nth derivative: $f^{(n)}(x)$ / $\frac{d^ny}{dx^n}$ / $y^{(n)}$

Given $f(x) = x^3 - 2x + 5$, find:

a $f(2)$

$= 2^3 - 2\times2+5$

$= 8-4+5 = 9$

b $f'(2)$

$f'(x) = 3x^2 - 2$

$f'(2) = 3\times2^2 - 2$

$= 12 - 2 = 10$

c $f''(2)$

$f''(x) = 6x$

$f''(2) = 6\times2$

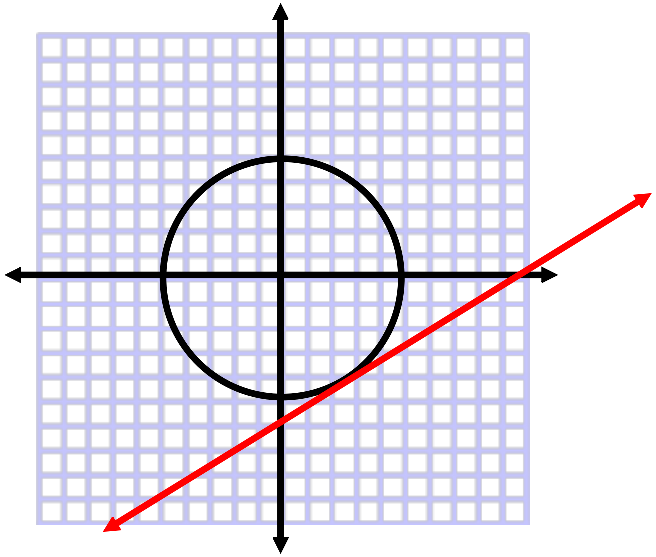
$= 12$

d $f^{(3)}(2)$

$f^{(3)}(x) = 6$

$f^{(3)}(2) = 6$

Find the slope of the tangent to the circle $x^2 + y^2 = 25$ at the point (3, -4).



1. $x^2 + y^2 = 25$

$y^2 = 25 - x^2$

$y = \pm\sqrt{25 - x^2}$

use $y = -\sqrt{25 - x^2} = -(25 - x^2)^{\frac{1}{2}}$

$y' = -\frac{1}{2}(25 - x^2)^{-\frac{1}{2}}(-2x)$

$y'\big|_{x=3} = 3(25 - 9)^{-\frac{1}{2}} = 3 \times \frac{1}{4} = \frac{3}{4}$

2. $\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(25)$

$2x + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{x}{y}$

$\frac{dy}{dx}\bigg|_{\substack{x=3 \\ y=-4}} = -\frac{3}{(-4)} = \frac{3}{4}$

$f'(x, y)$

Implicit Differentiation

$y = 3x + 4$ - defined explicitly in terms of x

$x^2 + y^2 = 25$ - defined implicitly

Example #2

If y is a function of x, find:

a) $\dfrac{d\left(y^3\right)}{dx}$

$= 3y^2 \dfrac{dy}{dx}$

b) $\dfrac{d\left(5y^4\right)}{dx}$

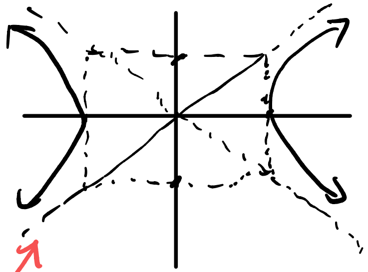
$= 20y^3 \dfrac{dy}{dx}$

c) $\dfrac{d\left(xy\right)}{dx}$

$= y + x\dfrac{dy}{dx}$

Example #3

Find $\frac{dy}{dx}$ for each of the following:



a) $x^2 - y^2 = 25$

$$2x - 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{x}{y}$$

b) $2x^5 + x^4 y + y^5 = 36$

$$10x^4 + 4x^3y + x^4\frac{dy}{dx} + 5y^4\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-(10x^4 + 4x^3y)}{x^4 + 5y^4}$$

$$\frac{dy}{dx} = -\frac{2x^3(5x+2y)}{x^4+5y^4}$$