

Bottomonium thermalization and master equations for quarkonium evolution

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in collaboration with

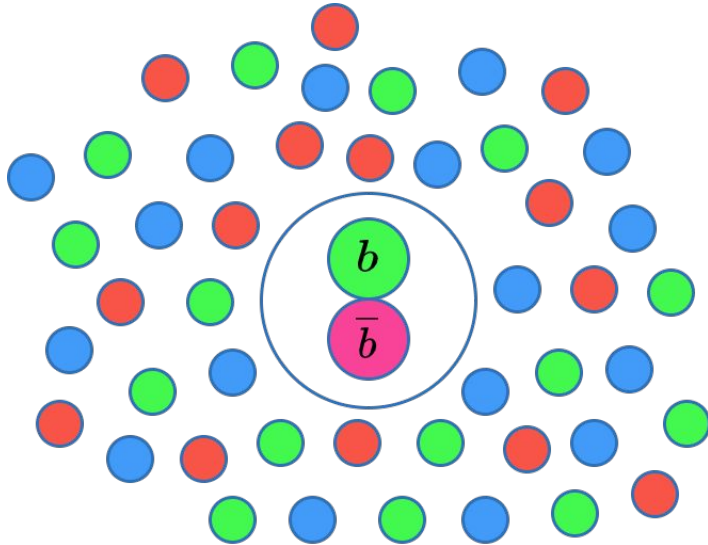
Nora Brambilla, Arthur Lin and Antonio Vairo



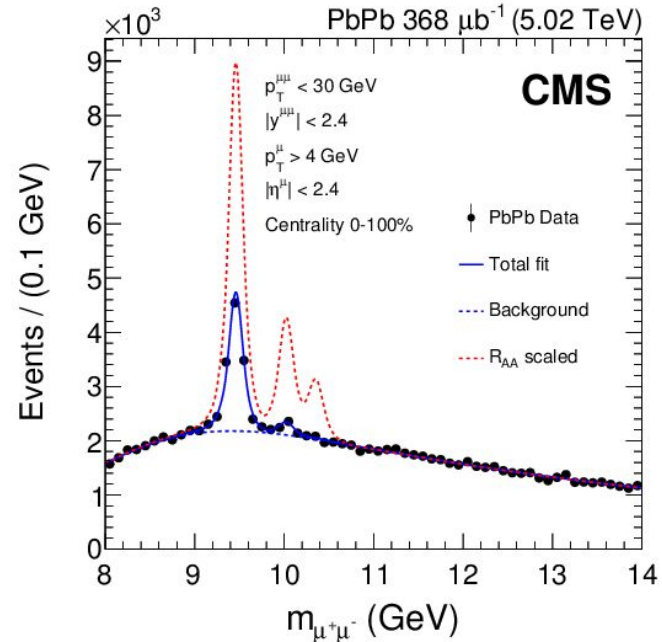
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Quarkonium is an important probe of the QGP

