When a ball is thrown, its height above the ground is given by $s(t) = 1.2 + 28.1t - 4.9t^2$ metres where t is the time in seconds.

a From what distance above the ground was the ball released? $s(0) = 1.2\text{m}$

b Find $s'(t)$ and state what it represents. $s'(t) = 28.1 - 9.8t \rightarrow$ velocity

c Find t when $s'(t) = 0$. What is the significance of this result? $9.8t = 28.1$, $t = 2.9\text{s}$ It has reached its peak.

d What is the maximum height reached by the ball?

e Find the ball's speed: **i** when released **ii** at $t = 2$ s **iii** at $t = 5$ s. State the significance of the sign of the derivative.

f How long will it take for the ball to hit the ground?

g What is the significance of $\frac{d^2s}{dt^2}$?

d) $s(2.9) = 1.2 + 28.1 \times 2.9 - 4.9 \times 2.9^2 = 41.5\text{s}$

e) $s'(0) = 28.1\text{ms}^{-1}$ $s'(2) = 8.5\text{ms}^{-1}$ $s'(5) = -20.9\text{ms}^{-1}$

f) $0 = 1.2 + 28.1t - 4.9t^2$

$$t = \frac{-28.1 \pm \sqrt{28.1^2 + 1.2 \times 4 \times 4.9}}{-9.8}$$

$t = -0.04$ and $\boxed{5.8\text{s}}$

g) $\frac{d^2s}{dt^2} = v'(t) = a(t)$ acceleration

RATES OF CHANGE

A **rate** is a comparison between two quantities with different units.

There are countless quantities in the real world that vary with time.

For example:
- temperature varies continuously
- the height of a tree varies as it grows
- the prices of stocks and shares vary with each day's trading.

Varying quantities can be modelled using functions of time.

For example, we could use:

- $s(t)$ to model the distance travelled by a runner
- $H(t)$ to model the height of a person riding in a Ferris wheel
- $C(t)$ to model the capacity of a person's lungs, which changes when the person breathes.

The quantity of a chemical in human skin which is responsible for its 'elasticity' is given by $Q = 100 - 10\sqrt{t}$ where t is the age of a person in years.

a Find Q at: **i** $t = 0$ **ii** $t = 25$ **iii** $t = 100$ years.

b At what rate is the quantity of the chemical changing at the age of:

i 25 years **ii** 50 years?

c Show that the rate at which the skin loses the chemical is decreasing for all $t > 0$.

a) $Q(0) = 100 - 10\sqrt{0} = 100$

$Q(25) = 100 - 10\sqrt{25} = 50$

$Q(100) = 100 - 10\sqrt{100} = 0$

b) $Q'(t) = -5t^{-\frac{1}{2}}$

$Q'(25) = -5(25)^{-\frac{1}{2}} = -1$ units/year

$Q'(50) = -5(50)^{-\frac{1}{2}} = \frac{-5}{5\sqrt{2}} = -\frac{1}{\sqrt{2}} = -0.71$ units/year

c) $Q'(t)$