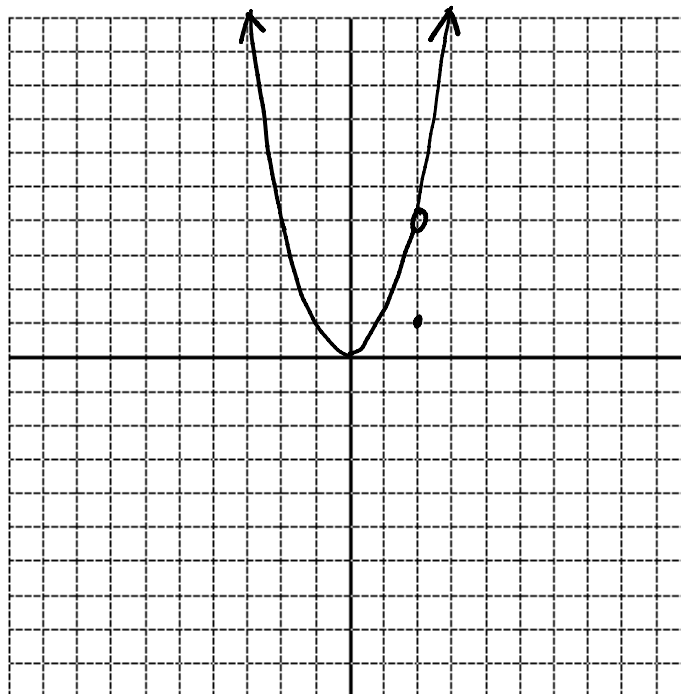


Graph the function:

$$f(x) = \begin{cases} x^2, & x \in \mathbb{R}, x \neq 2 \\ 1, & x = 2 \end{cases}$$

Calculate: $\lim_{x \rightarrow 2} f(x) = 4$

$$f(2) = 1$$



More Limits

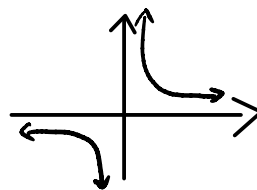
The limit $\lim_{x \rightarrow a} f(x)$ exists and is equal to the finite value A if $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = A$.

We say that $f(x)$ **converges** to A as x approaches a .

from left *from right*

Example #1

Calculate $\lim_{x \rightarrow 0} \frac{1}{x}$



$$x \rightarrow 0^- \quad f(x) \rightarrow -\infty$$

$$x \rightarrow 0^+ \quad f(x) \rightarrow +\infty$$

$\lim_{x \rightarrow 0} \frac{1}{x}$ does not exist

Infinite Limits

$$\lim_{x \rightarrow \infty} \frac{1}{x} = 0$$

as x gets larger and larger

$$\lim_{x \rightarrow -\infty} \frac{1}{x} = 0$$

Example #2 - Evaluate each of the following limits:

$$\begin{aligned} \text{a) } \lim_{x \rightarrow \infty} \frac{2x+3}{x-4} \\ &= \lim_{x \rightarrow \infty} \frac{\frac{2x}{x} + \frac{3}{x}}{\frac{x}{x} - \frac{4}{x}} \\ &= \lim_{x \rightarrow \infty} \frac{2 + \frac{3}{x}}{1 - \frac{4}{x}} \\ &= \frac{2+0}{1-0} = 2 \end{aligned}$$

$$\begin{aligned} \text{b) } \lim_{x \rightarrow \infty} \frac{x^2 - 3x + 2}{1 - x^2} \\ &= \lim_{x \rightarrow \infty} \frac{\frac{x^2}{x^2} - \frac{3x}{x^2} + \frac{2}{x^2}}{\frac{1}{x^2} - \frac{x^2}{x^2}} \\ &= \lim_{x \rightarrow \infty} \frac{1 - \frac{3}{x} + \frac{2}{x^2}}{\frac{1}{x^2} - 1} \\ &= \frac{1}{-1} = -1 \end{aligned}$$

horizontal asymptote $\begin{cases} \lim_{x \rightarrow \infty} \frac{5x^3 - 7x^2}{x^5} = 0 \\ \lim_{x \rightarrow \infty} \frac{3x^2 + 5x - 7}{2x - 7x^2} = -\frac{3}{7} \end{cases}$

oblique asymptote $\lim_{x \rightarrow \infty} \frac{7x^3 + 3}{5x^2} = \infty$