Bottomonium suppression from the 3-loop QCD potential

Tom Magorsch

Phys. Rev. D **109**, 114016

in collaboration with Nora Brambilla, Michael Strickland,
Antonio Vairo and Peter Vander Griend

SEW 2024

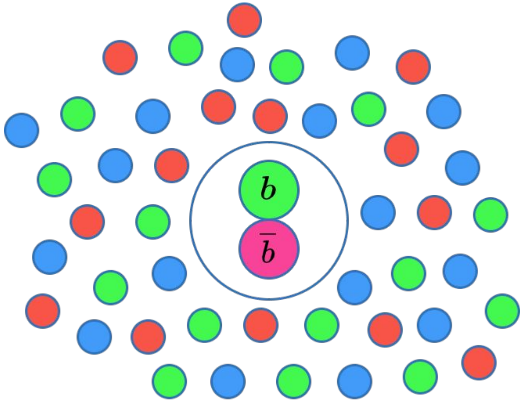
28.08.2024

Quarkonium suppression

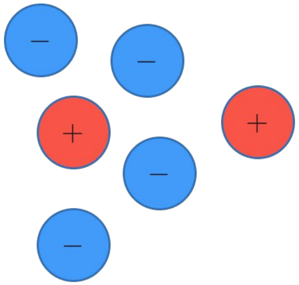
Veres, Gábor I. "Heavy ion physics at CMS and ATLAS: hard probes." *arXiv preprint arXiv:1905.10461* (2019).

Propagation through QGP

$T \approx O(100\mathrm{MeV})$



T. Matsui, H. Satz, Phys. Lett. B 178 (1986) 416

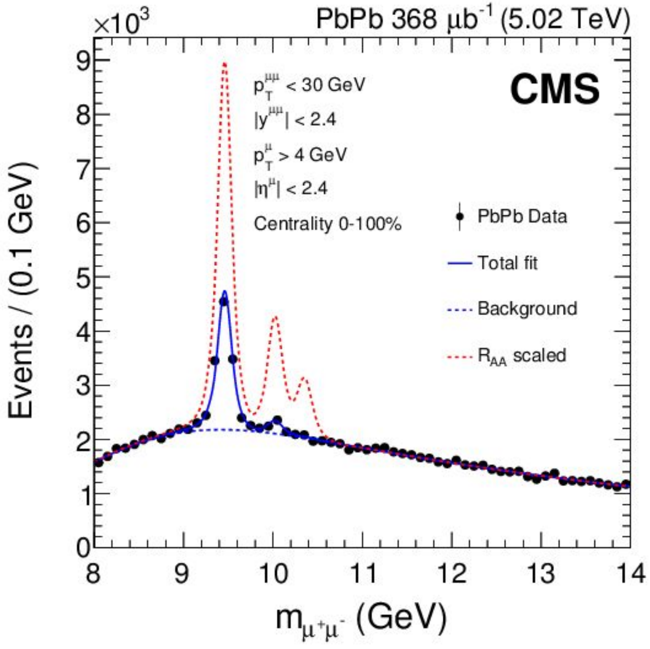


$$V(r) = -\frac{\alpha}{r}$$

$$\downarrow$$

$$V(r) = -\frac{\alpha}{r}\mathrm{e}^{-m_D r}$$

Debye-screening in medium

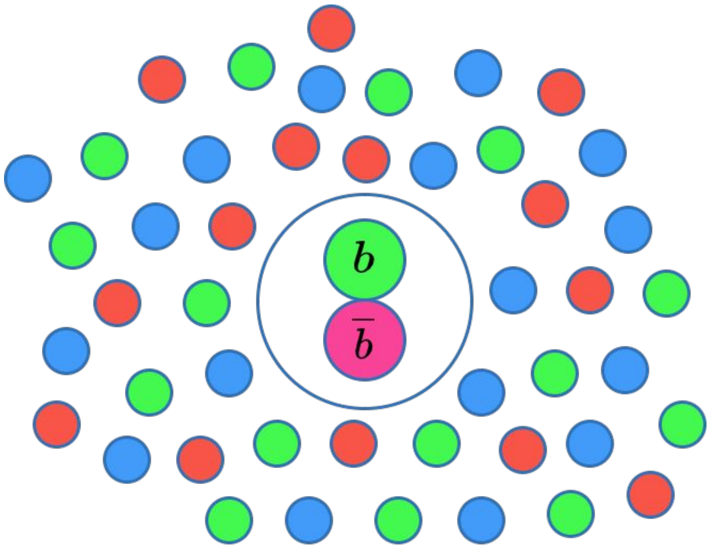


Quarkonium suppression from first principles

- We aim to describe this phenomenon from **first principles**
- Provide predictions for experiments
- We focus on **bottomonium** since the high mass allows for simplifications

Propagation through QGP

$T \approx O(100\mathrm{MeV})$



Open Quantum Systems

- Quantum system not isolated
- Split into System S and Environment E

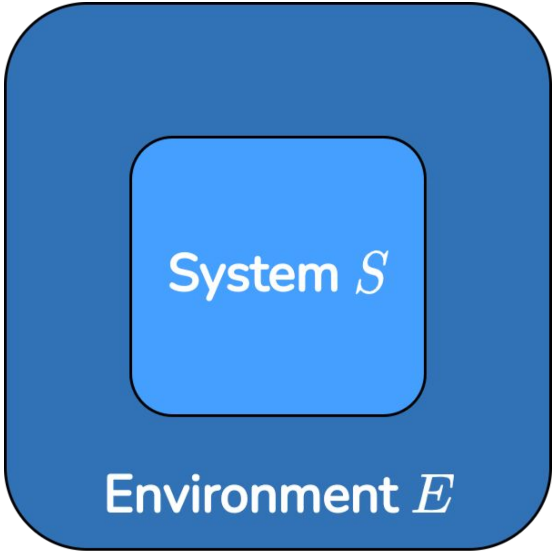
$$H = H_S \otimes I_E + I_S \otimes H_E + H_{\text{int}}$$

- Time evolution by Von-Neumann Equation

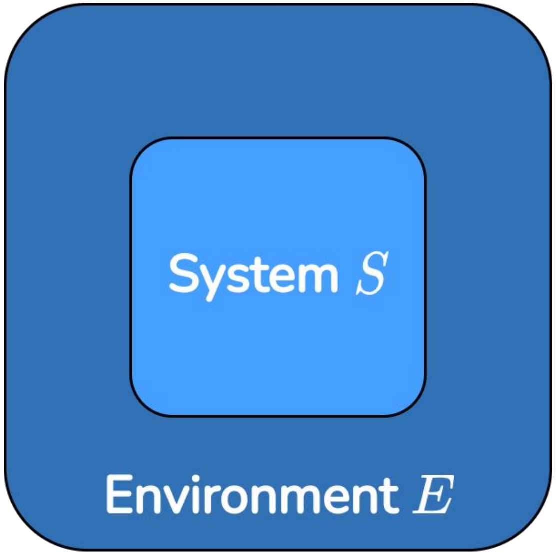
$$\frac{d}{dt}\rho = -i[H, \rho]$$

- Not interested in environmental d.o.f.: **Trace out**!

