Find the derivative of

a) $y = \log_3 x$

$$y = \frac{\ln(x)}{\ln(3)} = \ln(x)\ln(3)$$

$$\frac{dy}{dx} = \frac{1}{\ln(3)}\frac{1}{x} = \frac{1}{x\ln(3)}$$

b) $y = \log_5\left(4x^2+3\right)$

$$y = \frac{\ln(4x^2+3)}{\ln(5)}$$

$$\frac{dy}{dx} = \frac{1}{\ln(5)}\left(\frac{1}{4x^2+3}\right)(8x)$$

$$\frac{dy}{dx} = \frac{8x}{\ln(5)(4x^2+3)}$$

4. a) $y = -2\log_3(x)$

$$y' = -\frac{2}{x\ln 3}$$

b) $y = \log_2[(x+5)(x-1)]$

$$y' = \log_2(x+5) + \log_2(x-1)$$

$$y' = \frac{1}{\ln 2(x+5)} + \frac{1}{\ln 2(x-1)}$$

Use your calculator (use radians) to evaluate each of the following limits

a) $\lim_{\theta \to 0} \frac{\sin \theta}{\theta}$

$= 1$

b) $\lim_{\theta \to 0} \frac{\cos \theta - 1}{\theta}$

$= 0$

3.2	Identities	$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
	Pythagorean identities	$\cos^2\theta + \sin^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$
3.3	Compound angle identities	$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$ $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$ $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$
	Double angle identities	$\sin 2\theta = 2\sin\theta\cos\theta$ $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$ $\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

Derivatives of sinx and cosx

Example #1

Find the derivative of $f(x) = \sin x$ using first principles.

$$f'(x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x(\cos h - 1) + \cos x \sin h}{h}$$

$$= \lim_{h \to 0} \sin x \lim_{h \to 0} \frac{\cos h - 1}{h} + \lim_{h \to 0} \cos x \lim_{h \to 0} \frac{\sin h}{h}$$

$$= \sin x(0) + \cos x(1)$$

$$= \cos x$$

$$\boxed{\frac{d \sin x}{dx} = \cos x}$$

Example #2

Find the derivative of $f(x) = \cos x = \sin\left(x + \frac{\pi}{2}\right)$

$= \sin x \cos \frac{\pi}{2} + \cos x \sin \frac{\pi}{2}$

$f'(x) = \cos\left(x + \frac{\pi}{2}\right)$

$f'(x) = \cos x \cos \frac{\pi}{2} - \sin x \sin \frac{\pi}{2}$

$f'(x) = -\sin x$

Example #3

Differentiate

a) $y = \tan x = \frac{\sin x}{\cos x} = \sin x(\cos x)^{-1}$

$y' = \cos x(\cos x)^{-1} - (\cos x)^{-2}(-\sin x)(\sin x)$

$y' = 1 + \frac{\sin^2 x}{\cos^2 x}$

$y' = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$

$$\boxed{\frac{d \tan x}{dx} = \sec^2 x}$$

b) $y = \cot x = \frac{\cos x}{\sin x}$

$y' = \frac{(-\sin x)(\sin x) - \cos x(\cos x)}{\sin^2 x}$

$y' = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$

$y' = -\frac{1}{\sin^2 x} = -\csc^2(x) = -\operatorname{cosec}^2(x)$

$$\boxed{\frac{d \cot x}{dx} = -\csc^2 x}$$

c) $y = \sin 2x$

$y' = 2\cos 2x$

d) $y = \cos^2 x$

$y' = 2(\cos x)(-\sin x)$

$y' = -2\sin x \cos x$

$y' = -\sin(2x)$

e) $y=\sin^3(3x-1)$

$y' = 3\sin^2(3x-1)[\cos(3x-1)](3)$

$y' = 9\sin^2(3x-1)\cos(3x-1)$

f) $y = x^2\csc^2 x = \dfrac{x^2}{\sin^2 x}$

$$y' = \frac{2x\sin^2 x - (2\sin x\cos x)x^2}{\sin^4 x}$$

$$y' = \frac{2x\cancel{\sin x}(\sin x - x\cos x)}{\sin^{\cancel{4}\,3} x}$$

$$y' = \frac{2x(\sin x - x\cos x)}{\sin^3 x}$$