# ECON 210 Final

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## Question 1

Figure 1 shows ten simulations of the income process from my discretized approximation, and explains how I constructed the approximation. As expected, the series display a great deal of persistence around expected income, since $\rho = 0.95$ is close to 1, in the sense that if income is above the unconditional expectation today it is likely to also be above tomorrow's unconditional expectation.

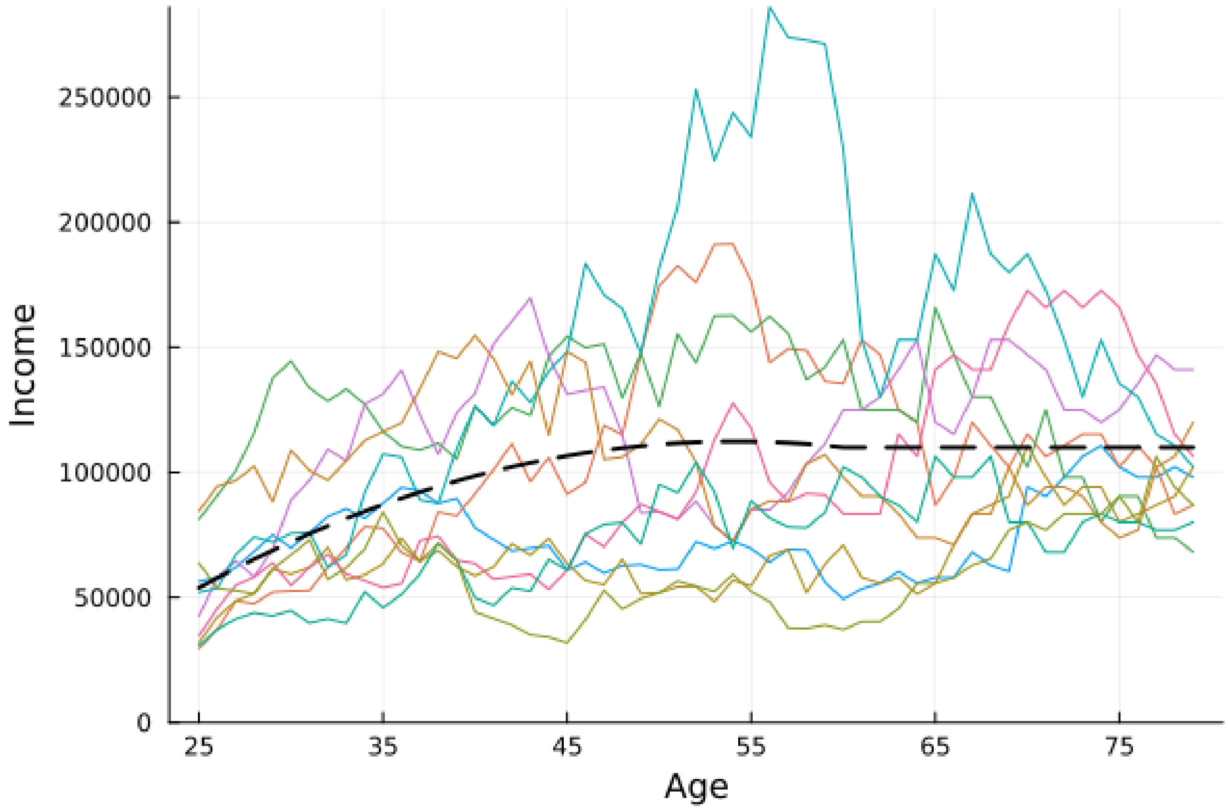


Figure 1: Ten simulations of the income process.

*Notes for Figure 1:* I draw the initial state randomly according to the probability of each state in the stationary distribution. I chose to draw with probabilities according to the stationary distribution since then expected income at age 25 is \$55,000 as mentioned in the question. I discretized the stochastic process $x_t$ using Tauchen's (1986) method with 101 states, going 5.1 standard deviations to either side so that the lowest income state at age 25 is just over \$7,000. The black dotted line is the income process that would be observed if the individual had the draw $\mathbb{E}[x_t]$ in every period, that is, it is the process $y_t = A_t\mathbb{E}[x_t]$. Please see *SimulateIncomeProcess.jl* for the corresponding code. For subsequent questions, I use only 15 states.