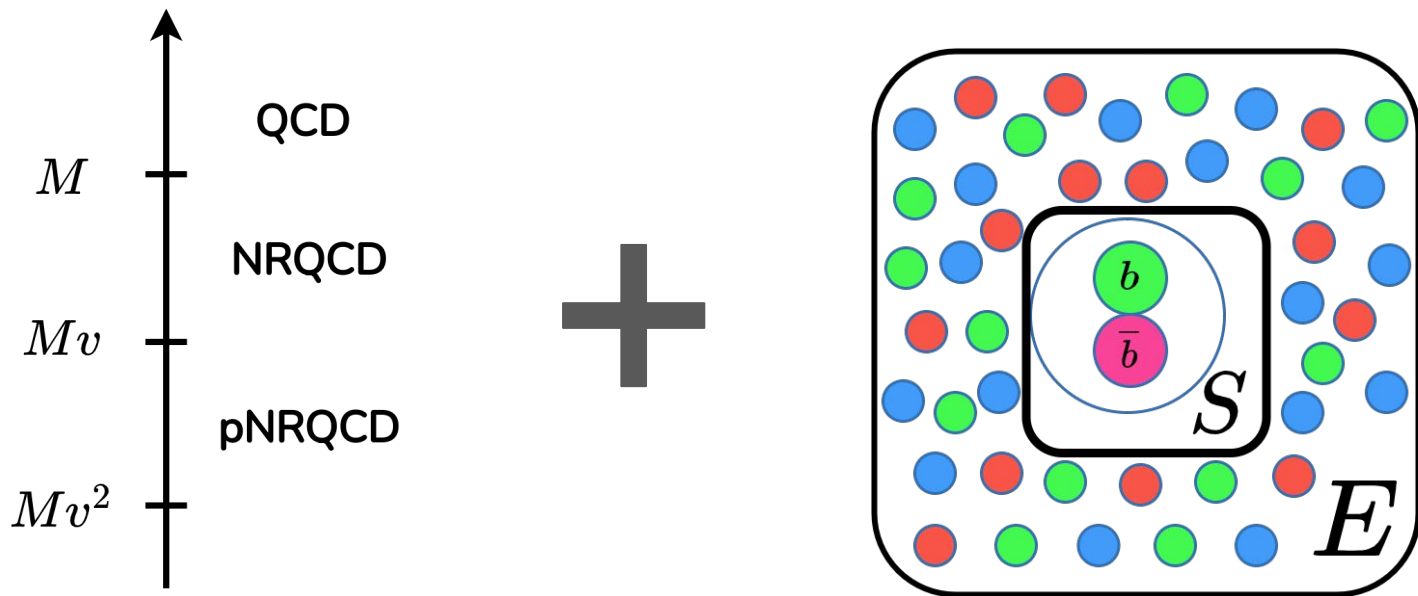
Propagation through QGP



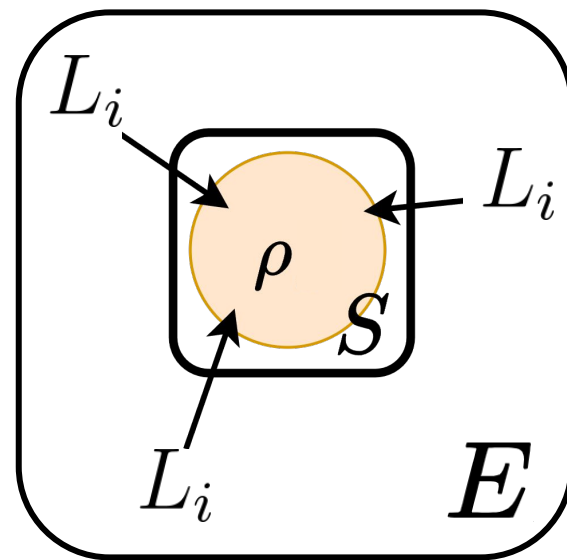
Veres, Gábor I. "Heavy ion physics at CMS and ATLAS: hard probes." *arXiv preprint arXiv:1905.10461* (2019).



The evolution can be described using Effective Field Theories + Open Quantum Systems

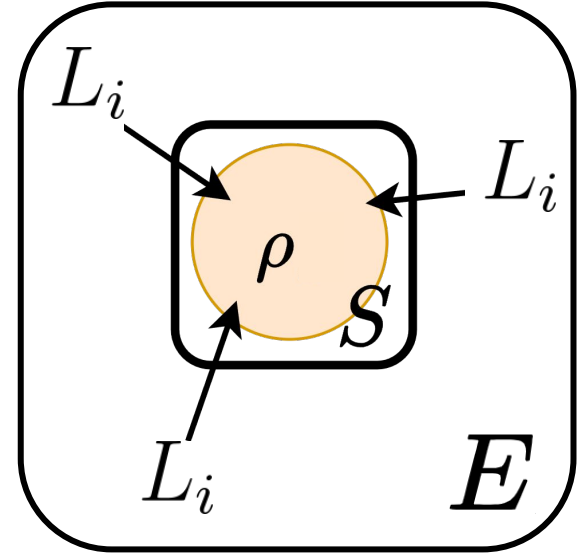


EFTs + OQS lead to a quantum master equation



EFTs + OQS lead to a quantum master equation

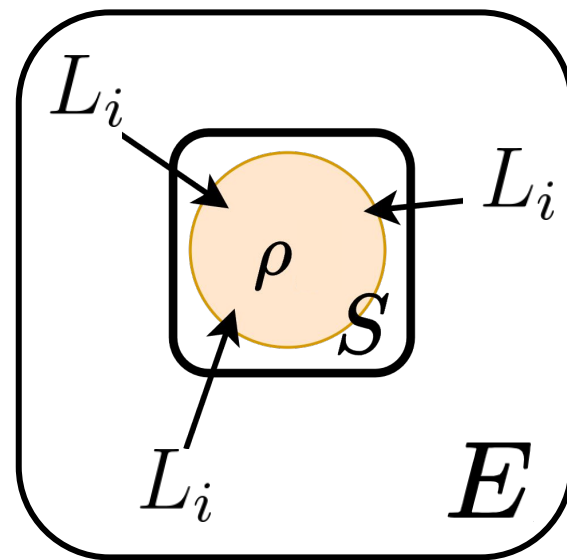
$$\frac{d\rho(t)}{dt} = -i [H, \rho(t)] + \sum_{nm} h_{nm} \left(L_i^n \rho(t) L_i^{m\dagger} - \frac{1}{2} \left\{ L_i^{m\dagger} L_i^n, \rho(t) \right\} \right)$$



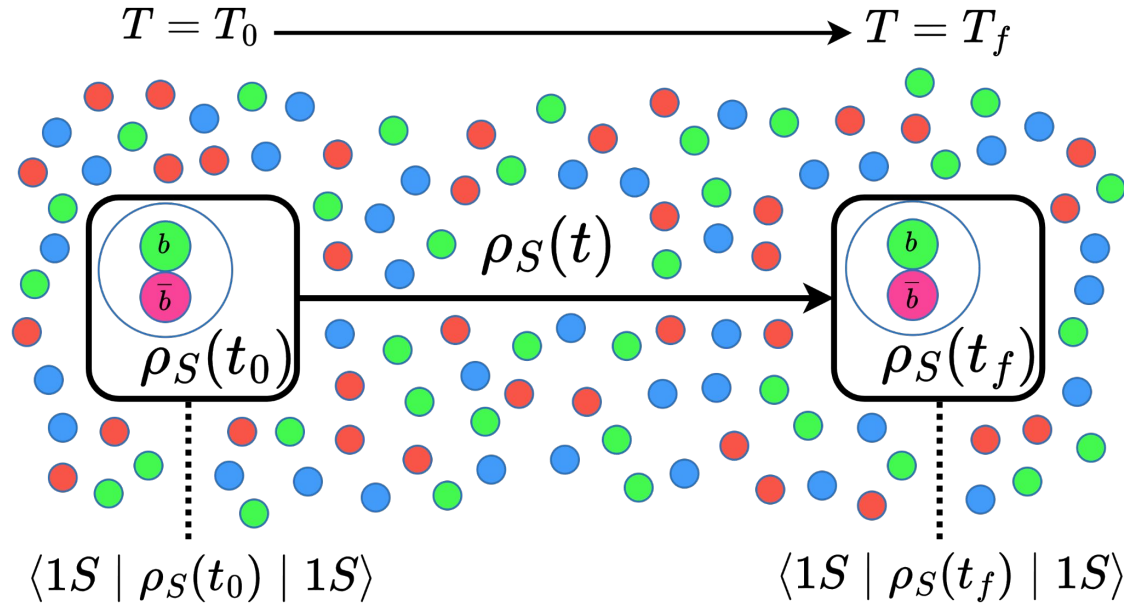
EFTs + OQS lead to a quantum master equation

$$\frac{d\rho(t)}{dt} = -i [H, \rho(t)] + \sum_{nm} h_{nm} \left(L_i^n \rho(t) L_i^{m\dagger} - \frac{1}{2} \left\{ L_i^{m\dagger} L_i^n, \rho(t) \right\} \right)$$

$$L_i \propto \int_0^\infty ds e^{-ih_us} r_i e^{ih_vs} \langle E_j^a(s) \Omega^{ab}(s, 0) E_j^b(0) \rangle$$



