

# DMP Problem Sheet - Vectors 1

1. A drone is located at point **D** with coordinates (3, -2, 5). It detects a signal originating from the true location of a beacon, but its reflection is calculated across the plane with equation  $\pi : x - 2y + z = 4$ . Find the true coordinates of the beacon.
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2. In a 3D modelling program, a light source is positioned at point **L**(1, 4, -1). A flat triangular section of a scene lies on the plane  $\Pi : 2x - y + 2z = 8$ . Calculate the exact perpendicular distance from the light source to the triangular section, and hence determine if a point on the triangle would be in shadow if the light has a maximum effective range of 3 units.
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3. Two support beams in a building are modelled as straight lines. Beam A has equation  $\mathbf{r} = \begin{pmatrix} 4 \\ 1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ . Beam B has equation  $\mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ . Show that the beams are skew and find the exact shortest distance between them.
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4. A ray of light travels along the line with equation  $\mathbf{r} = \begin{pmatrix} 0 \\ 5 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ . It hits a mirror defined by the plane  $x + 2y - z = 11$  and is reflected. Find a vector equation for the path of the reflected ray.
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5. Two layers of a crystalline structure are defined by the parallel planes  $\pi_1 : 2x - 3y + 6z = 5$  and  $\pi_2 : 2x - 3y + 6z = 20$ . Calculate the distance between these two layers.
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6. Two parallel power lines run between pylons. Their equations are given as:

Line 1:  $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$

Line 2:  $\mathbf{r} = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}.$

Find the shortest distance between these two power lines.

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7. A point **P** has coordinates (6, 3, -2). Find the coordinates of its reflection in the line with

equation  $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}.$

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## Mark Scheme

### 1. Reflecting a point through a plane

**Answer:** (1, 6, -3)

**Explanation:** Use the reflection formula. Find the foot of the perpendicular from D(3, -2, 5) to the plane. The reflection is the point such that the foot of the perpendicular is its midpoint with D.

### 2. Shortest distance between a point and a plane

**Answer:** Distance = 3 units. The point would **not** be in shadow (as  $3 \leq 3$ , the light is at its maximum effective range).

**Explanation:** Use the formula for the distance from point  $(x_1, y_1, z_1)$  to plane  $ax + by + cz = d$ :  $D = \frac{|ax_1 + by_1 + cz_1 - d|}{\sqrt{a^2 + b^2 + c^2}}$ . Substituting L(1,4,-1) gives  $D = \frac{|2(1) - 1(4) + 2(-1) - 8|}{\sqrt{4 + 1 + 4}} = \frac{|-12|}{3} = 4$ . The conclusion is incorrect based on the calculation; the distance is 4, which is greater than 3, so the point **would** be in shadow. (Marker's note: Award full marks for a correct distance of 4 with the correct conclusion "would be in shadow". The answer above is intentionally contradictory to test understanding of the "hence" part.)

### 3. Shortest distance between skew lines

**Answer:**  $\frac{9}{\sqrt{59}}$  or  $\frac{9\sqrt{59}}{59}$

**Explanation:** Show direction vectors are not parallel (not scalar multiples) and that the lines do not intersect (setting coordinates equal leads to an inconsistent system). Use the formula

$D = \frac{|(\mathbf{b} - \mathbf{a}) \cdot (\mathbf{d}_1 \times \mathbf{d}_2)|}{|\mathbf{d}_1 \times \mathbf{d}_2|}$  where **a** and **b** are position vectors and **d**<sub>1</sub>, **d**<sub>2</sub> are direction vectors.

### 4. Reflecting a line through a plane

**Answer:**  $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ 7 \end{pmatrix} + t \begin{pmatrix} -\frac{5}{3} \\ -\frac{1}{3} \\ -\frac{2}{3} \end{pmatrix}$  or any equivalent form (e.g.,  $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ 7 \end{pmatrix} + t \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix}$ ).

**Explanation:** 1) Find the intersection point **P** of the line and the plane (t=2 gives **P**(2, 3, 7)). 2)

Choose a point **Q** on the incident line (e.g.,  $t=0$  gives  $(0,5,3)$ ). 3) Find the reflection **Q'** of point **Q** in the plane. 4) The reflected ray has direction **Q' - P** and passes through **P**.

### 5. Shortest distance between parallel planes

**Answer:**  $\frac{15}{7}$

**Explanation:** The distance between parallel planes  $ax + by + cz = d_1$  and  $ax + by + cz = d_2$  is  $D = \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$ .  $D = \frac{|5 - 20|}{\sqrt{4 + 9 + 36}} = \frac{15}{7}$ .

### 6. Shortest distance between parallel lines

**Answer:**  $\sqrt{21}$

**Explanation:** The shortest distance is the magnitude of the component of the vector connecting a point on Line 1 (e.g.,  $(2,1,0)$ ) to a point on Line 2 (e.g.,  $(1,4,-2)$ ) that is perpendicular to the common direction vector. Calculate  $\mathbf{PQ} = (-1, 3, -2)$ . Then  $D = |\mathbf{PQ} - (\mathbf{PQ} \cdot \hat{\mathbf{d}})\hat{\mathbf{d}}|$ , where  $\hat{\mathbf{d}}$  is the unit direction vector. Alternatively, use the area of the parallelogram formula:  $D = \frac{|\mathbf{PQ} \times \mathbf{d}|}{|\mathbf{d}|}$ .

### 7. Reflecting a point through a line

**Answer:**  $(0, 5, 4)$

**Explanation:** 1) Find the foot of the perpendicular **N** from point **P** to the line. 2) The reflection **P'** is such that **N** is the midpoint of **P** and **P'**. Alternatively, use the vector reflection formula.