



Thermalization of Quarkonium in the QGP

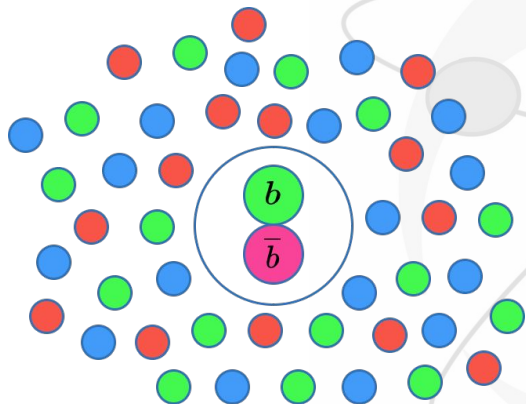
Decanting the Universe

09.11.24

Quarkonium suppression

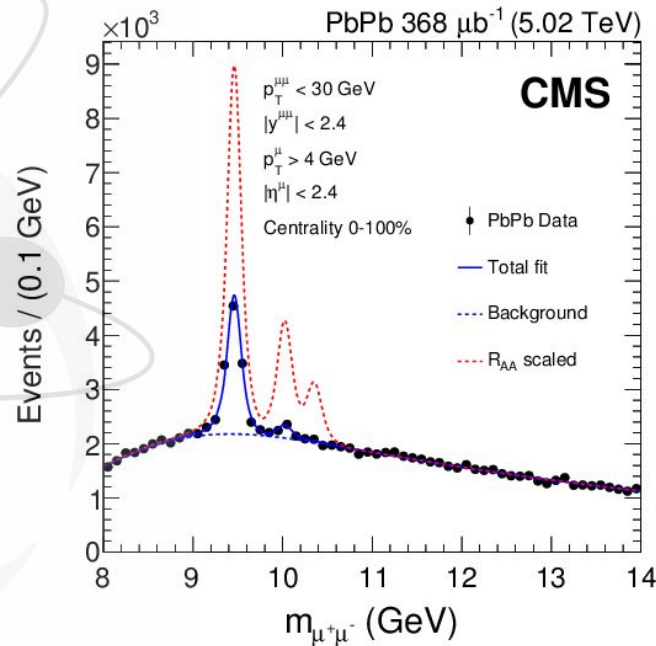
Propagation through QGP

$T \sim 1 \text{ M K}$



$$V(r) = -\frac{\alpha}{r}$$

$$V(r) = -\frac{\alpha}{r} e^{-m_D r}$$



Fundamental question: Thermalization

- Observation: All systems always evolve over long time to the same steady state: **Thermal state**
- Classically given by **Boltzmann distribution**

$$P(E) = Z e^{-E/T}$$

Fundamental question: Thermalization

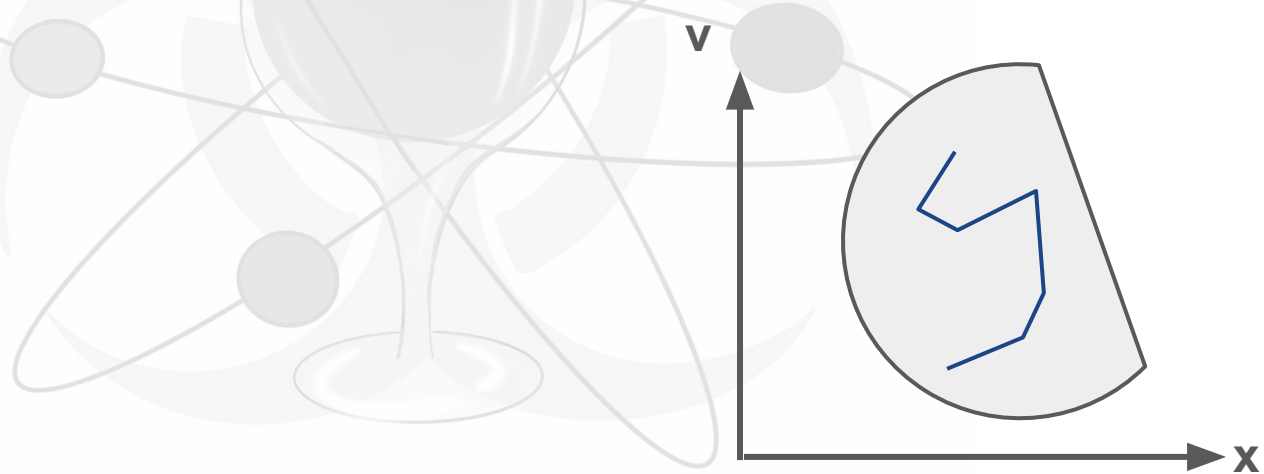
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Why does this happen?

