SRNN Model Parameter Table

This document describes the equations, state variables, derived quantities, and parameters used to generate Figure 2.

System Equations

$$\frac{dx_i}{dt} = \frac{-x_i + u_i + \sum_{j=1}^{N} w_{ij}\, b_j r_j}{\tau_d}$$

$$r_i = \phi\left(x_i - a_{0_i} - c\sum_{k=1}^{K} a_{ik}\right)$$

$$\frac{da_{ik}}{dt} = \frac{-a_{ik} + r_i}{\tau_{a_k}}$$

$$\frac{db_i}{dt} = \frac{1 - b_i}{\tau_{rec}} - \frac{b_i\, r_i}{\tau_{rel}}$$

Abbreviations: - **SRNN**: Stable Recurrent Nonlinear Network - **SFA**: Spike frequency adaptation - **STD**: Short-term synaptic depression

Table 1: Model Parameters

Symbol	Name	Value	Units	Description
State Variables				
x_i	Membrane potential	$(-\infty, \infty)$	arbitrary	Dendritic potential of neuron i
a_{ik}	Adaptation variable	$[0, 1]$	—	SFA state for neuron i, timescale k
b_i	Synaptic resource	$[0, 1]$	—	STD variable for neuron i (available resources)
Dependent Variables				
r_i	Firing rate	$[0, 1]$	—	Instantaneous firing rate, $r_i = \phi(\cdot)$
$b_i r_i$	Synaptic output	$[0, 1]$	—	Effective synaptic output with STD
u_i	External input	$[0, \infty)$	arbitrary	External drive to neuron i
Connection Weight parameters				
W	Connection matrix	—	—	$N \times N$ sparse weight matrix
N	Network size	300	—	Total number of neurons
f	Fraction excitatory	$\frac{1}{2}$ or (0.4–0.6)	—	Fraction of excitatory neurons, remainder are inhibitory; systematically varied from 0.4 to 0.6 to produce figure 2 x-zz
S	Sparsity mask	—	—	Binary mask, $S_{ij} \sim$ Bernoulli(α)
α	Connection probability	$\frac{1}{3}$	—	$\alpha = \text{indegree}/N = 100/300$
F	Default scaling factor	$\frac{1}{\sqrt{N\alpha(2-\alpha)}}$	—	Scaling factor which yields $R = 1$ if $\tilde{\mu}_E, \tilde{\mu}_I, \tilde{\sigma}_E,$ and $\tilde{\sigma}_I$ are equal.

Symbol	Name	Value	Units	Description
$\tilde{\mu}_E$	Mean excitatory weight	$3F$	—	Normalized mean of non-zero E weights
$\tilde{\mu}_I$	Mean inhibitory weight	$-4F$	—	Normalized mean of non-zero I weights. If $f = \frac{1}{2}$, then inhibition exceeds excitation ($-4F$ vs. $+3F$), creating a negative global outlier eigenvalue
$\tilde{\sigma}_E$	Std dev excitatory	F	—	Normalized std dev of non-zero E weights
$\tilde{\sigma}_I$	Std dev inhibitory	F	—	Normalized std dev of non-zero I weights
Time Constants				
τ_d	Dendritic time constant	100	ms	Membrane integration time constant
τ_{a_k}	SFA time constants	$[0.1, 1, 10]$	s	Logspaced, $K = 3$ timescales
τ_{rec}	STD recovery	1	s	Synaptic vesicle recovery time constant
τ_{rel}	STD release	$\frac{1}{2}$	s	Synaptic vesicle release time constant
Adaptation Strength				
c_E	SFA coupling	$\frac{1}{12}$	—	SFA strength per timescale
Activation Function				
ϕ	Piecewise sigmoid	$(-\infty, \infty) \to [0, 1]$	—	Hard sigmoid with rounded corners
a_ϕ	Linear fraction	0.9	—	Fraction of domain with slope 1
c_ϕ	Sigmoid center	0.4	—	Horizontal shift; $\phi(c_\phi) = \frac{1}{2}$
Stimulus Configuration				
n_{steps}	Number of periods	3	—	Total number of no-stim and stim periods
ρ_E	E input density	0.15	—	Fraction of E neurons receiving external input
ρ_I	I input density	0	—	Fraction of I neurons receiving external input
A	Input amplitude	$\frac{1}{2}$	—	Amplitude scale for Gaussian random step input
Simulation Settings				
f_s	Sampling frequency	400	Hz	ODE solver sampling rate
T	Simulation interval	$[-15, 45]$	s	Start and end time
ODE Integration				
—	Solver	RK45	—	MATLAB ode45 (Dormand-Prince)
—	Relative tolerance	10^{-9}	—	ODE solver RelTol
—	Absolute tolerance	10^{-9}	—	ODE solver AbsTol
—	Maximum step	2.5	ms	ODE solver MaxStep ($1/f_s$)

Symbol	Name	Value	Units	Description
Lyapunov Settings				
—	LLE method	—	—	Benettin rescaling shadow trace method
Δt_{lya}	Rescaling period	20	ms	Shadow trace rescaling interval
d_0	Perturbation norm	10^{-3}	—	Initial and rescaled perturbation magnitude
f_{corner}	LLE filter corner	0.25	Hz	corner frequency for lowpass filter of local LLE for plotting, 4th order bidirectional Butterworth

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> **Note on Zero Row Sum (ZRS):** Harris (2023) describes a Zero Row Sum condition (ZRS/SZRS) that controls "local" eigenvalue outliers escaping the spectral disc. In these simulations, we deliberately did not apply the ZRS condition in order to test whether adaptation mechanisms (SFA and STD) could fulfill a similar stabilizing role—effectively examining if adaptation can substitute for ZRS in constraining network dynamics.

Table 2: Adaptation Conditions

The four conditions are defined by enabling/disabling SFA and STD via the number of timescales ($K = n_{a_E}$ in the equations):

Condition	n_{a_E} (K)	n_{a_I}	n_{b_E}	n_{b_I}	Description
No Adaptation	0	0	0	0	Baseline
SFA Only	3	0	0	0	Spike-frequency adaptation enabled
STD Only	0	0	1	0	Short-term depression enabled
SFA + STD	3	0	1	0	Both mechanisms enabled

Effect on parameters:

- When $n_{a_E} = 0$: No SFA variables a_{ik}; the $c_E \sum_k a_{ik}$ term is zero.
- When $n_{a_E} = 3$: Three SFA timescales with $\tau_{a_k} \in \{0.1, 1, 10\}$ s and coupling $c_E = \frac{1}{12}$.
- When $n_{b_E} = 0$: No STD variable b_i; synaptic output equals r_i (equivalent to $b_i = 1$).
- When $n_{b_E} = 1$: STD enabled with $\tau_{rec} = 1$ s and $\tau_{rel} = \frac{1}{2}$ s.

Inhibitory neurons have no adaptation mechanisms ($n_{a_I} = n_{b_I} = 0$).

> **Implementation note:** When any of n_{a_E}, n_{a_I}, n_{b_E}, or n_{b_I} is set to zero, the corresponding state variables (a_{ik} or b_i) are excluded from the system state vector and the Jacobian matrix. This prevents spurious zero eigenvalues that would otherwise arise from including disabled adaptation dynamics.