Solving the master equation gives the evolution of the bottomonium



$$P_{\text{survival}}(1S) = \frac{\langle 1S | \rho(t_f) | 1S \rangle}{\langle 1S | \rho(t_0) | 1S \rangle}$$

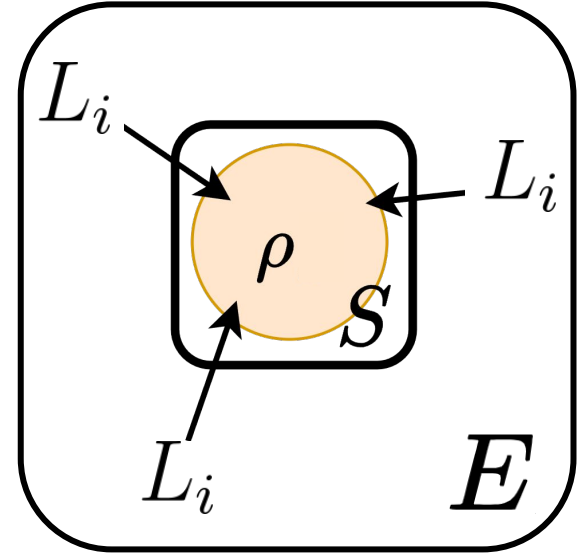
To simplify the equation one can expand in E/T

$$L_i \propto \int_0^\infty ds e^{-ih_us} r_i e^{ih_vs} \langle E_j^a(s) \Omega^{ab}(s, 0) E_j^b(0) \rangle$$

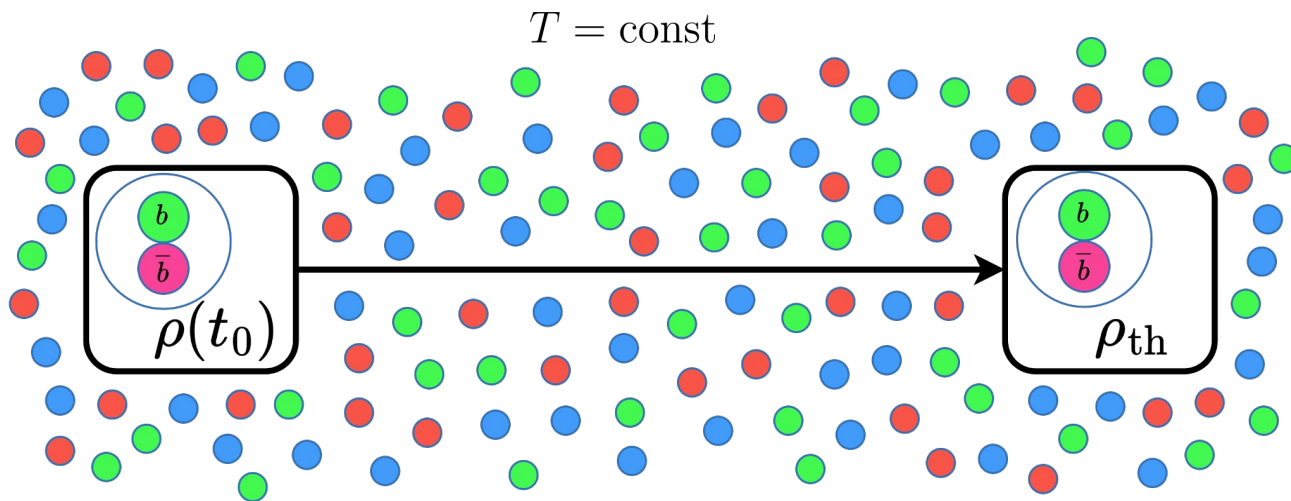
$$1 - ih_us + \mathcal{O}(E^2/T^2) \quad 1 - ih_vs + \mathcal{O}(E^2/T^2)$$

Next-to-Leading order in E/T

Leading order in E/T



We investigated the thermalization Bottomonium



thermal state?