$$\rho_S = \mathrm{Tr}_E[\rho]$$



Open Quantum Systems



- Time evolution by Von-Neumann Equation

$$\frac{d}{dt}\rho = -i[H, \rho]$$

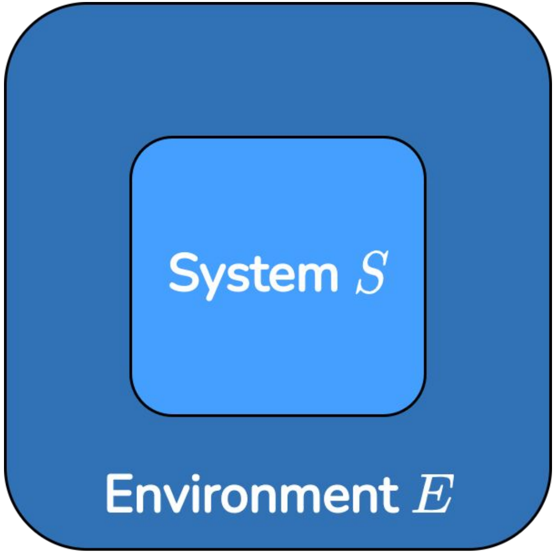
- Environmental d.o.f. not needed Trace out!

$$\rho_S = \mathrm{Tr}_E[\rho]$$

- “Master equation” for the System: **Lindblad Equation**

$$\frac{d\rho_S}{dt} = -i[H_S, \rho_S] + \sum_n \left(C_n \rho_S C_n^\dagger - \frac{1}{2} \{ C_n^\dagger C_n, \rho_S \} \right)$$

Open Quantum Systems



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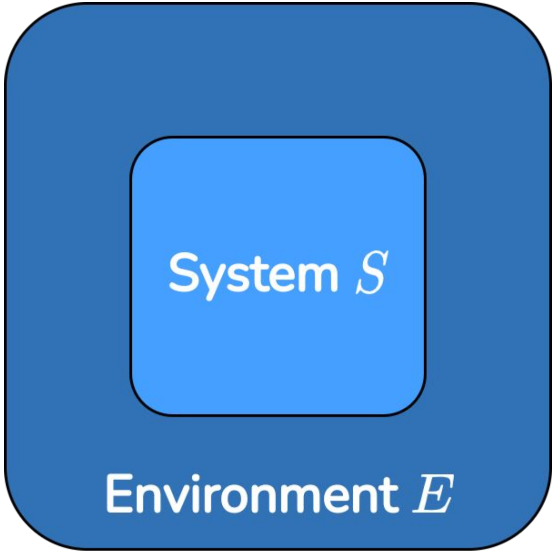
Open Quantum Systems



- Time evolution by Von-Neumann Equation

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- Environmental d.o.f. not needed Trace out!

$$\rho_S = \mathrm{Tr}_E[\rho]$$

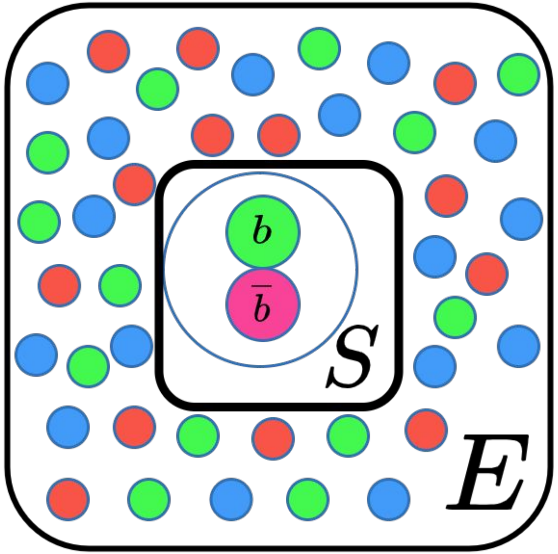
- “Master equation” for the System: **Lindblad Equation**

non-unitary

$$\frac{d\rho_S}{dt} = -i[H_S, \rho_S] + \sum_n \left(C_n \rho_S C_n^\dagger - \frac{1}{2}\left\{C_n^\dagger C_n, \rho_S\right\}\right)$$

OQS for quarkonium

- Quarkonium: System S
- QGP: Environment E



Aim to describe Quarkonium Suppression by a master equation for encoding the interaction with the QGP

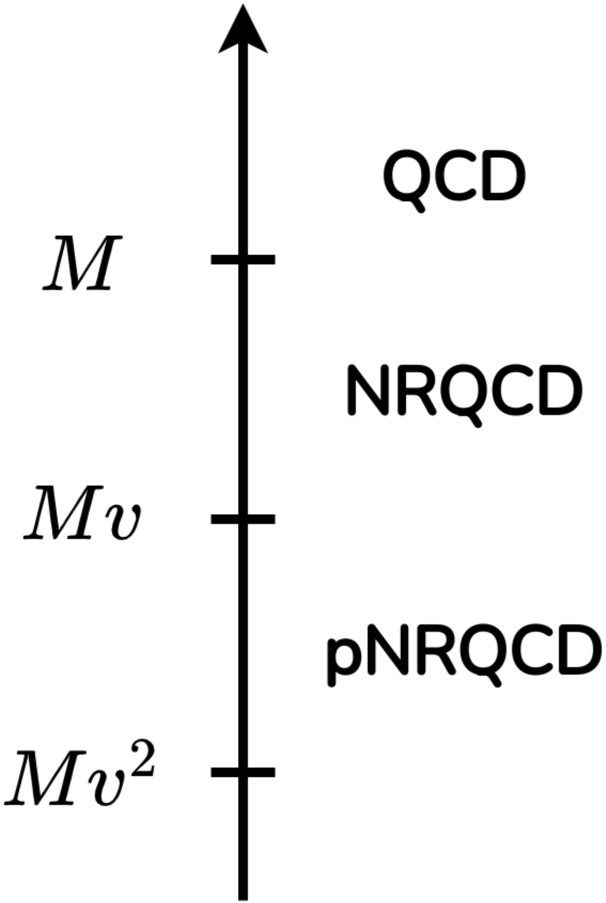
$$\frac{d\rho_S}{dt} = -i[H_S, \rho_S] + \sum_n \left(C_n \rho_S C_n^\dagger - \frac{1}{2}\left\{C_n^\dagger C_n, \rho_S\right\} \right)$$

pNRQCD

N. Brambilla, A. Pineda, J. Soto, and A. Vairo, Nuclear Physics B 566, 275 (2000)

- We use pNRQCD, an EFT from full QCD
- pNRQCD is obtained by integrating out the hard scale M and soft scale Mv
- Degrees of freedom: Singlet and octet bound states
- Using pNRQCD one can derive a master equation for the quarkonium density matrix

