

Bottomonium thermalization and master equations for quarkonium evolution

Tom Magorsch

in collaboration with

Nora Brambilla, Arthur Lin and Antonio Vairo

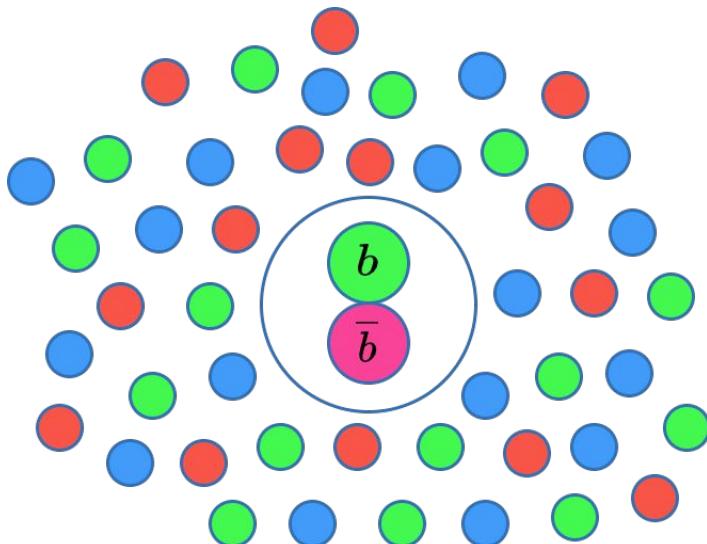


21.11.2025

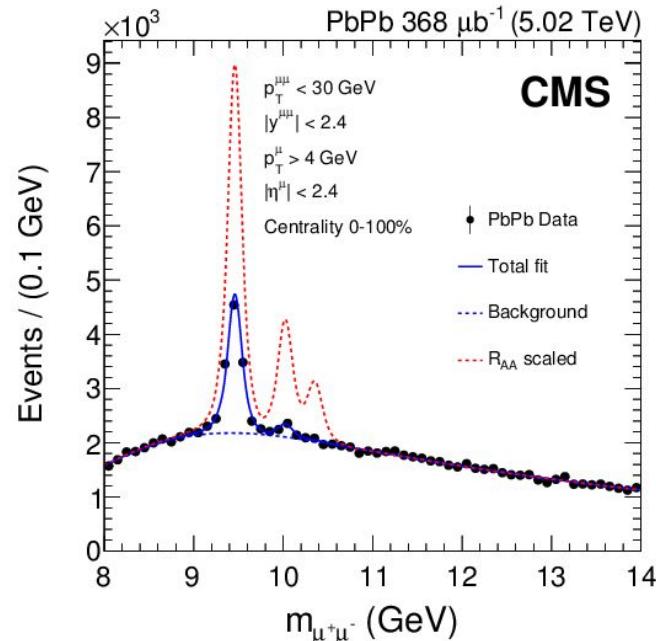


Quarkonium is an important probe of the QGP

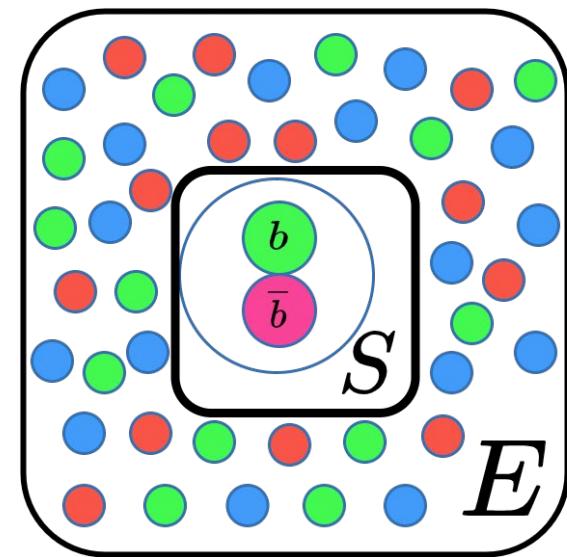
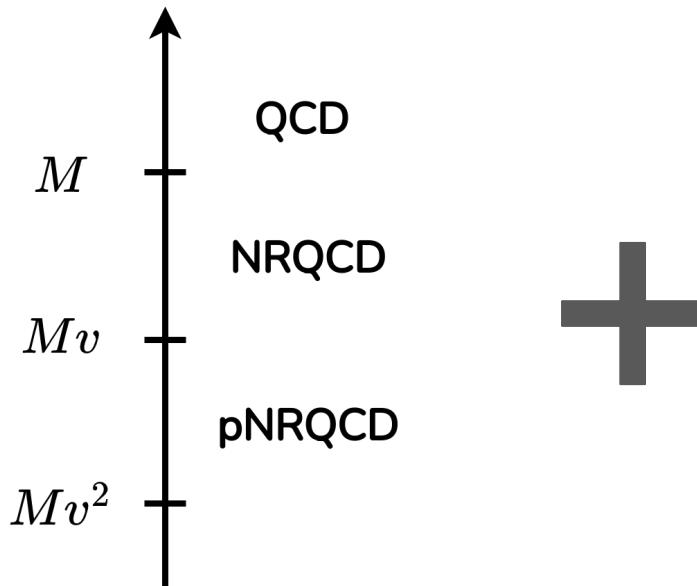
Propagation through QGP



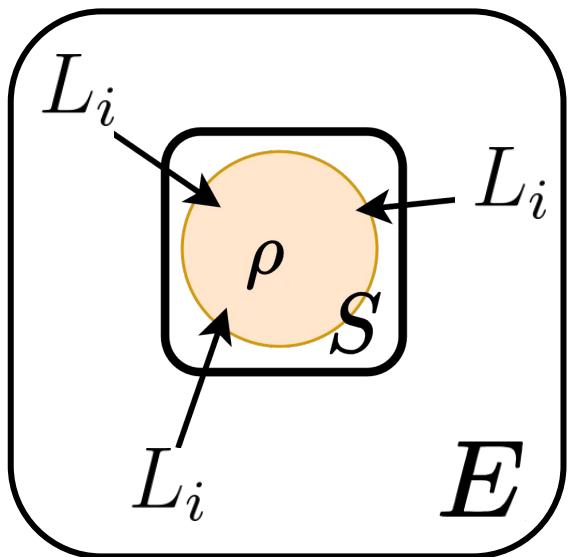
Veres, Gábor I. "Heavy ion physics at CMS and ATLAS: hard probes." *arXiv preprint arXiv:1905.10461* (2019).