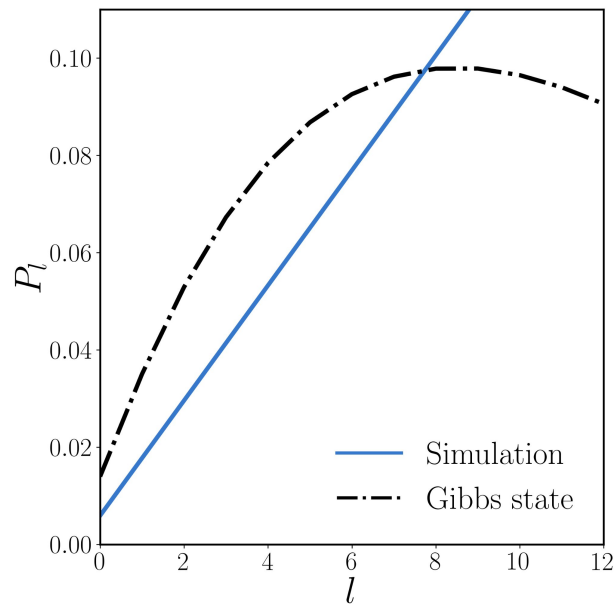
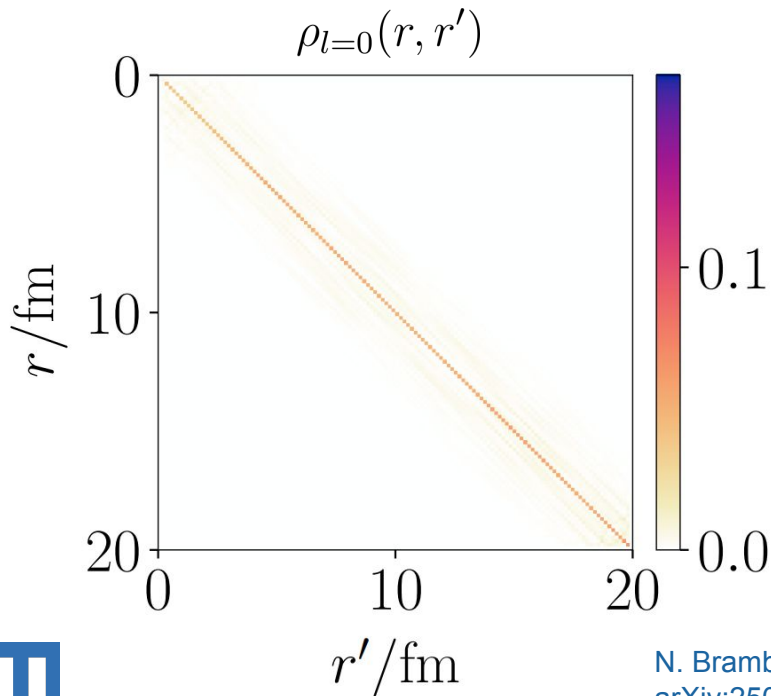
$$\rho_{\text{th}} = e^{-H/T}$$

thermalization
timescale ?

$$\tau_{\text{th}}$$

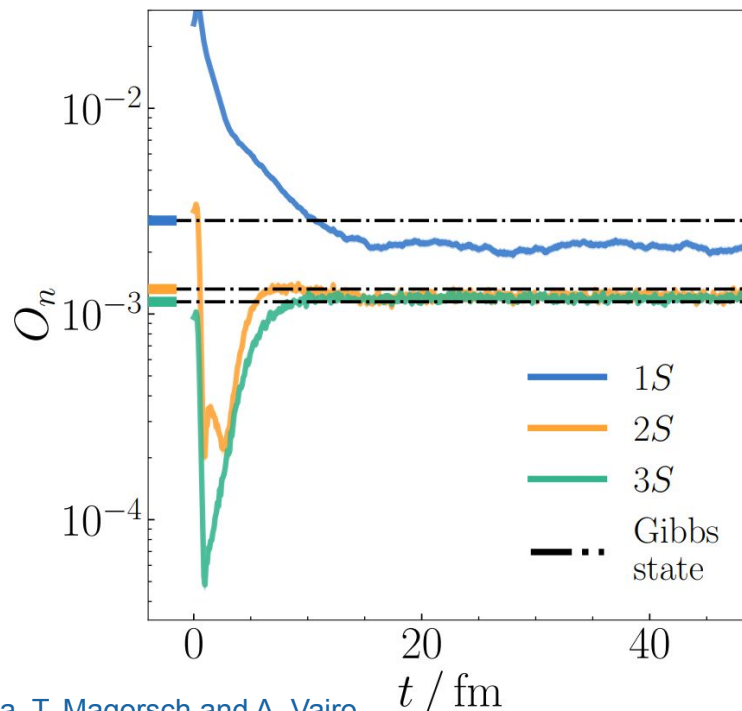
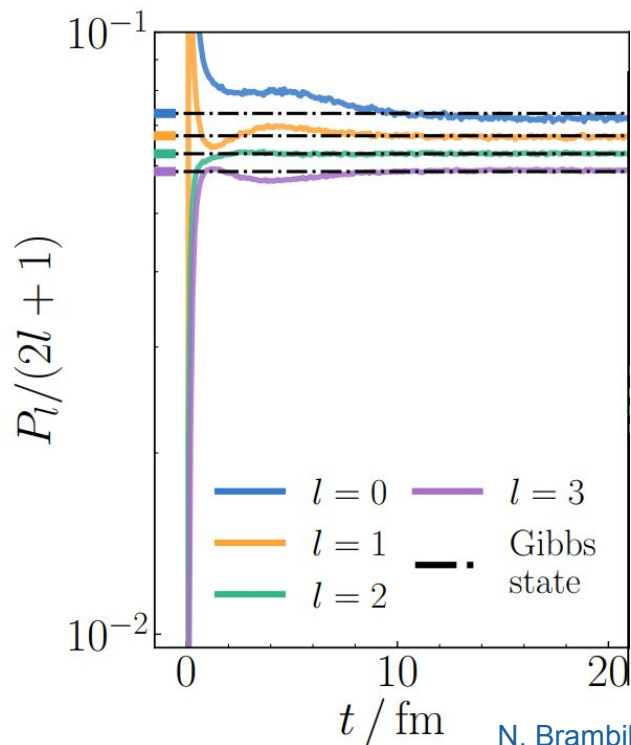
At leading order in E/T the steady state is trivial