Brambilla, Escobedo, Soto, Vairo: Phys. Rev. D 97 (2018) 7, 074009



$v \ll 1$: Relative Quark-Antiquark velocity

pNRQCD master equation

Brambilla, Escobedo, Soto, Vairo: Phys. Rev. D 97 (2018) 7, 074009

$$\frac{d\rho(t)}{dt} = -i\left[H, \rho(t)\right] + \sum_{nm} h_{nm}\left(L_i^n \rho(t) L_i^{m\dagger} - \frac{1}{2}\left\{L_i^{m\dagger} L_i^n, \rho(t)\right\}\right)$$

$$h = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$L_i^0 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} r_i, \qquad L_i^1 = \begin{pmatrix} 0 & 0 \\ 0 & \frac{N_c^2-4}{2(N_c^2-1)} A_i^{oo\dagger} \end{pmatrix}$$

$$L_i^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} r_i, \qquad L_i^3 = \begin{pmatrix} 0 & \frac{1}{N_c^2-1} A_i^{os\dagger} \\ A_i^{so\dagger} & 0 \end{pmatrix}$$

pNRQCD master equation

Brambilla, Escobedo, Soto, Vairo: Phys. Rev. D 97 (2018) 7, 074009

$$\frac{d\rho(t)}{dt} = -i\left[H, \rho(t)\right] + \sum_{nm} h_{nm}\left(L_i^n \rho(t) L_i^{m\dagger} - \frac{1}{2}\left\{L_i^{m\dagger} L_i^n, \rho(t)\right\}\right)$$

- In general h_{nm} not completely positive:

Master equation *not necessarily* of Lindblad type

$$L_i^0 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} r_i, \quad L_i^1 = \begin{pmatrix} 0 & 0 \\ 0 & \frac{N_c^2-4}{2(N_c^2-1)} A_i^{oo\dagger} \end{pmatrix}$$

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pNRQCD master equation

Brambilla, Escobedo, Soto, Vairo: Phys. Rev. D 97 (2018) 7, 074009

- Simplify using hierarchy of scales $\pi T \gg E$

$$A_i^{uv} = \frac{g^2}{6N_c} \int_0^\infty \mathrm{d}s e^{-ih_u s} r_i e^{ih_v s} \langle E_j^a(s) \Omega(s,0)^{ab} E_j^b(0) \rangle$$

- Expand exponentials in $E/(\pi T)$
- At **LO** in $E/(\pi T)$ we get

$$A_i^{uv} = \frac{g^2}{6N_c} \int_0^\infty \mathrm{d}s r_i \langle E_j^a(s) \Omega(s,0)^{ab} E_j^b(0) \rangle$$
$$= \frac{r_i}{2} (\kappa - i\gamma)$$

Transport coefficients

pNRQCD master equation

Brambilla, Escobedo, Soto, Vairo: Phys. Rev. D 97 (2018) 7, 074009

$$A_i^{uv} = \frac{g^2}{6N_c}\int_0^\infty \mathrm{d}s r_i \langle E_j^a(s)\Omega(s,0)^{ab}E_j^b(0)\rangle$$
$$= \frac{r_i}{2}(\kappa - i\gamma)$$

Transport coefficients

$$L_i^0 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} r_i, \qquad L_i^1 \propto \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} r_i,$$

$$L_i^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} r_i, \qquad L_i^3 \propto \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} r_i,$$

Linearly dependent: Lindblad possible!

pNRQCD master equation

$$\frac{d\rho(t)}{dt} = -i[H, \rho(t)] + \sum_n \left[C_i^n \rho(t) C_i^{n\dagger} - \frac{1}{2} \left\{ C_i^{n\dagger} C_i^n, \rho(t) \right\} \right],$$

$$H = \begin{pmatrix} h_s + \frac{r^2}{2}\gamma & 0 \\ 0 & h_o + \frac{N_c^2-2}{2(N_c^2-1)}\frac{r^2}{2}\gamma \end{pmatrix}$$

$$h_{s,o} = \vec{p}^2/M + V_{s,o}$$

pNRQCD master equation

$$\frac{d\rho(t)}{dt} = -i[H,\rho(t)] + \sum_n \left[C_i^n \rho(t) C_i^{n\dagger} - \frac{1}{2}\left\{ C_i^{n\dagger} C_i^n, \rho(t) \right\} \right],$$

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Quarkonium Potential

pNRQCD master equation

$$\frac{d\rho(t)}{dt} = -i[H,\rho(t)] + \sum_n \left[C_i^n \rho(t) C_i^{n\dagger} - \frac{1}{2}\left\{ C_i^{n\dagger} C_i^n, \rho(t) \right\} \right],$$

$$H = \begin{pmatrix} h_s + \frac{r^2}{2}\gamma & 0 \\ 0 & h_o + \frac{N_c^2-2}{2(N_c^2-1)}\frac{r^2}{2}\gamma \end{pmatrix}$$

$$C_i^0 = \sqrt{\frac{\kappa}{N_c^2-1}}\, r_i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2-1} & 0 \end{pmatrix},$$

$$C_i^1 = \sqrt{\frac{\kappa(N_c^2-4)}{2(N_c^2-1)}}\, r_i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$h_{s,o} = \vec{p}^{\,2}/M + V_{s,o}$$

pNRQCD master equation

$$\frac{d\rho(t)}{dt} = -i[H, \rho(t)] + \sum_n \left[C_i^n \rho(t) C_i^{n\dagger} - \frac{1}{2}\left\{ C_i^{n\dagger} C_i^n, \rho(t) \right\} \right],$$

$$H = \begin{pmatrix} h_s + \frac{r^2}{2}\gamma & 0 \\ 0 & h_o + \frac{N_c^2-2}{2(N_c^2-1)}\frac{r^2}{2}\gamma \end{pmatrix} \qquad C_i^0 = \sqrt{\frac{\kappa}{N_c^2-1}}\, r_i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2-1} & 0 \end{pmatrix},$$

$$C_i^1 = \sqrt{\frac{\kappa(N_c^2-4)}{2(N_c^2-1)}}\, r_i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$h_{s,o} = \vec{p}^{\,2}/M + V_{s,o}$$