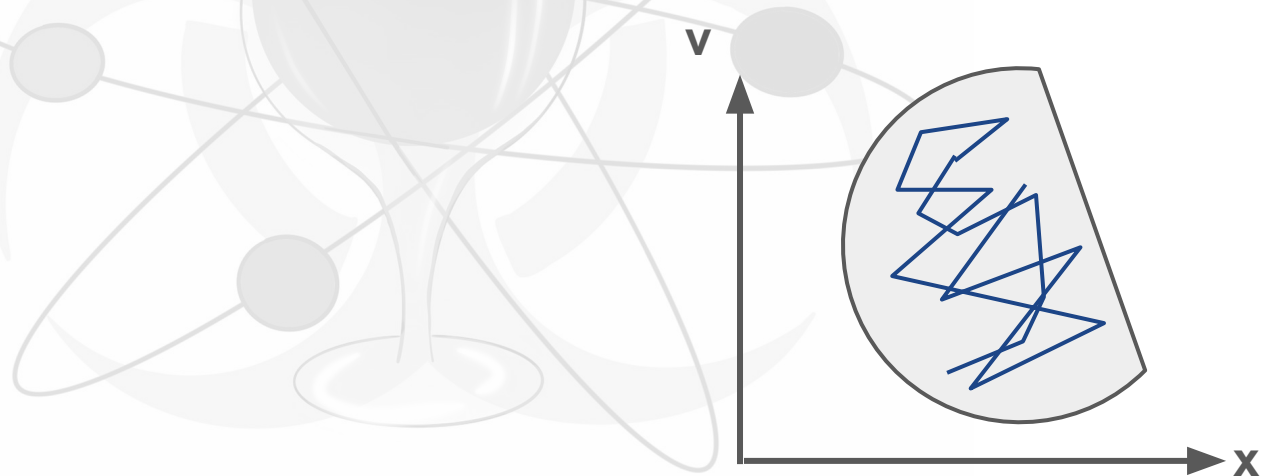
Fundamental question: Thermalization

- Usually systems are chaotic and show **ergodicity**
- Over long time all points in phase space will be visited



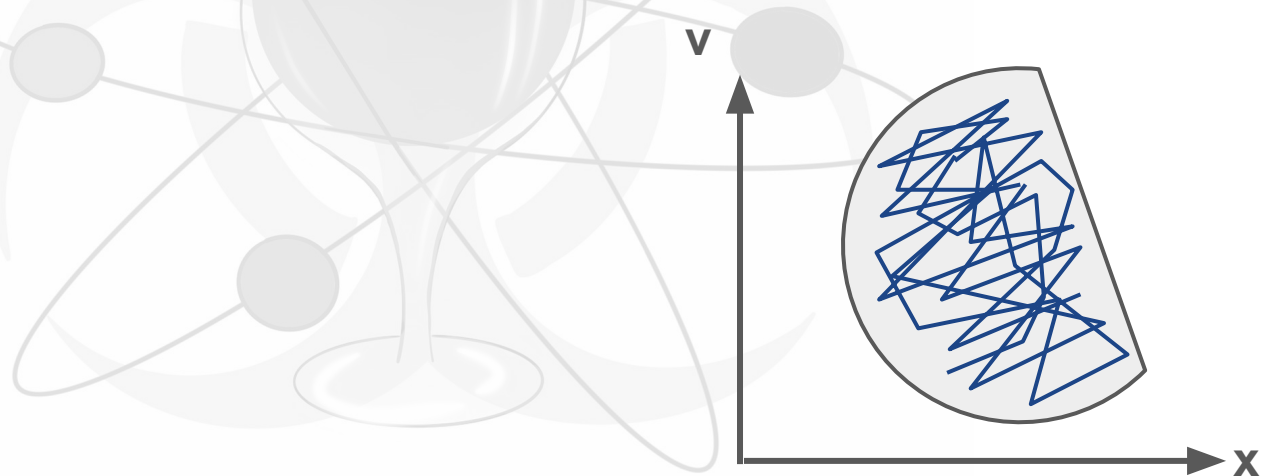
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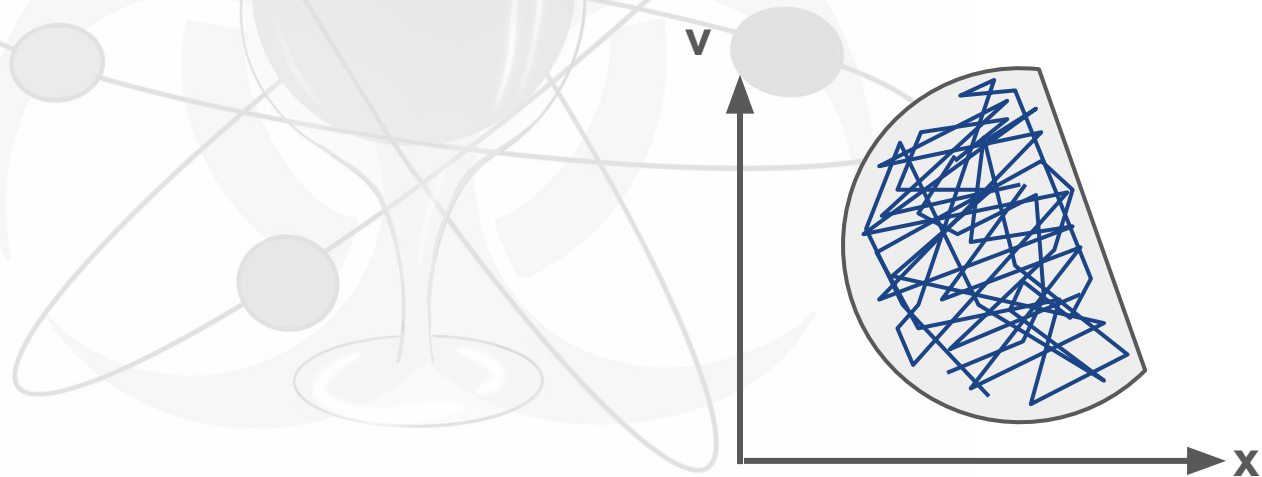
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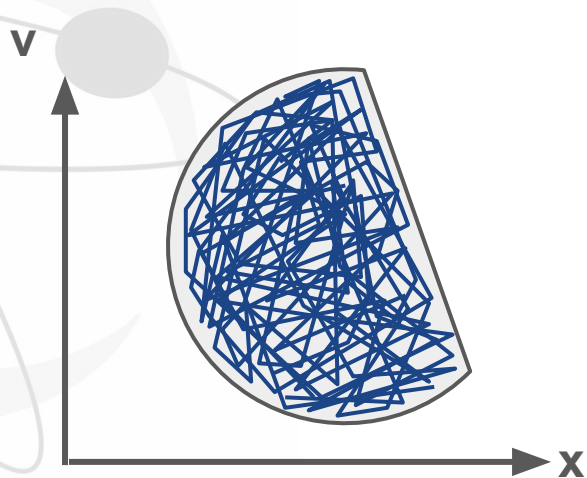
Fundamental question: Thermalization