## Question 2

Inducting backwards from the final period, we can express the continuation utility from cash-on-hand $w$ with income $y$ as $V_{t+1}(w, y)$. Given that the agent dies, with no bequest motive, in the final period, I set $V_{80}(w, y) = 0$ and therefore $C_{79}(w, y) = w$. (This corresponds to the agent dropping dead at age 80, and not being able to consume anything in that period.)

Since income is a state anyway, I prefer to set up the problem as depending on the state variables income and savings, which I denote $a$ for assets. I can later transform the policy functions and value function to express them as functions of cash on hand.

For any period $t$ we can set up a Bellman equation:

$$V_t(a, y) = \max_{a'} \{u(Ra + y - a') + \beta \mathbb{E}_t[V_{t+1}(a', y')]\}$$

subject to the constraint $a' \geq 0$. By the principle of optimality, solutions to this set of functional equations correspond to solutions to the sequence problem.

I solve the model using Carroll's (2006) endogenous grid method. For details, please see *Q2_EGM.jl* and *EGM_Helper_Functions.jl*.

Figure 2 shows the consumption functions for the four cases outlined in the question. Income matters over and on top of its effect on current disposable cash because the persistence parameter of the AR(1) income process is positive. (In particular, since $\rho = 0.95$, the series is highly persistent as shown in the simulations in Figure 1.) High income today implies income will probably also be high tomorrow. Since income tomorrow is a factor determining cash on hand tomorrow, income today matters through its effect on expected cash on hand tomorrow (holding consumption today fixed), and hence higher income today suggests that for any given value of consumption today, the expected marginal utility of consumption tomorrow will be lower. Therefore the agent wants to increase consumption today, saving less for the future, until the Euler equation is satisfied once again.

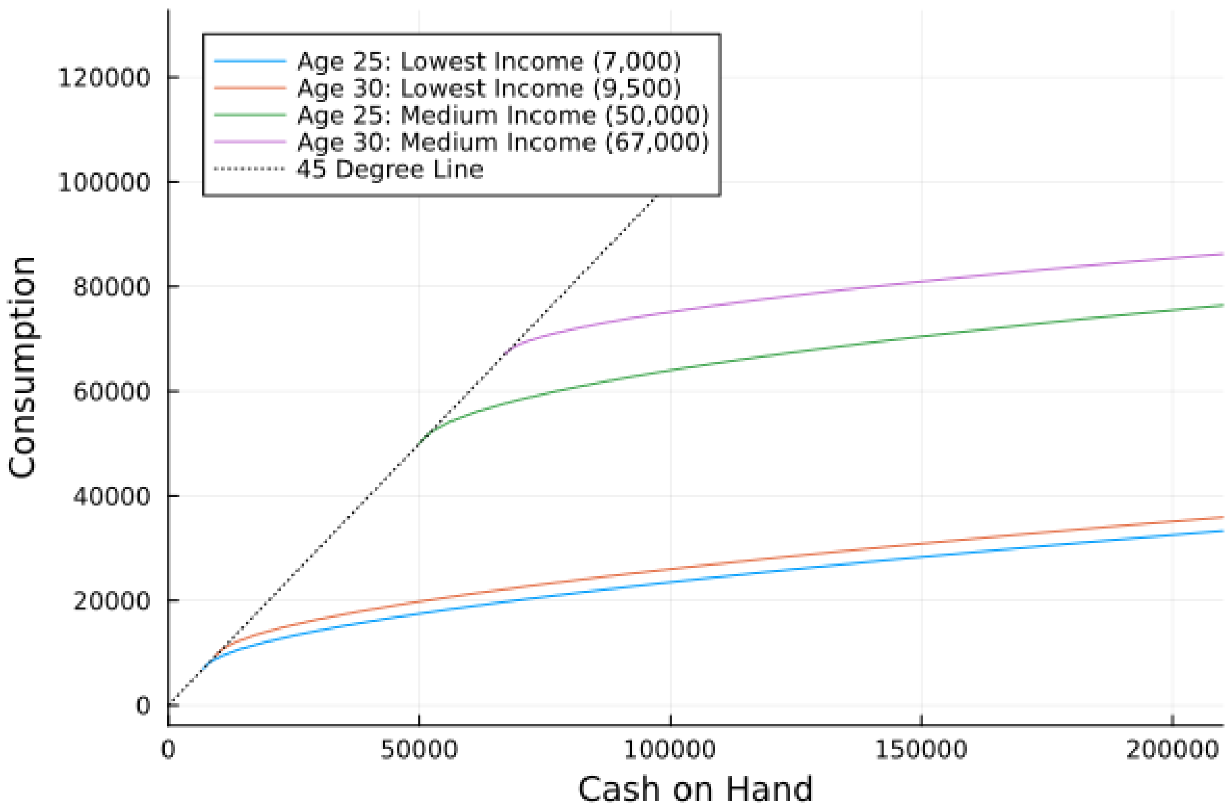


FIGURE 2: Consumption as a function of cash-on-hand and income, with no loans or taxes.

