$$U(r) = -\frac{2uR^2}{r^2\left[\left(\frac{r}{R}\right)^{\mu} + \left(\frac{R}{r}\right)^{\mu}\right]^2}$$

μ is the parameter to focus on.

Central force, $F = -\frac{dU_{eff}}{dr}$

$$U_{eff} = U(r) + \frac{ml^2}{2r^2}$$

where $\frac{ml^2}{2r^2}$ is the angular momentum barrier.

$$F = -\frac{dU_{eff}}{dr} = -\frac{d}{dr}\left(U(r) + \frac{1}{2}\cancel{m}r^2\dot{\theta}^2\right)$$, let $m = 1$.

Just to verify this coincides with the energy of the system:

$$
\begin{aligned}
E &= T + U(r) \\
&= \tfrac{1}{2}\dot{x}^2 + U(r) \\
&= \tfrac{1}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + U(r) \\
&= \tfrac{1}{2}\dot{r}^2 + \tfrac{1}{2}r^2\dot{\theta}^2 + U(r) = \tfrac{1}{2}m\dot{r}^2 + \frac{l^2}{2r^2} + U(r) \\
&= \frac{1}{2}m\dot{r}^2 + U_{eff}(r).
\end{aligned}
$$

Lagrangian of the system,

$$
\begin{aligned}
L &= T - U(r) \\
&= \tfrac{1}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - U(r)
\end{aligned}
$$

Angular momentum, $l = r^2\dot{\theta} = \text{const.}$

We use $l = \text{const.}$ as an initial condition.

In polar coordinates,

$$\hat{r} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}, \quad \hat{\theta} = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$$

$$\Leftarrow \vec{r} = (x, y) = r(\cos\theta, \sin\theta) \Rightarrow \vec{r} = r\hat{r}$$

$$\vec{v} = d\vec{r}/dt = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + \underbrace{(2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}}_{=0 \Leftarrow \vec{F} \text{ acts with only radial part for our case.}}$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)$$

$$\vec{F} = m\vec{a}$$

$$= m(\ddot{r} - r\dot{\theta}^2) = -\frac{dU_{eff}}{dr}, \text{ let } m = 1.$$

First equation of motion:

$$= -\frac{dU}{dr} + \frac{l^2}{r^3} \quad \boxed{\ddot{r} - r\dot{\theta}^2 = -\frac{dU}{dr}} = \ddot{r} - r\left(\frac{l^2}{r^4}\right) = \ddot{r} - \frac{l^2}{r^3}$$

should be Conz potential.

Hamiltonian in polar coordinates:

$$H = \frac{1}{2m}\left(p_r^2 + \frac{p_\theta^2}{r^2}\right) + U(r) \qquad p_\theta = \text{const.}$$

$$= \frac{1}{2m}(\dot{r}^2 + r^2\dot{\theta}^2) + U(r)$$

$$\frac{d\theta}{dt} = \frac{\partial H}{\partial p_\theta} = \frac{p_\theta}{mr^2} = \frac{l}{mr^2} = \dot{\theta}$$

Equation of motion for θ: $\boxed{\dot{\theta} = l/r^2}$

$$\frac{dr}{dt} = \frac{\partial H}{\partial p_r} = \frac{p_r}{m} = p$$

Equation of motion for r: $\boxed{\dot{r} = p}$

$$= \dot{r}^2$$