Find the sum of the following infinite series:

$$\underbrace{1+\frac{1}{4}}_{\frac{1}{4}}+\underbrace{\frac{1}{16}}_{\frac{1}{4}}+\underbrace{\frac{1}{64}}_{\frac{1}{4}}+\ldots$$

$$S=\frac{1}{1-\frac{1}{4}}$$

$$S=\frac{1}{\frac{3}{4}}=\frac{4}{3}$$

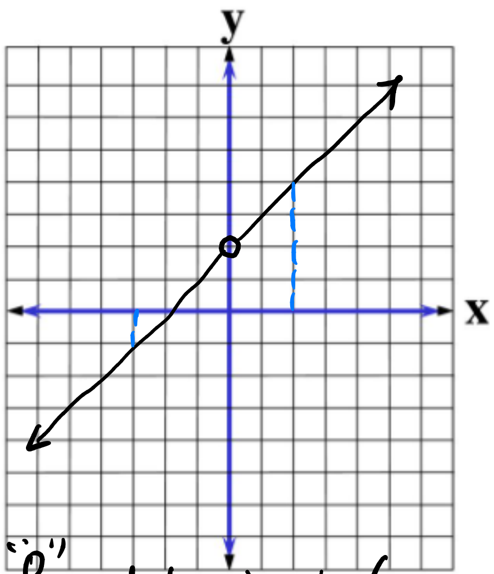
Limit

If $f(x)$ is as close as we like to some real number A for all x sufficiently close to (but not equal to) a, then we say that $f(x)$ has a **limit** of A as x approaches a, and we write $\lim_{x \to a} f(x) = A$.

Example #1

a) Graph the function $y = \dfrac{x^2 + 2x}{x} = \dfrac{\cancel{x}(x+2)}{\cancel{x}}$

$x \neq 0$



b) Evaluate the following limits:

i) $\lim_{x \to 2} \dfrac{x^2 + 2x}{x} = \dfrac{2^2 + 2 \times 2}{2} = 4$

ii) $\lim_{x \to -3} \dfrac{x^2 + 2x}{x} = \dfrac{(-3)^2 + 2 \times (-3)}{-3}$

$= \dfrac{9-6}{-3} = -1$

iii) $\lim_{x \to 0} \dfrac{x^2 + 2x}{x} = \dfrac{\text{“}0\text{”}}{0}$ indeterminant form

$= \lim_{x \to 0} \dfrac{\cancel{x}(x+2)}{\cancel{x}} = 2$

THE LIMIT LAWS

The following **limit laws** will be useful in our study of limits:

Consider functions $f(x)$ and $g(x)$ for which $\lim\limits_{x \to a} f(x) = l$ and $\lim\limits_{x \to a} g(x) = m$, where $a, l, m \in \mathbb{R}$.

- $\lim\limits_{x \to a} (f(x) \pm g(x)) = l \pm m$
- $\lim\limits_{x \to a} f(x)\, g(x) = lm$
- $\lim\limits_{x \to a} \left(\dfrac{f(x)}{g(x)}\right) = \dfrac{l}{m}$ provided $m \neq 0$

Example #2

Evaluate each of the following limits.

a) $\lim\limits_{x \to 2} \left(2xe^x\right)$

$$= \lim_{x \to 2} (2x) \lim_{x \to 2} (e^x)$$

$$= 4e^2$$

b) $\lim\limits_{x \to -1} \dfrac{x+3}{3x^2}$

$$= \frac{\lim\limits_{x \to -1} (x+3)}{\lim\limits_{x \to -1} (3x^2)}$$

$$= \frac{2}{3}$$

Example #3

Evaluate each of the following limits:

a) $\lim_{x\to 5}\left(x^2+2x+3\right)$

$= 5^2+2\times5+3$

$= 25+10+3 = 38$

b) $\lim_{x\to 2}\frac{x^3-8}{x^2-3x+2}$

$= \lim_{x\to 2}\frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}(x-1)}$

$= \frac{2^2+2\times2+4}{2-1}$

$= \frac{4+4+4}{1} = 12$

$$\begin{array}{r} x^2+2x+4 \\ x-2\,\big)\,\overline{x^3-8} \\ x^3-2x^2 \\ \hline 2x^2-8 \\ 2x^2-4x \\ \hline 4x-8 \end{array}$$