*Notes for Figure 2:* The assets grid begins at zero, hence each series starts at the relevant income value. I plot the 45 degree line as well (that is, the line where consumption is equal to cash on hand) since that shows when the agent is up against the borrowing constraint.

## Question 3

Figure 3 shows the consumption functions for the four cases outlined in the question, where the individual must make a fixed loan repayment in each of the first ten periods. Please see *Q3_EGM.jl* for the corresponding code.

Comparing Figures 3 and 2, we see that for a given level of cash on hand (where I have defined cash on hand in Figure 3 to not account for the fixed payment $L_t$, so as to facilitate comparison to Figure 2) and a given level of income, individuals consume less at age 25 and 30 when they must make loan repayments. This makes sense. In effect, the loan repayment acts to reduce income in the first ten periods, and money that could have been spent on consumption in Question 2 is now spent on loan repayments instead. Due to the presence of the borrowing constraint, agents cannot borrow against future income, which effectively remains unchanged in expectation beyond the loan repayment period, so this decline in effective income early on in the agent's life leads to a noticeable decline in early consumption as well relative to the case with no loan repayments.

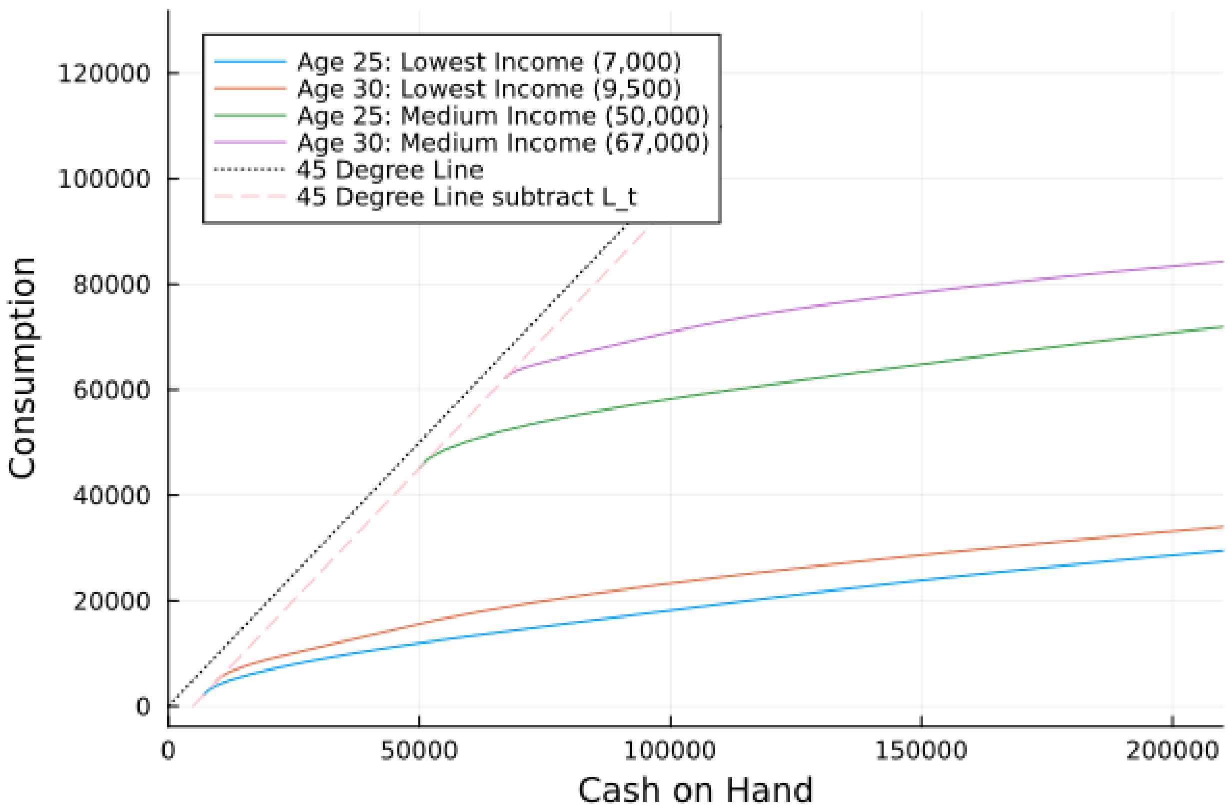


FIGURE 3: Consumption as a function of cash-on-hand and income, under fixed early-life loan repayments.

