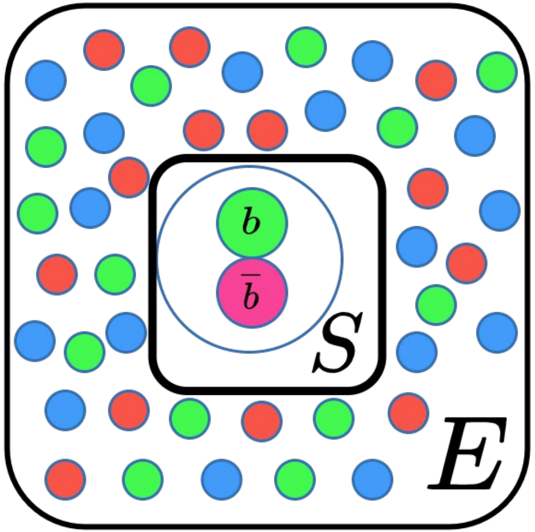
$$V(r) = -\frac{\alpha}{r}\mathrm{e}^{-m_D r}$$

Debye-screening in medium

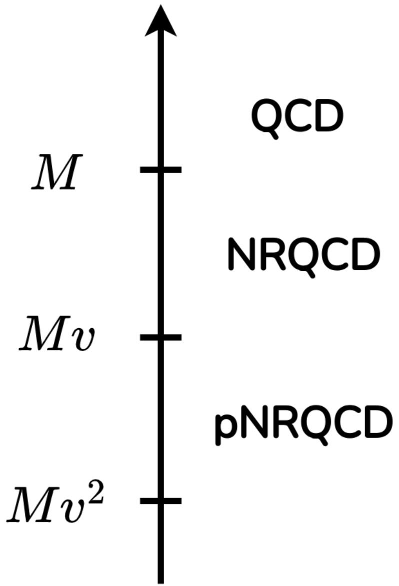


Quarkonium suppression

Yukinao Akamatsu, Prog.Part.Nucl.Phys. 123 (2022), 103932
Xiaojun Yao, Int.J.Mod.Phys.A 36 (2021) 20, 2130010



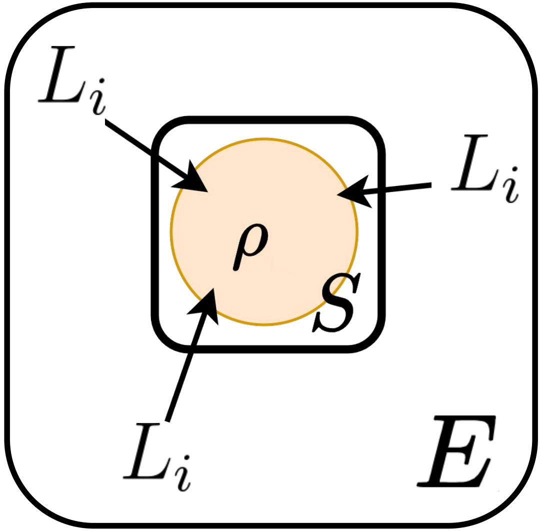
+



Brambilla, Escobedo, Soto, Vairo: Phys. Rev. D 97 (2018) 7, 074009

Quarkonium suppression

$$\frac{d\rho(t)}{dt} = -i\left[H, \rho(t)\right] + \sum_{nm} h_{nm} \left(L_i^n \rho(t) L_i^{m\dagger} - \frac{1}{2}\left\{ L_i^{m\dagger} L_i^n, \rho(t) \right\} \right)$$



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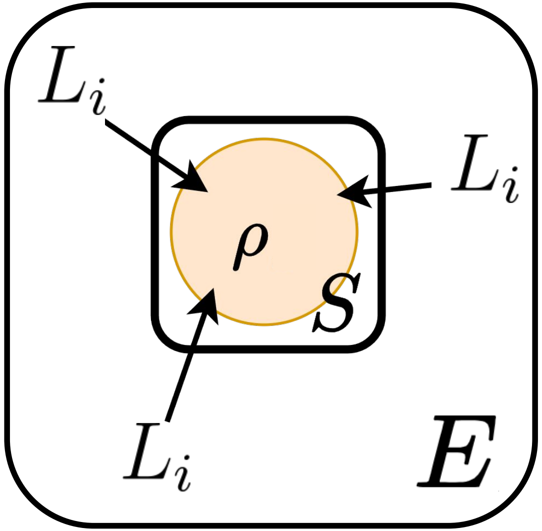
Quarkonium suppression



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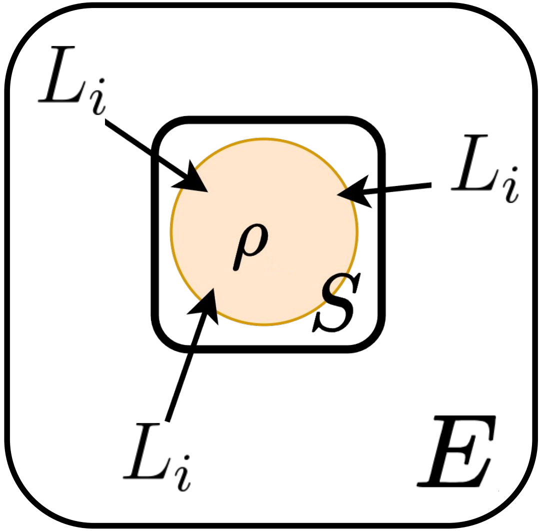
$$h = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$L_i^0 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} r_i, \qquad L_i^1 = \begin{pmatrix} 0 & 0 \\ 0 & \frac{N_c^2-4}{2(N_c^2-1)} A_i^{oo\dagger} \end{pmatrix}$$

$$L_i^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} r_i, \qquad L_i^3 = \begin{pmatrix} 0 & \frac{1}{N_c^2-1} A_i^{os\dagger} \\ A_i^{so\dagger} & 0 \end{pmatrix}$$

Brambilla, Escobedo, Soto, Vairo: Phys. Rev. D 97 (2018) 7, 074009

Quarkonium suppression



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$$A_i^{uv} = \frac{g^2}{6N_c} \int_0^\infty \mathrm{d}s e^{-ih_u s} r_i e^{ih_v s} \langle E_j^a(s) \Omega(s,0)^{ab} E_j^b(0) \rangle$$

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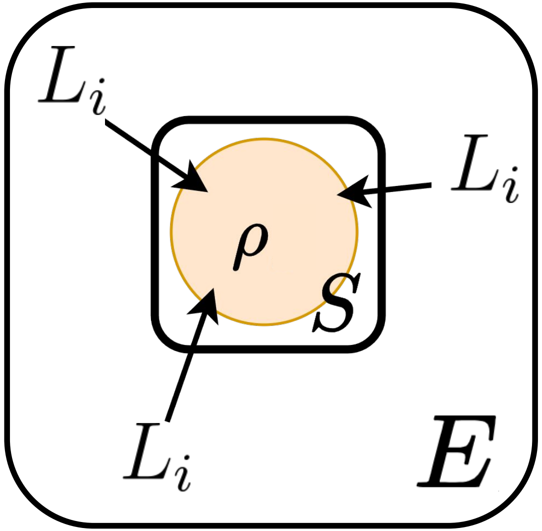
Quarkonium suppression



