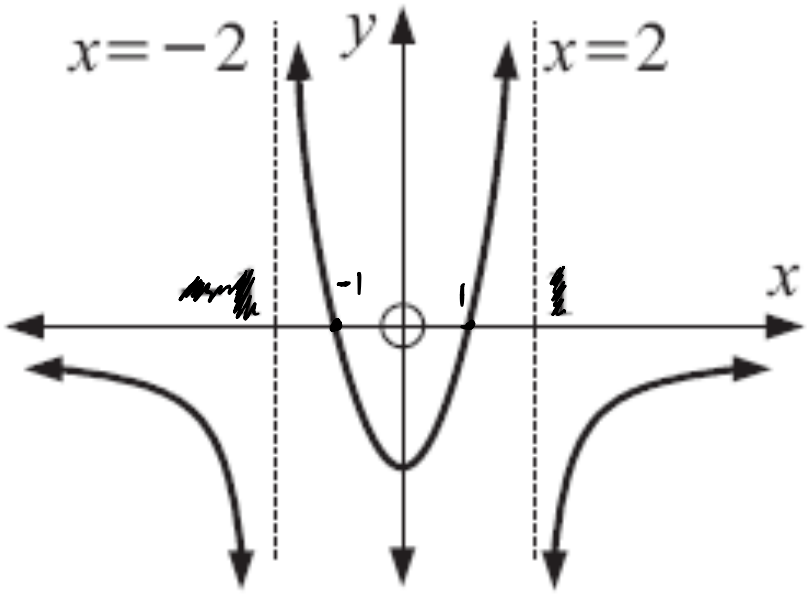
A function has the following graph:

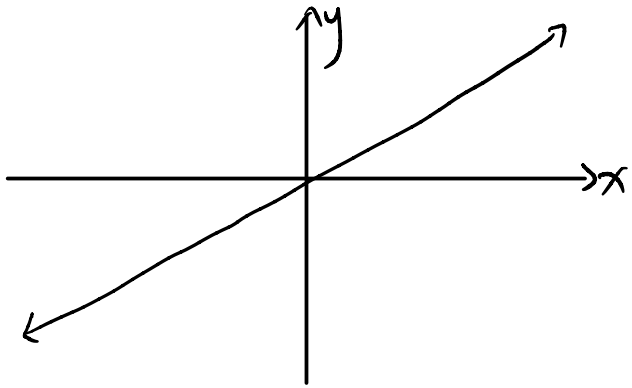


Where is the function positive? Negative?

positive $\{x \in \mathbb{R} \mid 1 \leq x < 2 \text{ , } -2 < x \leq -1\}$

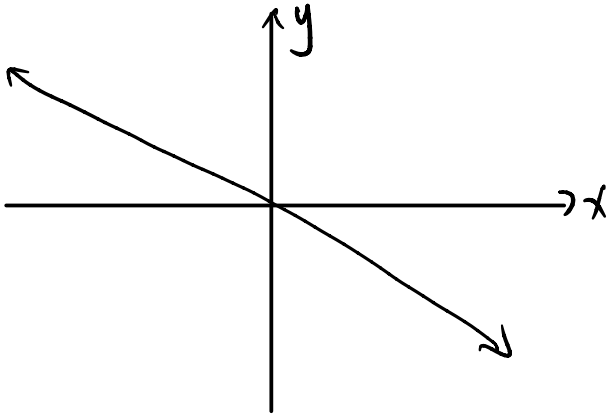
negative $\{x \in \mathbb{R} \mid -1 < x < 1 \text{ , } x < -2 \text{ , } x > 2\}$

Sketch a function that always increases.



- going up as we move from left to right

Sketch a function that always decreases.



- going down as we move from left to right

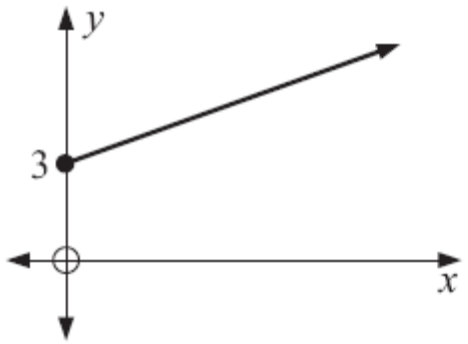
- any function that increases or decreases only is called monotonic.