*Notes for Figure 3*: The assets grid begins at zero, hence each series starts at the relevant income value. Here I do not include the payment $L_t$ when defining cash-on-hand so as to facilitate comparison to Figure 2. As a result, the relevant borrowing constraint is now that consumption must be less than cash on hand subtract the fixed loan repayment $L_t$, and this borrowing constraint is indicated by the dashed pink line.

## Question 4

We can rearrange the expression to isolate $\varepsilon_a(w, y)$.

$$\begin{aligned} \tilde{V}_a/V_a &= (1+\varepsilon)^{1-\gamma} \\ \implies \left(\tilde{V}_a/V_a\right)^{\frac{1}{1-\gamma}} &= 1+\varepsilon \\ \implies \left(\tilde{V}_a/V_a\right)^{\frac{1}{1-\gamma}} - 1 &= \varepsilon \end{aligned}$$

I compute the set of functions $\tilde{V}_a$ and $V_a$ analytically using the corresponding consumption functions. Please see *Construct_Value_Functions.jl* for the specifics of the implementation.

Figure 4 plots $\varepsilon_a(w, y)$ by age, cash on hand, and income.

The individuals who gain most from debt relief:

1. **are younger**. Younger individuals have not yet repaid most of the loan, and since the policy in this model is complete debt forgiveness across all agents, younger agents in effect receive a greater transfer of resources from the government relative to older individuals. (For example, individuals above 35 don't benefit at all from the policy since they have already paid back all of their loan.)

2. **have lower incomes**. Young individuals with low income are really punished by debt repayments since the elasticity of substitution $\gamma$ is relatively high in this problem (relative to, for example, the meta-analysis by Havránek (2015)), and the borrowing constraint prevents these individuals from borrowing against future income in order to smooth consumption by increasing consumption in the early periods. These households really suffer as a result of these front-loaded (relative to the entire life cycle) loan repayments, since they lead to a highly uneven consumption profile, and hence these people benefit greatly from debt forgiveness as reflected by the high $\varepsilon$.

3. **have lower levels of cash on hand**. To isolate an effect separate from the effect of income described above consider two agents with the same age and income but different levels of saving. The agent with high levels of saving can dip into these savings in order to smooth consumption early on, whereas the agent with low savings is likely unable to smooth consumption effectively as a result of the borrowing constraint, and hence benefits more relatively from debt relief which allows the agent to pursue a smoother consumption path.