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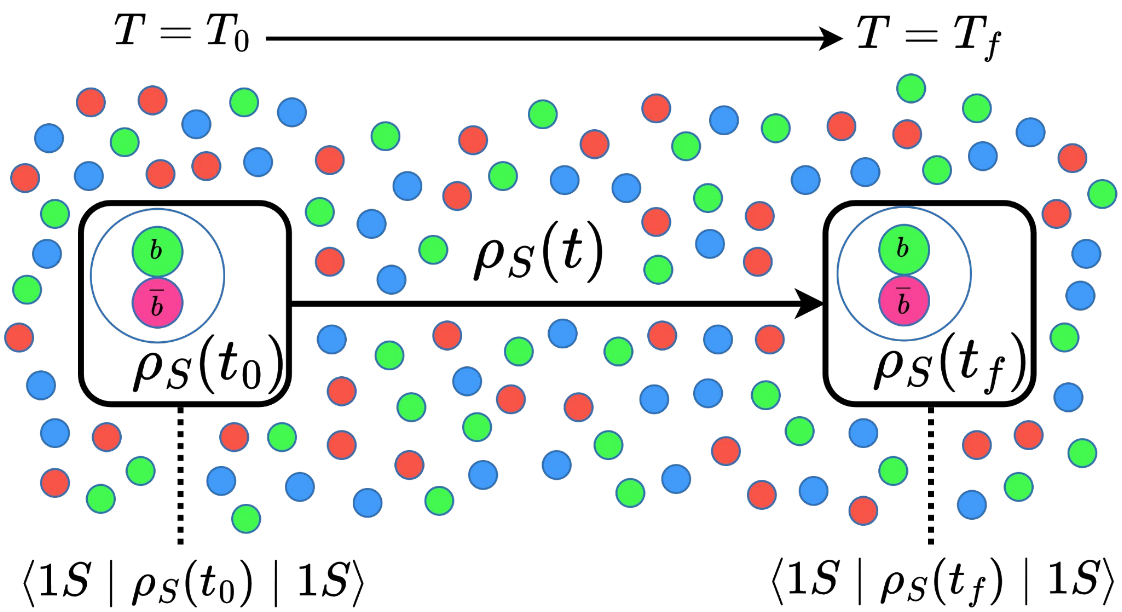
$$h_u = \frac{p^2}{M} + V_u(r)$$

Solving the master equation gives the evolution of the bottomonium



$$P_{\text{survival}}(1S) = \frac{\langle 1S|\rho(t_f)|1S\rangle}{\langle 1S|\rho(t_0)|1S\rangle}$$

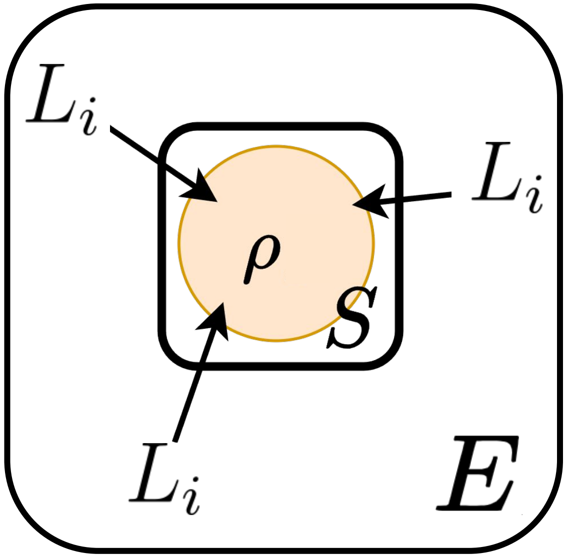
To simplify the equation one can expand in E/T

$$A_i^{uv} = \int_0^\infty \mathrm{d}s\, e^{-ih_u s} r_i e^{ih_v s} \langle E_j^a(s)\Omega^{ab}(s,0)E_j^b(0)\rangle$$

$$1 - ih_u s + \mathcal{O}(E^2/T^2) \qquad 1 - ih_v s + \mathcal{O}(E^2/T^2)$$

Next-to-Leading order in E/T

Leading order in E/T



Brambilla, N., Escobedo, M. Á., Islam, A., Strickland, M., Tiwari, A., Vairo, A., & Vander Griend, P. (2022). *Journal of High Energy Physics*, *2022*(8), 1-39.

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Transport coefficients

$$A_i^{uv} = \underbrace{\frac{r_i}{2}(\kappa - i\gamma)}_{\text{LO}} + \underbrace{\kappa\left(-\frac{ip_i}{2MT} + \frac{\Delta V_{uv}}{4T} r_i\right)}_{\text{NLO}}$$

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The master equation is not positive

$$\frac{d\rho(t)}{dt} = -i\,[H, \rho(t)] + \sum_{nm} h_{nm} \left(L_i^n \rho(t) L_i^{m\dagger} - \frac{1}{2} \left\{ L_i^{m\dagger} L_i^n, \rho(t) \right\} \right)$$

$$L_i \longrightarrow L_i'$$

$$h = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \longrightarrow h' = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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$$L_i \longrightarrow L_i'$$

$$\mathcal{O}(E^2/T^2)$$

$$h = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \longrightarrow h' = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Neglect negative terms as they are suppressed!

Projection on spherical harmonics

$$\frac{d\rho(t)}{dt} = -i[H, \rho(t)] + \sum_{n=0}^{\cancel{1}\ 6} \left(L_{\cancel{i}}^{n} \rho(t) L_{\cancel{i}}^{n\dagger} - \frac{1}{2}\{L_{\cancel{i}}^{n\dagger} L_{\cancel{i}}^{n}, \rho(t)\} \right)$$

Hilbert space:

$$H_c \otimes H_l \otimes H_r$$

c = s,o

l = 0,1,2,...

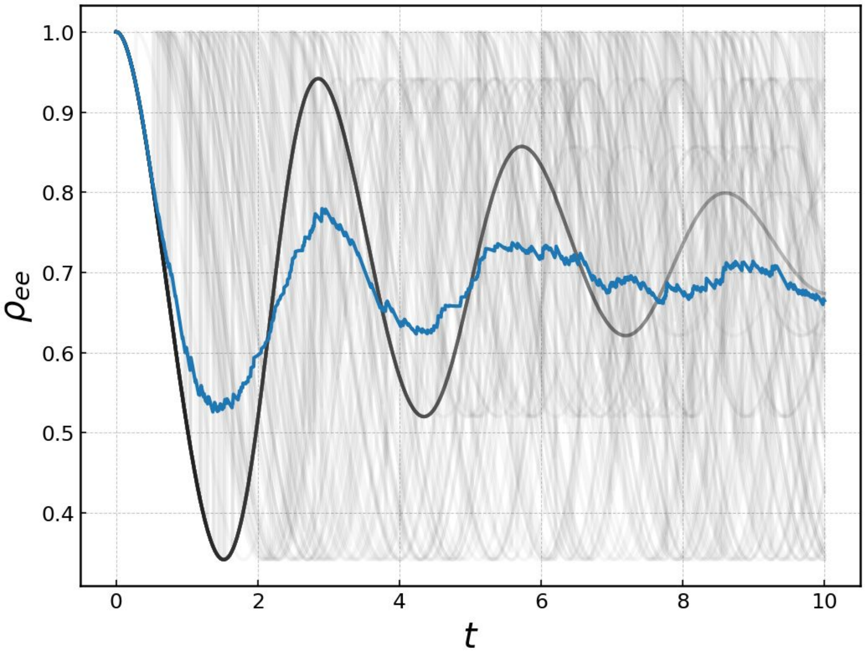
Radial wavefunction

$$\rho = \begin{pmatrix} \rho_s^0 & 0 & \dots & 0 & 0 & \dots \\ 0 & \rho_s^1 & \dots & 0 & 0 & \dots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \\ 0 & 0 & \dots & \rho_o^0 & 0 & \dots \\ 0 & 0 & \dots & 0 & \rho_o^1 & \dots \\ \vdots & \vdots & & \vdots & \vdots & \ddots \end{pmatrix}$$

Quantum trajectory algorithm

J. Dalibard, Y. Castin, and K. Mølmer, Wave-function approach to dissipative processes in quantum optics, Phys. Rev. Lett. 68 (1992), pp. 580–583.

