

A function $f(x)$ has a derivative function $F(x) = 2x + 2$.
What is $f(x)$?

$$f(x) = x^2 + 2x + C$$

Integration

An integral is the opposite to a derivative.

Integrating is the opposite operation of differentiating.

$$\text{if } F'(x) = f(x) \text{ then } \int f(x) dx = F(x) + c.$$

Example #1

Find each of the following:

$$\begin{aligned} \text{a) } \int 4x dx \\ = 2x^2 + c \end{aligned}$$

$$\begin{aligned} \text{b) } \int (4x^3 - 6x^2 + 11) dx \\ = x^4 - 2x^3 + 11x + c \end{aligned}$$

$$\begin{aligned} \text{c) } & \int \sqrt{x} dx \\ &= \frac{2}{3} x^{\frac{3}{2}} + C \end{aligned}$$

$$\begin{aligned} \text{d) } & \int \frac{-1}{3x^2} dx \\ &= \frac{1}{3} x^{-1} + C \\ &= \frac{1}{3x} + C \end{aligned}$$

$$\begin{aligned} \text{e) } & \int 2 \sin 2x dx \\ &= -\cos 2x + C \end{aligned}$$

$$\begin{aligned} \text{f) } \int e^{3x} dx \\ = \frac{1}{3} e^{3x} + C \end{aligned}$$

$$\begin{aligned} \text{g) } \int \frac{1}{2x-1} dx \\ = \frac{1}{2} \ln|2x-1| + C \end{aligned}$$

$$\begin{aligned} \text{h) } \int \frac{-6}{\sqrt{1-x^2}} dx \\ = \int -6(1-x^2)^{-\frac{1}{2}} dx \\ = -6 \arcsin x + C \\ \text{or} \\ 6 \arccos x + C \end{aligned}$$

Standard Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$$

$$\int a^x dx = \frac{1}{\ln a} a^x + C$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin\left(\frac{x}{a}\right) + C, |x| < a$$

$$\int \cos x dx = \sin x + C$$

$$\int e^x dx = e^x + C$$