The evolution can be described using Effective Field Theories + Open Quantum Systems

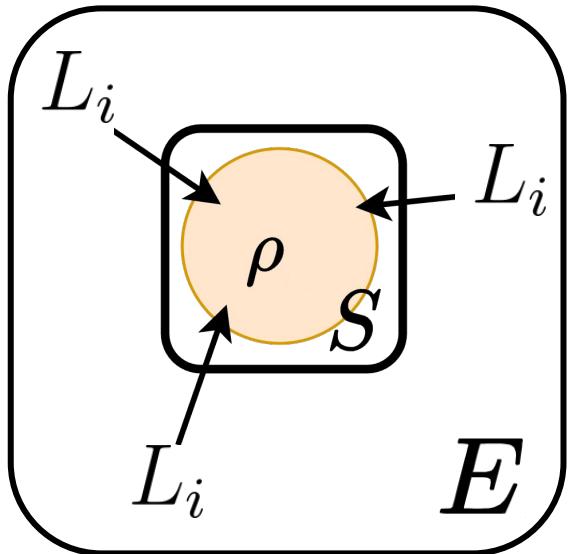


EFTs + OQS lead to a quantum master equation



EFTs + OQS lead to a quantum master equation

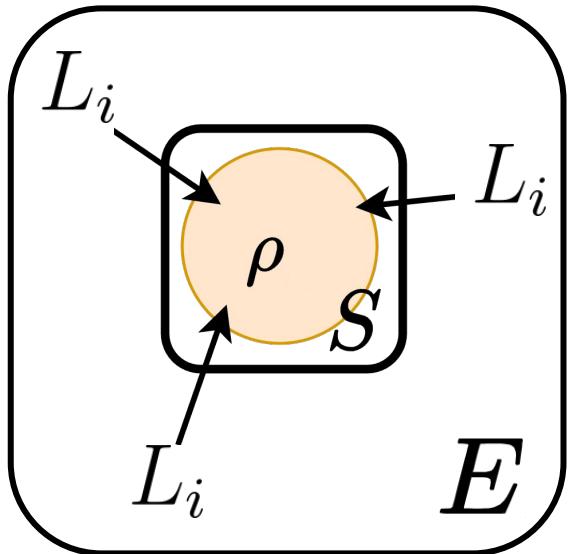
$$\frac{d\rho(t)}{dt} = -i [H, \rho(t)] + \sum_{nm} h_{nm} \left(L_i^n \rho(t) L_i^{m\dagger} - \frac{1}{2} \{ L_i^{m\dagger} L_i^n, \rho(t) \} \right)$$



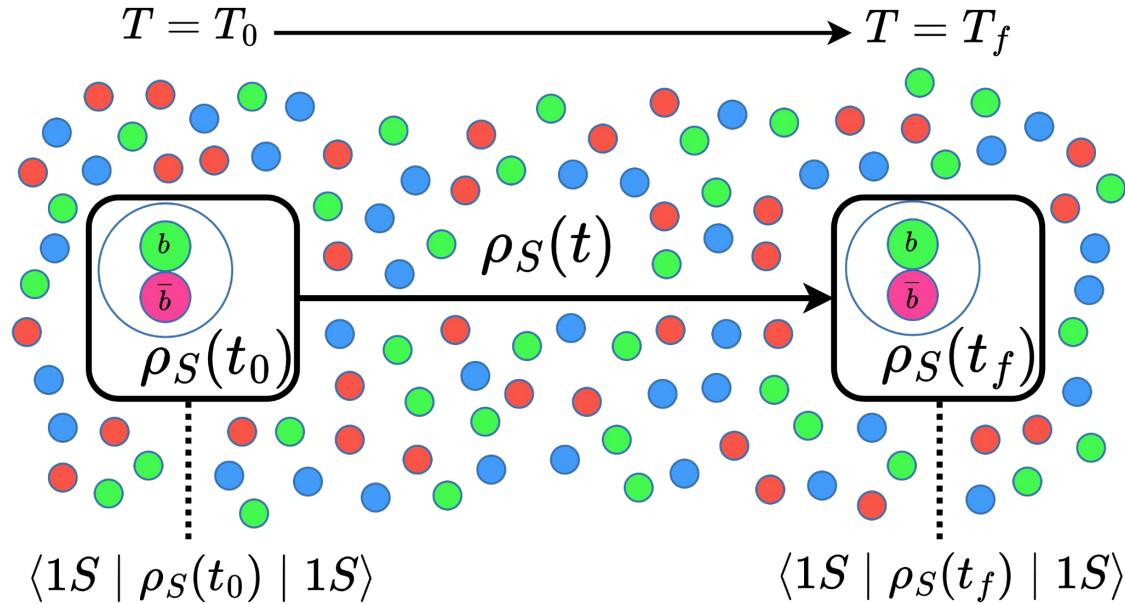
EFTs + OQS lead to a quantum master equation

$$\frac{d\rho(t)}{dt} = -i [H, \rho(t)] + \sum_{nm} h_{nm} \left(L_i^n \rho(t) L_i^{m\dagger} - \frac{1}{2} \{ L_i^{m\dagger} L_i^n, \rho(t) \} \right)$$

$$L_i \propto \int_0^\infty ds e^{-ih_u s} r_i e^{ih_v s} \langle E_j^a(s) \Omega^{ab}(s, 0) E_j^b(0) \rangle$$