- Idea:
  1. Evolve individual trajectories $|\psi(t)\rangle$ **stochastically**
  2. Calculate observables by averaging over trajectories
  $$\langle A \rangle = \mathbb{E}\left[\langle \psi(t)|A|\psi(t)\rangle\right]$$



Quantum Trajectories

e.g.: $U = 1 - iH_{\text{eff}}\delta t$

$$\psi_0$$

1. Evolve state $|\psi(t)\rangle$ with $H_{\text{eff}} = H - \frac{i}{2}C^\dagger C$

$$|\psi'(t+\delta t)\rangle = U|\psi(t)\rangle$$

Quantum Trajectories

e.g.: $U = 1 - iH_{\text{eff}}\delta t$

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2. Compute norm

$$\langle\psi'(t+\delta t)|\psi'(t+\delta t)\rangle = 1 - \delta p < 1$$

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$$\langle\psi'(t+\delta t)|\psi'(t+\delta t)\rangle = 1 - \delta p < 1$$

3. Apply jump operator C with probability δp

$$P\left(|\psi(t+\delta t)\rangle = C|\psi(t)\rangle\right) = \delta p$$
$$P\left(|\psi(t+\delta t)\rangle = |\psi'(t+\delta t)\rangle\right) = 1 - \delta p$$

Quantum Trajectories

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4. Normalize $|\psi(t+\delta t)\rangle$

Quantum trajectory algorithm

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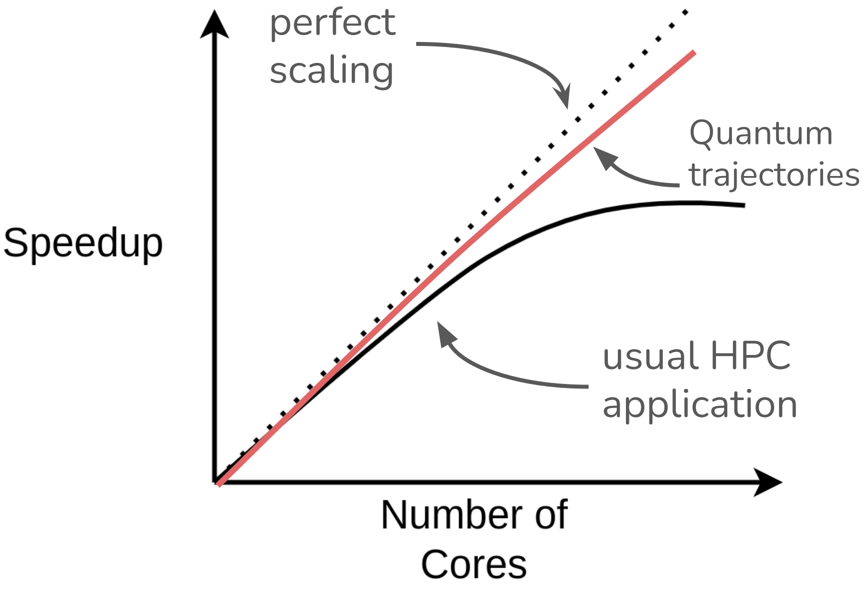


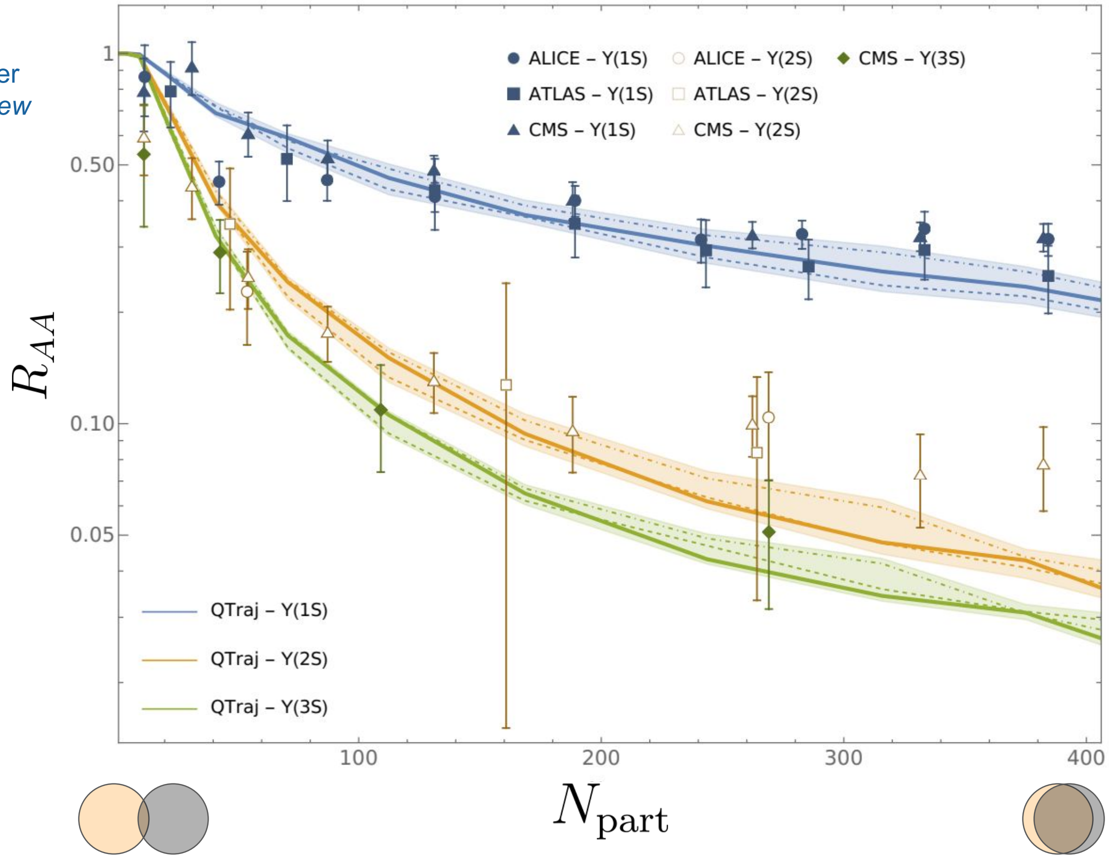
Omar, H. B., Escobedo, M. Á., Islam, A., Strickland, M., Thapa, S., Vander Griend, P., & Weber, J. H. (2022). *Computer Physics Communications*, *273*, 108266.