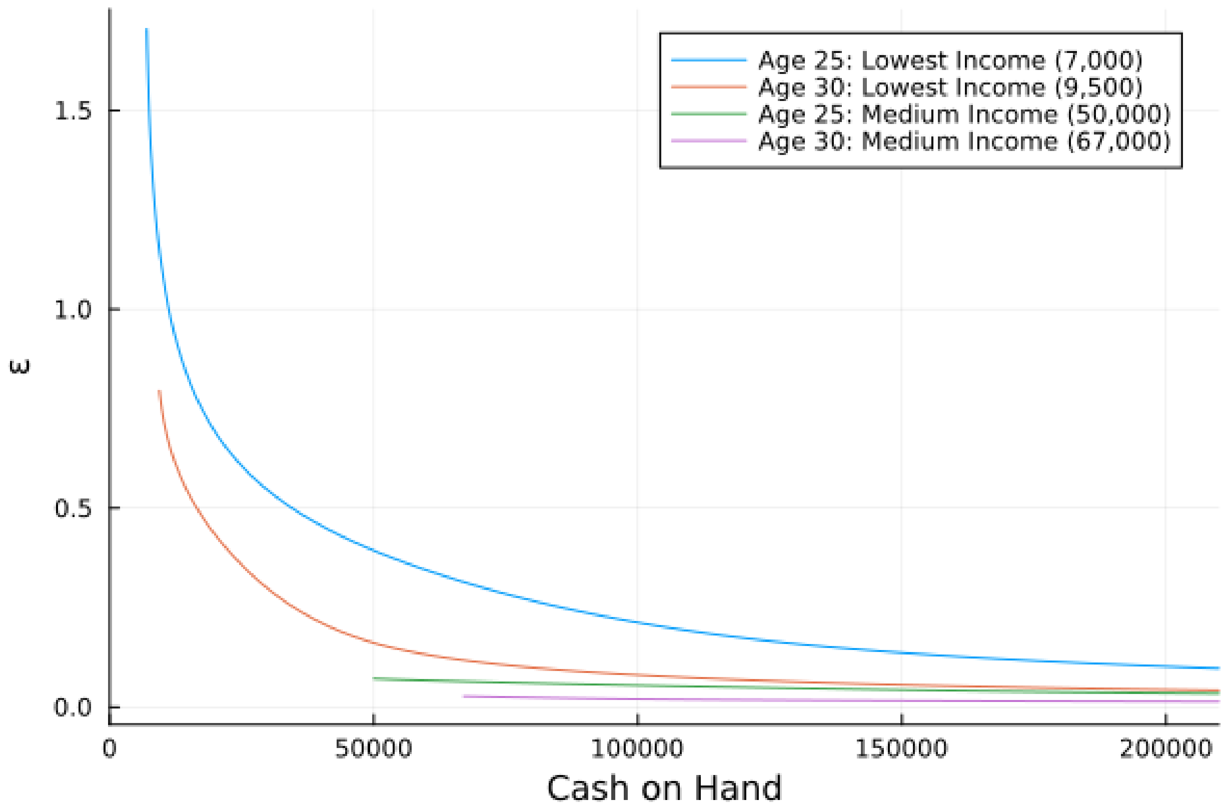


FIGURE 4: $\varepsilon_a(w, y)$ by cash-on-hand, income, and age.

*Notes for Figure 4:* The assets grid begins at zero, hence each series starts at the relevant income value.

## Question 5

I think this is a useful model that helps to articulate and formalize one particular aspect of the student loan debt relief debate—specifically that the welfare effect of relief is greater for individuals who graduate college with low income/low savings beyond just the fact that these individuals have a higher marginal value of wealth across their whole liftetime since debt relief gives these individuals smoother consumption profiles, especially relative to individuals who graduate college with large amounts of savings who may well have been able to pursue a relatively smooth consumption profile even prior to the debt relief. Indeed, I believe that this result is ex-post very intuitive and likely to generalize to more elaborate models.

One key feature that I think could be added to the model is to nest this consumption-savings problem within a broader macroeconomic model. This may allow us to think formally about the effects of this policy on, for example, inflation. In our high-inflation environment, such a large transfer ($40,000 is over half of US GDP per capita) to a relatively large set of individuals may in some sense be "throwing more fuel on the fire", provided such spending is financed via a deficit and not via tax increases or spending cuts elsewhere. The Federal Reserve may even attempt to counteract the aggregate effects of such a policy by, for example, increasing rates above where they would otherwise have been in order to stop inflation moving even further from target. Such interest rates themselves have distributional effects by reallocating from borrowers to savers (given these individuals were borrowers in the first place, it seems likely to me that such an interest rate increase could somewhat offset many of the distributional aspects of this policy) and via asset price changes (Fagereng et al., 2022).

It is also possible to endogenize the labor supply choice of these individuals. Debt relief can, via the income effect, lead individuals to choose to work less, affecting aggregate supply in the economy, GDP, and tax revenue. That said, my understanding is that estimates of the income effect on labor supply in the literature are generally modest (e.g., Cesarini et al. (2017)), so perhaps this channel is not so important to focus on.

There are other effects that the above model misses, but which are probably harder to identify and incorporate into a model. For example, today's teenagers may see today's debt relief and infer that something similar will happen again in the future, hence be more likely to attend college since the expected cost has fallen. Such effects of this could go beyond increased earnings for these individuals since college attendance may have social spillovers by, for example leading to a more informed and educated population (hopefully…).

When thinking about the distribution of effects across the economy, this model only contains agents who did in fact attend college. In the 2019 ACS, 37.1% of US individuals aged 25-44 had a four-year college degree.[1] Since college-attendees tend to be both more wealthy than non-college attendees and have higher incomes, the fact that the policy's welfare effects are progressive within the class of college attendees does not mean the policy's welfare effects are progressive over the class of all individuals in the economy. Incorporating non-college attendees into the model would be simple in theory but would require some data on the joint distribution of income/wealth and student debt.[2]

Finally, although this is maybe not a macro topic per se, I think the above model can only really tell you about the distribution of partial equilibrium welfare gains from the policy. However I believe it would be possible to calculate a welfare metric along the lines of the marginal value of public funds procedure suggested by Hendren and Sprung-Keyser (2020) and Finkelstein and Hendren (2020). Whether or not this