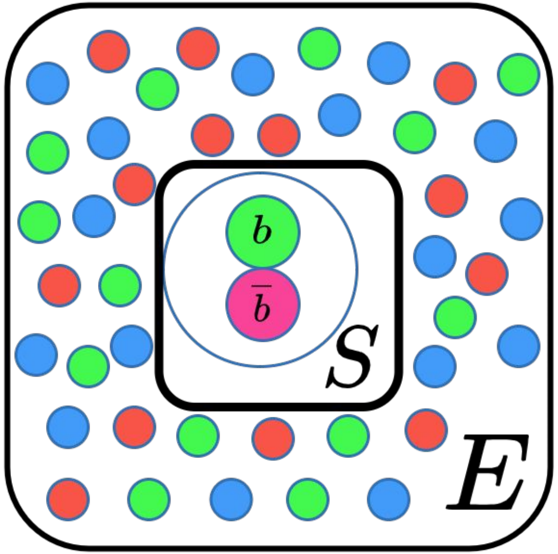
Transport coefficients

pNRQCD master equation

- Projection onto spherical harmonics
- Hilbert Space:
    - Singlet and octet states

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$

    - Discretizing radial wavefunction
    - Angular momentum quantum numbers
- **Very large Hilbert space**



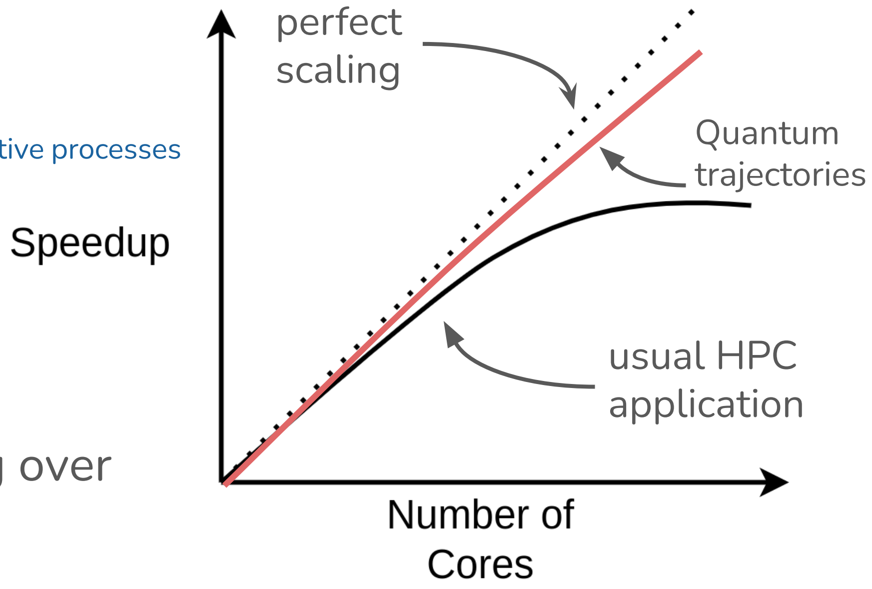
$$\frac{d\rho_S}{dt} = -i[H_S, \rho_S] + \sum_n \left(C_n \rho_S C_n^\dagger - \frac{1}{2} \left\{ C_n^\dagger C_n, \rho_S \right\} \right)$$

Quantum trajectory algorithm

J. Dalibard, Y. Castin, and K. Mølmer, Wave-function approach to dissipative processes in quantum optics, Phys. Rev. Lett. 68 (1992), pp. 580–583.

- Idea:
  1. Evolve individual trajectories $|\phi(t)\rangle$ stochastically
  2. Calculate observables by averaging over trajectories $\overline{\langle\phi(t)|A|\phi(t)\rangle}$

can evolve to arbitrary l



Advantages:

- Evolve vector of size N_H instead N_H^2 density matrix
- Simulation of individual trajectories is **embarrassingly parallel**

Previous work

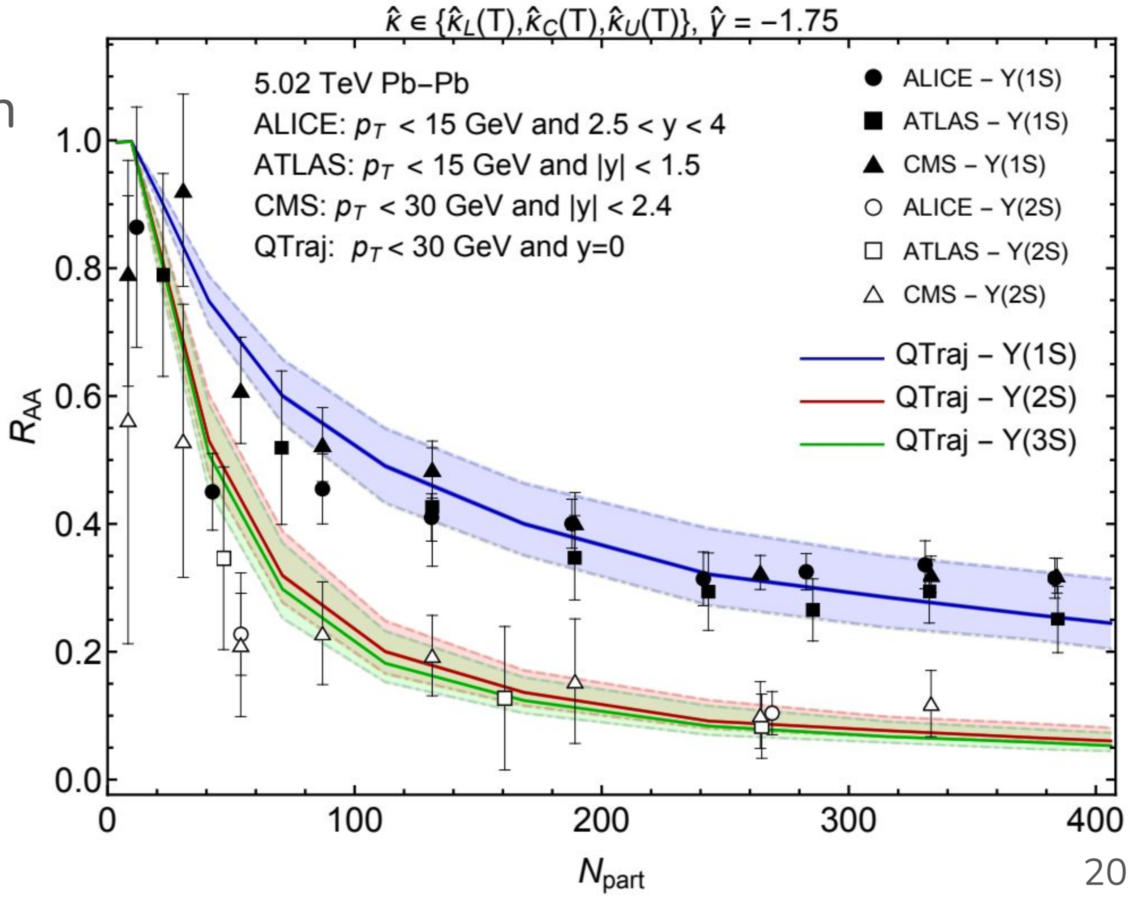
N. Brambilla, M. A. Escobedo, M. Strickland, A. Vairo, P. Vander Griend and J. H. Weber, Phys. Rev. D 104 (2021) 094049

