Lagrangian, $L = K - V$, $K > 0$

$= \frac{1}{2} m \dot{\theta}^2 - V(\theta)$, generally.

$E = \frac{1}{2} m \dot{\theta}^2 + V(\theta)$ (Note $\partial_t E = \text{const.}$ ∴ E conserved)

Integrating: $\int E \, dt = \int \frac{1}{2} m \dot{\theta}^2 + V(\theta) \, d\theta$

$$\Rightarrow \quad t = \int \frac{d\theta}{\sqrt{(E - V(\theta))\, 2/m}} \qquad \left[\frac{dx}{dt} = \left(2(E-V)/m\right)^{1/2} \right.$$

$$= \left(\frac{m}{2}\right)^{1/2} \int \frac{d\theta}{(E - V(\theta))^{1/2}} + c, \quad c = \text{const.}$$

limits of integration are roots of $E(t, \theta, \dot{\theta}) = V(\theta)$. In general, this ~~[illegible]~~ results from $K > 0$. Here, we look at a time period T between $+\theta_0$ and $+\theta_1$.

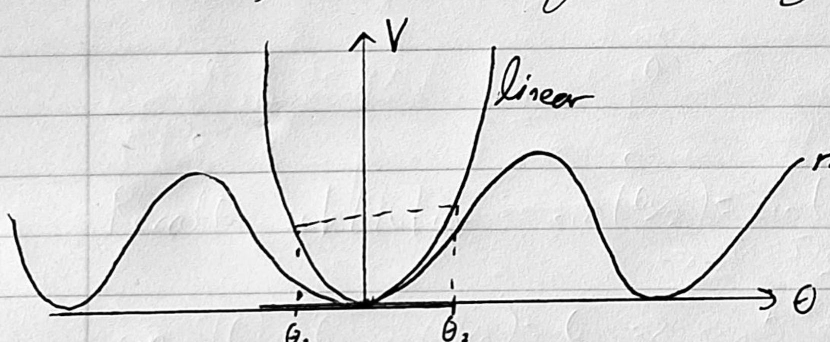
Motion bound on two sides ⇒ finite ⇒ oscillatory.

For simple pendulum, $T = 2 t_{\theta_0 \to \theta_1}$

$$T = 2 \left(\frac{m}{2}\right)^{1/2} \int_{\theta_0}^{\theta_1} \frac{d\theta}{(E - V(\theta))^{1/2}}$$

$$T = \sqrt{2m} \int_{\theta_0}^{\theta_1} \frac{d\theta}{(E - V(\theta))^{1/2}}$$

Solve this as an integral equation. We let $\theta_0(E)$, $\theta_1(E)$ be functions of energy total.



Both are bijective. nonlinear has more than one minimum, so more solutions to verify.

Split the integral along the V axis: consider linear case.

$$T = \sqrt{2m} \left(\int_0^E \frac{dV}{(E-V)^{1/2}} \frac{d\theta(V)}{dV} + \int_E^0 \frac{dV}{(E-V)^{1/2}} \frac{d\theta(V)}{dV} \right.$$

Energy integral.

$$= \sqrt{2m}\int_0^E \left(\frac{d\theta_2}{dV} - \frac{d\theta_1}{dV}\right)\frac{dV}{(E-V)^{1/2}}$$

(From Landau, Lifschitz mechanics), introduc parameter α divide by $(\alpha - E)^{1/2}$:

Paramaratized integral equation

$$\int_0^\alpha \frac{T(E)\,dE}{(\alpha-E)^{1/2}} = (2m)^{1/2}\int_0^\alpha \cancel{\int_0^E} \left(\frac{d\theta_2}{dV} - \frac{d\theta_1}{dV}\right)dV \underbrace{\int_V^\alpha \frac{dE}{((\alpha-E)(E-V))^{1/2}}}_{=\pi,\ \text{standard.}}$$

$$= (2m)^{1/2}(\theta_2(\alpha) - \theta_1(\alpha))^{1/2}\,\pi$$

$$\Rightarrow \quad \theta_2(V) - \theta_1(V) = \frac{1}{\pi\sqrt{2m}}\int_0^V \frac{T(E)\,dE}{(\alpha-E)^{1/2}}$$

T must be known.

Potential is symmetric about V.

$$\Rightarrow \quad \theta(V) = \frac{1}{\sqrt{2m}\,2\pi}\int_0^V \frac{T(E)\,dE}{(V-E)^{1/2}}$$