Advantages:

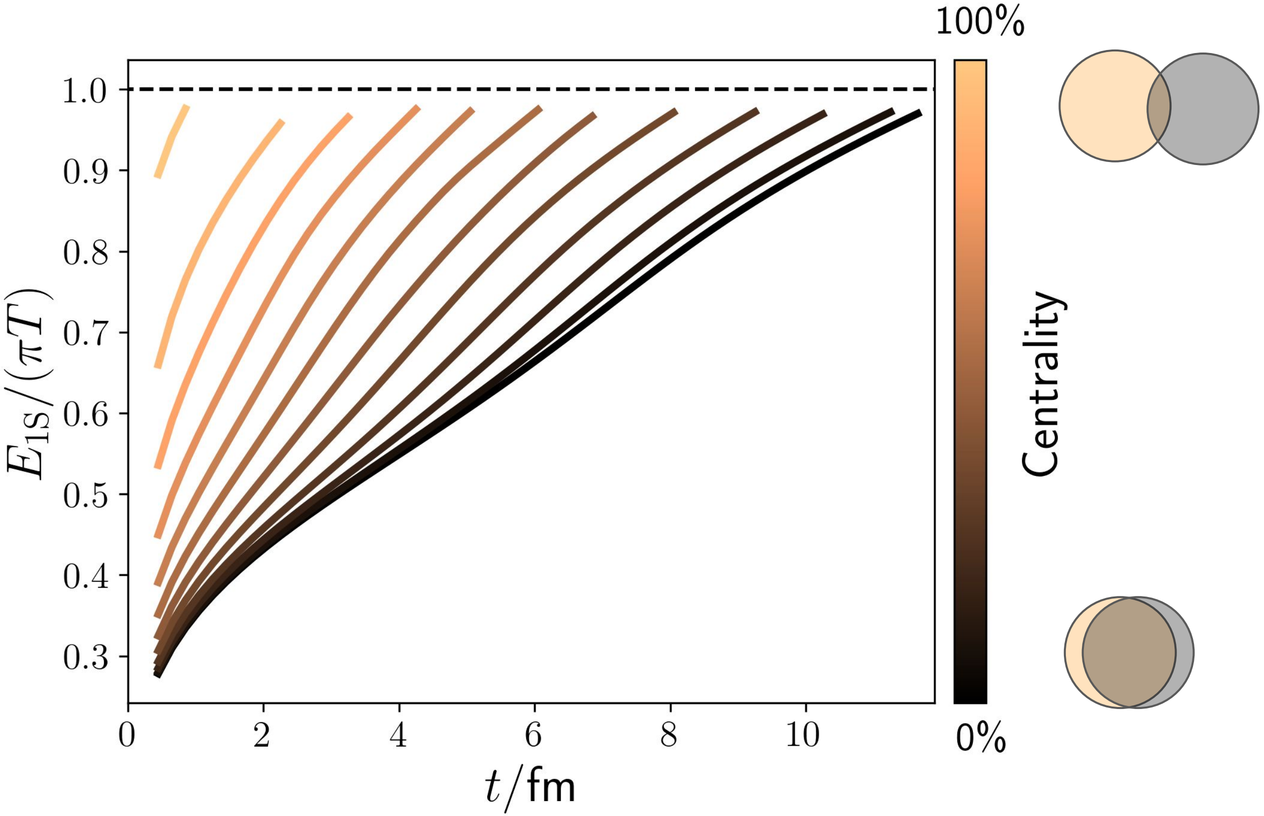
- Evolve vector of size N_H instead N_H^2 density matrix
- Simulation of individual trajectories is **embarrassingly parallel**

Overlaps lead to phenomenological predictions

Brambilla, N., Magorsch, T., Strickland, M., Vairo, A., & Vander Griend, P. (2024). *Physical Review D*, *109*(11), 114016.



At low temperatures the E/T expansion converges slowly



The original master equation is not positive

$$\frac{d\rho(t)}{dt} = -i\left[H, \rho(t)\right] + \sum_{nm} h_{nm}\left(L_i^n \rho(t) L_i^{m\dagger} - \frac{1}{2}\left\{L_i^{m\dagger} L_i^n, \rho(t)\right\}\right)$$

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?

$$h' = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad L_i^n \propto \frac{g^2}{6N_c} \int_0^\infty \mathrm{d}s e^{-ih_u s} r_i e^{ih_v s} \langle E_j^a(s)\Omega(s,0)^{ab} E_j^b(0)\rangle$$

Non-positive master equation

Pseudo Lindblad Quantum Trajectories

Becker, T., Netzer, C., & Eckardt, A. (2023). *Physical Review Letters*, *131*(16), 160401.

$$\frac{d}{dt}\rho(t) = -i[H,\rho(t)] + \left(L_+\rho L_+^\dagger - \frac{1}{2}\{L_+^\dagger L_+, \rho(t)\}\right) - \left(L_-\rho L_-^\dagger - \frac{1}{2}\{L_-^\dagger L_-, \rho(t)\}\right)$$

Pseudo Lindblad Quantum Trajectories

Becker, T., Netzer, C., & Eckardt, A. (2023). *Physical Review Letters*, *131*(16), 160401.

two jump operators

$$L_+, L_-$$

$$\psi_0 \quad s(0) = 1$$

$$\downarrow$$

1. Calculate rates r_σ

$$r_\sigma = \frac{||L_\sigma|\psi(t)\rangle||^2}{|||\psi(t)\rangle||^2}$$

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2. With probability $r_+\delta t$

$$|\psi(t+\delta t)\rangle = \frac{L_+|\psi(t)\rangle}{\sqrt{r_+(t)}} \qquad s(t+\delta t) = s(t)$$

With probability $r_-\delta t$

$$|\psi(t+\delta t)\rangle = \frac{L_-|\psi(t)\rangle}{\sqrt{r_-(t)}} \qquad s(t+\delta t) = -s(t)$$

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With probability $1 - \sum_\sigma r_\sigma \delta t$

$$|\psi(t+\delta t)\rangle = \frac{(1 - i\delta t H_{\mathrm{Heff}})|\psi(t)\rangle}{\sqrt{1 - \sum_\sigma r_\sigma(t)\delta t}} \qquad s(t+\delta t) = s(t)$$

Pseudo Lindblad Quantum Trajectories

Becker, T., Netzer, C., & Eckardt, A. (2023). *Physical Review Letters*, *131*(16), 160401.

$$\rho(t) = \mathbb{E}\left[s(t)|\psi(t)\rangle\langle\psi(t)|\right]$$

Pseudo Lindblad Quantum Trajectories

$$\psi_0 \quad s(0) = 1$$

1. Draw $p_1 \in [0,1]$. While $R(t) > p_1$ evolve

$$R(t') = \exp\left(-\int_t^{t'} \sum_i r_i(s)ds\right) R(t) \qquad |\psi(t')\rangle = \frac{\exp\left(-i\int_t^{t'} H_{\text{eff}}(s)ds\right)}{\sqrt{\exp\left(-\int_t^{t'} \sum_i r_i(s)ds\right)}}$$

$$r_i(t) = \frac{||L_i|\psi(t)\rangle||^2}{||\,|\psi(t)\rangle||^2} \qquad s(t') = s(t)$$

