State the definition of each of the following:

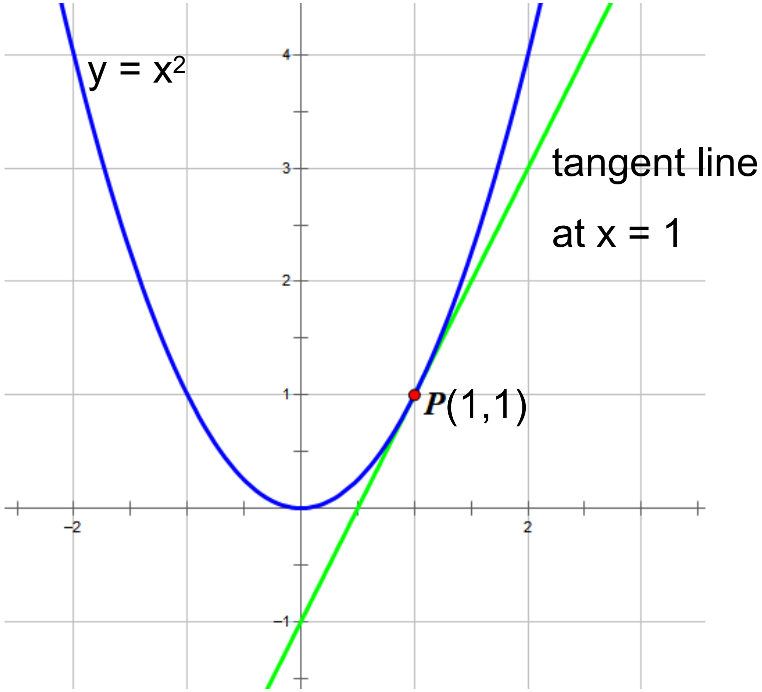
a) a line tangent to a curve

A line that crosses a curve at one local point without cutting the graph

b) a line secant to a curve

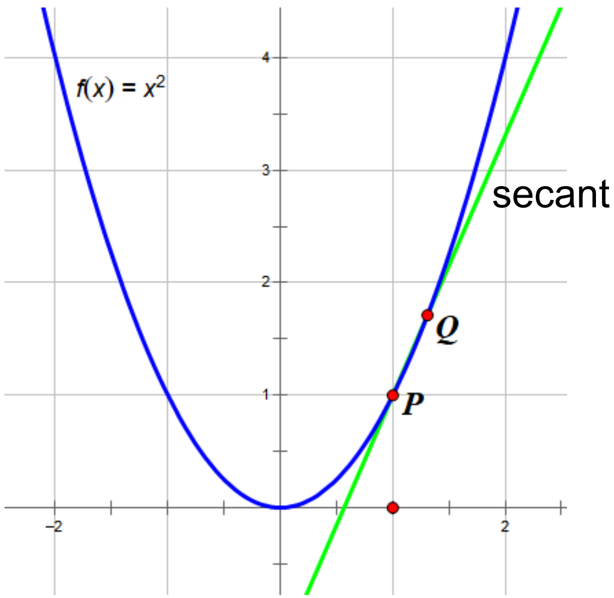
A line that connects two points on a curve.

- two intersection points on a curve



Find the slope of the tangent to the parabola $y = x^2$ when $x = 1$.

- only know 1 point.



$m = 2$

Complete the following chart for each point Q, given P is (1, 1).

Point	Slope
Q(1.5, 2.25)	$m_{PQ} = \frac{2.25-1}{1.5-1} = \frac{1.25}{0.5} = 2.5$
Q(1.1, 1.21)	$m_{PQ} = \frac{1.21-1}{1.1-1} = \frac{0.21}{0.1} = 2.1$
Q(1.01, 1.01^2)	$m_{PQ} = \frac{1.01^2-1}{1.01-1} = \frac{0.0201}{0.01} = 2.01$
Q(1.001, 1.001^2)	$m_{PQ} = \frac{1.001^2-1}{1.001-1} = 2.001$
Q(0.5, 0.5^2)	$m_{PQ} = \frac{0.5^2-1}{0.5-1} = \frac{-0.75}{-0.5} = 1.5$
Q(0.9, 0.81	$m_{PQ} = \frac{0.81-1}{0.9-1} = 1.9$
Q(0.99, 0.99^2	$m_{PQ} = \frac{0.99^2-1}{0.99-1} = 1.99$
Q(0.999, 0.999^2	$m_{PQ} = \frac{0.999^2-1}{0.999-1} = 1.999$

Example #1

Find the slope of the tangent to the parabola y = x^2 when x = 1.

$P(1, 1)$

$Q(1+h, (1+h)^2)$

make h really small

$$m = \lim_{h \to 0} \frac{(1+h)^2 - 1}{1+h-1}$$

$$= \lim_{h \to 0} \frac{1+2h+h^2-1}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2+h)}{\cancel{h}} = 2+0 = 2$$

Example #2

Given the curve $y = x^3$, find the slope of the tangent when $x = 2$.

$(2, 8)$

$(2+h, (2+h)^3)$

$$m = \lim_{h \to 0} \frac{(2+h)^3 - 8}{2+h-2}$$

$$= \lim_{h \to 0} \frac{(2+h)(4+4h+h^2) - 8}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{8} + 8h + 2h^2 + 4h + 4h^2 + h^3 \cancel{-8}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(8 + 2h + 4 + 4h + h^2)}{\cancel{h}}$$

$$= 8 + 0 + 4 + 0 + 0 = 12$$

Example #3

Given the function f(x), find the slope, $f'(x)$, of the tangent when x = a.

$(a, f(a))$

$(a+h, f(a+h))$

$$m = \boxed{f'(x) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

first principles

Example #4

Use first principles to find the slope of the tangent to $y = 5x - 7$ when x is 0.

$(0, -7)$

$(h, 5h-7)$

$$f'(x) = \lim_{h \to 0} \frac{5h - 7 + 7}{h}$$

$$= \lim_{h \to 0} \frac{5h}{h}$$

$$= 5$$