$$\rho_{\text{LO}} \rightarrow \mathbb{1} \neq e^{-H/T}$$



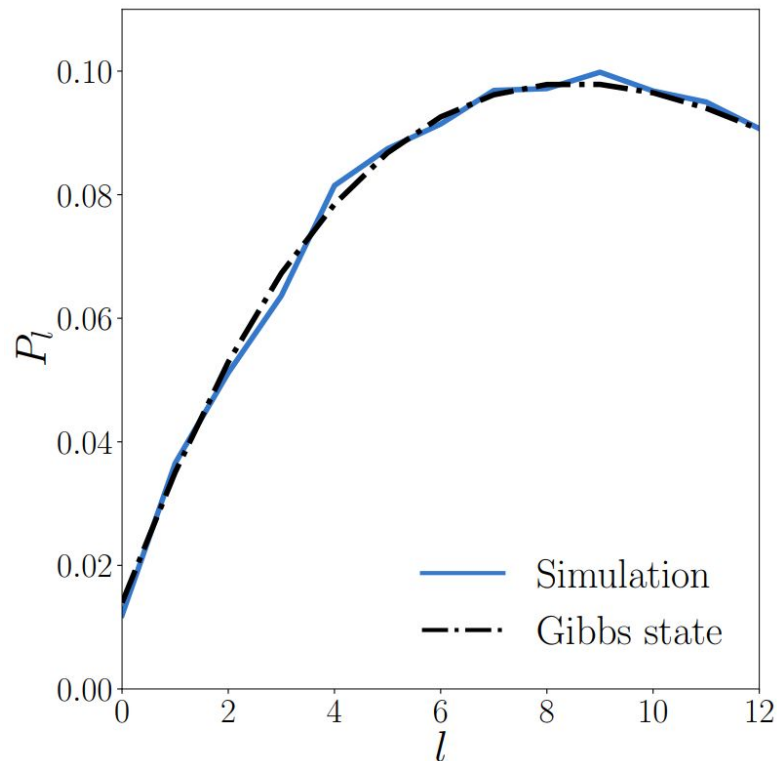
N. Brambilla, T. Magorsch and A. Vairo
[arXiv:2508.11743 \[hep-ph\]](https://arxiv.org/abs/2508.11743).

At next-to-leading order in E/T bottomonium approximately thermalizes within 10-20fm



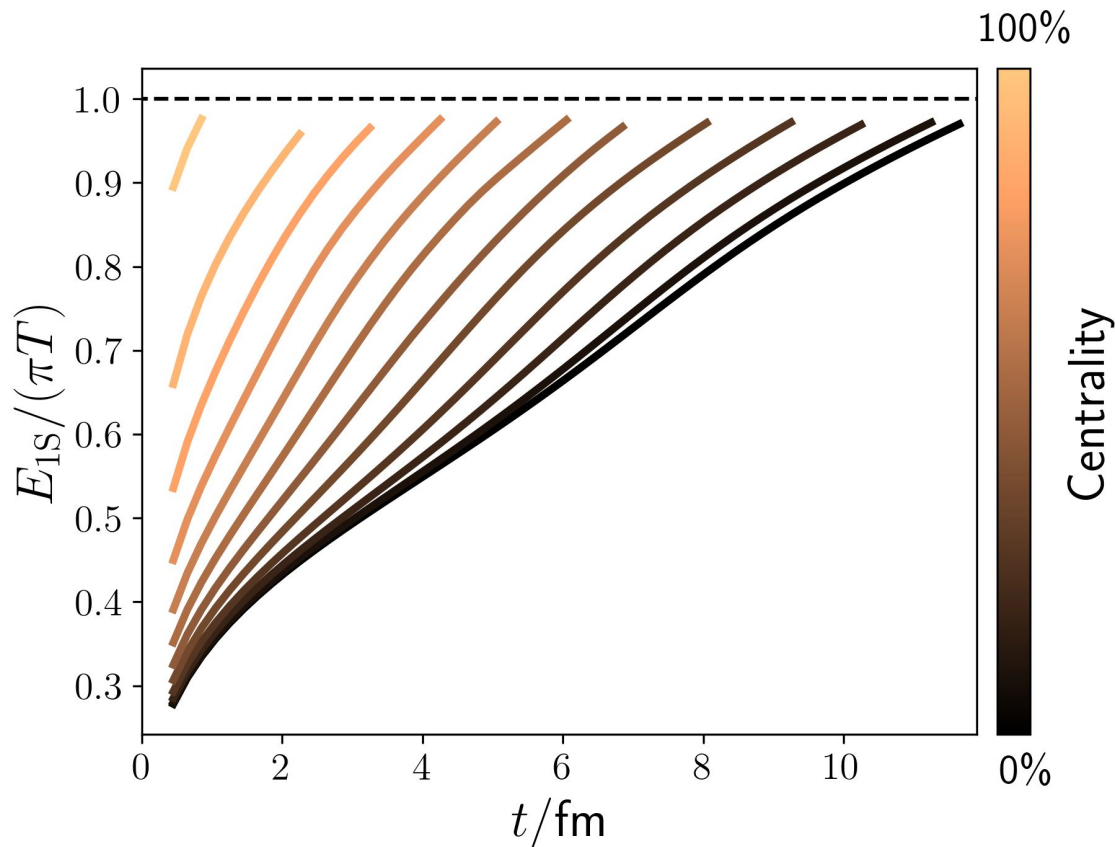
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At next-to-leading order in E/T bottomonium approximately thermalizes within 10-20fm



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At low temperatures the E/T expansion converges slowly



The original master equation is not positive

$$\frac{d}{dt}\rho(t) = -i[H, \rho(t)] \oplus \left(L_+ \rho L_+^\dagger - \frac{1}{2} \{ L_+^\dagger L_+, \rho(t) \} \right) \ominus \left(L_- \rho L_-^\dagger - \frac{1}{2} \{ L_-^\dagger L_-, \rho(t) \} \right)$$

At NLO the negative term is suppressed by E/T

$$\underbrace{\frac{d}{dt}\rho(t) = -i[H, \rho(t)] \oplus \left(L_+ \rho L_+^\dagger - \frac{1}{2} \{ L_+^\dagger L_+, \rho(t) \} \right) \ominus \left(L_- \rho L_-^\dagger - \frac{1}{2} \{ L_-^\dagger L_-, \rho(t) \} \right)}_{\text{“Lindblad equation”}} \underbrace{\quad}_{\mathcal{O}(E^2/T^2)}$$