2. Draw Jump operator i with probability $\propto r_i(t)$ and perform jump

$$|\psi(t)\rangle \leftarrow \frac{L_i|\psi(t)\rangle}{\sqrt{r_i(t)}}$$

3. Flip the sign bit **if the applied jump operator is a negative one**

$$s(t) \leftarrow -s(t)$$

Pseudo Lindblad Quantum Trajectories

$$\rho(t) = \mathbb{E}\left[s(t)|\psi(t)\rangle\langle\psi(t)|\right]$$

$\bar{s}(t) \to 0$ **Sign problem**

$$\overline{s}(t) = \exp\left[-2\int_0^t \sum_i r_{i,-}(s)ds\right]$$

Minimize negative rates

$$r_i(t) = \frac{||L_i|\psi(t)\rangle||^2}{||\,|\psi(t)\rangle||^2}$$

Operator optimizations

$$\frac{d}{dt}\rho(t) = -i[H, \rho(t)] + \left(L_+\rho L_+^\dagger - \frac{1}{2}\{L_+^\dagger L_+, \rho(t)\}\right) - \left(L_-\rho L_-^\dagger - \frac{1}{2}\{L_-^\dagger L_-, \rho(t)\}\right)$$

Operator optimizations

$$\begin{pmatrix} L'_+ \\ L'_- \end{pmatrix} = \begin{pmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{pmatrix} \begin{pmatrix} L_+ \\ L_- \end{pmatrix}$$

$$\frac{d}{dt}\rho(t) = -i[H, \rho(t)] + \left(L_+ \rho L_+^\dagger - \frac{1}{2}\{L_+^\dagger L_+, \rho(t)\} \right) - \left(L_- \rho L_-^\dagger - \frac{1}{2}\{L_-^\dagger L_-, \rho(t)\} \right)$$

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$$\frac{d}{dt}\rho(t) = -i[H, \rho(t)] + \left(L'_+ \rho L'^{\dagger}_+ - \frac{1}{2}\{L'^{\dagger}_+ L'_+, \rho(t)\} \right) - \left(L'_- \rho L'^{\dagger}_- - \frac{1}{2}\{L'^{\dagger}_- L'_-, \rho(t)\} \right)$$

$$L'_+(w, \phi) = e^{i\phi/2} \cosh(w) L_+ + e^{-i\phi/2} \sinh(w) L_-$$
$$L'_-(w, \phi) = e^{i\phi/2} \sinh(w) L_+ + e^{-\phi/2} \cosh(w) L_-$$

Operator optimizations

Global optimization

$$\underset{\omega,\,\phi}{\operatorname{argmin}} \left|\left| L'_{-} \right|\right|^2$$

$$\left|\left| L_\sigma \right|\right|^2 = \operatorname{Tr}\left[L_\sigma^\dagger L_\sigma \right]$$

Local optimization

$$\underset{\omega,\,\phi}{\operatorname{argmin}} \langle \psi | L'^\dagger_{-} L'_{-} | \psi \rangle$$

Operator optimizations

Global optimization

$$\underset{\omega,\,\phi}{\operatorname{argmin}} \left|\left|L'_-\right|\right|^2$$

$$\phi = \pi - \arg\left[\mathrm{Tr}(L_+ L_-^\dagger)\right]$$

$$w = \frac{1}{2}\operatorname{arctanh}\left(\frac{2\,\mathrm{Tr}(L_+ L_-^\dagger)}{\mathrm{Tr}(L_+ L_+^\dagger) + \mathrm{Tr}(L_- L_-^\dagger)}\right)$$

Local optimization

$$\underset{\omega,\,\phi}{\operatorname{argmin}} \langle\psi|L_-'^\dagger L'_-|\psi\rangle$$

$$\phi = \pi - \arg\left[\langle\psi|L_-^\dagger L_+|\psi\rangle\right]$$

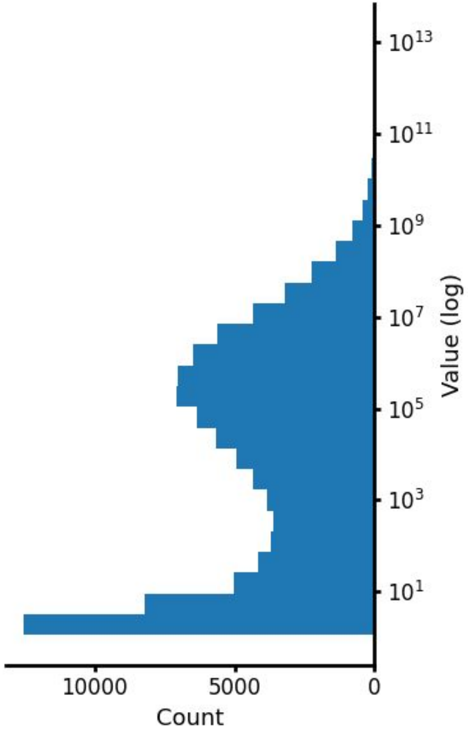
$$w = \frac{1}{2}\operatorname{arctanh}\left(\frac{2\left|\langle\psi|L_-^\dagger L_+|\psi\rangle\right|}{||L_+|\psi\rangle||^2 + ||L_-|\psi\rangle||^2}\right)$$

Because trajectories can have large norm, we are sampling a heavy tailed distribution

$$|\psi(t')\rangle = \frac{\exp\left(-i\int_t^{t'} H_{\text{eff}}(s)ds\right)}{\sqrt{\exp\left(-\int_t^{t'}\sum_i r_i(s)ds\right)}}$$

Norm can grow

$$\mathrm{Tr}[A\rho(t)] = \mathbb{E}\left[s(t)\langle\psi(t)|A|\psi(t)\rangle\right]$$



Tailed distribution: Takes many samples to converge

Truncation

$$\frac{d}{dt}\rho(t) = -i[H,\rho(t)] + \left(L_+\rho L_+^\dagger - \frac{1}{2}\{L_+^\dagger L_+, \rho(t)\}\right) - \left(L_-\rho L_-^\dagger - \frac{1}{2}\{L_-^\dagger L_-, \rho(t)\}\right)$$

Truncation

$$\underbrace{\frac{d}{dt}\rho(t) = -i[H,\rho(t)] + \left(L_+\rho L_+^\dagger - \frac{1}{2}\{L_+^\dagger L_+, \rho(t)\}\right)}_{\text{“Lindblad equation”}} - \underbrace{\left(L_-\rho L_-^\dagger - \frac{1}{2}\{L_-^\dagger L_-, \rho(t)\}\right)}_{\mathcal{O}(E^2/T^2)}$$

Truncation

$$\begin{pmatrix} L'_+ \\ L'_- \end{pmatrix} = \begin{pmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{pmatrix} \begin{pmatrix} L_+ \\ L_- \end{pmatrix}$$

$$\underbrace{\frac{d}{dt}\rho(t) = -i[H, \rho(t)] + \left(L'_+ \rho L'^{\dagger}_+ - \frac{1}{2}\{L'^{\dagger}_+ L'_+, \rho(t)\} \right)}_{\text{“Lindblad equation”}} - \underbrace{\left(L'_- \rho L'^{\dagger}_- - \frac{1}{2}\{L'^{\dagger}_- L'_-, \rho(t)\} \right)}_{\text{minimal}}$$

Becker, T., & Eckardt, A. (2025). *Physical Review E*, *111*(3), 034132.

