Find $y = f(x)$ given that $\frac{dy}{dx} = 3x^2 - 4$ and $f(-1) = 2$.

$y = \int (3x^2 - 4)\, dx$

$y = x^3 - 4x + c$

$2 = -1 + 4 + c$

$c = -1$

$y = x^3 - 4x - 1$

More Integration

Example #1

Find an expression for y given that $\dfrac{dy}{dx} = (1 - e^x)^2$, and that the graph has y-intercept 4.

$$y = \int (1 - e^x)^2 \, dx$$

$$y = \int (1 - 2e^x + e^{2x}) \, dx$$

$$y = x - 2e^x + \frac{1}{2}e^{2x} + c$$

$$4 = 0 - 2 + \frac{1}{2} + c$$

$$c = \frac{11}{2}$$

$$\therefore \; y = x - 2e^x + \frac{1}{2}e^{2x} + \frac{11}{2}$$

Example #2

Find:

a) $\int 3^{2x-1}\,dx$

$\int (e^{\ln 3(2x-1)})\,dx$

$= \frac{1}{2\ln 3}(3^{2x-1}) + C$

$= \frac{3^{2x-1}}{2\ln 3} + C$

trig identity

b) $\int \sin^2 x\,dx$

$\cos 2x = 1 - 2\sin^2 x$

$2\sin^2 x = 1 - \cos 2x$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$\int \left(\frac{1}{2} - \frac{\cos 2x}{2}\right) dx$

$= \frac{x}{2} - \frac{\sin 2x}{4} + C$

$= \frac{2x - \sin 2x}{4} + C$

ln

Long division

c) $\int \frac{1}{2x-1}\,dx$

$= \frac{\ln|2x-1|}{2} + c$

d) $\int \frac{3x-1}{x+2}\,dx.$

$$\begin{array}{r} 3 \\ x+2 \overline{) 3x-1} \\ 3x+6 \\ \hline -7 \end{array}$$

$\frac{3x-1}{x+2} = 3 + \frac{-7}{x+2}$

$\int \left(3 - \frac{7}{x+2}\right) dx$

$= 3x - 7\ln|x+2| + c$

complete the square

e) $\int \frac{3}{(1-x)\ln 2}\,dx$

$= -\frac{3}{\ln 2}\left(\ln|1-x|\right) + C$

$= -\frac{3\ln|1-x|}{\ln 2} + C$

f) $\int \frac{-2}{\sqrt{-x^2+10x-24}}\,dx$

$= \int \left(\frac{-2}{\sqrt{-((x-5)^2-1)}}\right) dx$

$= \int \left(\frac{-2}{\sqrt{1-(x-5)^2}}\right) dx = -2\int \left(\frac{1}{\sqrt{1-(x-5)^2}}\right) dx$

$= -2\arcsin(x-5) + C$