Complete the square and state the vertex and the maximum/minimum point for:

$$y = x^2 + 2x + 3$$

$$= x^2 + 2x + 1 - 1 + 3$$

$$= (x+1)^2 + 2$$

vertex $(-1, 2)$

Parabola	Complete the Square	Vertex	Maximum or Minimum Y- Value	y'	y' = 0
$y = x^2 - 2x + 5$	$y=(x-1)^2+1$	$(1,5)$	min	$y'=2x-2$	$x=1$
$y = -x^2 + 6x - 3$ $-(x^2-6x)-3$	$y=-(x-3)^2+6$	$(3,6)$	max	$y'=-2x+6$	$x=3$
$y = 3x^2 + 6x - 2$ $3(x^2+2x)-2$	$y=3(x+1)^2-5$	$(-1,-5)$	min	$y'=6x+6$	$x=-1$
$y = 4x^2 - 12x + 7$ $4(x^2-3x)+7$	$y=4(x-1.5)^2-2$	$(1.5,-2)$	min	$y'=8x-12$	$x=\frac{3}{2}$
$y = -x^2 + 2x - 2$ $-(x^2-2x)-2$	$y=-(x-1)^2-1$	$(1,-1)$	max	$y'=-2x+2$	$x=1$
$y = -5x^2 + 10x$ $-5(x^2-2x)$	$y=-5(x-1)^2+5$	$(1,5)$	max	$y'=-10x+10$	$x=1$

A max or a min occurs when $f'(x)=0$

Maximum

A local maximum occurs when an increasing interval is followed by a decreasing interval.

Minimum

A local minimum occurs when a decreasing interval is followed by an increasing interval.

Stationary Points

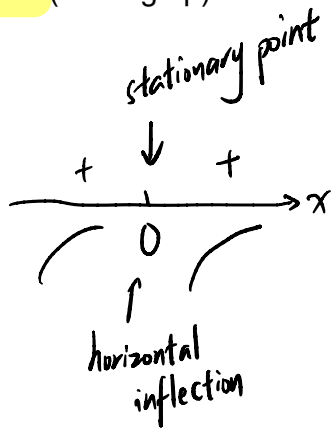
A stationary point occurs when $f'(x) = 0$

Stationary points may be local maximums, local minimums or horizontal (stationary) inflections (coming up).

$y = x^3$

$y' = 3x^2 = 0$

$x = 0$



Example #1

The tangents at points A, O, and B are horizontal.

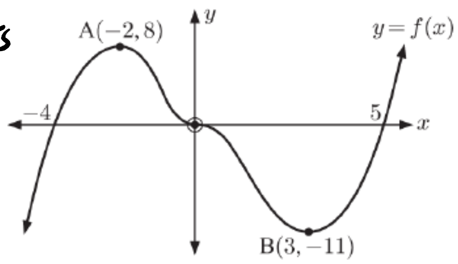
a Classify points A, O, and B. - stationary points

b Draw a sign diagram for the gradient function $f'(x)$ for all x.

c State intervals where $y = f(x)$ is:

  i increasing **ii** decreasing.

d Draw a sign diagram for $f(x)$ for all x.



b) $f'(x)$: + | − | − | + → x (at −2, 0, 3); −2 max, 3 min

c) increasing $x \leq -2$, $x \geq 3$

decreasing $-2 \leq x \leq 3$

d) $f(x)$: − | + | − | + → x (at −4, 0, 5)

Example #2

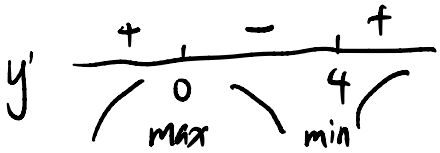
Find all stationary points and classify each for

$$y = x^3 - 6x^2 - 4$$

$y' = 3x^2 - 12x$

$0 = 3x(x-4)$

$x = 0, \quad x = 4 \quad (0,-4), \ (4,-36)$



max: $(0,-4)$

min: $(4,-36)$

Example #3

Find and classify all stationary points of

$$y = 3x^4 + 4x^3 - 12x^2$$

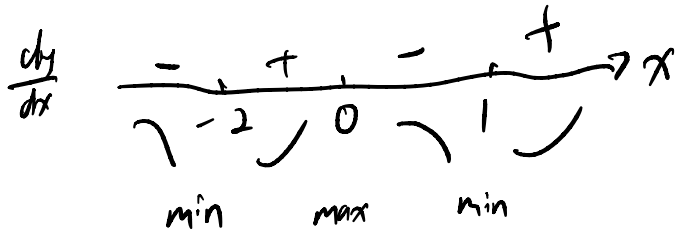
$\frac{dy}{dx} = 12x^3 + 12x^2 - 24x$

$0 = 12x(x^2 + x - 2)$

$0 = 12x(x+2)(x-1)$

$x = 0, -2, 1$

$(0,0), (-2,-32), (1,-5)$



minimum points: $(-2,-32), (1,-5)$

maximum point: $(0,0)$