Use first principles to find the slope of the tangent of y = 3x² - 4 at any point.

$(x, 3x^2 - 4)$

$(x+h, 3(x+h)^2 - 4)$

$$f'(x) = \lim_{h \to 0} \frac{3(x+h)^2 - 4 - 3x^2 + 4}{h}$$

$$= \lim_{h \to 0} \frac{3(x^2 + 2hx + h^2) - 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(6x + 3h)}{\cancel{h}}$$

$$= 6x + 3 \times 0 = 6x$$

Some Derivative Notes:

slope of the tangent

If we are given a function $f(x)$ then $f'(x)$ represents the derivative function.

If we are given y in terms of x then we use the **Leibniz notation** $\frac{dy}{dx}$ to represent the derivative.

$\frac{dy}{dx}$ reads "dee y by dee x" or "the derivative of y with respect to x".

If a function is not continuous at $x = a$, then it is not differentiable at $x = a$.

$f'(x)$, y', $\frac{dy}{dx}$ — limit has to exist at $x=a$

$\frac{d(k)}{dx} = 0$ k is a constant

Homework

Differentiate each of the following using first principles:

a) $y = x^2 - 3x + 5$

b) $y = \sqrt{x-1}$

c) $y = \frac{-2}{x-3}$

d) $y = x^n$

a) $\frac{dy}{dx} = \lim_{h\to 0} \frac{(x+h)^2 - 3(x+h) + 5 - x^2 + 3x - 5}{h}$

$= \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{3x} - 3h + \cancel{5} - \cancel{x^2} + \cancel{3x} - \cancel{5}}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(2x+h-3)}{\cancel{h}}$

$= 2x - 3$

b) $\frac{dy}{dx} = \lim_{h\to 0} \frac{\sqrt{x+h-1} - \sqrt{x-1}}{h}$

$= \lim_{h\to 0} \frac{\cancel{x}+h\cancel{-1}\cancel{-x}\cancel{+1}}{h(\sqrt{x+h-1} + \sqrt{x-1})}$

$= \lim_{h\to 0} \frac{1}{\sqrt{x+h-1} + \sqrt{x-1}}$

$= \frac{1}{2\sqrt{x-1}}$

c) $\frac{dy}{dx} = \lim_{h\to 0} \frac{\frac{-2}{x+h-3} - \frac{-2}{x-3}}{h}$

$= \lim_{h\to 0} \frac{\frac{\cancel{-2x}\cancel{+6} - (\cancel{-2x} - 2h \cancel{+6})}{(x+h-3)(x-3)}}{\cancel{h}}$

$= \lim_{h\to 0} \frac{2}{(x+h-3)(x-3)}$

$= \frac{2}{(x-3)^2}$

d) $\frac{dy}{dx} = \frac{(x+h)^n - x^n}{h}$

$= \lim_{h\to 0} \frac{\cancel{x^n} + n\cancel{h}x^{n-1} + \binom{n}{2} h^{\cancel{2}} x^{n-2} \dots - \cancel{x^n}}{\cancel{h}}$

$= \lim_{h\to 0} nx^{n-1} + \frac{n(n-1)}{2} h x^{n-2} \dots = nx^{n-1}$

General Rule for Differentiation

If $f(x) = x^n$, then $f'(x) = nx^{n-1}$

Find the derivative for each of the following functions:

a) $f(x) = -5x$

$f'(x) = -5$

b) $f(x) = 3$

$f'(x) = 0$

c) $f(x) = 10x^5$

$f'(x) = 50x^4$

d) $f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

e) $f(x) = 3x^{\frac{1}{3}}$

$f'(x) = x^{-\frac{2}{3}}$

$= \frac{1}{x^{\frac{2}{3}}}$

f) $f(x) = 2x^3 - 5x + 3$

$f'(x) = 6x^2 - 5$