L'hop it !!!

Calculate each of the following limits:

a) $\lim_{x\to 0}\frac{2^x-1}{x} = \text{"}\frac{0}{0}\text{"}$ indetermined

b) $\lim_{x\to\infty} x^2 e^{-x} = \lim_{x\to\infty} x^2 \lim_{x\to\infty}\frac{1}{e^x} = \text{"}\infty\times 0\text{"}$ indetermined

c) $\lim_{x\to\infty}\frac{\ln x}{x} = \frac{\lim_{x\to\infty}\ln x}{\lim_{x\to\infty} x} = \text{"}\frac{\infty}{\infty}\text{"}$ indetermined

a) $\lim_{x\to 0}\frac{2^x-1}{x}$

b) $\lim_{x\to\infty} x^2e^{-x}$ "$\infty\times 0$"

c) $\lim_{x\to\infty}\frac{\ln x}{x}$ "$\frac{\infty}{\infty}$"

L'HÔPITAL'S RULE

Suppose $f(x)$ and $g(x)$ are differentiable and $g'(x) \neq 0$ on an interval that contains the point $x = a$.
If $\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = 0$, or, if as $x \to a$, $f(x) \to \pm\infty$ and $g(x) \to \pm\infty$, then
$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$ provided the limit on the right exists.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Same

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a) = 0$$

$$\lim_{x \to a^-} g(x) = \lim_{x \to a^+} g(x) = g(a) = 0$$

$\therefore f(a) = g(a) = 0$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \frac{\lim_{x \to a} f(x) - f(a)}{\lim_{x \to a} g(x) - g(a)} = \frac{\lim_{x \to a} \frac{f(x) - f(a)}{x - a}}{\lim_{x \to a} \frac{g(x) - g(a)}{x - a}} = \frac{f'(x)}{g'(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Example #1

Use L'hopitals Rule to evaluate:

a) $\lim_{x\to 0}\frac{2^x-1}{x}$

$= \lim_{x\to 0}\frac{2^x\ln 2}{1}$

$= \ln 2$

b) $\lim_{x\to 0} x^2 e^{-x}$

$= \lim_{x\to\infty} 2x(-e^{-x})$ "$\infty \times 0$"

$= \lim_{x\to\infty} 2(e^{-x})$

$= 0$

c) $\lim_{x\to\infty}\frac{\ln x}{x}$

$= \lim_{x\to\infty}\frac{\frac{1}{x}}{1}$

$= 0$

d) $\lim\limits_{x\to 0^+}\left(\dfrac{1}{x}-\dfrac{1}{\sin x}\right)$

$= \lim\limits_{x\to 0^+}\left(\frac{1}{x}\right) - \lim\limits_{x\to 0^+}\left(\frac{1}{\sin x}\right)$ "$\infty-\infty$" indeterminant

$= \lim\limits_{x\to 0^+}\left(\dfrac{\sin x - x}{x\sin x}\right)$ "$\frac{0}{0}$"

$= \lim\limits_{x\to 0^+}\left(\dfrac{\cos x - 1}{\sin x + x\cos x}\right)$ "$\frac{0}{0}$"

$= \lim\limits_{x\to 0^+}\left(\dfrac{-\sin x}{\cos x - x\sin x + \cos x}\right)$

$= \lim\limits_{x\to 0^+}\left(\dfrac{-\sin x}{2\cos x - x\sin x}\right)$

$= \dfrac{0}{2} = 0$