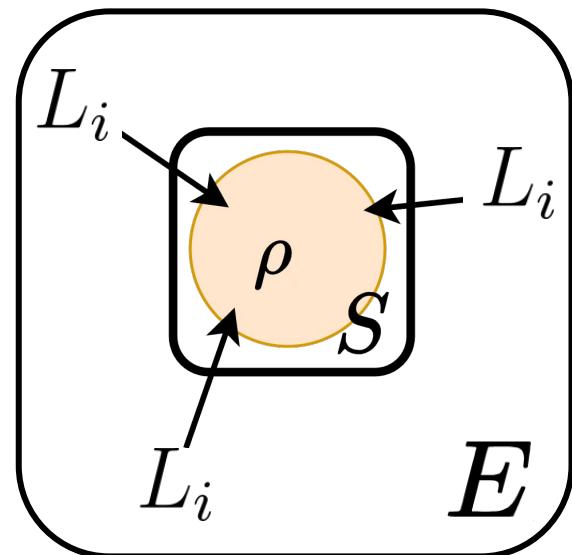
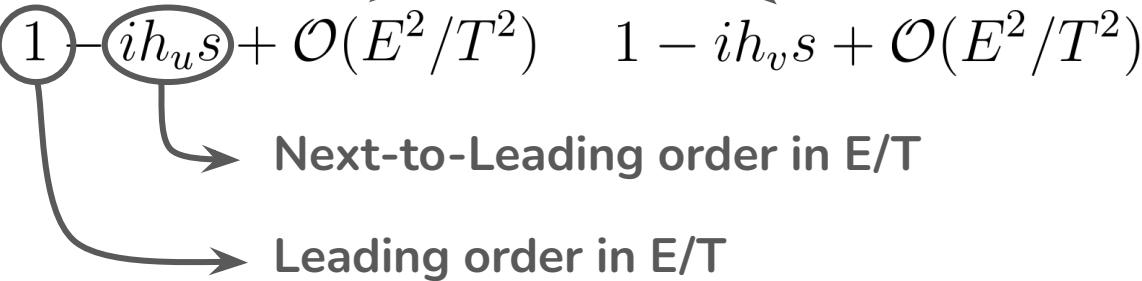
Solving the master equation gives the evolution of the bottomonium



$$P_{\text{survival}}(1S) = \frac{\langle 1S | \rho(t_f) | 1S \rangle}{\langle 1S | \rho(t_0) | 1S \rangle}$$

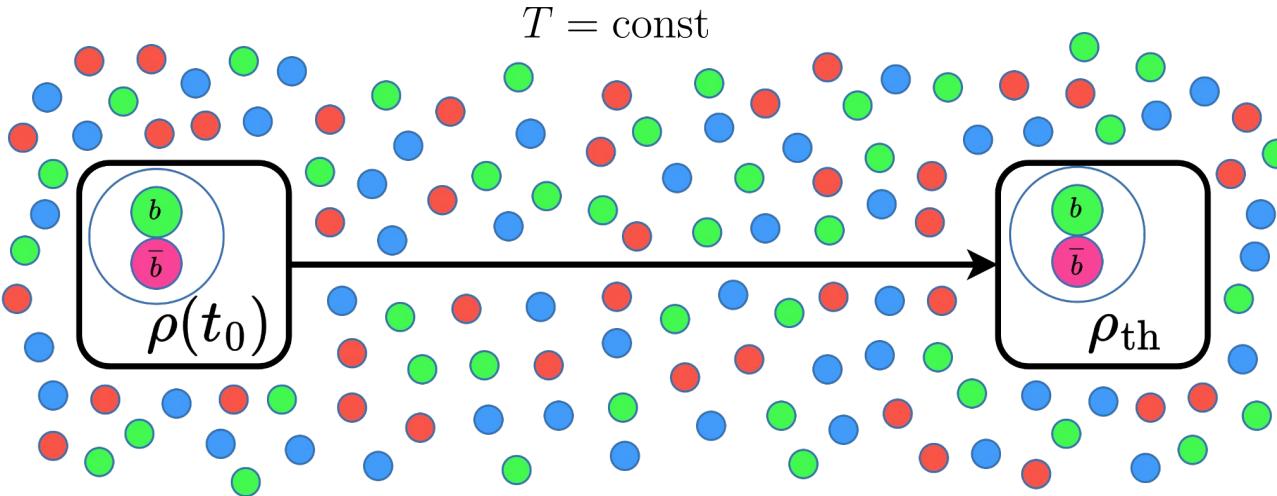
To simplify the equation one can expand in E/T

$$L_i \propto \int_0^\infty ds e^{-ih_u s} r_i e^{ih_v s} \langle E_j^a(s) \Omega^{ab}(s, 0) E_j^b(0) \rangle$$



Brambilla, N., Escobedo, M. Á., Islam, A., Strickland, M., Tiwari, A., Vairo, A., & Vander Giend, P. (2022). *Journal of High Energy Physics*, 2022(8), 1-39.

We investigated the thermalization Bottomonium



thermal state?