[1] Source: https://ncses.nsf.gov/indicators/states/indicator/bachelors-degree-holders-per-25-44-year-olds

[2] Such data is available for UK researchers with the relevant permissions. (Source)

policy is a wise use of government funds could then be evaluated by comparing the MVPF from this policy to the MVPFs of other policies the government could have pursued instead. My strong suspicion is that the debt relief policy will evaluate very poorly compared to, say, subsidising college for the next generation of students since then you also have the effect of inducing additional kids to enter college, although of course there is debate on the extent to which the observed college premium reflects value-added of college attendance, particularly for marginal students, versus selection.

## Question 6

First I calculate $\tau$. As above, I assume we start in the stationary distribution, since then average income at 25 is \$55,000 as in the question. $\tau$ is then defined by:

$$\tau = \frac{40,000}{\sum_{i=0}^{9} \frac{A_i}{R^i} \mathbb{E}[x_i]} = 5.8\%$$

I confirmed this is the correct value for my setup with a simulation. Please see *Q6_EGM.jl* for more details on the implementation.

Figure 5 plots consumption in this setting as a function of age, cash on hand, and income.

Figure 6 plots $\varepsilon_{25}(w, y)$ comparing the tax-based repayment and fixed repayment settings. We see that low-income agents benefit most from the move to a tax-based repayments system, since their expected payments are lower than they would be under a fixed repayments system. Conversely, high income agents prefer the fixed repayments system to the tax-based system because they pay more under the tax-based system than the fixed system—more by enough to compensate for the insurance value of the tax-based system over the fixed system, which I discuss in the next paragraph. (This insurance value is why medium income households prefer the tax-based system to the fixed system.)

Within the low and medium income bands, we see that agents with low initial levels of cash on hand prefer the tax-based system than the fixed-repayments system. When cash on hand is low, you have very little buffer when hit by shocks that reduce your future income, leading to large disutility from having particularly low consumption in one period. Under the tax-system, you repay less when your income is low, but that is precisely when the marginal value of cash-on-hand is highest. In a sense, the tax-based repayment system, at least relative to fixed repayments, is like having an asset that pays out positive amounts when your income is low (and hence marginal value of wealth/marginal utility of consumption is high) and negative amounts when your income is high (when the marginal value of wealth/marginal utility of consumption) is low anyway. Perhaps an intuitive way to think about it is that the state-dependence of the repayments under the tax system provide insurance value that is not provided under the fixed repayments system. Another intuitive way to think about this is that the tax-system reduces risk in post-payment income relative to the fixed repayments system.

There is also an additional effect that, since expected income increases over time, even for a given initial income draw an agent repays less when he is in his late-20s and income is low versus when income is higher in the agent's early-30s. Again, this has value to the agent because of the agent's desire to follow a smooth consumption profile (holding fixed average consumption over time).

