Find the derivative of $y = \ln x$

$$y = \ln x$$

$$e^y = e^{\ln x}$$

$$e^y = x$$

$$e^y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{e^y}$$

$$\frac{dy}{dx} = \frac{1}{x}$$

Example #4

Differentiate

a) $y = \ln 2x$

$$\frac{dy}{dx} = \frac{1}{\cancel{2}x}(\cancel{2})$$

$$\frac{dy}{dx} = \frac{1}{x}$$

b) $y = \ln 5x$

$$\frac{dy}{dx} = \frac{1}{\cancel{5}x}(\cancel{5})$$

$$\frac{dy}{dx} = \frac{1}{x}$$

c) $y = 2\ln x$

$$\frac{dy}{dx} = \frac{2}{x}$$

$$y = \ln kx$$

$$\frac{dy}{dx} = \frac{1}{x}$$

d) $y = x\ln x$

$$\frac{dy}{dx} = \ln x + \frac{x}{x}$$

$$\frac{dy}{dx} = \ln x + 1$$

e) $y = \ln\left(x^2 + x + 5\right)$

$$\frac{dy}{dx} = \frac{1}{x^2+x+5}\left(2x+1\right)$$

$$\frac{dy}{dx} = \frac{2x+1}{x^2+x+5}$$

f) $y = \left(\ln x\right)^4$

$$\frac{dy}{dx} = 4(\ln x)^3\left(\frac{1}{x}\right)$$

$$\frac{dy}{dx} = \frac{4(\ln x)^3}{x}$$

g) $y = \ln\left(\dfrac{x}{\sqrt{x+1}}\right)$

$$\frac{dy}{dx} = \frac{1}{\frac{x}{\sqrt{x+1}}}\left(\frac{\sqrt{x+1} - \frac{1}{2}(x+1)^{-\frac{1}{2}}(x)}{x+1}\right)$$

$$\frac{dy}{dx} = \frac{\sqrt{x+1}}{x}\left(\frac{\frac{1}{2}(x+1)^{-\frac{1}{2}}\left(2(x+1) - x\right)}{x+1}\right)$$

$$\frac{dy}{dx} = \frac{\cancel{\sqrt{x+1}}}{x} \times \frac{\cancel{2x}+2-\cancel{x}}{2\cancel{\sqrt{x+1}}(x+1)}$$

$$\frac{dy}{dx} = \frac{x+2}{2x(x+1)}$$

$$\boxed{\begin{aligned} y &= \ln(x) - \ln(\sqrt{x+1}) \\ y &= \ln(x) - \tfrac{1}{2}\ln(x+1) \\ y &= \frac{1}{x} - \frac{1}{2}\left(\frac{1}{x+1}\right) \\ y &= \frac{1}{x} - \frac{1}{2(x+1)} \\ y &= \frac{2x+2-x}{2x(x+1)} = \frac{x+2}{2x(x+1)} \end{aligned}}$$

Better

Example #5

Find the equation of the tangent to $f(x) = \frac{\ln x}{x}$ $= \ln x \, x^{-1}$

where $x = e$.

$$f'(x) = \frac{1}{x} x^{-1} + (-1) x^{-2} \ln x$$

$$= \frac{1}{x^2} - \frac{\ln x}{x^2} = \frac{1 - \ln x}{x^2}$$

$$f'(e) = \frac{1 - \ln e}{e^2} = \frac{1-1}{e^2} = 0$$

$$f(e) = \frac{\ln e}{e} = \frac{1}{e}$$

$$y = ax + b$$

$$\frac{1}{e} = b$$

$$y = \frac{1}{e}$$