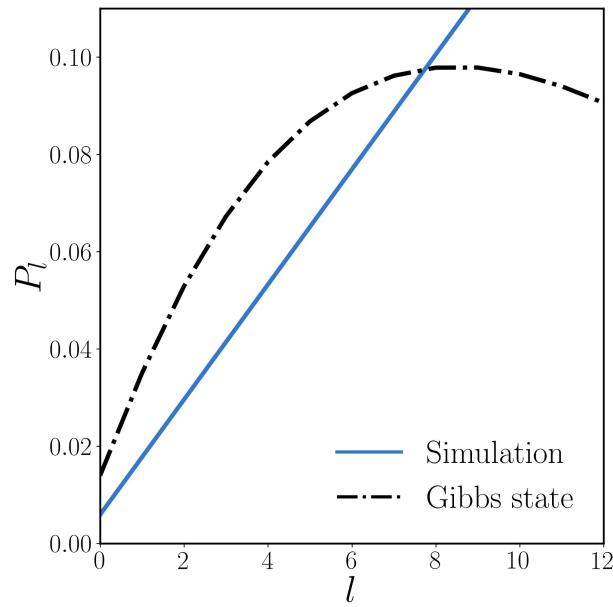
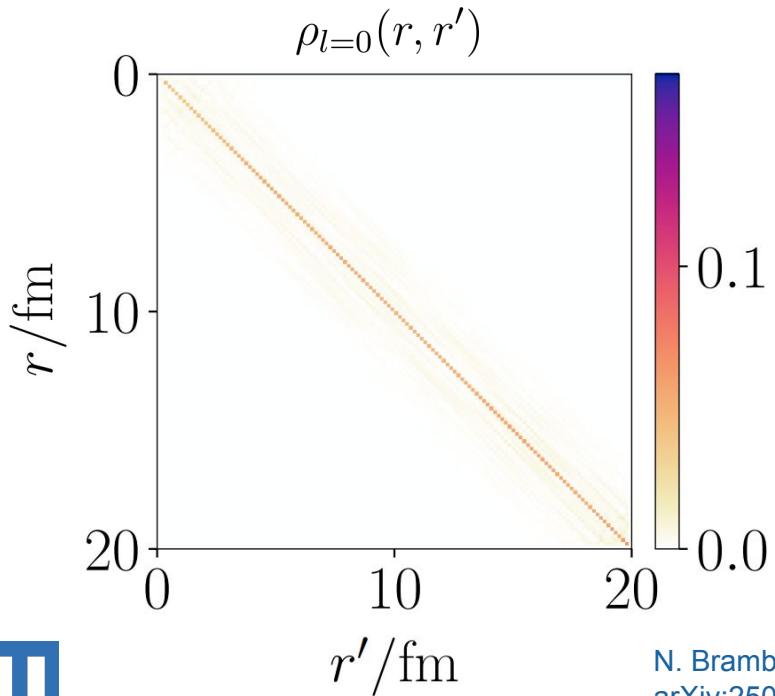
thermalization
timescale ?

