$$\ddot{\theta} + \frac{g}{l}\sin(\theta) = 0$$

2nd order homogeneous non-linear ODE

$$\ddot{\theta}\dot{\theta} + \frac{g}{l}\sin(\theta)\dot{\theta} = 0$$

[e.g. $\frac{d}{dt}\dot{\theta}^2 = 2\dot{\theta}\ddot{\theta}$ $\Leftarrow \dot{\theta}(t)$ and chain rule]

$$\Rightarrow \quad \ddot{\theta}\dot{\theta} = \frac{1}{2}\frac{d}{dt}\dot{\theta}^2 \,,$$

Integrating

$$\ddot{\theta}\dot{\theta} + \frac{g}{l}\sin(\theta)\dot{\theta} = 0$$

$$\frac{1}{2}\frac{d}{dt}\dot{\theta}^2 - \frac{g}{l}\frac{d}{dt}(\cos\theta) = 0$$

$$\frac{d}{dt}\left[\frac{1}{2}\dot{\theta}^2 - \frac{g}{l}\cos\theta\right] = 0$$

$$\frac{d}{dt}\left[\frac{1}{2}\dot{\theta}^2 - \frac{g}{l}\cos\theta\right] = 0$$

$$\Rightarrow \quad \frac{1}{2}\dot{\theta}^2 - \frac{g}{l}\cos\theta = C \quad , \quad \dot{\theta}^2 - \frac{2g}{l}\cos\theta = C$$

EXPLICIT: Initial conditions:

$$\theta(0) = \theta_0$$
$$\dot{\theta}(0) = 0$$
$$\Rightarrow \quad -\frac{g}{l}\cos\theta_0 = C$$
$$C = -\frac{2g}{l}\cos\theta_0$$

$$\cancel{\frac{d}{dt}\left(\frac{1}{2}\dot{\theta}^2 - \frac{g}{l}\cos\theta\right)}$$

$$\frac{1}{2}\dot{\theta}^2 - \frac{g}{l}\cos\theta = -\frac{g}{l}\cos\theta_0$$

$$\frac{1}{2}\dot{\theta}^2 - \frac{g}{l}(\cos\theta - \cos\theta_0) = 0$$ [Apply $\cos\theta = 1 - 2\sin$

$$\frac{1}{2}\dot{\theta}^2 - \frac{g}{l}\left(1 - 2\sin^2\left(\theta/2\right) - 1 + 2\sin^2\left(\theta_0/2\right)\right) = 0$$

$$\frac{1}{2}\dot{\theta}^2 - \frac{2g}{l}\left(\sin^2\left(\frac{\theta_0}{2}\right) - \sin^2\left(\theta/2\right)\right) = 0$$

$$\frac{1}{2}\dot{\theta}^2 = \frac{2g}{l}\left(\sin^2\left(\theta_0/2\right) - \sin^2\left(\theta/2\right)\right)$$

$$\dot{\theta}^2 = \frac{4g}{l}\left(\sin^2\left(\frac{\theta_0}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)\right)$$

$= g/L$

$$\dot{\theta} = 2\sqrt{\frac{g}{l}}\left(\sin^2\left(\frac{\theta_0}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)\right)^{1/2}$$

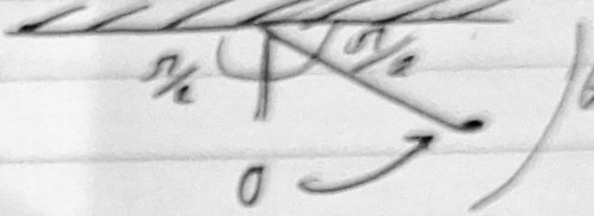
Integrate:

$$\int \dot{\theta}\, dt = 2\sqrt{g/l}\int\left(\sin^2\frac{\theta_0}{2} - \sin^2\frac{\theta}{2}\right)^{1/2} dt$$

$$\int \frac{\cancel{(d\theta/dt)\ dt}}{\left(\sin^2\theta_0/2 - \sin^2\theta/2\right)^{1/2}}\, d\theta = 2\sqrt{\frac{g}{l}}\int dt$$

$$\int \frac{d\theta}{\left(\sin^2(\theta_0/2) - \sin^2(\theta/2)\right)^{1/2}} = 2\sqrt{\frac{g}{l}}\int dt$$

can we do $\theta > \pi/2$?



$$\int \frac{d\theta}{\left(\sin^2\left(\frac{\theta_0}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)\right)^{1/2}} = \int dt \left(2\sqrt{\frac{g}{l}}\right)$$

$$\int_0^{\theta_0} \frac{d\theta}{\left(\sin^2\left(\frac{\theta_0}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)\right)^{1/2}} = \int_0^{T/4} dt \left(2\sqrt{\frac{g}{l}}\right)$$

$$\int_0^{\theta_0} \frac{d\theta}{\left(\sin^2\left(\frac{\theta_0}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)\right)^{1/2}} = \left(\frac{T}{4}\right)\left(2\sqrt{\frac{g}{l}}\right)$$

$$T = \frac{2}{\omega} \int_0^{\theta_0} \frac{d\theta}{\left(\sin^2\left(\frac{\theta_0}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)\right)^{1/2}}$$

T