At NLO the negative term is suppressed by E/T

$$\underbrace{\frac{d}{dt}\rho(t) = -i[H, \rho(t)] + \left(L_+ \rho L_+^\dagger - \frac{1}{2} \{ L_+^\dagger L_+, \rho(t) \} \right)}_{\text{“Lindblad equation”}} - \underbrace{\left(L_- \rho L_-^\dagger - \frac{1}{2} \{ L_-^\dagger L_-, \rho(t) \} \right)}_{\mathcal{O}(E^2/T^2)}$$

Contribution of jump terms:

$$||L_\sigma||^2 = \text{Tr} [L_\sigma^\dagger L_\sigma]$$

There are operator rotations, which leave the equation invariant

$$\frac{d}{dt}\rho(t) = -i[H, \rho(t)] + \left(L_+\rho L_+^\dagger - \frac{1}{2}\{L_+^\dagger L_+, \rho(t)\} \right) - \left(L_-\rho L_-^\dagger - \frac{1}{2}\{L_-^\dagger L_-, \rho(t)\} \right)$$

$$\begin{pmatrix} L'_+ \\ L'_- \end{pmatrix} = \begin{pmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{pmatrix} \begin{pmatrix} L_+ \\ L_- \end{pmatrix}$$

There is an optimal rotation which minimizes the negative terms

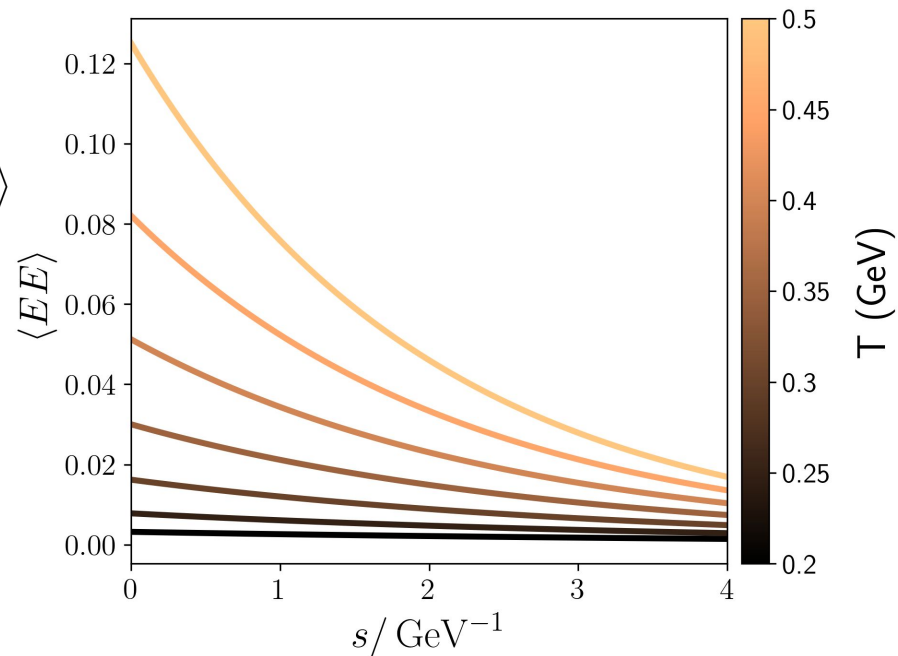
$$\underbrace{\frac{d}{dt}\rho(t) = -i[H, \rho(t)] + \left(L'_+ \rho L'^{\dagger}_+ - \frac{1}{2}\{L'^{\dagger}_+ L'_+, \rho(t)\} \right)}_{\text{"Lindblad equation"}} - \underbrace{\left(L'_- \rho L'^{\dagger}_- - \frac{1}{2}\{L'^{\dagger}_- L'_-, \rho(t)\} \right)}_{\text{minimal}}$$

$$\operatorname{argmin}_W ||L'_-||^2 \longrightarrow L'_+, L'_-$$

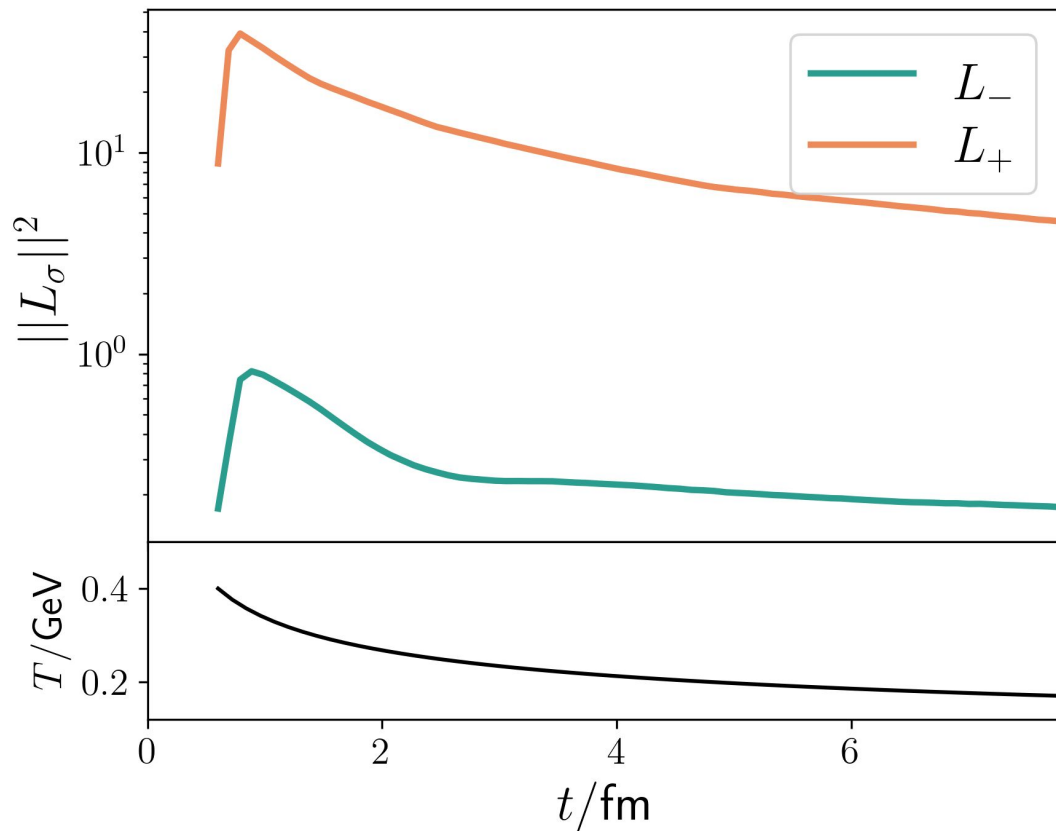
Optimization enables numerical evaluation