$$\rho_{\text{th}} = e^{-H/T}$$

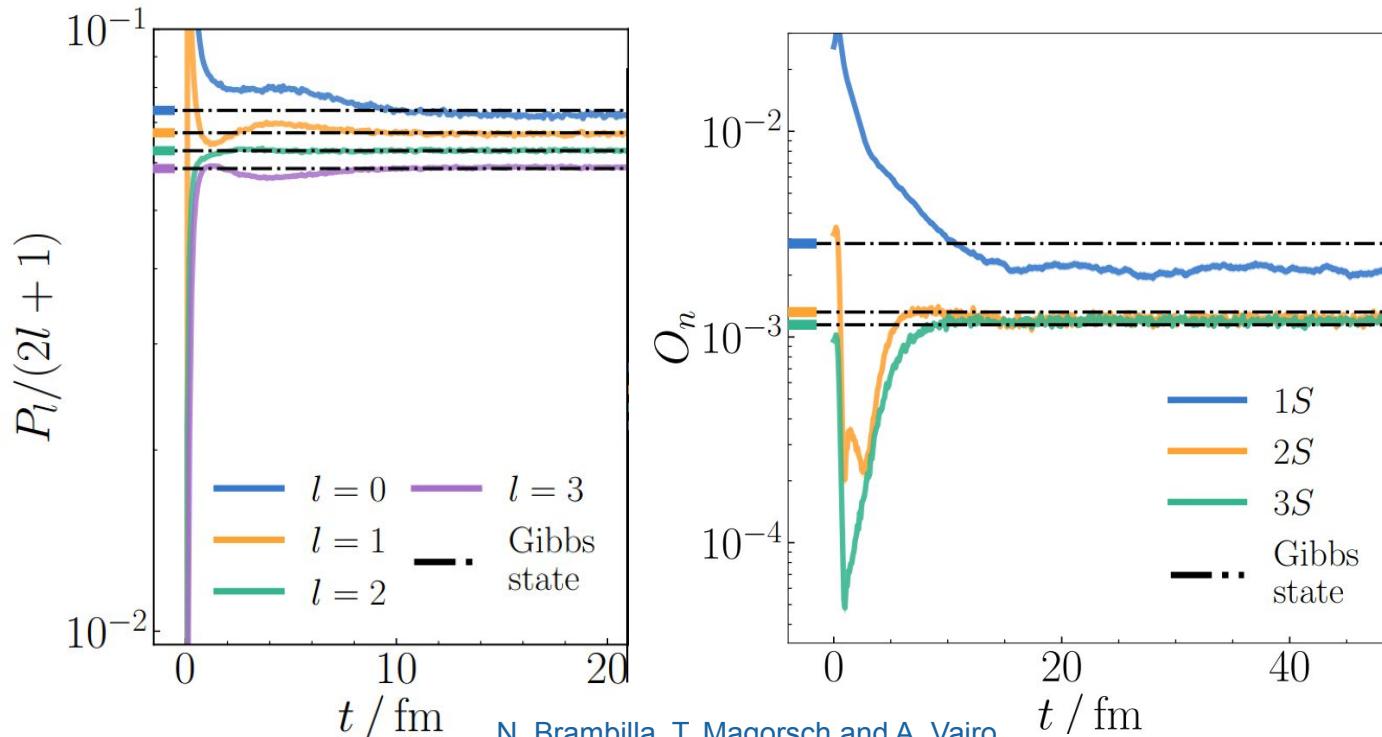
$$\tau_{\text{th}}$$

At leading order in E/T the steady state is trivial

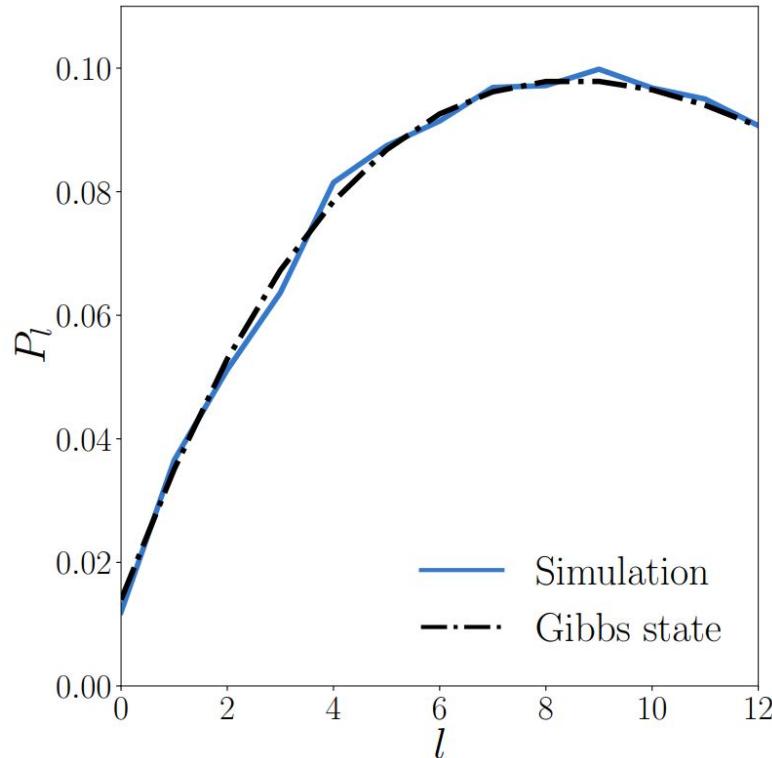
$$\rho_{\text{LO}} \rightarrow \mathbf{1} \neq e^{-H/T}$$



At next-to-leading order in E/T bottomonium approximately thermalizes within 10-20fm

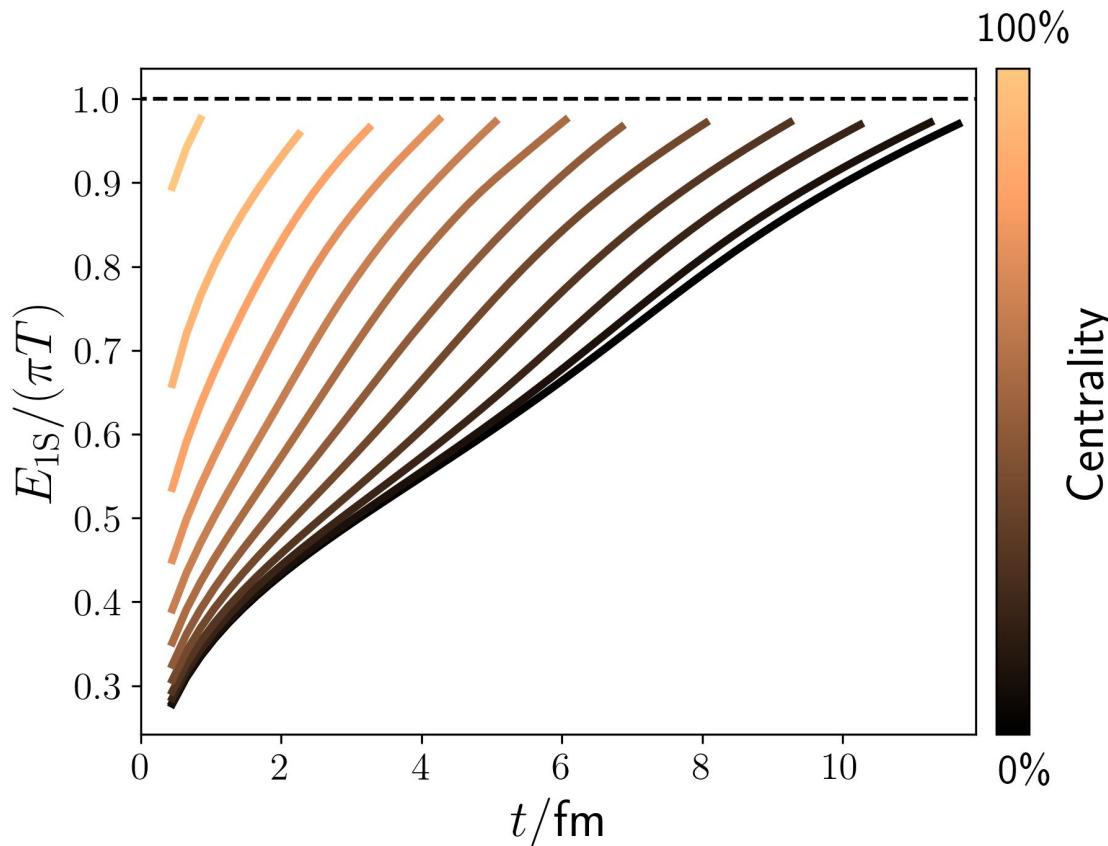


At next-to-leading order in E/T bottomonium approximately thermalizes within 10-20fm



N. Brambilla, T. Magorsch and A. Vairo
arXiv:2508.11743 [hep-ph].

At low temperatures the E/T expansion converges slowly



The original master equation is not positive

$$\frac{d}{dt}\rho(t) = -i[H, \rho(t)] + \left(L_+ \rho L_+^\dagger - \frac{1}{2} \{L_+^\dagger L_+, \rho(t)\} \right) - \left(L_- \rho L_-^\dagger - \frac{1}{2} \{L_-^\dagger L_-, \rho(t)\} \right)$$

At NLO the negative term is suppressed by E/T

$$\frac{d}{dt}\rho(t) = -i[H, \rho(t)] + \left(L_+ \rho L_+^\dagger - \frac{1}{2} \{L_+^\dagger L_+, \rho(t)\} \right) - \left(L_- \rho L_-^\dagger - \frac{1}{2} \{L_-^\dagger L_-, \rho(t)\} \right)$$

