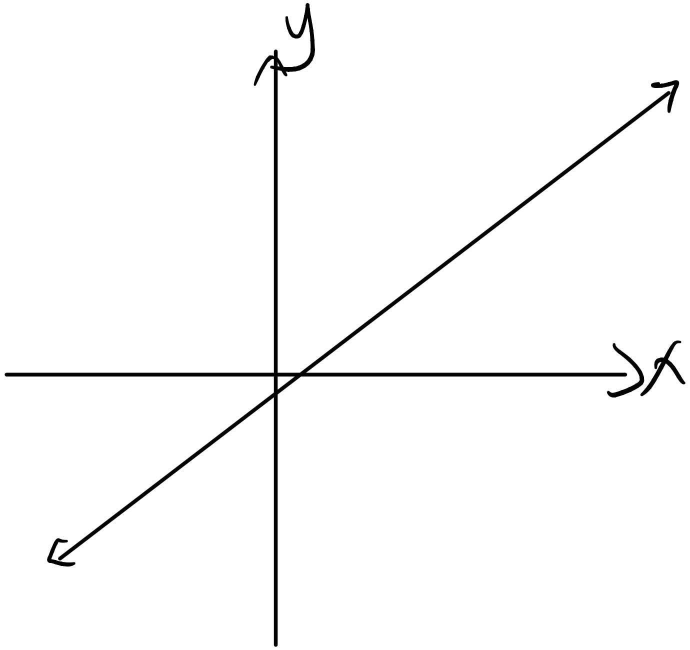
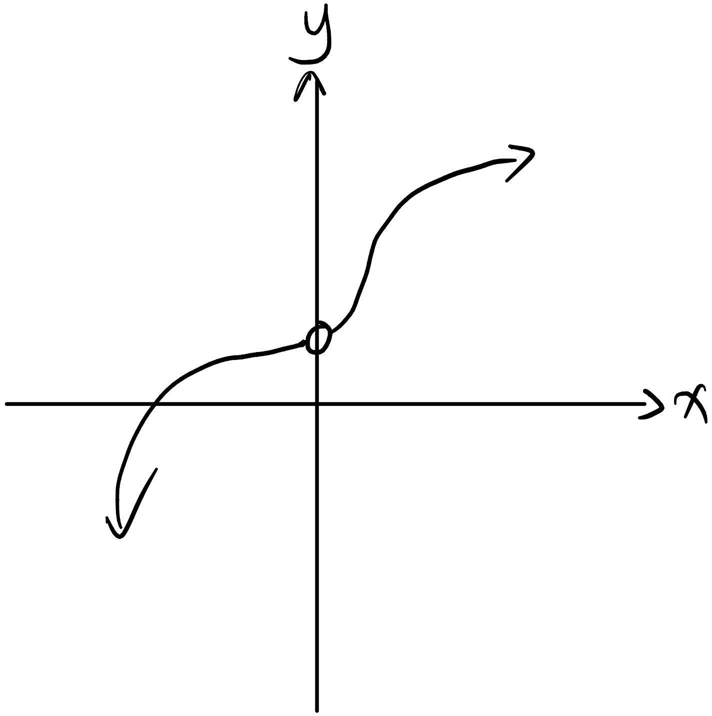
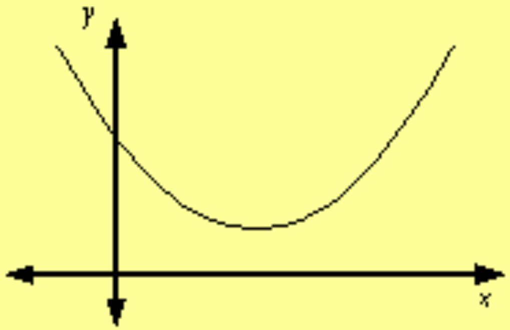
Draw a function that we would call continuous.

Draw a function that we would call discontinuous.

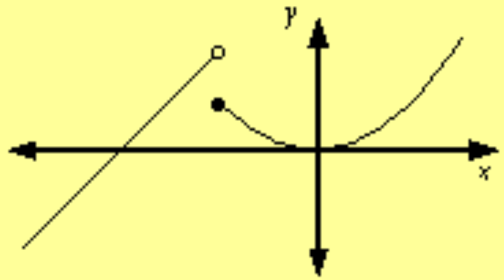




Continuous Function



Discontinuous Function



Definition of Continuity: A function is continuous at $x = a$ if

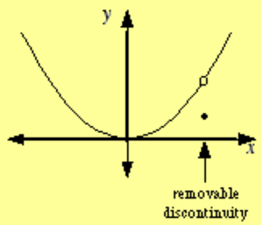
1. $\lim_{x \to a} f(x)$ exists
2. $f(a)$ exists
3. $\lim_{x \to a} f(x) = f(a)$

fails removable continuity

Removable Discontinuity
Hole

A hole in a graph. That is, a discontinuity that can be "repaired" by filling in a single point. In other words, a removable discontinuity is a point at which a graph is not connected but can be made connected by filling in a single point.

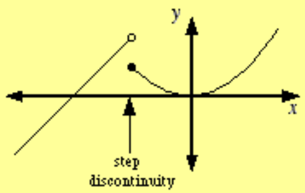
Formally, a removable discontinuity is one at which the limit of the function exists but does not equal the value of the function at that point; this may be because the function does not exist at that point.



Essential Discontinuity

Any discontinuity that is not removable. That is, a place where a graph is not connected and cannot be made connected simply by filling in a single point. Step discontinuities and vertical asymptotes are two types of essential discontinuities.

Formally, an essential discontinuity is a discontinuity at which the limit of the function does not exist.



Example #1

Is $f(x)=\dfrac{x^2-3x-4}{x-4}$ continuous at x=4? Explain.

$$\lim_{x\to 4}\frac{\cancel{(x-4)}(x+1)}{\cancel{x-4}}$$

$$=\lim_{x\to 4} x+1$$

$$=4+1=5$$

f(4) doesn't exist.

Example #2

Is $f(x) = \begin{cases} x - 3, & x > 0 \\ 1, & x \leq 0 \end{cases}$ continuous? Explain.

$\lim_{x \to 0^-} f(x) = 1 \qquad \lim_{x \to 0^+} f(x) = -3$

$\because 1 \neq -3$

$\therefore$ not continuous at $x = 0$.

Example #3

Find the value of c that makes f(x) continuous?

$$f(x) = \begin{cases} x^2 - c^2, & x < 4 \\ cx + 20, & x \geq 4 \end{cases}$$

$$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x) = f(4)$$

$$16 - c^2 = 4c + 20$$

$$c^2 + 4c + 4 = 0$$

$$(c+2)^2 = 0$$

$$c = -2$$