Find the equation of the line that is tangent to the curve $y = x^2$ when $x = 1$

$y' = 2x$

$y' = 2 \cdot 1 = 2$

$y = 2x + b$

$y = 1^2 = 1$

$(1, 1)$

$1 = 2 + b$

$b = -1$

$\boxed{y = 2x - 1}$

Find the equation of the normal to the curve when $x = 1$.

$y = -\frac{1}{2}x + b$

$1 = -\frac{1}{2} + b$

$b = \frac{3}{2}$

$\boxed{\therefore y = \frac{1}{2}x + \frac{3}{2}}$

Normals and Tangents of Curves

Example #1

The line y = 8x + b is tangent to the curve y = 2x ^{2}.
Determine the point of tangency and the value of *b*.

$y' = 4x$

$4x = 8$

$x = 2$

$y = 2 \times 2^2 = 8$

$(2, 8)$

$y = 8x + b$

$8 = 8 \times 2 + b$

$b = -8$

A **normal** to a curve is a line which is perpendicular to the tangent at the point of contact.

Example #2

Find the equation of the normal to the curve y = f(x) when x = a.

$$y' = f'(x)$$

$$y = f'(a)x + b$$

$$f(a) = f'(a)a + b$$

$$b = f(a) - f'(a)a$$

tangent: $y = f'(a)x + f(a) - f'(a)a = f'(a)(x-a) + f(a)$

$$y = -\frac{1}{f'(a)}x + c$$

$$f(a) = -\frac{1}{f'(a)}a + c$$

$$c = f(a) + \frac{a}{f'(a)}$$

normal: $y = -\frac{1}{f'(a)}x + f(a) + \frac{a}{f'(a)}$

$$y = -\frac{1}{f'(a)}(x-a) + f(a)$$