The equation is shown with two curly braces. The left brace groups the first term and the first two terms of the equation (the Lindblad part). The right brace groups the second term and the third term (the correction term). Red circles with plus and minus signs are placed around the first two terms to indicate they are retained.
“Lindblad equation” $\mathcal{O}(E^2/T^2)$

At NLO the negative term is suppressed by E/T

$$\frac{d}{dt}\rho(t) = -i[H, \rho(t)] + \underbrace{\left(L_+ \rho L_+^\dagger - \frac{1}{2} \{L_+^\dagger L_+, \rho(t)\} \right)}_{\text{“Lindblad equation”}} - \underbrace{\left(L_- \rho L_-^\dagger - \frac{1}{2} \{L_-^\dagger L_-, \rho(t)\} \right)}_{O(E^2/T^2)}$$

Contribution of jump terms:

$$||L_\sigma||^2 = \text{Tr} [L_\sigma^\dagger L_\sigma]$$

There are operator rotations, which leave the equation invariant

$$\frac{d}{dt}\rho(t) = -i[H, \rho(t)] + \left(L_+ \rho L_+^\dagger - \frac{1}{2} \{L_+^\dagger L_+, \rho(t)\} \right) - \left(L_- \rho L_-^\dagger - \frac{1}{2} \{L_-^\dagger L_-, \rho(t)\} \right)$$

$$\begin{pmatrix} L'_+ \\ L'_- \end{pmatrix} = \begin{pmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{pmatrix} \begin{pmatrix} L_+ \\ L_- \end{pmatrix}$$

There is an optimal rotation which minimizes the negative terms

$$\frac{d}{dt}\rho(t) = -i[H, \rho(t)] + \underbrace{\left(L'_+ \rho L'^\dagger_+ - \frac{1}{2} \{L'^\dagger_+ L'_+, \rho(t)\} \right)}_{\text{"Lindblad equation"}} - \underbrace{\left(L'_- \rho L'^\dagger_- - \frac{1}{2} \{L'^\dagger_-, L'_-, \rho(t)\} \right)}_{\text{minimal}}$$

“Lindblad equation”

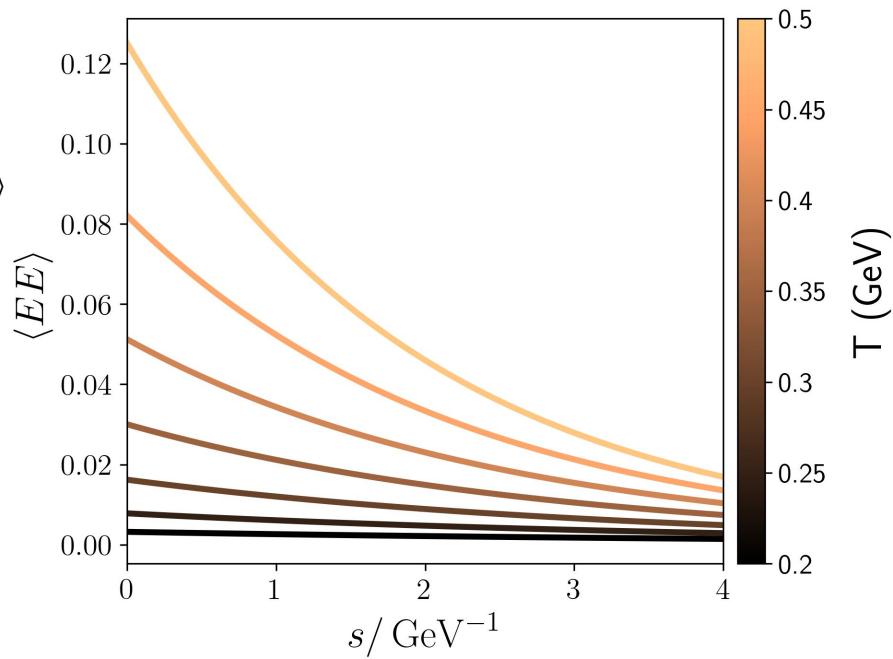
minimal

$$\operatorname{argmin}_W ||L'_-||^2 \longrightarrow L'_+, L'_-$$

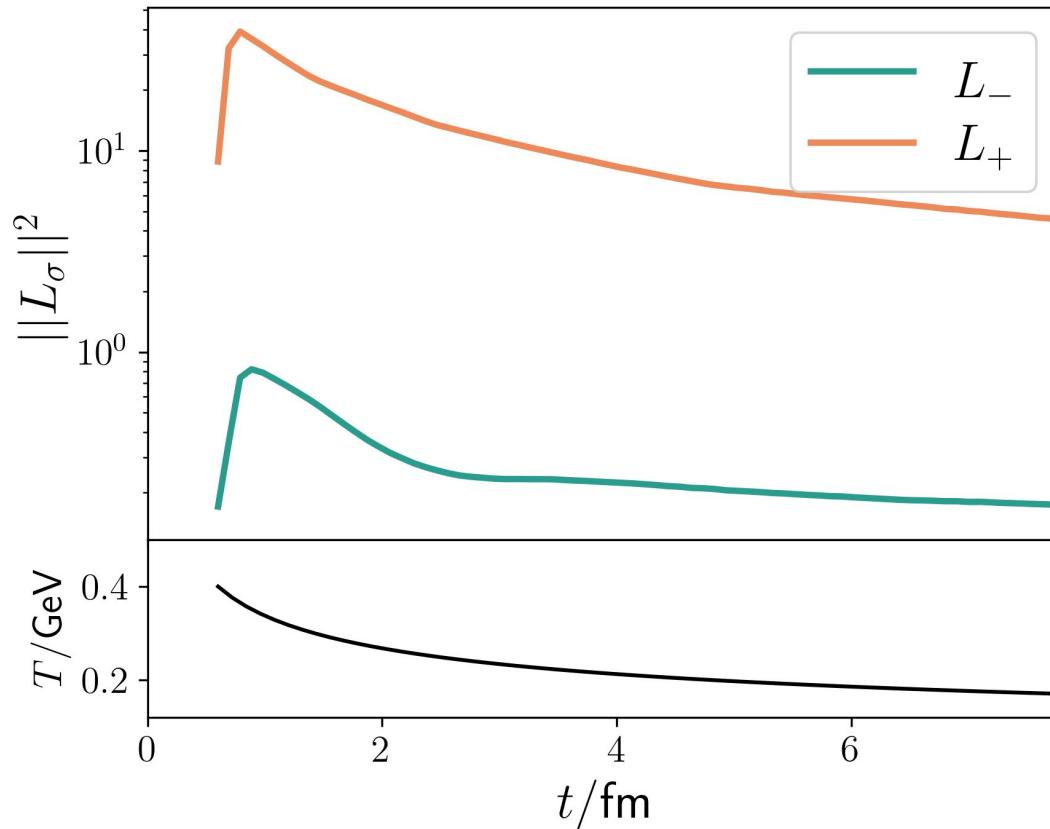
Optimization enables numerical evaluation