- temperature evolution from hydrodynamics simulation

M. Alqahtani and M. Strickland, The European Physical Journal C 81 (2021)

$$\text{Survival Probability} = \frac{\langle \psi(t) | 1S \rangle}{\langle \psi(0) | 1S \rangle}$$

- Including Feed-down from PDG data



Previous work

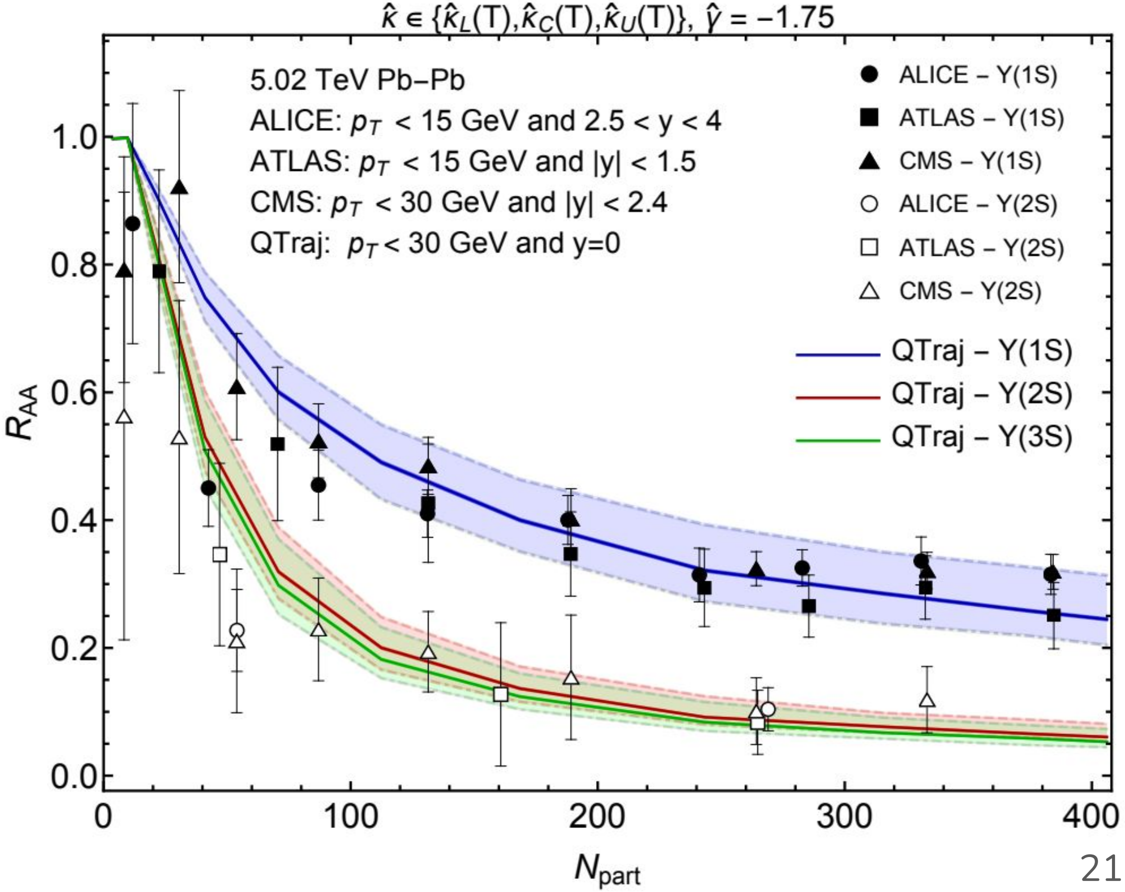
N. Brambilla, M. A. Escobedo, M. Strickland, A. Vairo, P. Vander Griend and J. H. Weber, Phys. Rev. D 104 (2021) 094049

- Coulomb potential

$$V_s = -C_f \alpha_s / r$$
$$V_o = \alpha_s / (2N_c r)$$

- Temperature dependent $\hat{\kappa}$

$$\hat{\gamma} = -1.75$$



New Potential

Kiyo, Y., Pineda, A., & Signer, A. (2010). *Nuclear Physics B*, *841*(1-2), 231-256.

- Motivation: Implement a higher order potential with a more realistic spectrum

$$V_s^{3\mathrm{L}}(r) = V_s^{\mathrm{pert}}(r) + V_s^{\mathrm{non\text{-}pert}}(r)$$

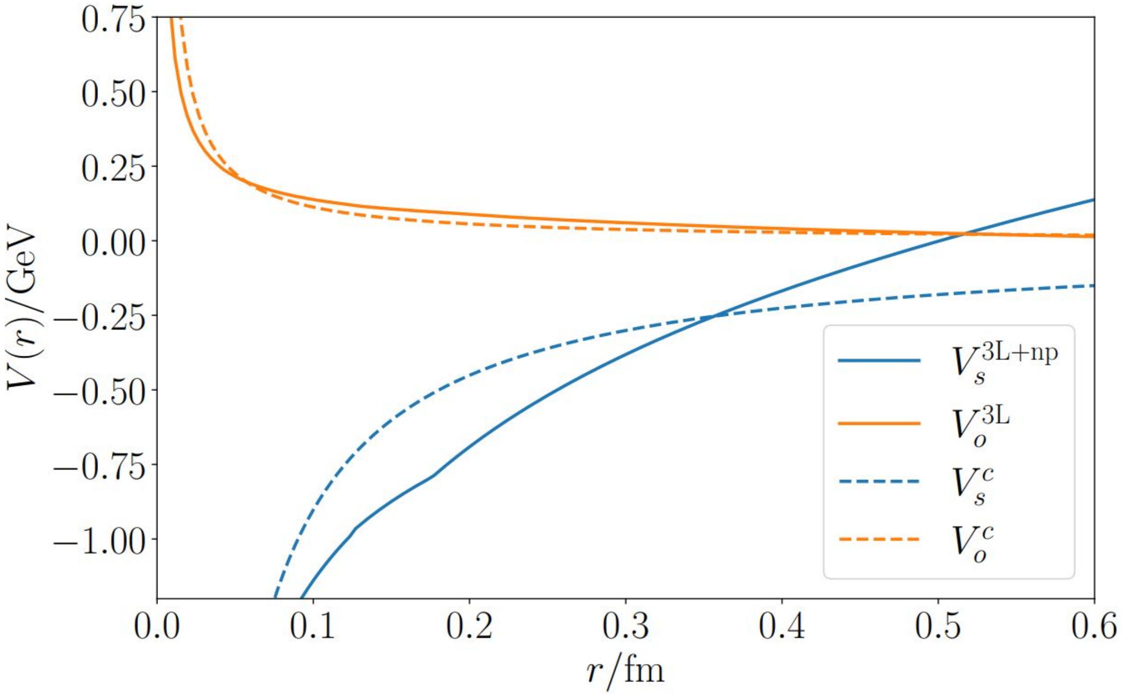
$$V_s^{\mathrm{pert}}(\nu, \nu_r, r) = \begin{cases} \sum_{k=0}^{3} V_{s,\mathrm{RS}'}^{(k)} \alpha_s^{k+1}(1/r) & \text{if } r < \nu_r^{-1} \\ \sum_{k=0}^{3} V_{s,\mathrm{RS}'}^{(k)} \alpha_s^{k+1}(\nu) & \text{if } r > \nu_r^{-1} \end{cases}$$

three loop pNRQCD

$$\mathrm{Re}\left(V_s^{\mathrm{non\text{-}pert}}(r)\right) = \frac{\gamma}{2} r^2$$

leading non-perturbative correction

New Potential



Spectrum:

	PDG	V_s^c	V_s^{3L}
$M(1S)/\,\mathrm{GeV}$	9.445	9.445	9.445
$M(2S)/\,\mathrm{GeV}$	10.017	9.635	10.042
$M(3S)/\,\mathrm{GeV}$	10.355	9.670	10.395
$M(1P)/\,\mathrm{GeV}$	9.888	9.635	9.887
$M(2P)/\,\mathrm{GeV}$	10.251	9.670	10.279

Determination of transport coefficients

- Require the transport coefficients

$$\kappa = \hat{\kappa} T^3$$

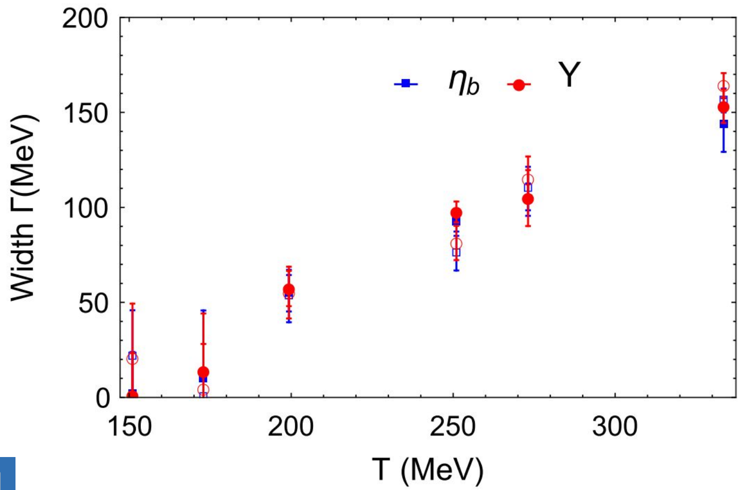
no vacuum part

$$\gamma = \gamma(T = 0) + \hat{\gamma} T^3$$

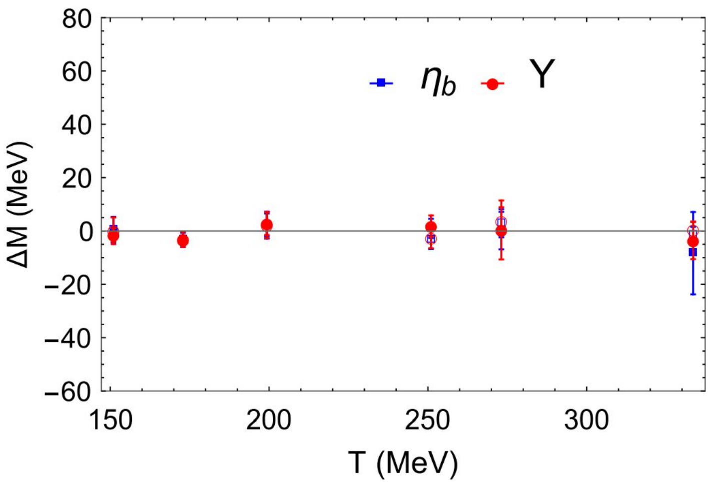
Determination of transport coefficients

- Indirectly determine $\hat{\kappa}$ and $\hat{\gamma}$ from lattice measurements of the **in medium width** Γ and **mass shift** δm

$$\Gamma \propto \hat{\kappa}$$



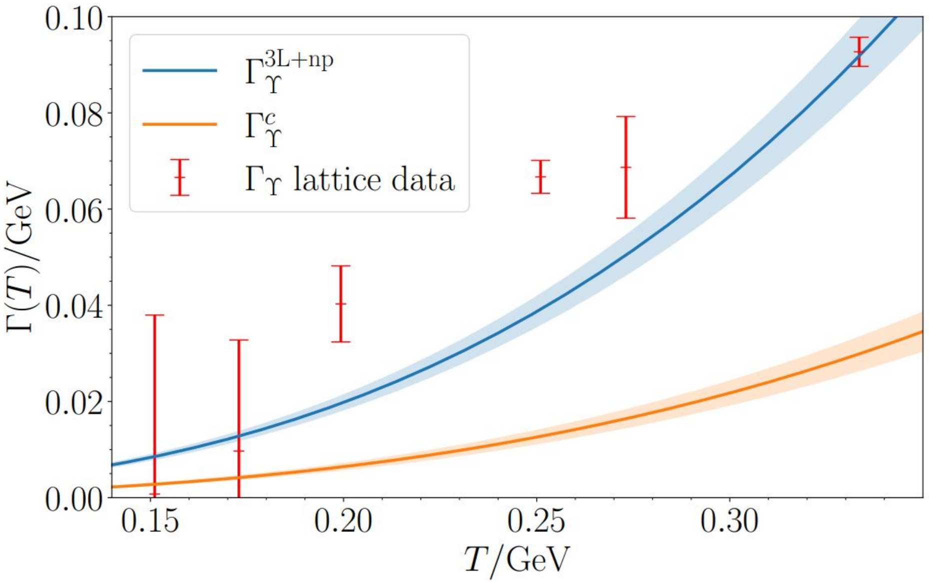
$$\delta m \propto \hat{\gamma} \Rightarrow \hat{\gamma} = 0$$

