Find and classify all points of inflection for:

$$y = \frac{x^2}{1-x^2}$$

$$y' = \frac{2x(1-x^2) + (2x)x^2}{(1-x^2)^2} = \frac{(2x)(1-x^2+x^2)}{(1-x^2)^2}$$

$$= \frac{2x}{(1-x^2)^2}$$

$$y'' = \frac{2(1-x^2)^2 - 2x(2)(1-x^2)(-2x)}{(1-x^2)^4}$$

$$y'' = \frac{2\cancel{(1-x^2)}(1-x^2+4x^2)}{(1-x^2)^{\cancel{4}3}}$$

$$y'' = \frac{2(1+3x^2)}{(1-x^2)^3} \qquad x \neq \pm 1$$

$1+3x^2 = 0$

$3x^2 = 0$

$x^2 = -\frac{1}{3}$

no solution

$\therefore$ no pts of inflections

$-$ $+$ $-$ $\rightarrow x$

$\cap$ -1 $\cup$ 1 $\cap$

concave up $-1 < x < 1$

Concave down $x < -1$ and $x > 1$

Types of Asymptotes

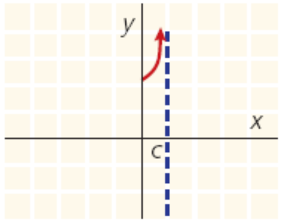
Vertical Asymptotes

A rational function of the form $f(x) = \frac{p(x)}{q(x)}$ has a vertical asymptote $x = c$ if $q(c) = 0$ and $p(c) \neq 0$.

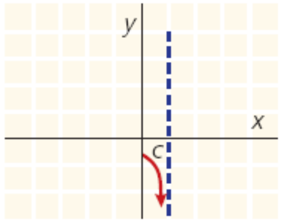
Vertical Asymptotes
The graph of $f(x)$ has a vertical asymptote $x = c$ if one of the following limit statements is true:

$$\lim_{x \to c^-} f(x) = +\infty \quad \lim_{x \to c^-} f(x) = -\infty \quad \lim_{x \to c^+} f(x) = +\infty \quad \lim_{x \to c^+} f(x) = -\infty$$

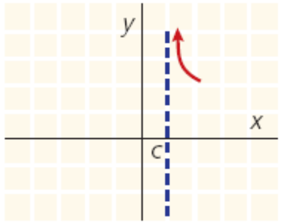
The following graphs correspond to each limit statement.



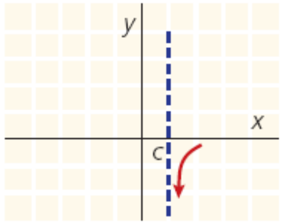
$\lim_{x \to c^-} f(x) = +\infty$



$\lim_{x \to c^-} f(x) = -\infty$



$\lim_{x \to c^+} f(x) = +\infty$



$\lim_{x \to c^+} f(x) = -\infty$

Example#1

State the vertical asymptotes for $f\left(x\right)=\dfrac{x}{x^2+x-2}$

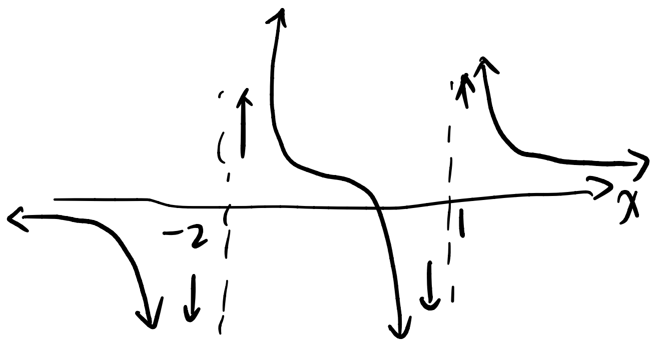
$$\frac{x}{(x+2)(x-1)}$$

$(x+2)(x-1)=0$

$x=-2,\ 1$

vertical asymptotes: $x=-2$, $x=1$

$\lim\limits_{x\to -2^+} f(x)=\infty$ $\quad \lim\limits_{x\to -2^-} f(x)=-\infty$ $\quad \lim\limits_{x\to 1^+} f(x)=+\infty$ $\quad \lim\limits_{x\to 1^-}=-\infty$



Horizontal Asymptotes

- occur when the degree of the numerator is less than or equal to the degree of the denominator

$$\lim_{x \to +\infty} \frac{1}{x} = 0 \quad \text{and} \quad \lim_{x \to -\infty} \frac{1}{x} = 0$$

Horizontal Asymptotes

If $\lim_{x \to +\infty} f(x) = L$ or $\lim_{x \to -\infty} f(x) = L$, we say that the line $y = L$ is a horizontal asymptote of the graph of $f(x)$.

Example#2

State the horizontal asymptotes for $f(x)=\dfrac{x}{x^2+x-2}$

$$\lim_{x\to\infty}\frac{x}{x^2+x-2}=0$$

$$y=\frac{x^2+1}{3x^2+2} \quad H.A:\ y=\frac{1}{3}$$

Oblique Asymptote

- straight lines that functions approach infinitely closer and are not parallel to either axes
- occur when the degree of the numerator is larger than the degree of the denominator

Example #3

Find the oblique asymptote of the function $f(x)=\dfrac{x^3-2}{x^2+1}$

$$\begin{array}{r} x \\ x^2+1 \overline{)\, x^3-2} \\ \underline{x^3+x} \\ -x-2 \end{array}$$

oblique asymptote is $y=x$

$$y=\frac{(x^2+1)x-x-2}{x^2+1}$$

$$y=x-\frac{x+2}{x^2+1}$$

$$\lim_{x\to\infty}\frac{x+2}{x^2+1}=0$$

$$y=x$$

Example #4

$\frac{1}{2}$

Determine the equations of all asymptotes of the graph of $f(x)=\dfrac{2x^2+3x-1}{x+1}$

vertical asymptote: $x=-1$

$\lim_{x\to -1^+} f(x) = +\infty$

$\lim_{x\to -1^-} f(x) = -\infty$

oblique

$$\begin{array}{r} 2x+1 \\ x+1 \overline{\big) 2x^2+3x-1} \\ \underline{2x^2+2x} \\ x-1 \\ \underline{x+1} \\ -2 \end{array}$$

$y=2x+1$

horizontal asymptote: $\lim_{x\to\infty} f(x) = \infty$

none