$$L_i \propto \int_0^\infty ds e^{-ih_us} r_i e^{ih_vs} \langle E_j^a(s) \Omega^{ab}(s, 0) E_j^b(0) \rangle$$

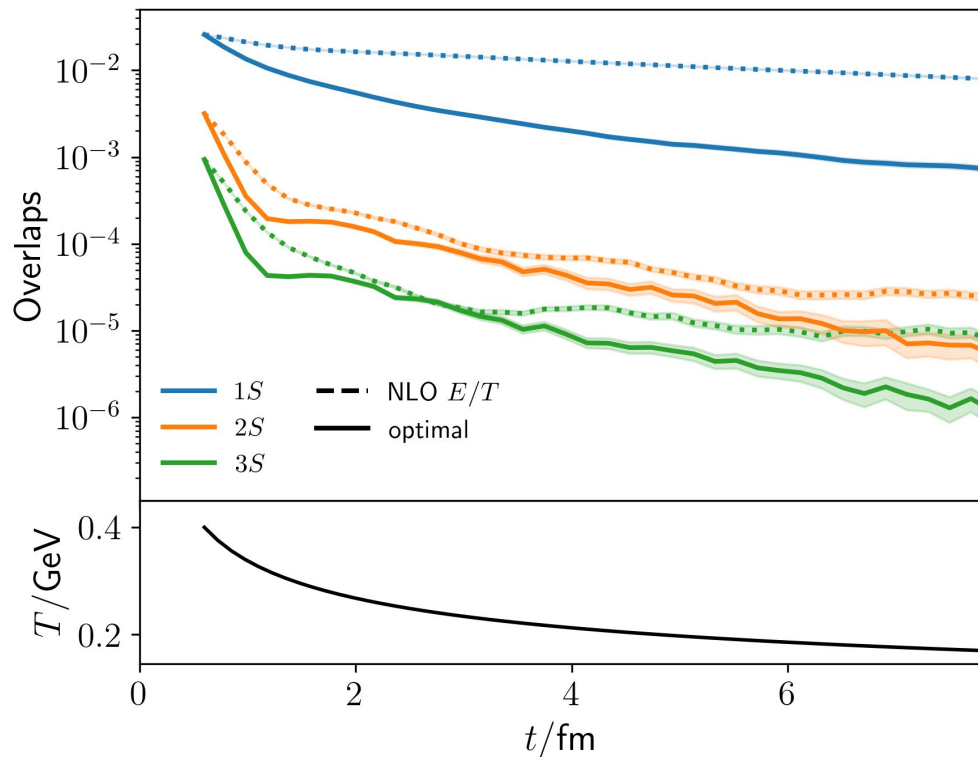
$$\langle E_j^a(s) \Omega^{ab}(s, 0) E_j^b(0) \rangle \propto \frac{\kappa T^4}{2} e^{-sT}$$



The truncation to the optimal form is efficient

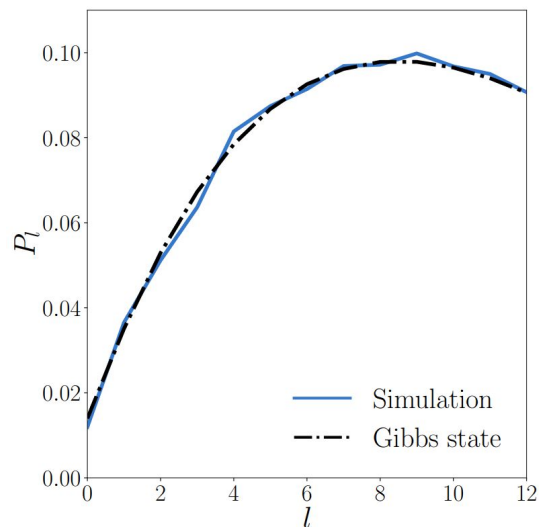


For high temperatures, we find agreement for the 2S and 3S and corrections for the 1S

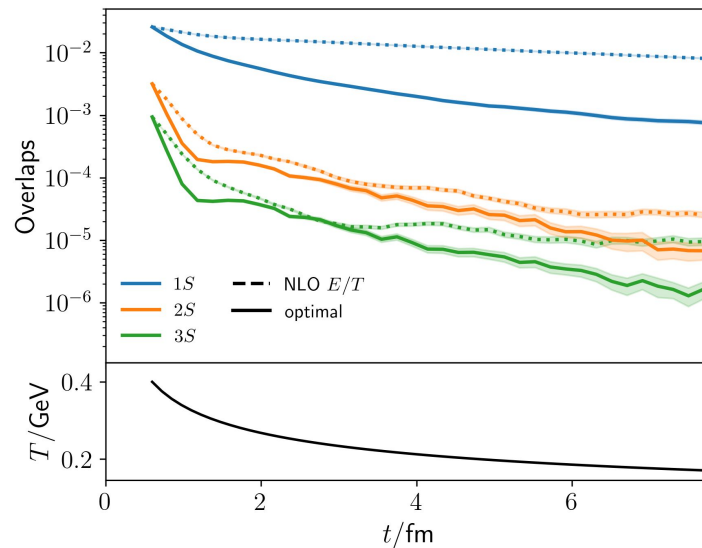


Summary

Thermalization at NLO



Lindblad beyond E/T expansion



- Outlook:
- Impact of correlator shape
 - Phenomenological studies low temperature regime