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Time Averages = Ensemble Averages

Probability of visiting each state given by its energy: **Boltzmann distribution**

$$P(E) = Z e^{-E/T}$$



How about quantum mechanics ?

- Similar!

matrix which gives the
state of the system

$$\rho = Z e^{-E_n/T}$$

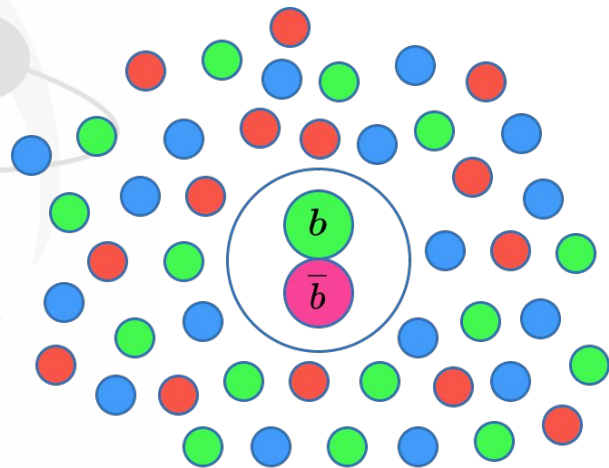
$$|\psi\rangle = \uparrow \uparrow \downarrow \uparrow \downarrow \downarrow \uparrow \uparrow \downarrow \uparrow \uparrow \uparrow \downarrow \uparrow$$

$$P(\psi) \propto \exp(-E_\psi/T)$$

Back to Quarkonium

- Question: What happens to the quarkonium after very long time?
- Many people use models and assume:

$$P(\psi) \propto \exp(-E_\psi/T)$$



Example: Dark matter

Disclaimer: I don't know much about this

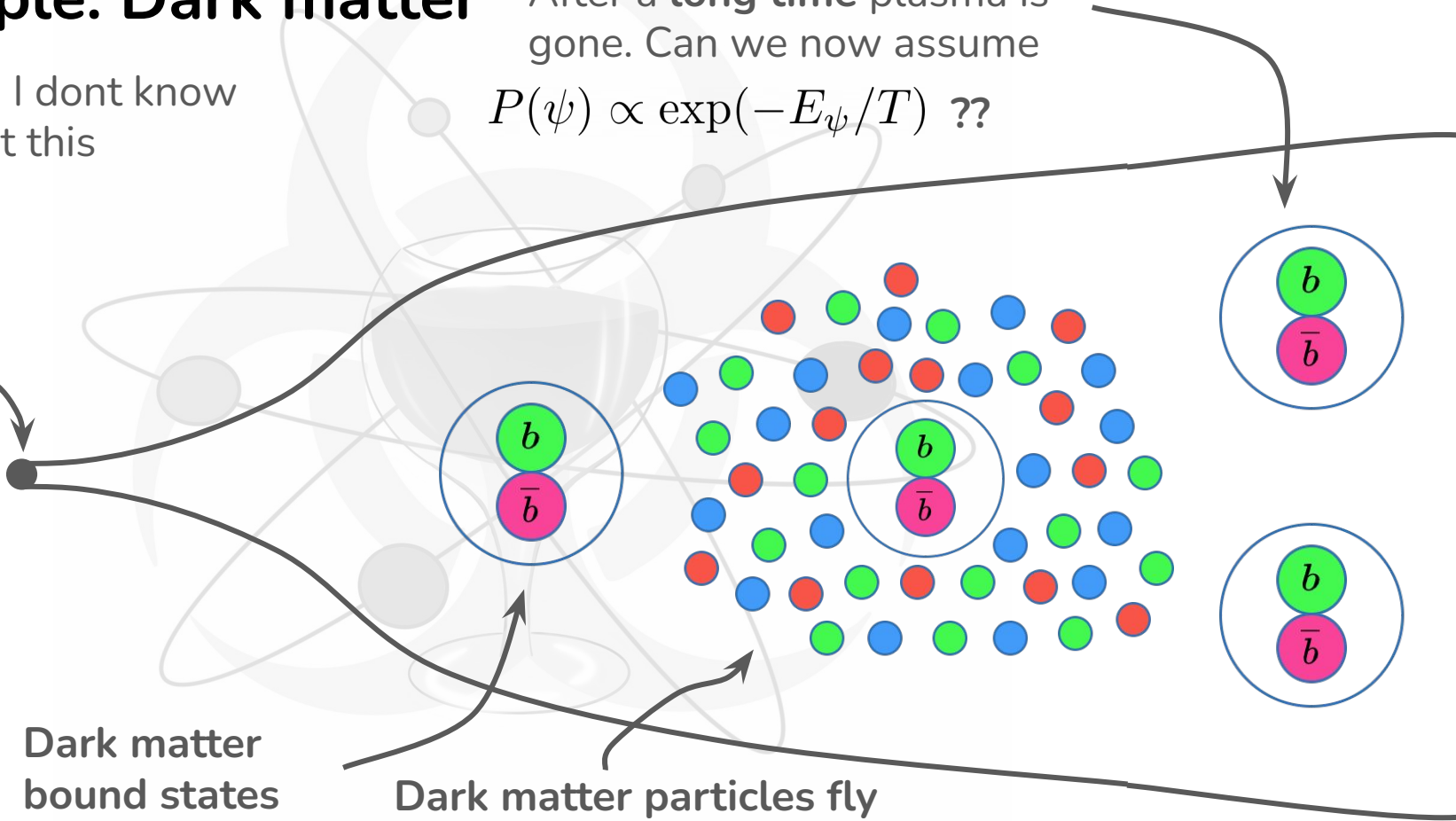
After a long time plasma is gone. Can we now assume

