Divisibility

When we talk about the "divisibility" of a whole number, we're just talking about the whole numbers that divide evenly into it. As an example, 5 divides evenly into 15 three times, since $15 \div 5 = 3$, which means we can say that 15 is "divisible" by 5.

Or put another way, 15 can be cut evenly into three equal pieces of size 5. When we can cut some X into equally sized pieces of size Y, using up the entire X without any remainder, then X is divisible by Y.

If we want to be technical, 15 is divisible by 5 because when we do the division $15 \div 5$, the result 3 is a whole number. That's the technical definition of divisibility: The result of the division must be a whole number.

Another way to say this is that we must get a remainder of 0. Since we often think of 0 as "nothing," we sometimes say that there's **no** remainder when the remainder is 0. So a third way to say that one whole number is divisible by another is that we don't get a remainder when we do the division.

Example

Is 56 divisible by 8?

If we do the division, we get $56 \div 8 = 7$. Since 7 is a whole number, we can say that 56 is divisible by 8.

The counterexample

Here's a counterexample. Is 9 divisible by 4? If we divide 9 by 4, we know 4 goes into 9 two times, and that gets us up to 8, but then we have a remainder of 1. In other words, because we have a remainder (other than 0), our answer isn't a whole number.

We get a whole number as the answer for $8 \div 4$ (the answer is the whole number 2), and we get a whole number as the answer for $12 \div 4$ (the answer is the whole number 3), but we don't get a whole number as the answer for $9 \div 4$. Therefore, 9 is not divisible by 4. But as we just saw, 8 and 12 are both divisible by 4.

Divisibility rules

The following list of divisibility rules are a shorthand way of determining if a particular integer is divisible by another, without actually performing the division.

Divisible by 2 if the last digit is $0, 2, 4, 6, 8$

Ex: 24 divisible by 2 because the last digit is 4.

Divisible by 3 if the sum of the digits is divisible by 3

Ex: 123 divisible by 3 because the sum of the digits is $1 + 2 + 3 = 6$. Since 6 is divisible by 3, 123 is also divisible by 3.

Divisible by 4 if the last two digits are divisible by 4

Ex: $3{,}448$ divisible by 4 because the last two digits form the number 48. Since 48 is divisible by 4, $3{,}448$ is also divisible by 4.

Divisible by 5 if the last digit is 0, 5

Ex: $2{,}360$ divisible by 5 because the last digit is 0.

Divisible by 6 if divisible by 2 and 3

Ex: 330 divisible by 6 because 330 is divisible by 2, because the last digit is 0. The sum of the digits is $3 + 3 + 0 = 6$. Since 6 is divisible by 3, 330 is also divisible by 3.

Divisible by 7 if $5 \times$ last digit + rest of the number is divisible by 7, or if subtracting twice the last digit from the rest of the number is divisible by 7

Ex: 256 is not divisible by 7 because multiplying the last digit 6 by 5 gives 30. When we take the 6 off of 256, we're left with 25. Adding 30 to 25 gives 55, which is not divisible by 7.

Ex: 256 is not divisible by 7 because the last digit is 6. Multiplying 6 by 2 gives $6 \times 2 = 12$, and when we take the 6 off of 256, we're left with 25. Subtracting 12 from 25 gives 13, which is not divisible by 7.

Divisible by 8 if the last three digits are divisible by 8

Ex: $34,256$ is divisible by 8 because the last three digits are 256, and 256 is divisible by 8.

Divisible by 9 if the sum of the digits is divisible by 9

Ex: $3,254$ is not divisible by 9 because the sum of the digits is $3+2+5+4=14$. Since 14 is not divisible by 9, $3,254$ is not divisible by 9.

Divisible by 10 if the last digit is 0

Ex: 125 is not divisible by 10 because the last digit is 5, not 0.