Increasing/Decreasing Functions

Example #1 (Exercise 18C#1)

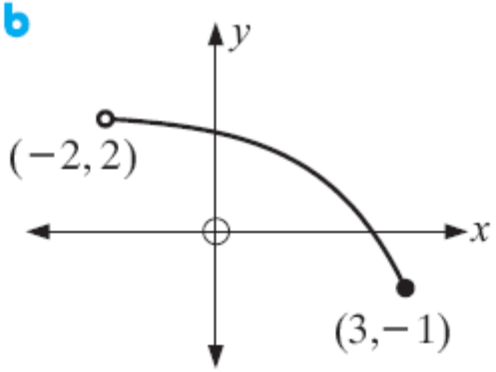
For what intervals do each of the following graphs increase or decrease?

a



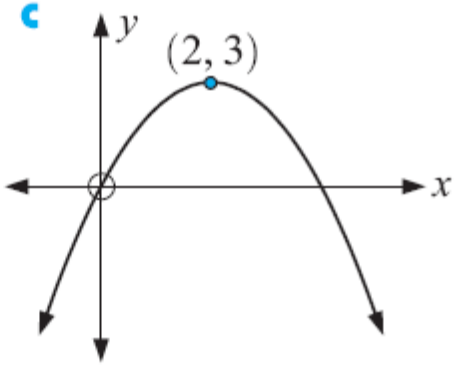
increase $\{x \in \mathbb{R} \mid x \geq 0\}$

b



decrease $\{x \in \mathbb{R} \mid -2 < x \leq 3\}$

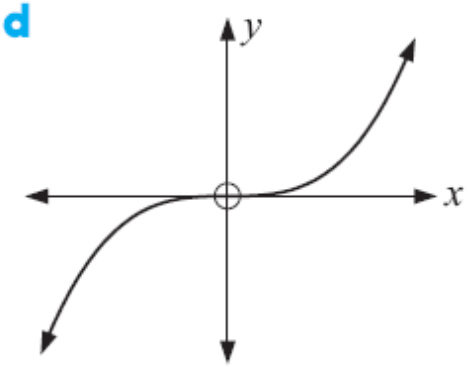
c



increase $\{x \in \mathbb{R} \mid x \leq 2\}$

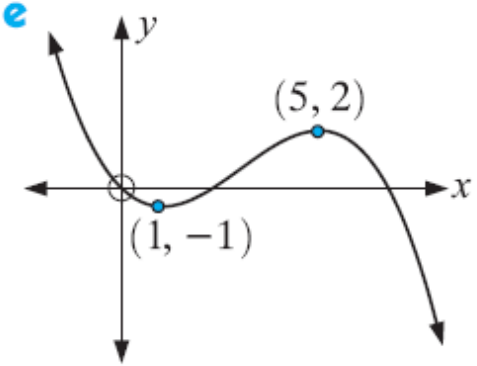
decrease $\{x \in \mathbb{R} \mid x \geq 2\}$

d



increasing $\{x \in \mathbb{R}\}$

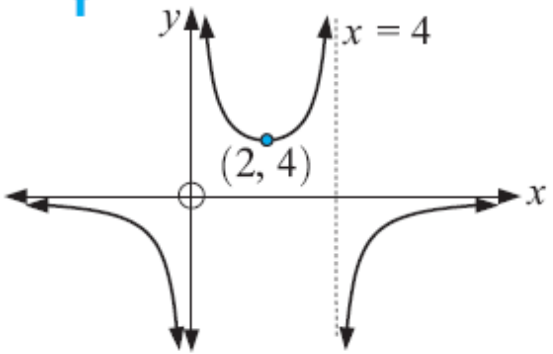
e



decreasing $\{x \in \mathbb{R} \mid x \leq 1, x \geq 5\}$

increasing $\{x \in \mathbb{R} \mid 1 \leq x \leq 5\}$

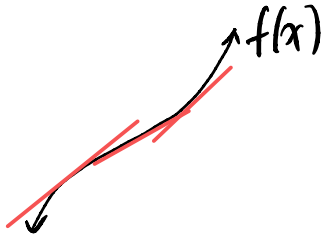
f



decreasing $\{x \in \mathbb{R} \mid x \leq 2, x \neq 0\}$

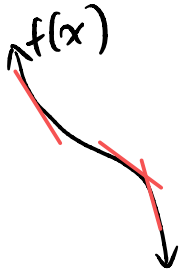
increasing $\{x \in \mathbb{R} \mid x \geq 2, x \neq 4\}$

For an increasing interval, what can be said about the slopes of the tangent?



$m \geq 0$ $f'(x) \geq 0$

For a decreasing interval, what can be said about the slopes of the tangent?



$m \leq 0$ $f'(x) \leq 0$

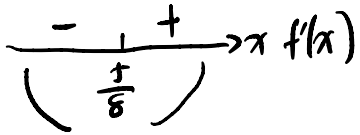
Example #2

Find the intervals on which the function $f(x) = 1 - 5x + 4x^2$ is increasing and decreasing.

$f'(x) = -5 + 8x$

$0 = 8x - 5$

$x = \frac{5}{8}$



decreasing for $x \leq \frac{5}{8}$

increasing for $x \geq \frac{5}{8}$

Example #3

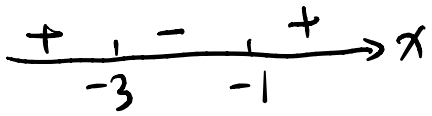
Where is the function $y = x^3 + 6x^2 + 9x + 2$ increasing?

$$\frac{dy}{dx} = 3x^2 + 12x + 9$$

$$0 = 3(x^2 + 4x + 3)$$

$$0 = (x+1)(x+3)$$

$$x = -1, \; x = -3$$



increasing: $x \geq -1, \; x \leq -3$