Truncation

Global optimization

$$\underset{\omega,\phi}{\operatorname{argmin}} \left|\left|L'_-\right|\right|^2$$

Local optimization

$$\underset{\omega,\phi}{\operatorname{argmin}} \langle\psi|L'^{\dagger}_- L'_-|\psi\rangle$$

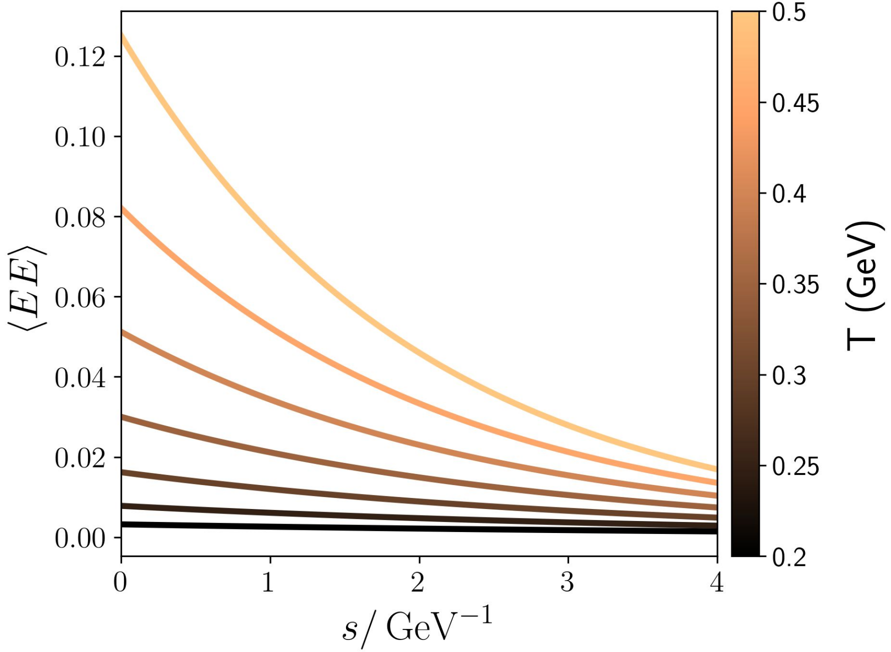
$$\underbrace{\frac{d}{dt}\rho(t) = -i[H,\rho(t)] + \left(L'_+\rho L'^{\dagger}_+ - \frac{1}{2}\{L'^{\dagger}_+ L'_+, \rho(t)\}\right)}_{\text{“Lindblad equation”}} - \underbrace{\left(L'_-\rho L'^{\dagger}_- - \frac{1}{2}\{L'^{\dagger}_- L'_-, \rho(t)\}\right)}_{\text{minimal}}$$

Becker, T., & Eckardt, A. (2025). *Physical Review E*, *111*(3), 034132.

Numerical study

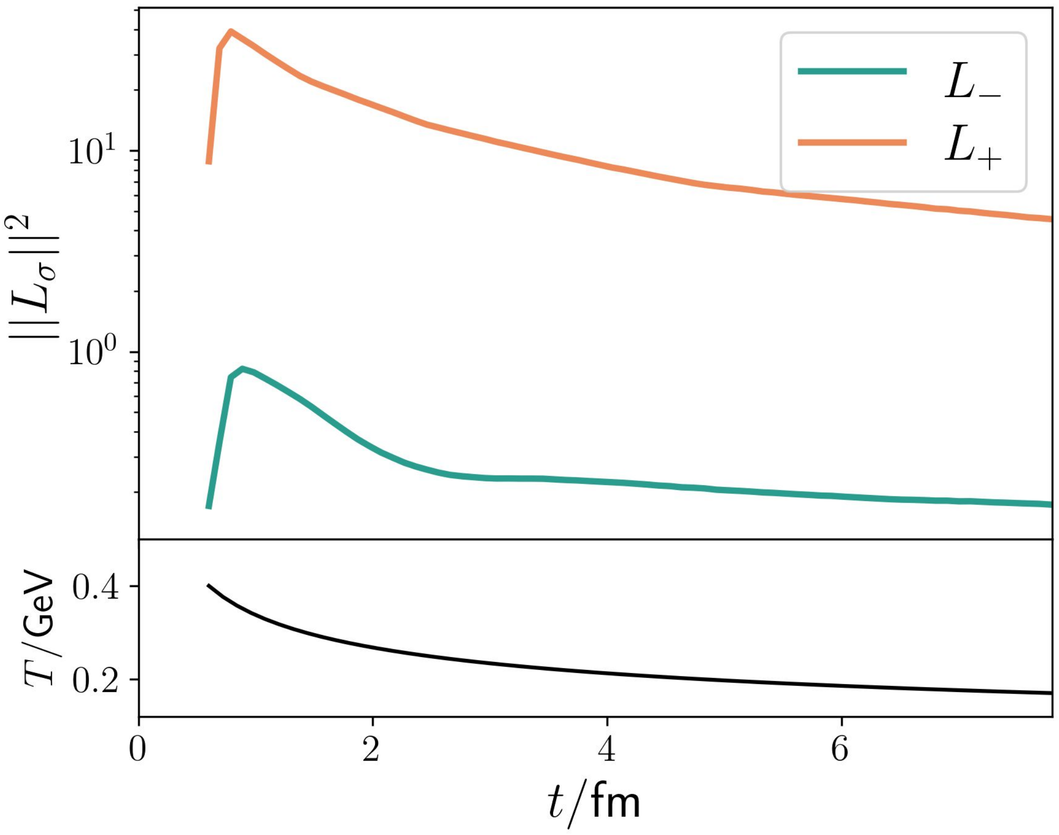
$$L_i \propto \int_0^\infty \mathrm{d}s e^{-ih_u s} r_i e^{ih_v s} \langle E_j^a(s)\Omega^{ab}(s,0)E_j^b(0)\rangle$$

$$\langle E_j^a(s)\Omega^{ab}(s,0)E_j^b(0)\rangle \propto \frac{\kappa T^4}{2} e^{-sT}$$