$$L_i \propto \int_0^\infty ds e^{-ih_u s} r_i e^{ih_v s} \langle E_j^a(s) \Omega^{ab}(s, 0) E_j^b(0) \rangle$$

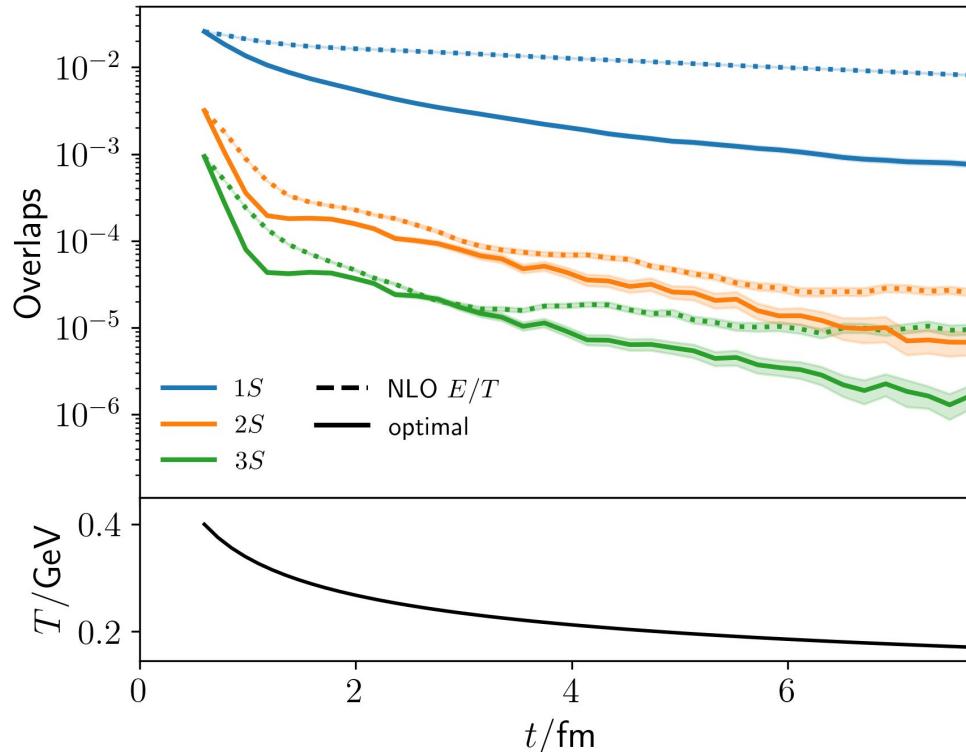
$$\langle E_j^a(s) \Omega^{ab}(s, 0) E_j^b(0) \rangle \propto \frac{\kappa T^4}{2} e^{-sT}$$



The truncation to the optimal form is efficient

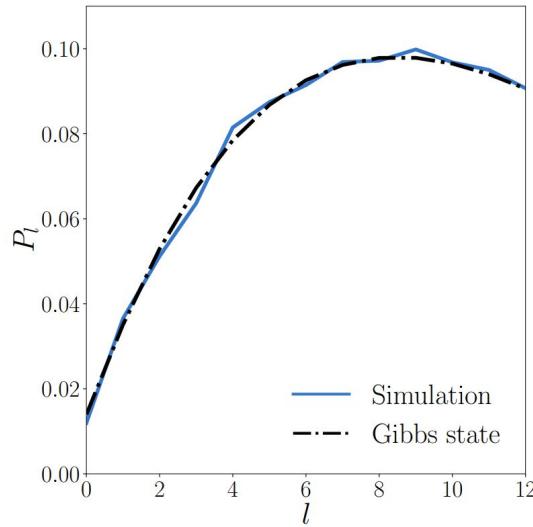


For high temperatures, we find agreement for the 2S and 3S and corrections for the 1S

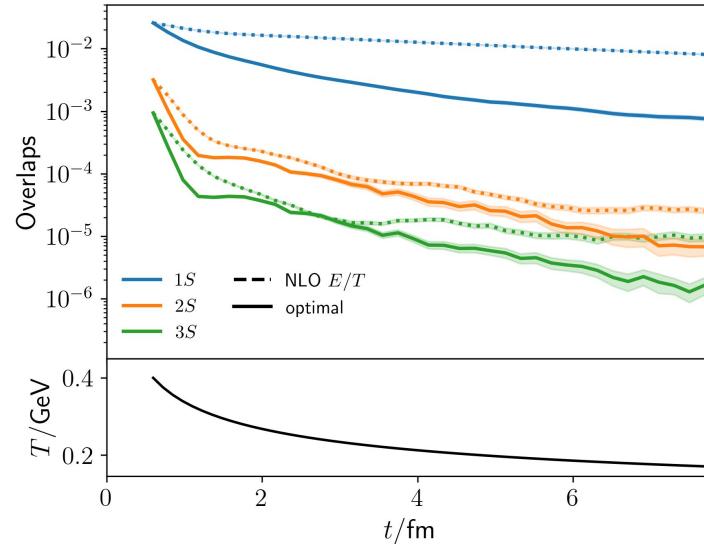


Summary

Thermalization at NLO



Lindblad beyond E/T expansion



Outlook:

- Impact of correlator shape
- Phenomenological studies low temperature regime

Backup



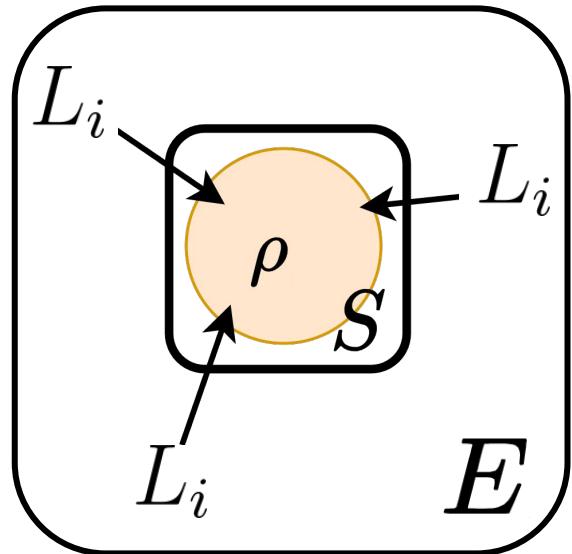
EFTs + OQS lead to a quantum master equation

$$\frac{d\rho(t)}{dt} = -i [H, \rho(t)] + \sum_{nm} h_{nm} \left(L_i^n \rho(t) L_i^{m\dagger} - \frac{1}{2} \{ L_i^{m\dagger} L_i^n, \rho(t) \} \right)$$

$$h = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad L_i^0 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} r_i, \quad L_i^1 = \begin{pmatrix} 0 & 0 \\ 0 & \frac{N_c^2-4}{2(N_c^2-1)} A_i^{oo\dagger} \end{pmatrix}$$

$$L_i^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} r_i, \quad L_i^3 = \begin{pmatrix} 0 & \frac{1}{N_c^2-1} A_i^{os\dagger} \\ A_i^{so\dagger} & 0 \end{pmatrix}$$

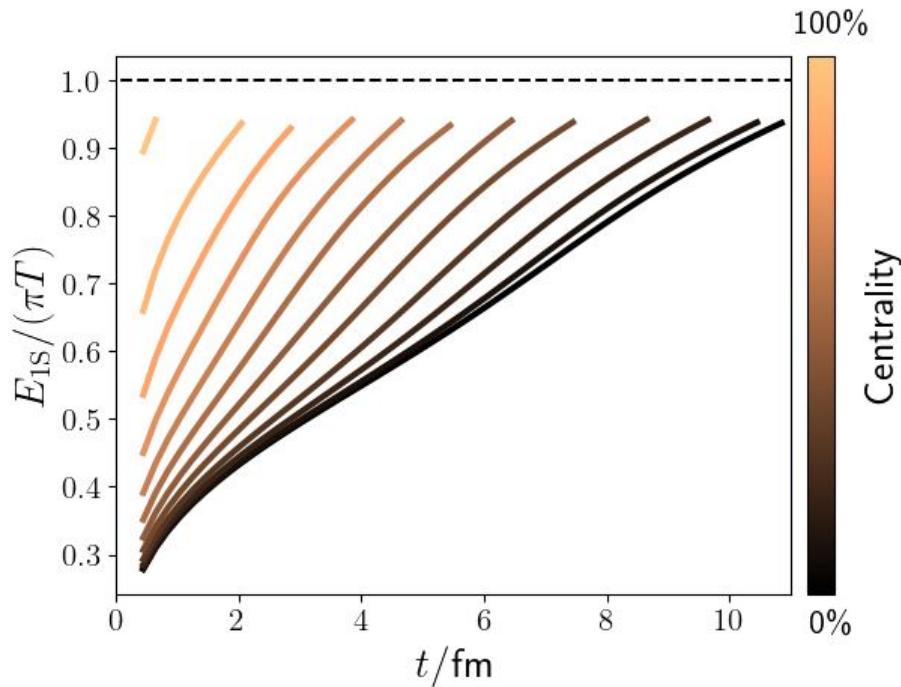
$$A_i^{uv} \propto \int_0^\infty ds e^{-ih_us} r_i e^{ih_vs} \langle E_j^a(s) \Omega^{ab}(s, 0) E_j^b(0) \rangle$$



At low temperatures the E/T expansion converges slowly

$$A_i^{uv} \propto \int_0^\infty ds e^{-ih_u s} r_i e^{ih_v s} \langle E_j^a(s) E_j^a(0) \rangle$$

$1 - ih_u s + \mathcal{O}(E^2/T^2)$ $1 - ih_v s + \mathcal{O}(E^2/T^2)$



$$\begin{aligned} A_+(w, \phi) &= e^{i\phi/2} \cosh(w)L_+ + e^{-i\phi/2} \sinh(w)L_- \\ A_-(w, \phi) &= e^{i\phi/2} \sinh(w)L_+ + e^{-\phi/2} \cosh(w)L_-. \end{aligned}$$

$$\phi = \pi - \arg \left[\text{tr}(L_+ L_-^\dagger) \right]$$

$$w = \frac{1}{2} \operatorname{arctanh} \left(\frac{2 \operatorname{tr}(L_+ L_-^\dagger)}{\operatorname{tr}(L_+ L_+^\dagger) + \operatorname{tr}(L_- L_-^\dagger)} \right)$$