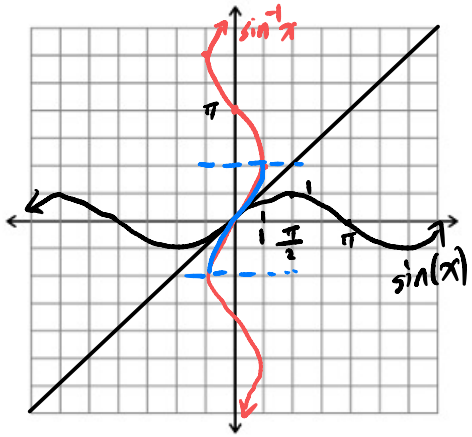
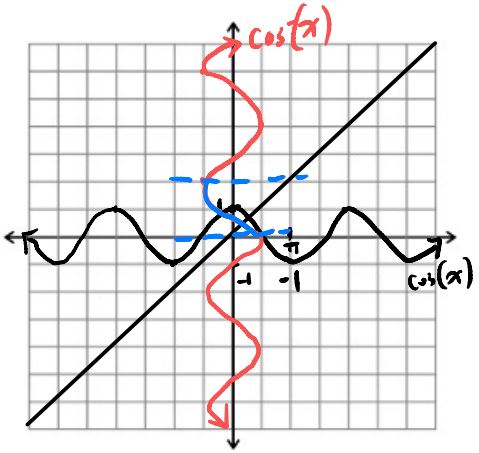
Graph each of the following curves and their inverses.

a) y = sin x



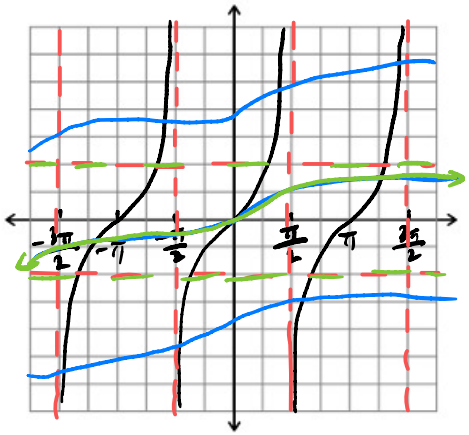
y = arcsin x

b) y = cos x



y = arccos x

c) y = tan x



y = arctan x

The **inverse trigonometric functions** are defined as:

Function	*Definition*	*Range*
$y = \arcsin x$	$x = \sin y, \quad -1 \leqslant x \leqslant 1$	$-\frac{\pi}{2} \leqslant y \leqslant \frac{\pi}{2}$
$y = \arccos x$	$x = \cos y, \quad -1 \leqslant x \leqslant 1$	$0 \leqslant y \leqslant \pi$
$y = \arctan x$	$x = \tan y, \quad x \in \mathbb{R}$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

Example #1

Calculate each of the following:

a) $\arcsin(1)$

$\sin x = 1$

$x = \frac{\pi}{2}$

b) $\arccos(-1)$

$\cos x = -1$

$x = \pi$

c) $\arctan\left(\frac{\sqrt{3}}{1}\right)$

$\tan x = \sqrt{3}$

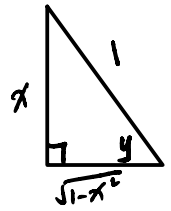
$x = \frac{\pi}{3}$

Derivatives of Inverse Trig Functions

Find the derivatives of each of the following:

a) $y = \arcsin x$

$x = \sin y$



or

$\sin^2 y + \cos^2 y = 1$

$x^2 + \cos^2 y = 1$

$\cos y = \sqrt{1-x^2}$

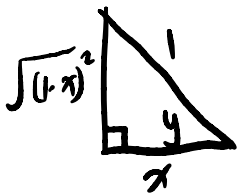
$1 = \cos y \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$

$\frac{d(\arcsin x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

b) $y = \arccos x$

$x = \cos y$



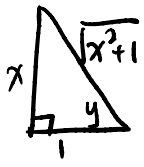
$1 = -\sin y \frac{dy}{dx}$

$\frac{dy}{dx} = -\frac{1}{\sin y} = -\frac{1}{\sqrt{1-x^2}}$

$\frac{d(\arccos x)}{dx} = -\frac{1}{\sqrt{1-x^2}}$

c) $y = \arctan x$

$\tan y = x$



$1 = \sec^2 y \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \cos^2 y = \left(\frac{1}{\sqrt{x^2+1}}\right)^2 = \frac{1}{x^2+1}$

d) $y = \arcsin(5x)$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(5x)^2}}\,(5)$$

$$\frac{dy}{dx} = \frac{5}{\sqrt{1-25x^2}}$$

e) $y = \arccos\left(\frac{x}{2}\right)$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\left(\frac{1}{2}\right)$$

$$\frac{dy}{dx} = -\frac{1}{2\sqrt{1-\frac{x^2}{4}}} = -\frac{1}{2\sqrt{\frac{4-x^2}{4}}} = -\frac{1}{2\left(\frac{1}{2}\right)\sqrt{4-x^2}}$$

$$= -\frac{1}{\sqrt{4-x^2}}$$