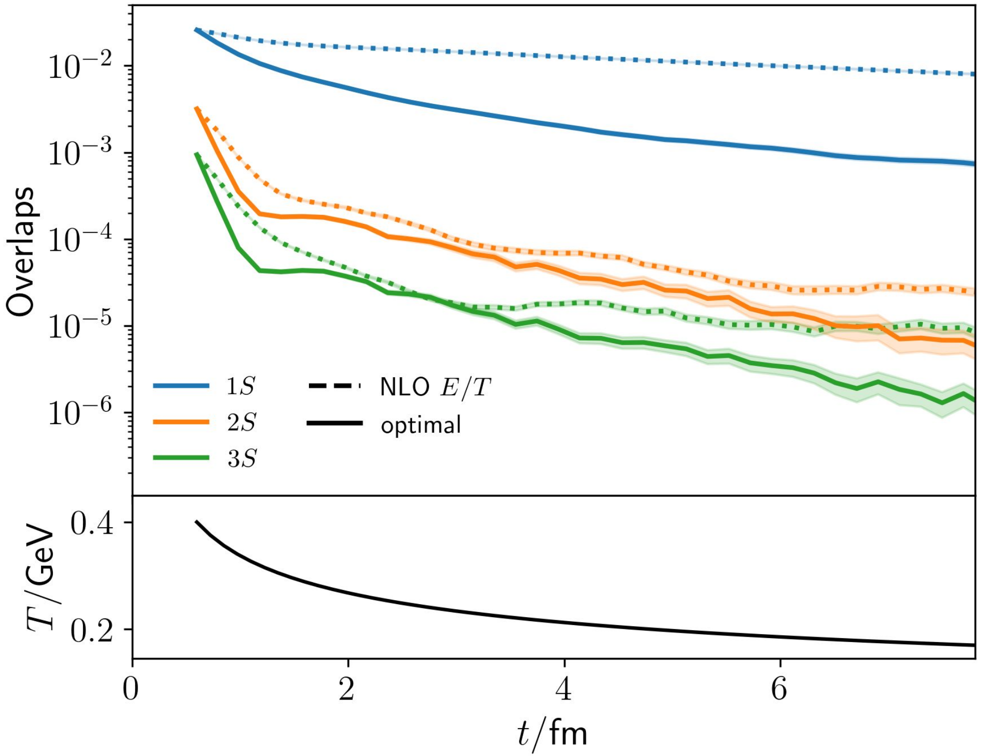


VyshakhB, R., & Sharma, R. (2025). 2504.19348

The truncation to the optimal form is efficient

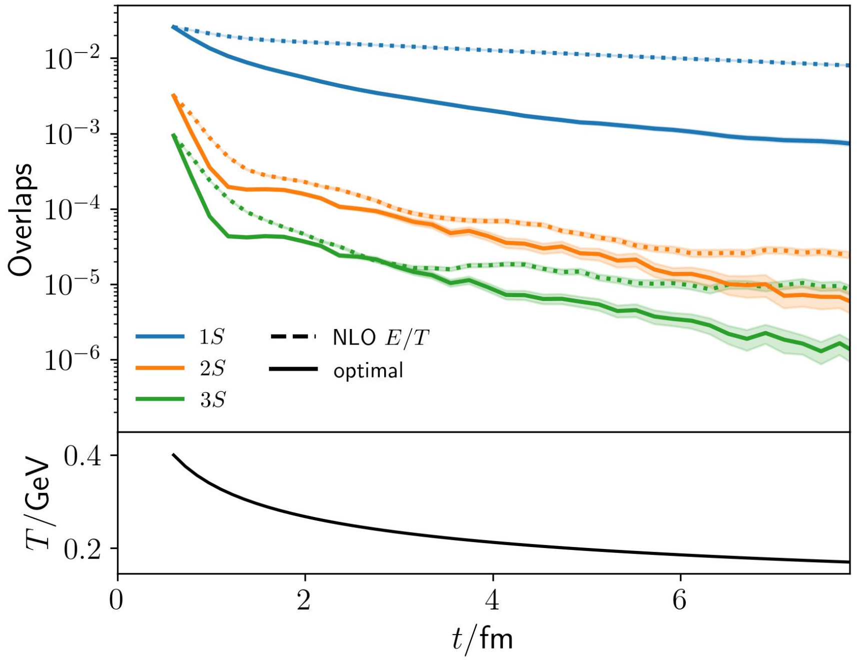


For high temperatures, we find agreement for the 2S and 3S and corrections for the 1S



Summary

- At low temperatures our master equation is not positive
- Operator optimizations lead to efficient Lindblad truncation
- Efficient simulation of quarkonium dynamics at low temperatures



Outlook: **- Correlator study**

- Phenomenology (RHIC, small systems)

Backup

Connecting to phenomenology

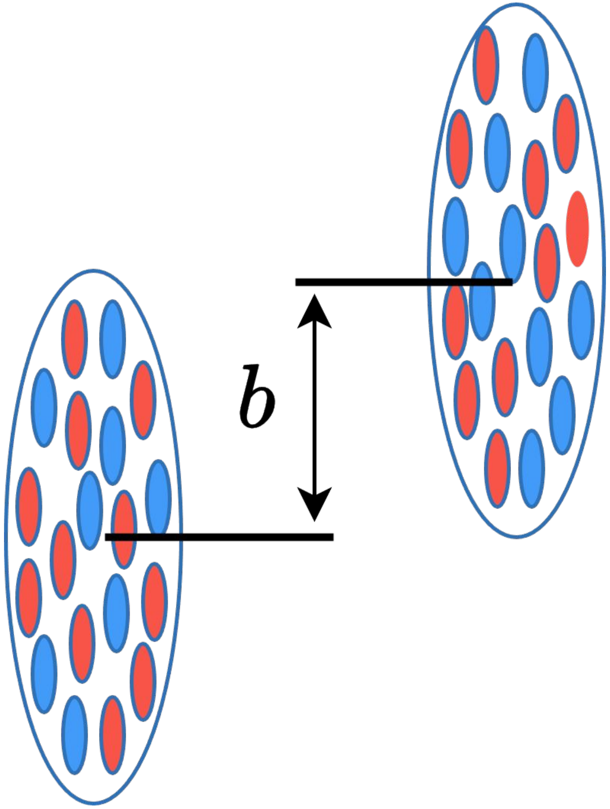
$$R^i_{AA}(c) = \frac{\left(F \cdot S(c) \cdot \vec{\sigma}_{\text{direct}}\right)^i}{\sigma^i_{\text{exp}}}$$

$$S_{ii} = P_{\text{survival}}(i) = \frac{\langle i|\rho(t_f)|i\rangle}{\langle i|\rho(t_0)|i\rangle}$$

Connecting to phenomenology

centrality

$$R^i_{AA}(c) = \frac{\left(F \cdot S(c) \cdot \vec{\sigma}_{\text{direct}}\right)^i}{\sigma^i_{\text{exp}}}$$



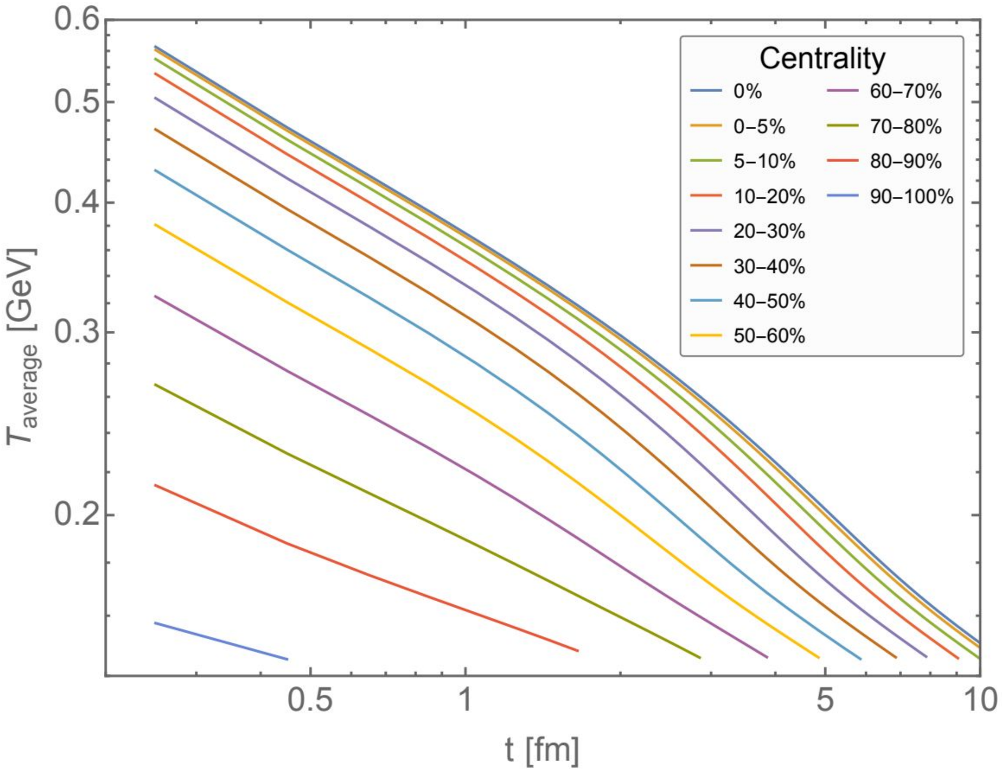
b: Impact parameter

Connecting to phenomenology

Brambilla, Nora, et al. *Journal of High Energy Physics* 2021.5 (2021): 1-47.