$$P(\psi) \propto \exp(-E_\psi/T) \quad ??$$

Big Bang

Dark matter bound states form

Dark matter particles fly through plasma





So this is my research project:

**What happens to the
quarkonium in the plasma in
the infinite time limit?**



Split into three sub-questions:

1. Is there a steady state it is evolving to ?
2. How long does it take to reach this steady state?
3. How does this steady state look like?
it the Boltzmann distribution?

Is

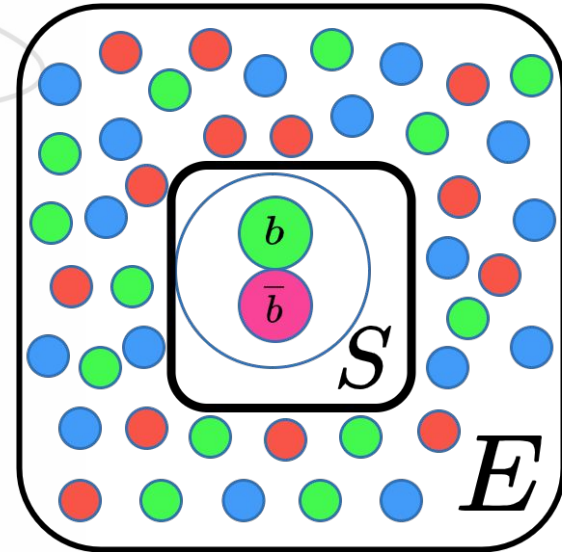
Our framework: Open quantum systems

Evolution given by differential equation

$$\frac{d\rho_S}{dt} = -i[H_S, \rho_S] + \sum_n \left(C_n \rho_S C_n^\dagger - \frac{1}{2} \{ C_n^\dagger C_n, \rho_S \} \right)$$
$$\frac{d\rho_S}{dt} = \mathcal{L}[\rho_S]$$

ρ_S Matrix encoding the state of the Quarkonium

H_S, C_n Matrices that don't depend on time



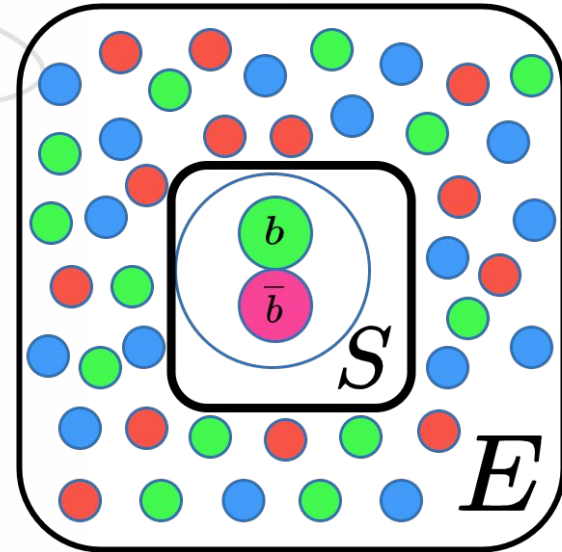
1. Is there a steady state it is evolving to ?

