The radius of a circle is growing at a rate of 2 cm/s. How fast is the area of the circle growing when the radius is 10 cm?

$\frac{dr}{dt} = 2$ $\qquad \frac{dA}{dt} = ?$ $\qquad r = 10cm$ $\qquad \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ → sub $r = 10$, $\frac{dr}{dt} = 2$

$\frac{dA}{dt} = 2\pi (10)(2) = 40\pi \ cm^2/s$

Related Rates

- related rates occur when time is involved in a change of rate

Example #1

Water is pouring into a cylindrical rain barrel with radius of 30 cm at a rate of 500 cm^3/min. How fast is the water level in the barrel rising?

$\frac{dh}{dt} = ?$ $\frac{dV}{dt} = 500\ cm^3/min$ $r = 30cm$

$\pi r^2 = 900\pi\ cm^2$

$V = \pi r^2 h = 900\pi h$

$h = \frac{V}{900\pi}$

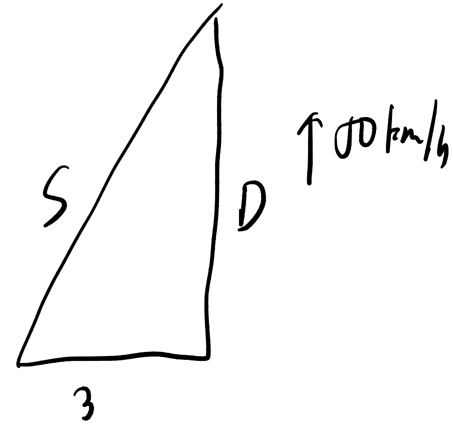
$\frac{dh}{dt} = \frac{\frac{dV}{dt}}{900\pi}$

$\frac{dh}{dt} = \frac{500}{900\pi} = \frac{5}{9\pi}\ cm/min$

Example #2

A taxi drives 3 blocks east, and then turns north, travelling at 50 km/h. How fast is the distance between the cab and the starting point increasing when the taxi has driven 4 blocks north?

$$\frac{dD}{dt} = 50 \text{ km/h}$$



$$\frac{dS}{dt} = ?$$

$$S^2 = 3^2 + D^2$$

$$2S\frac{dS}{dt} = 2D\frac{dD}{dt}$$

when $D = 4$ blocks, $S = 5$ blocks

$$\frac{dS}{dt} = \frac{4(50)}{5} = 40 \text{ km/h}$$