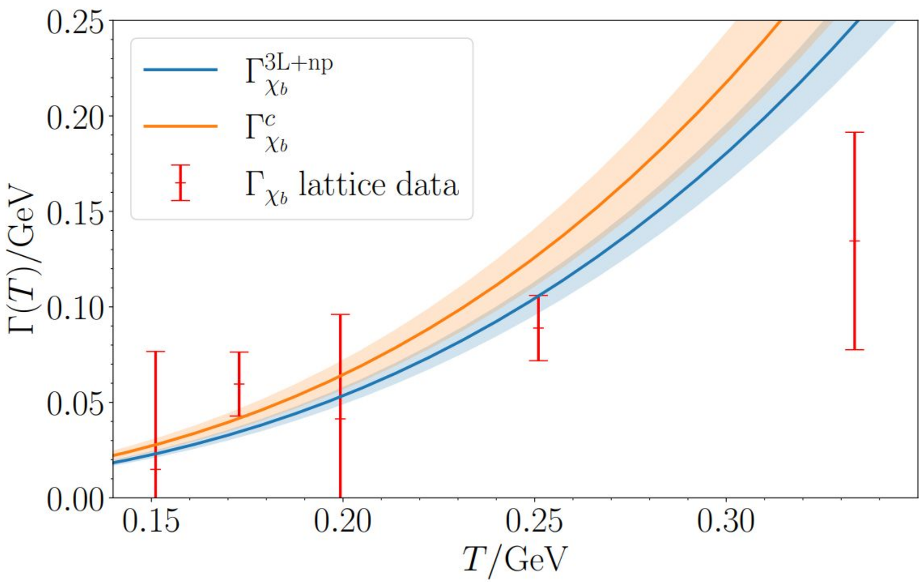


R. Larsen, S. Meinel, S. Mukherjee, and P. Petreczky, Physical Review D 100 (2019)

Determination of transport coefficients

- Obtain $\hat{\kappa}$ from fits to $1S$ and $1P$ data and average





Coulomb: $\hat{\kappa} = 0.33 \pm 0.04$

New potential: $\hat{\kappa} = 1.88 \pm 0.16$

Determination of transport coefficients

- Indirectly determine $\hat{\kappa}$ and $\hat{\gamma}$ from lattice measurements of the **in medium width** Γ and **mass shift** δm

$$\kappa = \hat{\kappa} T^3$$

no vacuum part

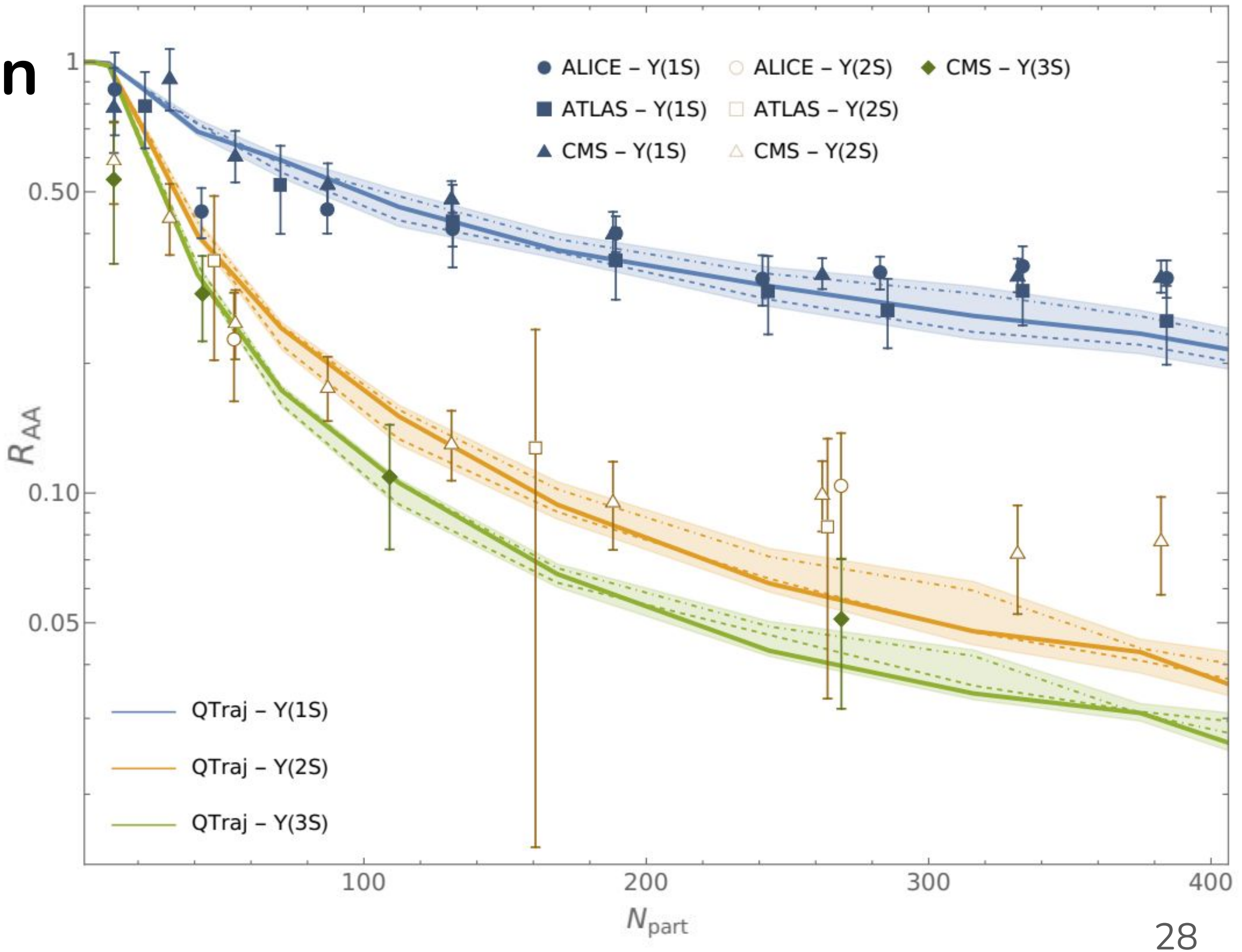
$$\gamma = \gamma(T=0) + \hat{\gamma} T^3$$

Assume simple model

$$\gamma(T=0) = 0.017 \mathrm{GeV}^3$$

Nuclear modification factor results

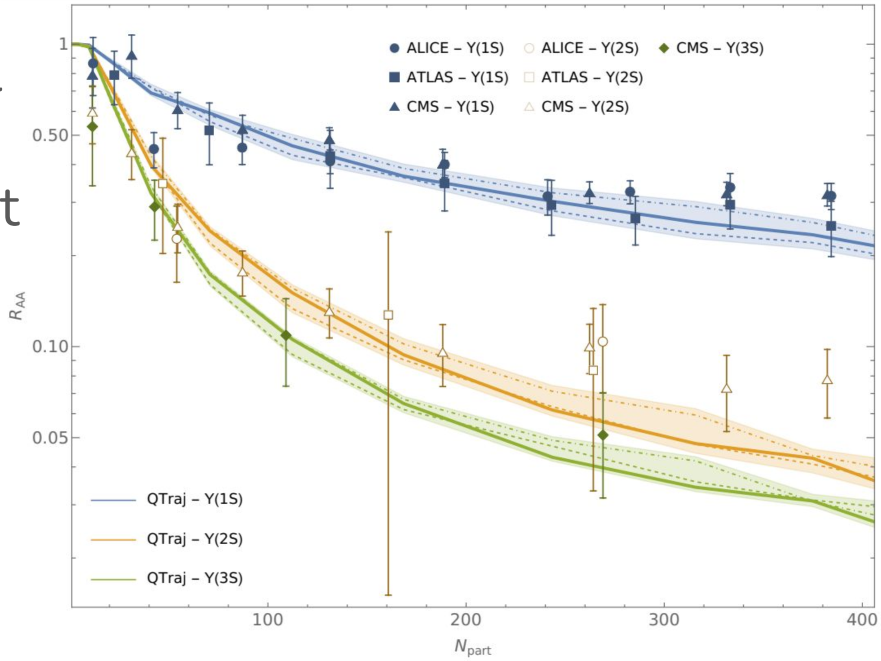
- New potential can describe the experimental data
- Coulomb potential with $\hat{\kappa} = 0.33 \pm 0.04$ can not describe the data