- Steady state defined as

$$\frac{d\rho_S}{dt} = \mathcal{L}[\rho_S] = 0$$

There is a single attractive steady state if the only matrices commuting with all H_S, C_n are proportional to identity

commutation of A & B means $[A, B] = AB - BA$

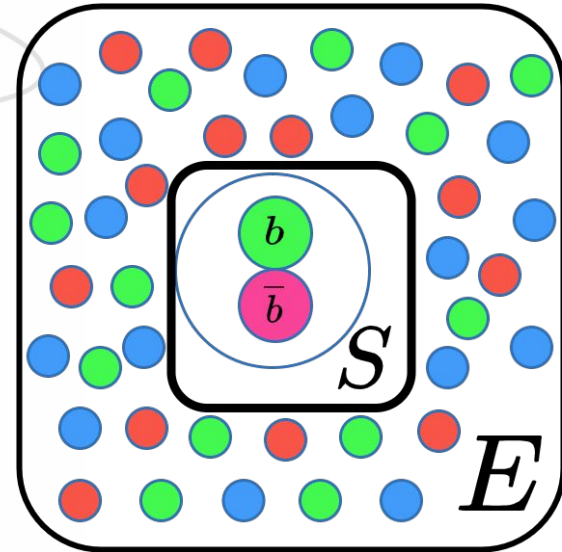


1. Is there a steady state it is evolving to ?

- Steady state defined as

$$\frac{d\rho_S}{dt} = \mathcal{L}[\rho_S] = 0$$

I was able to proof that!
There has to be a single steady
state this is evolving to!



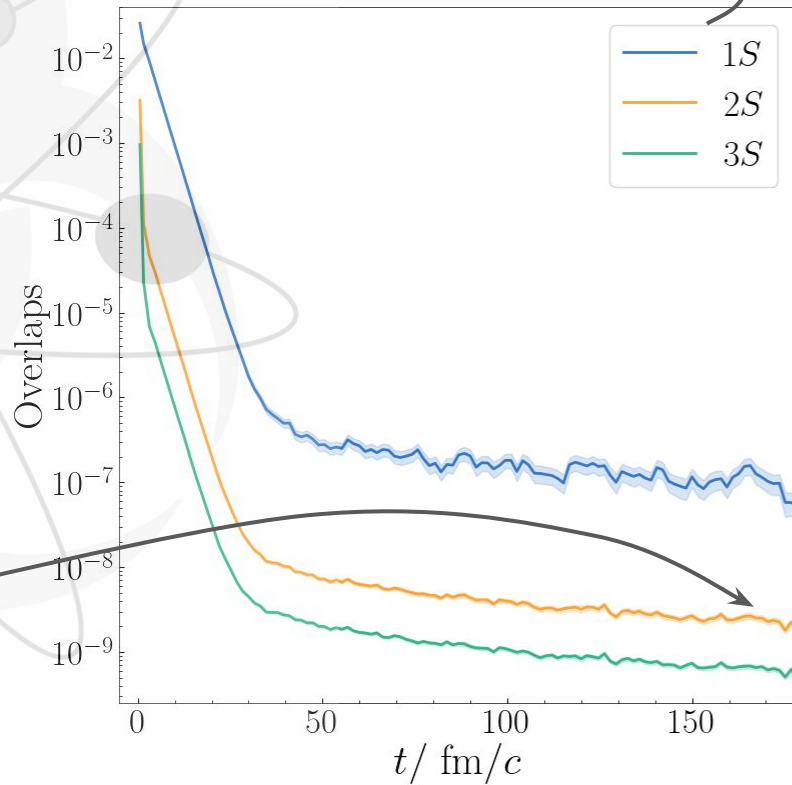
2. How long does it take to reach this steady state?

- Perform **very** demanding simulations

can be seen as
three entries of ρ_S

Unfortunately still negative
slope...

Is this a steady
state?



