# Product Rule

## Finding the derivative of $h(x) = f(x)g(x)$

Method #1 - Expand the function first.

- might be worth doing for easy expansions, but for harder expansions, not worth while

Method #2 - Find a new way using **first principles**!

$$h'(x) = \lim_{h\to 0} \frac{h(x+h) - h(x)}{h} = \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x+h)g(x) + f(x+h)g(x) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)[g(x+h) - g(x)] + g(x)[f(x+h) - f(x)]}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)[g(x+h) - g(x)]}{h} + \lim_{h\to 0} \frac{g(x)[f(x+h) - f(x)]}{h}$$

$$= \lim_{h\to 0} f(x+h) \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} + \lim_{h\to 0} g(x) \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= f(x)g'(x) + g(x)f'(x)$$

Differentiate each of the following:

**a)** $h(x) = (x^2+1)(x^2+2)$

$h'(x) = 2x(x^2+2) + 2x(x^2+1)$

$h'(x) = 2x^3 + 4x + 2x^3 + 2x$

$= 4x^3 + 6x = 2x(2x^2+3)$

**b)** $y=(3x-2)^3(2x+5)^5$

$$\frac{dy}{dx}=3(3x-2)^2(3)(2x+5)^5+5(2x+5)^4(2)(3x-2)^3$$

$$\frac{dy}{dx}=9(3x-2)^2(2x+5)^5+10(3x-2)^3(2x+5)^4$$

$$\frac{dy}{dx}=(3x-2)^2(2x+5)^4(9(2x+5)+10(3x-2))$$

$$\frac{dy}{dx}=(3x-2)^2(2x+5)^4(48x+25)$$

## Quotient Rule

Find the derivative of $F(x)=\dfrac{x^2+2x-3}{x^2+1}$

Given $h(x)=\dfrac{f(x)}{g(x)}$, find $h'(x)$

$$f(x) = g(x)h(x)$$

$$f'(x) = g'(x)h(x) + h'(x)g(x)$$

$$h'(x)g(x) = f'(x) - g'(x)h(x)$$

$$h'(x) = \frac{f'(x) - g'(x)h(x)}{g(x)}$$

$$h'(x) = \frac{f'(x) - g'(x)\frac{f(x)}{g(x)}}{g(x)}$$

$$h'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$$

## Example #2

**Differentiate each of the following:**

a) $y = \dfrac{x^2 + 2x - 3}{x^2 + 1}$

$$y' = \frac{(2x+2)(x^2+1) - 2x(x^2+2x-3)}{(x^2+1)^2}$$

$$y' = \frac{2(x+1)(x^2+1) - 2x(x-1)(x+3)}{(x^2+1)^2}$$

$$y' = \frac{2[(x+1)(x^2+1) - x(x^2+2x-3)]}{(x^2+1)^2}$$

$$y' = \frac{2(\cancel{x^3}+x^2+x+1-\cancel{x^3}-2x^2+3x)}{(x^2+1)^2} = \frac{2(4x-x^2+1)}{(x^2+1)^2}$$

$$= -\frac{2(x^2-4x-1)}{(x^2+1)^2}$$

$$\frac{-2(x^2-4x-1)}{(x+1)^2(x-1)^2}$$

$$\frac{4(2x+1)}{(x^2+1)^2}$$

**b)**

$$y=\frac{(x-3)^3}{(5-2x)^5}$$

$$y'=\frac{3(x-3)^2(5-2x)^5-5(5-2x)^4(-2)(x-3)^3}{(5-2x)^{10}}$$

$$y'=\frac{(x-3)^2\cancel{(5-2x)^4}\left[3(5-2x)+10(x-3)\right]}{(5-2x)^{\cancel{10}\,6}}$$

$$y'=\frac{(x-3)^2(4x-15)}{(5-2x)^6}$$

**c)** $y = \dfrac{(2x-1)^3}{(3x+1)^5}$

$$\frac{dy}{dx} = \frac{3(2x-1)^2(2)(3x+1)^5 - 5(3x+1)^4(3)(2x-1)^3}{(3x+1)^{10}}$$

$$\frac{dy}{dx} = \frac{3(2x-1)^2\cancel{(3x+1)^4}\left[2(3x+1) - 5(2x-1)\right]}{(3x+1)^{\cancel{10}\,6}}$$

$$\frac{dy}{dx} = \frac{3(2x-1)^2(7-4x)}{(3x+1)^6}$$