Differentiate each of the following:

**a)** $y = x^2 + 4x$

$\frac{dy}{dx} = 2x + 4$

**b)** $y = 3x^5 - 6x^4 + 2$

$\frac{dy}{dx} = 15x^4 - 24x^3$

**c)** $f(x) = x^2 - \frac{2}{x^2}$

$f'(x) = 2x + 4x^{-3}$

**d)** $g(x) = \sqrt{x} + \sqrt[3]{x} + \sqrt[5]{x}$

$g'(x) = \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{3}x^{-\frac{2}{3}} + \frac{1}{5}x^{-\frac{4}{5}}$

$= \frac{1}{2\sqrt{x}} + \frac{1}{3x^{\frac{2}{3}}} + \frac{1}{5x^{\frac{4}{5}}}$

# A composite function has notation:

$$(f \circ g)(x) = f(g(x))$$ - f evaluated at $g(x)$

f composite g

**Example #1**

Given $f : x \mapsto 2x + 1$ and $g : x \mapsto 3 - 4x$ find in simplest form:

**a** $(f \circ g)(x)$

$= f(g(x))$

$= f(3-4x)$

$= 2(3-4x)+1$

$= 6-8x+1$

$= 7-8x$

**b** $(g \circ f)(x)$

$= g(f(x))$

$= g(2x+1)$

$= 3-4(2x+1)$

$= 3-8x-4$

$= -1-8x$

## Example #2

Given $y=(3x-2)^2$ $f(g(x))$

State: a) $f(x)$

$f(x)=x^2$

b) $g(x)$

$g(x)=3x-2$

If $h(x) = f(g(x))$ then $h'(x) = f'(g(x))g'(x)$

# Example #1

Differentiate each of the following:

a) $f(x) = (3x^2 - 5)^5$

$f'(x) = 5(3x^2-5)^4(6x)$

$f'(x) = 30x(3x^2-5)^4$

b) $f(x) = \sqrt{2x^2 + 3} = (2x^2+3)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(2x^2+3)^{-\frac{1}{2}}(4x)$

$f'(x) = \dfrac{2x}{\sqrt{2x^2+3}}$

# Leibniz's Rule

If $y = f(u)$ where $u = u(x)$ then $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$.

$y = f(x) = \left(3x^2 - 5\right)^5$

let $u = 3x^2 - 5$

$y = u^5$

$\dfrac{dy}{du} = 5u^4 \qquad \dfrac{du}{dx} = 6x$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 5u^4(6x) = 5(3x^2-5)^4 6x$$

$$= 30x(3x^2-5)^4$$

# Example #2

Find the gradient of the tangent to $y = 3(2x+1)^4$

when x = 2

$$\frac{dy}{dx} = 12(2x+1)^3(2)$$

$$\frac{dy}{dx} = 24(2x+1)^3$$

$$\frac{dy}{dx} = 24(2\times 2+1)^3 = 24\times 5^3 = 24\times 125 = 3000$$