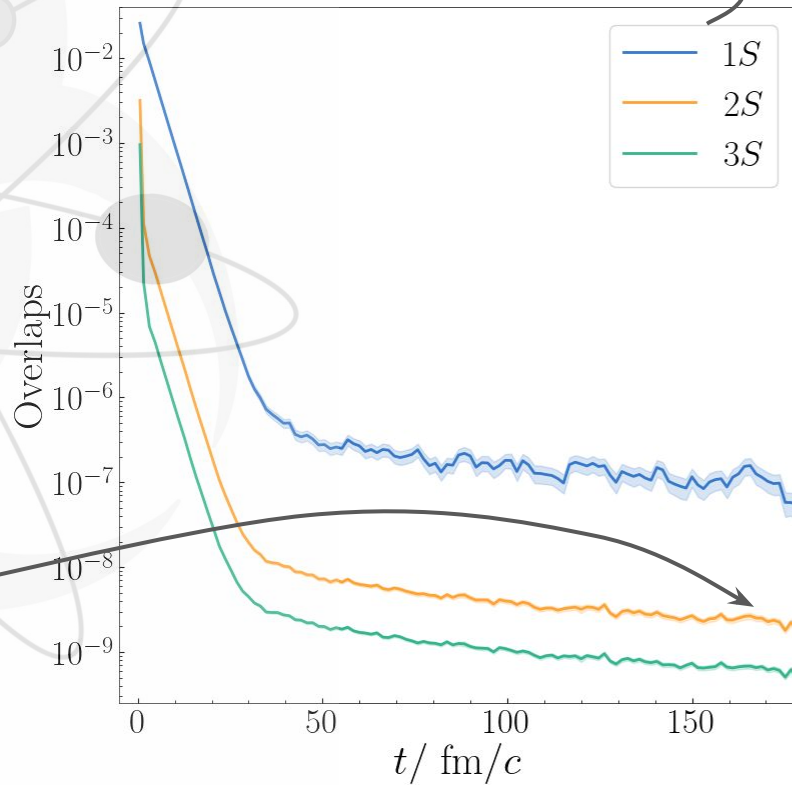
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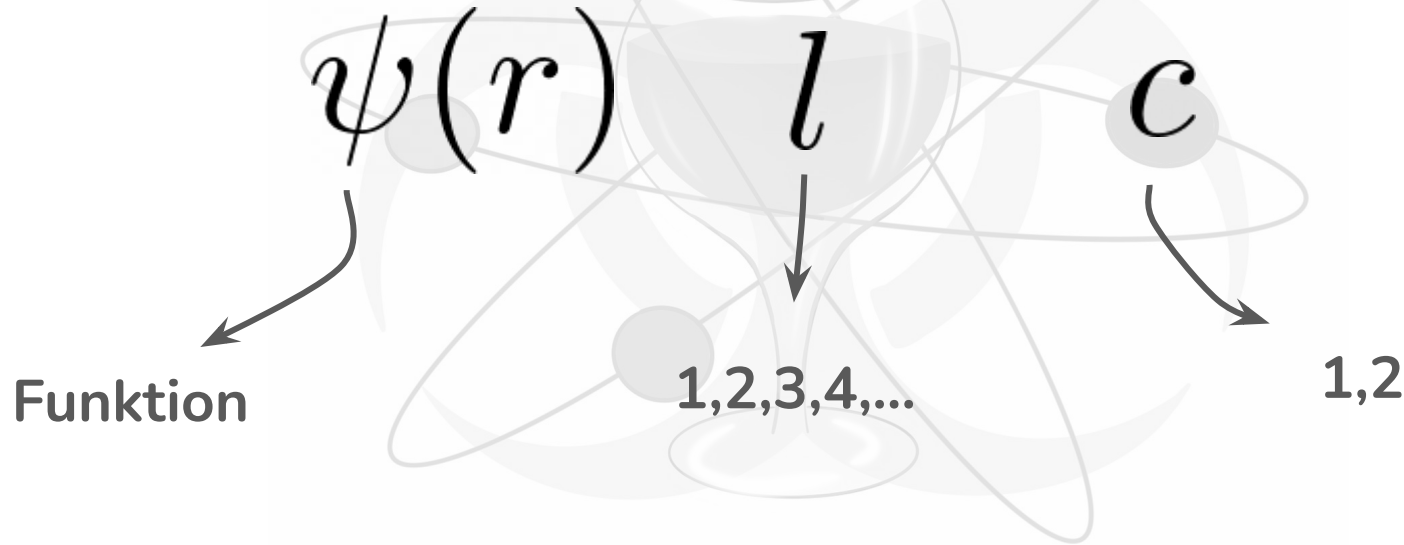
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Is this a steady
state?



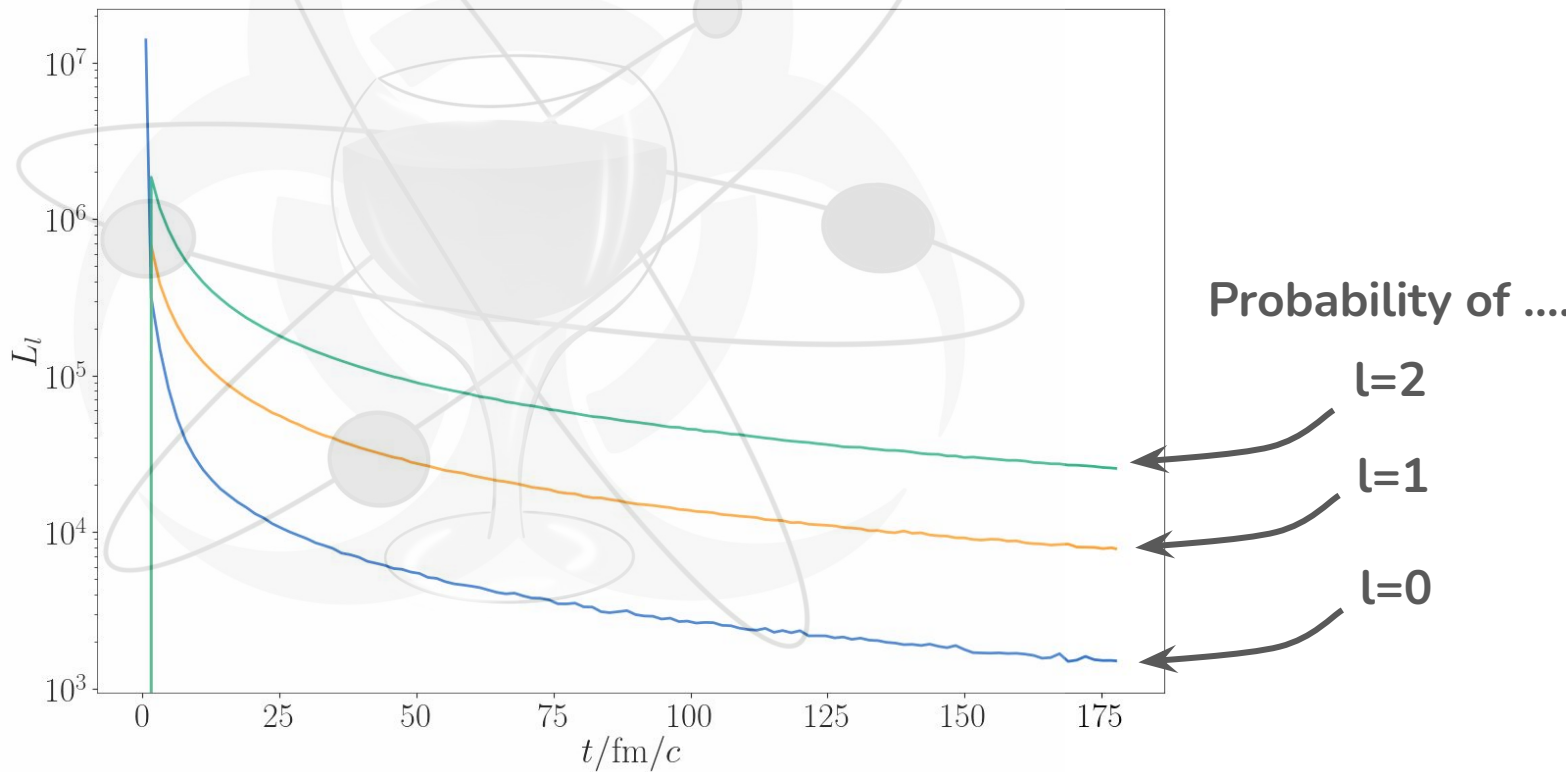
2. How long does it take to reach this steady state?