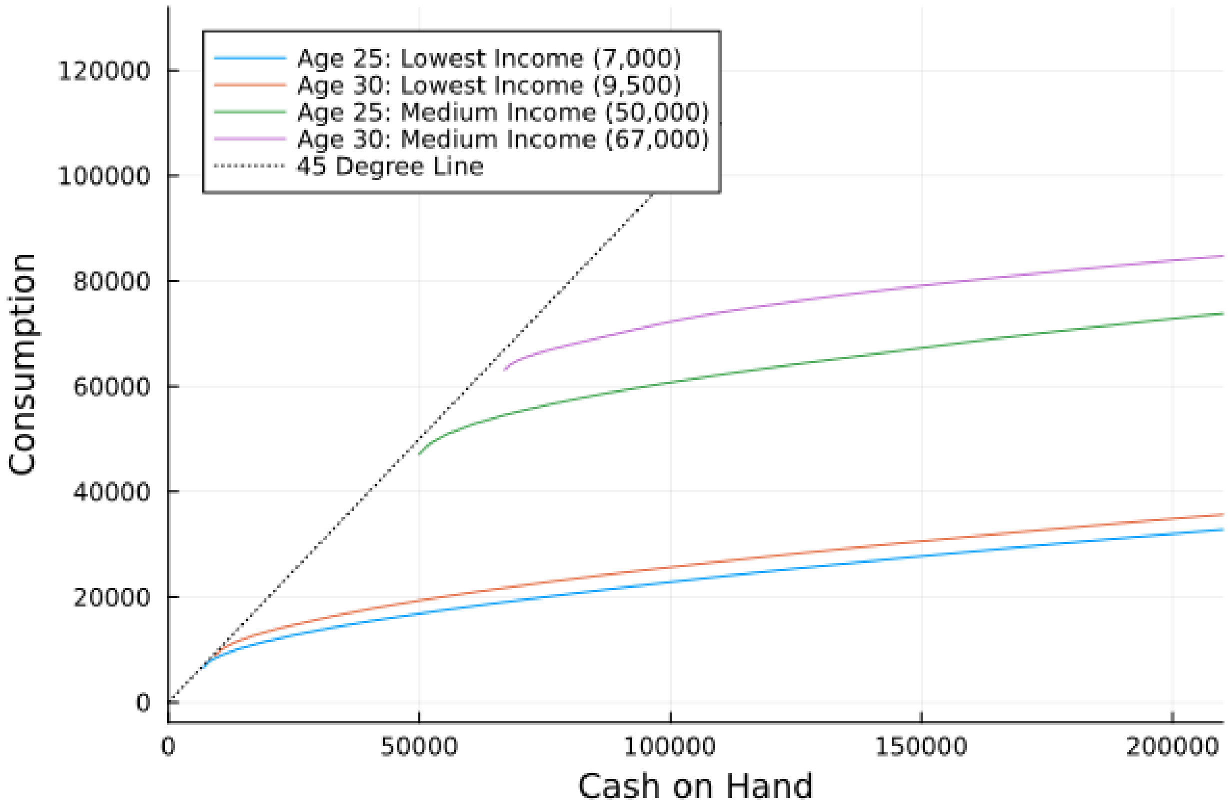


FIGURE 5: Consumption as a function of cash-on-hand and income, under early-life income tax $\tau$.

*Notes for Figure 5:* The assets grid begins at zero, hence each series starts at the relevant income value. I define cash-on-hand using pre-tax income, as opposed to post-tax income, in order to facilitate comparison with Figures 2 and 3. The 45 degree line is plotted as well (that is, the line where consumption is equal to cash on hand) since that shows when the agent is up against the borrowing constraint. The relevant borrowing constraint at these ages, however, is not that consumption is weakly than cash on hand but instead that consumption is weakly less than savings plus post-tax income. This is strictly less than cash-on-hand since income is always positive. Although it is not clear from the graph, the blue and orange lines do lie strictly below the 45 degree line, as we expect given the revised nature of the borrowing constraint.