Summary and outlook

- We implemented a new potential which gives a realistic spectrum
- We extracted transport coefficient values from lattice data
- Our results agree well with the experimental data

- Future: Extend analysis to NLO description in $E/(\pi T)$ expansion



Backup slides

Quantum Trajectories

$$U(\Theta) = 1 - iH_{\text{eff}}\delta t$$

$$\psi_0$$

1. Evolve state $|\psi(t)\rangle$ with $U(\Theta)$

$$|\psi(t+\delta t)\rangle = U(\Theta)|\psi(t)\rangle$$

2. Compute norm

$$\langle\psi(t+\delta t)|\psi(t+\delta t)\rangle = 1 - \delta p(\Theta) < 1$$

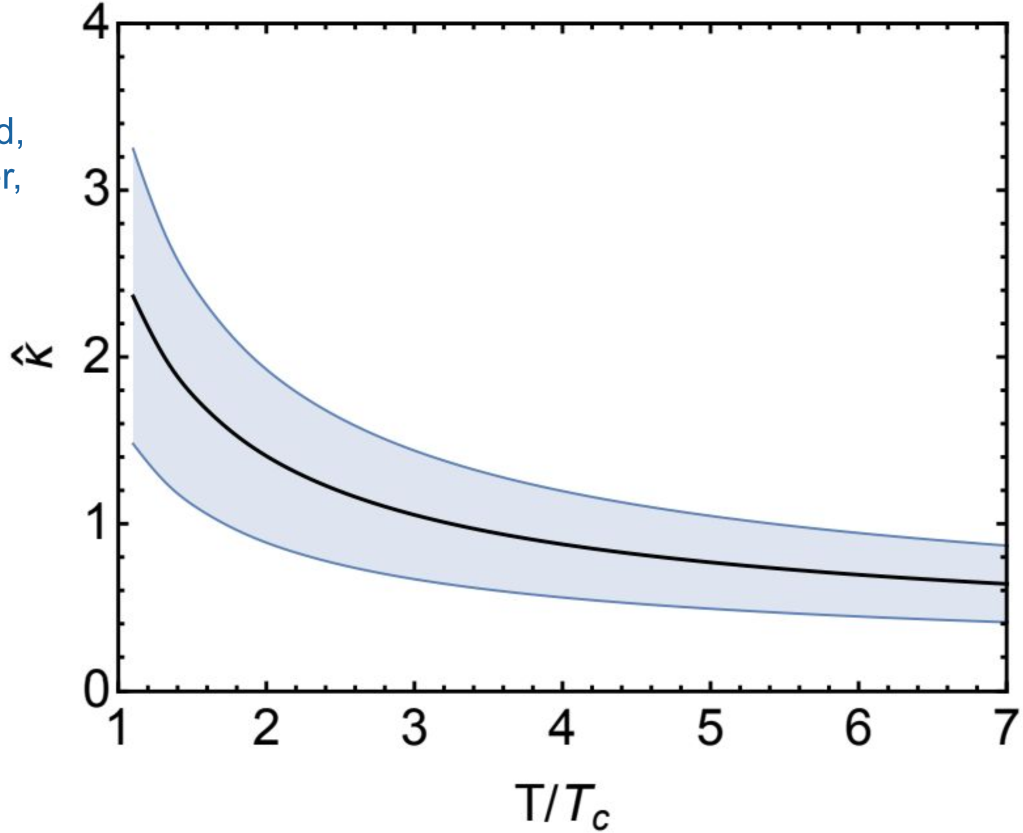
3. Apply jump operator $C(\Theta)$ with probability $\delta p(\Theta)$

$$|\psi(t+\delta t)\rangle = C(\Theta)|\psi(t)\rangle$$

4. Normalize $|\psi(t+\delta t)\rangle$

Non perturbative correction

N. Brambilla, M. A. Escobedo, M. Strickland, A. Vairo, P. Vander Griend, and J. H. Weber, JHEP 05, 136 (2021), 2012.01240



Heavy quark diffusion coefficient

$$V_s^{\text{non-pert}}(r) = -i\frac{g^2 T_F}{3N_c} r^2 \int_0^\infty dt \langle E^a(t)\Omega(t,0)^{ab}E^b(0)\rangle$$

$$\gamma = \frac{g^2}{3N_c}\mathrm{Im}\int_0^\infty dt\,\langle E^a(t)\Omega(t,0)^{ab}E^b(0)\rangle$$

In medium width

- Width given by collapse operators

$$\Gamma = \sum_n C_n^\dagger C_n$$

- At LO in E/T

$$\Gamma = \hat{\kappa} T^3 r^2$$

Determination of transport coefficients

- Indirectly determine $\hat{\kappa}$ and $\hat{\gamma}$ from lattice measurements of the **in medium width** Γ and **mass shift** δm

$$\kappa = \hat{\kappa} T^3$$

no vacuum part

$$\gamma = \gamma(T=0) + \hat{\gamma} T^3$$

- Assume simple model for the vacuum part $\gamma(T=0)$

$$\langle E^a(t)\Omega(t,0)^{ab}E^b(0)\rangle = \langle E^2(0)\rangle \mathrm{e}^{-i\Lambda_E t}$$

G. S. Bali and A. Pineda, Physical Review D 69 (2004)

$$\langle g^2 E^2(0)\rangle = -0.2\mathrm{GeV}^4$$

$$\Lambda_E = 1.25\mathrm{GeV}$$

S. Narison, Qcd parameters and sm-high precisions from $e_+e_-\to$hadrons :Summary(2023),2309.05342

EFTs for Quarkonium Suppression

- Use NREFTs to exploit hierarchy of scales

$$M \gg 1/a_0 \gg \pi T \gg E$$

- Inverse radius:

$$1/a_0 \approx 1.2\text{GeV}$$

- Temperature regime:

$$250\text{MeV} < T < 425\text{MeV}$$

- Binding Energy:

$$E \sim 0.4\text{GeV}$$