State of the system consists of three parts:



2. How long does it take to reach this steady state?

Look only at l part



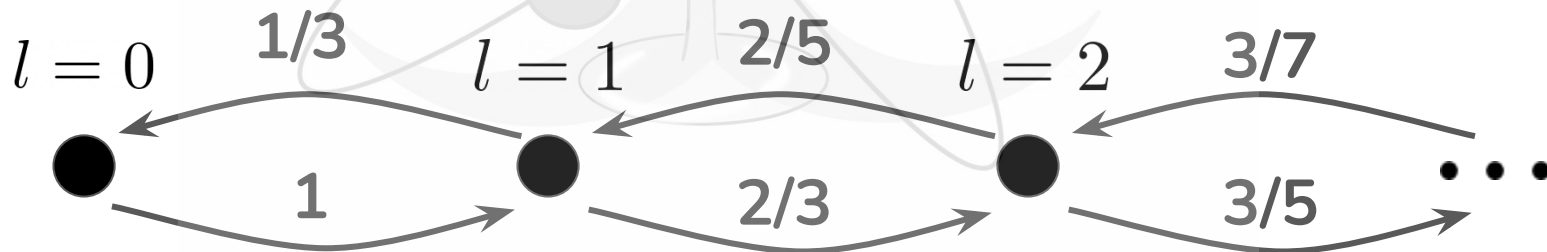
2. How long does it take to reach this steady state?

This part is not equilibrating...

Looking at the algorithm this makes sense:

Probability of increasing l with time $\Gamma^\uparrow = \frac{l+1}{2l+1}$

Probability of decreasing l with time $\Gamma^\downarrow = \frac{l}{2l+1}$

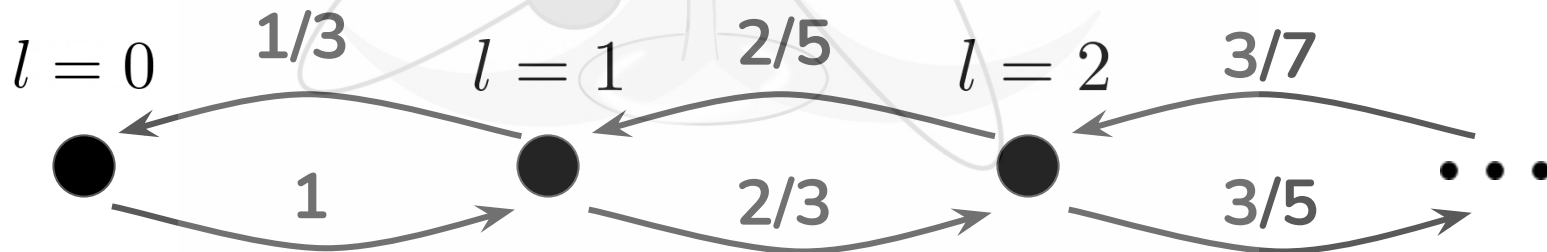


2. How long does it take to reach this steady state?

This part is not equilibrating...

Looking at the algorithm this makes sense:

Since l can be infinite it will increase for ever!

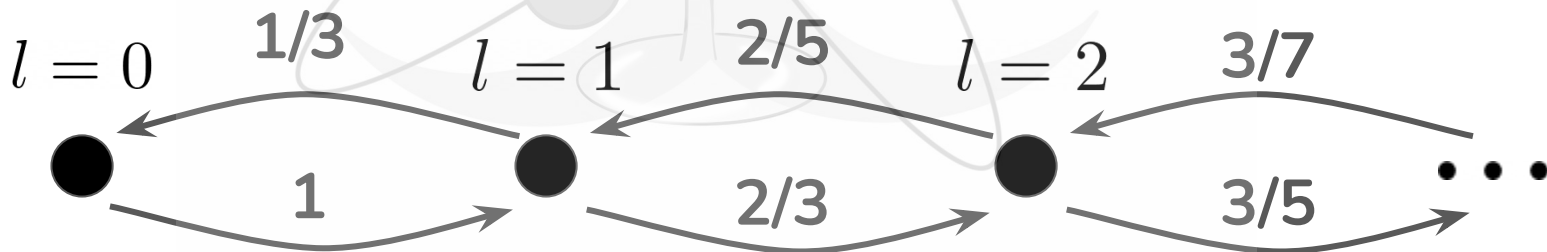


2. How long does it take to reach this steady state?

This part is not equilibrating...