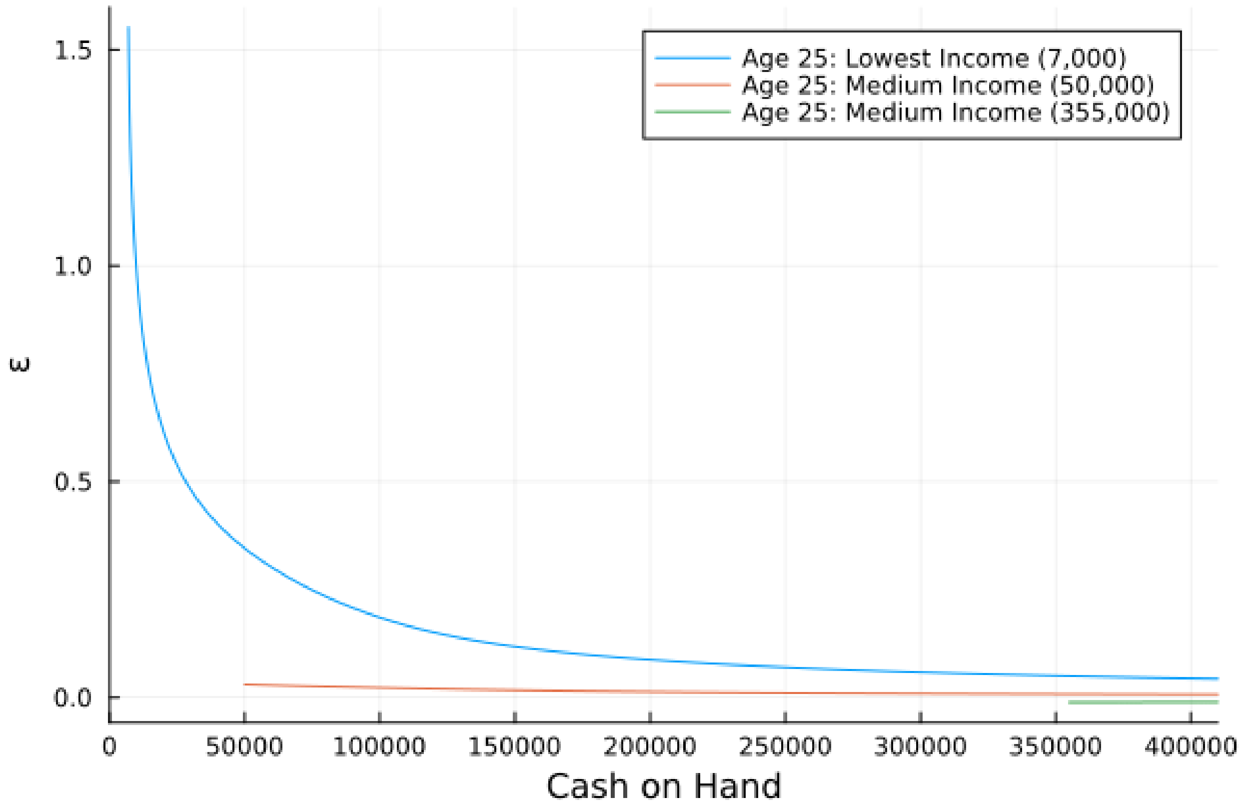


FIGURE 6: $\varepsilon_{25}(w, y)$ comparing the tax-based setting to the fixed-repayments-based setting for 25 year olds, by cash on hand and income.

*Notes for Figure 6:* The assets grid begins at zero, hence each series starts at the relevant income value. I define cash-on-hand using pre-tax income/pre-repayment income in order to give $\varepsilon$ a clean interpretation. Although it is not especially clear from the graph, the orange (medium-income) series lies above zero whereas the green (high-income) series lies below zero.

## Question 7

Figure 7 shows the ex-ante values of $\varepsilon$ regarding the tax-based versus fixed repayments scheme. Please see *Q7.jl* for the details of the calculation.

The intuititon for the shape of Figure 7 is very similar to the intuition I provided in Question 6. Ex-ante, the individual expects to make the same repayments on average under both the tax scheme and the fixed repayments scheme. But because under the tax scheme the agent repays less when income is low (both due to the uncertain nature of income and due to the fact that expected income is increasing in agent age between 25–34), which is exactly when the marginal value of wealth is high, ex ante agents prefer the tax-based repayments system to the fixed repayments system.


Figure 7: Ex-ante $\varepsilon$, when period 0 income is unknown, by savings available in period 0.

## Question 8

As discussed above, equity financing of student loans has some clear advantages over a fixed repayments system, stemming mainly from how they can reduce the risk involved in human capital investment. Such arguments go back to at least Friedman (1955).

While it is unlikely that private attempts to provide equity financing for human capital would succeed without large subsidies, due to the potential for adverse selection and subsequent unravelling found from survey data by Herbst and Hendren (2021), a mandatory government "graduate tax", as suggested by the question, could circumvent this issue. However, if the tax is not perfectly enforceable (for example as a result of emigration) it could still be possible for this unravelling to take place.

I think the main drawback of this policy is that, in a model with endogenous labor supply (at both the intensive and extensive margin), this tax is distortionary and via the substitution effect acts to reduce labor supply. If the tax does reduce labor supply, it will have a knock-on effect on government finances via reducing the revenue from the conventional income tax. (This distortionary criticism applies not just to the labor supply choice but also, for example, to the choice of what major to choose at university.) In the model we used above, every agent works, there is no intesive margin, and the presence of the tax does not affect pre-tax income. In the presence of endogenous responses to the presence of the tax such as changes in labor supply and major choice, the actual level of the tax may have to be much higher than 5.8% in order to break even, which would make the tax even more distortionary and exacerbate the above issues.

A somewhat related criticism is that it could also create perverse incentives for students to complete most of their course then drop out prior to graduating. Such a situation indicates that the tax would probably have to be proportional therefore to the amount of time you spend in education, but then such a tax would now involve detailed record-keeping of exactly how long everyone spent in education, what costs they accrued, and the corresponding tax rate they should pay which will vary each year depending on their income, so is likely to instead be a complicated function they will have to know instead of a given rate, and so on.

Additionally, the tax should probably still allow separate universities to charge separate fees, to encourage competition on costs between universities, which then adds to the complexity of the tax and precludes the "one rate fits all" policy suggested in the question. Putting all of this together, a graduate tax that is practical is probably quite complex and costly to administer.

## References


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