centrality

$$R_{AA}^{i}(c) = \frac{\left(F \cdot S(c) \cdot \vec{\sigma}_{\text{direct}}\right)^{i}}{\sigma_{\text{exp}}^{i}}$$

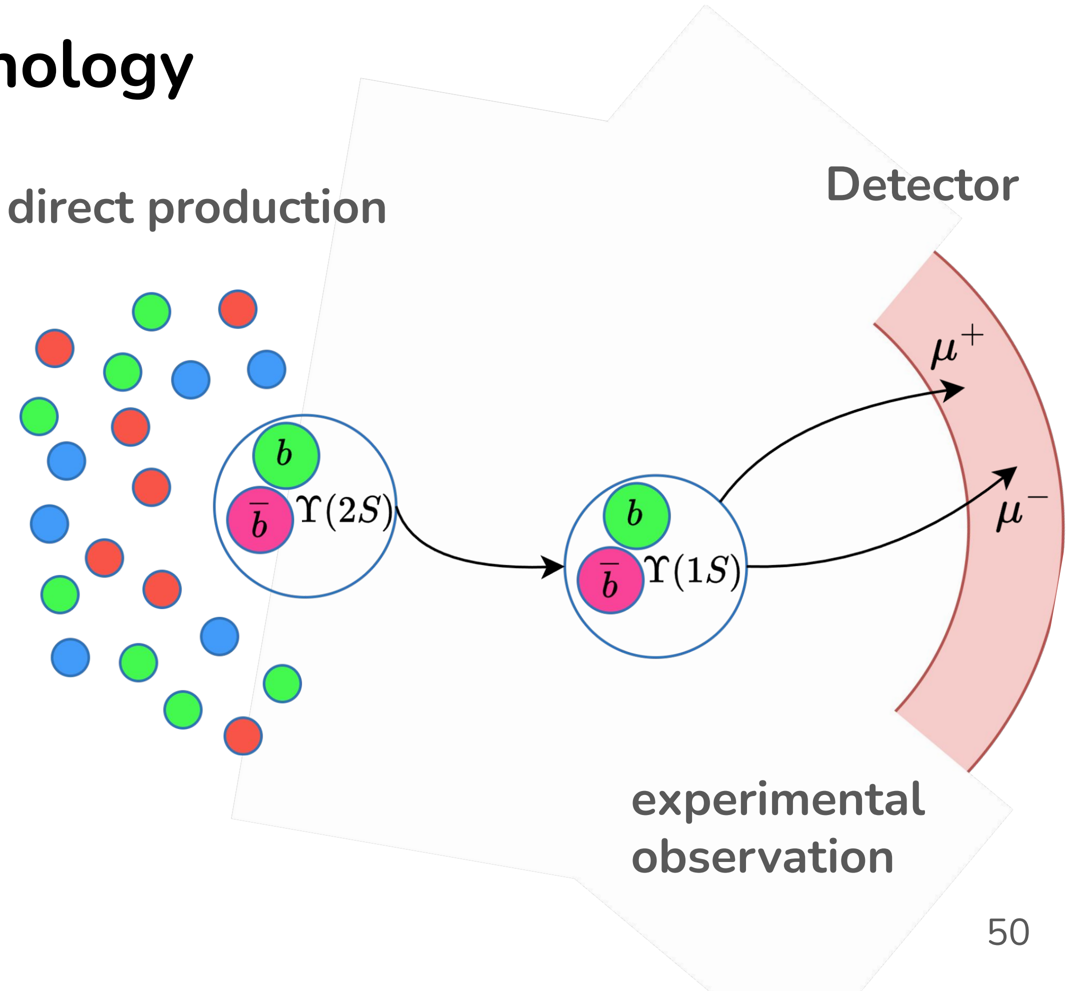


Connecting to phenomenology

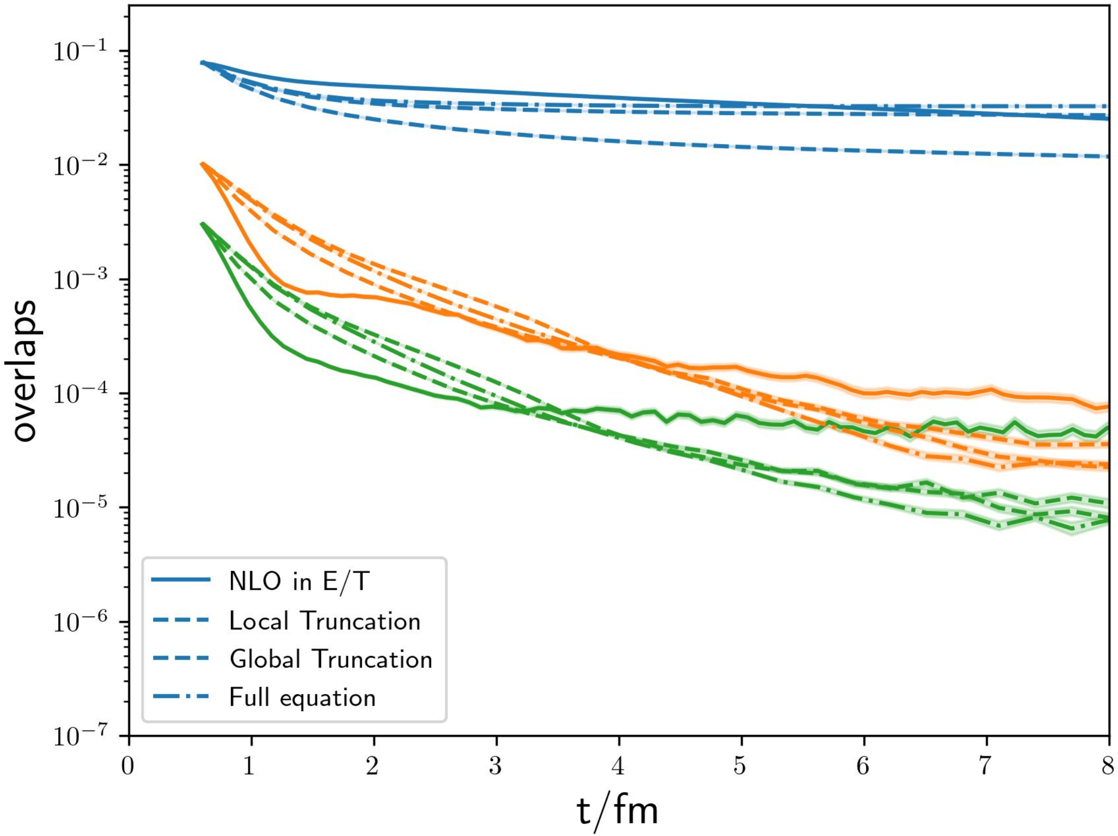
Feeddown matrix

$$R^i_{AA}(c) = \frac{\left(F \cdot S(c) \cdot \vec{\sigma}_{\mathrm{direct}}\right)^i}{\sigma^i_{\mathrm{exp}}}$$

$$\vec{\sigma}_{\mathrm{direct}} \;=\; F^{-1}\vec{\sigma}_{\mathrm{exp}}$$



N=128



10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
overlaps
0
1
2
3
4
5
6
7
8
t/fm
NLO in E/T
Local Truncation
Global Truncation
Full equation