Looking at the algorithm this makes sense:

But I thought there was an attractive steady state?
Yes! But it can only be reached in infinite time!



3. How does this steady state look like? Is it the Boltzmann distribution?

- Since the equilibration takes infinite time I cant see it in simulations
- However I can try to make some statements:
 - a. **It will not be the Boltzmann distribution**
 - b. Angular momentum is going to infinity
 - c. I can qualitatively show what $\psi(r)$ looks like

based on
 $\mathcal{L}[\rho_S] \neq 0$



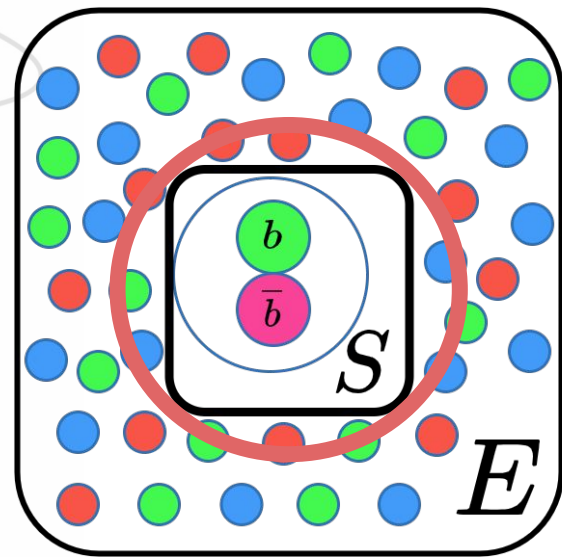
3. How does this steady state look like? Is it the Boltzmann distribution?

- What is confusing about this?

Many people think the distribution should look like the Boltzmann distribution

$$P(\psi) \propto \exp(-E_\psi / T)$$

But since we are looking at a subsystem there is no reason for that!



3. How does this steady state look like? Is it the Boltzmann distribution?

- What is confusing about this?

I thought it should at least have finite corrections
to the boltzmann distribution

$$P(\rho_S) = Ze^{-E_n/T} + \lambda \rho^{(1)} + \lambda^2 \rho^{(2)} + \dots$$

Boltzmann
limit $\lambda \rightarrow 0$ should lead to boltzmann

$\ll 1$

my main
question
right now

But: Boltzmann predicts $P(l \rightarrow \infty) \rightarrow 0$ while I find the opposite!

3. How does this steady state look like? Is it the Boltzmann distribution?

- What is confusing about this?

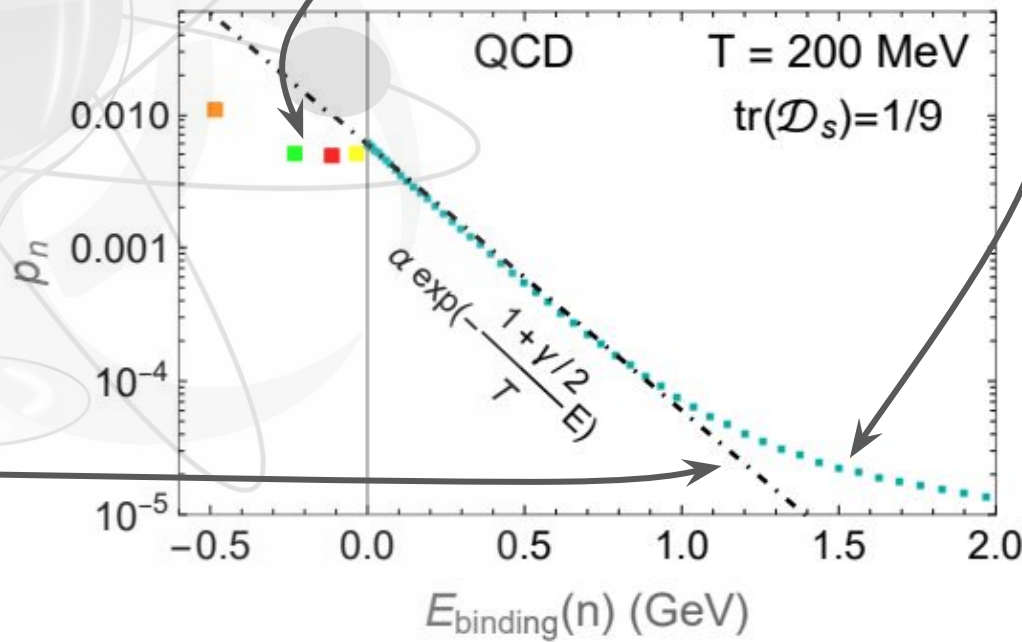
Other people find results which are at least close to the Boltzmann distribution

But they have simpler equations!

I can reproduce their results with simple equations

Boltzmann

Their results



And now ?

- In my opinion that's just what it is
- In an open system nothing guarantees you a boltzmann or whatever..
- Is this physical ? I'm not sure...

I would like to conclude this and
move on to other work..