Backup



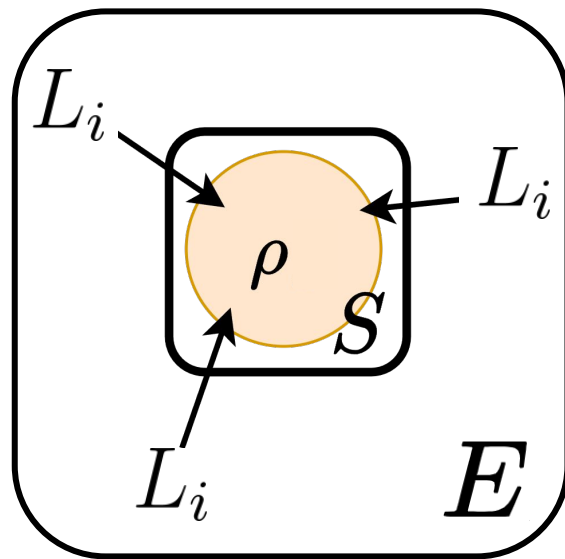
EFTs + OQS lead to a quantum master equation

$$\frac{d\rho(t)}{dt} = -i [H, \rho(t)] + \sum_{nm} h_{nm} \left(L_i^n \rho(t) L_i^{m\dagger} - \frac{1}{2} \left\{ L_i^{m\dagger} L_i^n, \rho(t) \right\} \right)$$

$$h = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad L_i^0 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} r_i, \quad L_i^1 = \begin{pmatrix} 0 & 0 \\ 0 & \frac{N_c^2 - 4}{2(N_c^2 - 1)} A_i^{oo\dagger} \end{pmatrix}$$

$$L_i^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} r_i, \quad L_i^3 = \begin{pmatrix} 0 & \frac{1}{N_c^2 - 1} A_i^{os\dagger} \\ A_i^{so\dagger} & 0 \end{pmatrix}$$

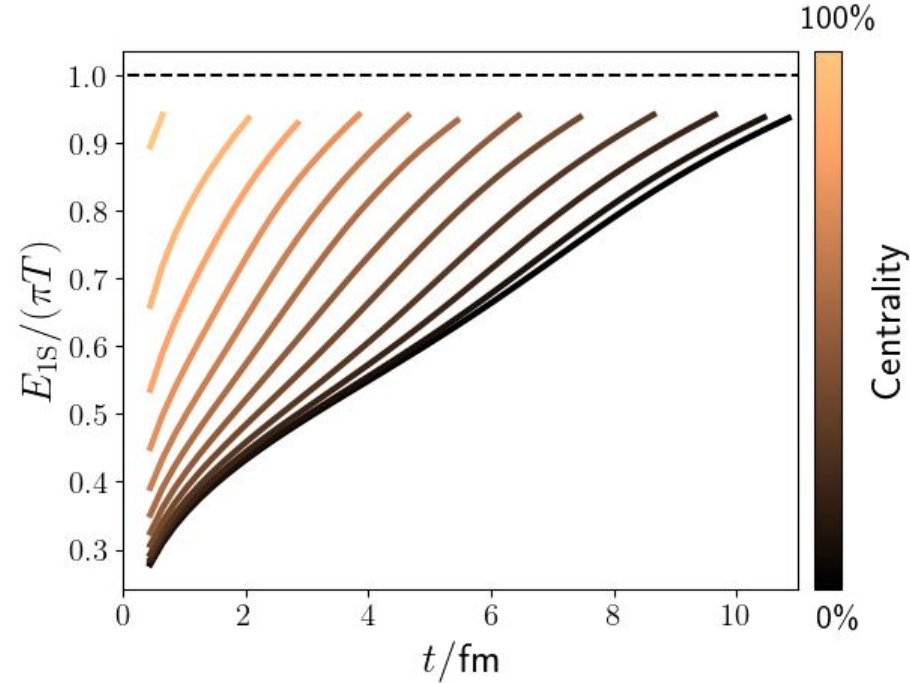
$$A_i^{uv} \propto \int_0^\infty ds e^{-ih_u s} r_i e^{ih_v s} \langle E_j^a(s) \Omega^{ab}(s, 0) E_j^b(0) \rangle$$



At low temperatures the E/T expansion converges slowly

$$A_i^{uv} \propto \int_0^\infty ds e^{-ih_us} r_i e^{ih_vs} \langle E_j^a(s) E_j^a(0) \rangle$$

$$1 - ih_us + \mathcal{O}(E^2/T^2) \quad 1 - ih_vs + \mathcal{O}(E^2/T^2)$$



$$A_+(w, \phi) = e^{i\phi/2} \cosh(w) L_+ + e^{-i\phi/2} \sinh(w) L_-$$

$$A_-(w, \phi) = e^{i\phi/2} \sinh(w) L_+ + e^{-i\phi/2} \cosh(w) L_-.$$

$$\phi = \pi - \arg \left[\text{tr}(L_+ L_-^\dagger) \right]$$

$$w = \frac{1}{2} \text{arctanh} \left(\frac{2 \text{tr}(L_+ L_-^\dagger)}{\text{tr}(L_+ L_+^\dagger) + \text{tr}(L_- L_-^\dagger)} \right)$$