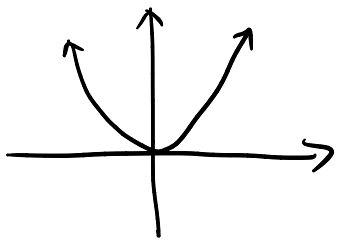
Find the second derivative of

a) $y = x^2$

$y' = 2x$

$y'' = 2$

concave up

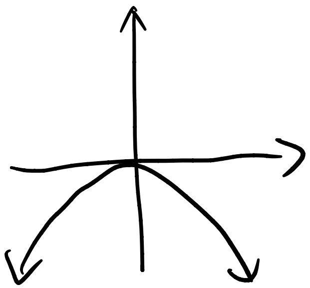


$y'' \geq 0$

b) $y = -x^2$

$y' = -2x$

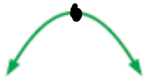
$y'' = -2$



concave down

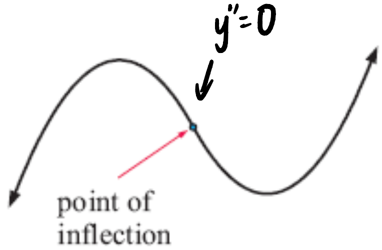
$y'' \leq 0$

Concavity and Points of Inflection



concave downwards

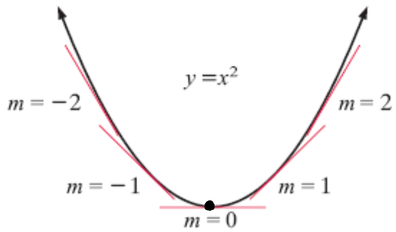
concave upwards



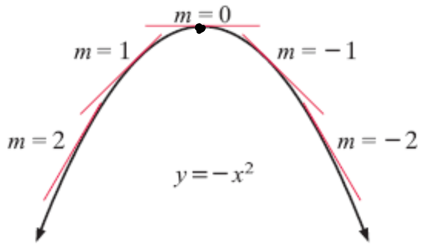
$f(x)$ - positive / negative

$f'(x)$ - increasing / decreasing
max/mins

$f''(x)$ - concavity

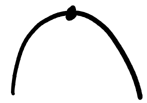


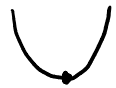
$y''>0$



$y''<0$

1st Derivative Test

If f '(x) = 0 and f "(x) < 0 then f(x) is a maximum. 

If f '(x) = 0 and f "(x) > 0 then f(x) is a minimum. 

Summary

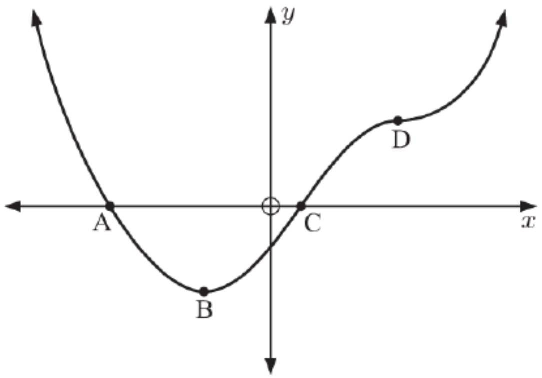
If f(x) = 0, x is a root/intercept/zero.

If f '(x) = 0, x is a stationary point, possible max/min

If f "(x) = 0, x is an inflection point, concave up or down

Example #1

In the diagram alongside, each labelled point corresponds to a zero of $f(x)$, $f'(x)$, or $f''(x)$.



a Complete the table by indicating whether each value is zero, positive, or negative:

Point	$f(x)$	$f'(x)$	$f''(x)$
A	0	negative	positive
B	negative	0	positive
C	0	positive	0
D	positive	0	0

b Describe the turning point of $y = f(x)$.

B- min

c Describe the inflection points of $y = f(x)$.

C, D.

Example #2

Given $y = x^3 - 3x^2 + 4x - 5$

a) Find all points of inflection.

b) Determine where the curve is concave up or down.

a) $y' = 3x^2 - 6x + 4$

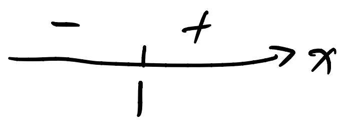
$y'' = 6x - 6$

$0 = 6(x-1)$

$x = 1$ $\qquad 1^3 - 3\times1^2 + 4\times1 - 5 = 1 - 3 + 4 - 5 = -3$

$(1, -3)$

b)



concave up for $x \in \mathbb{R} \mid x \geq 1$

concave down for $x \in \mathbb{R} \mid x \leq 1$

Example #3 $(x^2+1)^{-1}$

Given $f(x)=\dfrac{1}{x^2+1}$, find all points of inflection and state concavity.

$$f'(x) = -(x^2+1)^{-2}(2x) = -\frac{2x}{(x^2+1)^2}$$

$$f''(x) = \frac{-2(x^2+1)^{\cancel{2}} - 2\cancel{(x^2+1)}(2x)(-2x)}{(x^2+1)^{\cancel{4}\,3}}$$

$$f''(x) = \frac{-2(x^2+1-4x^2)}{(x^2+1)^3}$$

$$f''(x) = \frac{-2(1-3x^2)}{(x^2+1)^3}$$

$$0 = \frac{-2(1-3x^2)}{(x^2+1)^3}$$

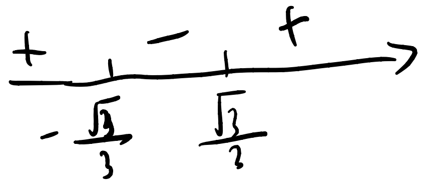
$$x^2 = \frac{1}{3}$$

$$x = \pm\frac{\sqrt{3}}{3}$$

$$y = \frac{1}{\frac{3}{9}+1} = \frac{1}{\frac{12}{9}} = \frac{9}{12} = \frac{3}{4}$$

$\left(-\frac{\sqrt{3}}{3}, \frac{3}{4}\right)$

$\left(\frac{\sqrt{3}}{3}, \frac{3}{4}\right)$



concave up for $x \in \mathbb{R} \mid x \geq \frac{\sqrt{3}}{3}, x \leq -\frac{\sqrt{3}}{3}$

concave down for $x \in \mathbb{R} \mid -\frac{\sqrt{3}}{3} \leq x \leq \frac{\sqrt{3}}{3}$