Pre-Algebra Workbook Solutions

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MATH

NUMBER SETS

■ 1. The number 0 is included in all the number sets except ___________ numbers.

Solution:

natural and irrational

■ 2. Positive and negative whole numbers are called ___________.

Solution:

integers

■ 3. Fractions and decimals can be considered ___________ numbers.

Solution:

real

■ 4. The number set $\{2,4,6,8\}$ shows a set of ____________ numbers.

Solution:

even

■ 5. What is the real number that's halfway between 1 and 2?

Solution:

$$1\frac{1}{2}$$

■ 6. The number sets that include negative numbers are ____________, ____________, ____________, and ____________ numbers.

Solution:

real, rational, irrational, integer

IDENTITY NUMBERS

■ 1. Find the sum.

$$4 + 0 =$$

Solution:

4

■ 2. Find the product.

$$15 \cdot 1$$

Solution:

15

■ 3. The identity number for addition is 0 because when we add 0 to a number the value does ____________ change.

Solution:

not

■ 4. The ____________ number for multiplication is 1 because when we multiply a number by 1, the value does not change.

Solution:

identity

■ 5. Given the problem $10 + 0 = 10$, the 0 is the identity number for ____________.

Solution:

addition

■ 6. Given the problem $20 \cdot 1 = 20$, the 1 is the identity number for ____________.

Solution:

multiplication

OPPOSITE OF A NUMBER

■ 1. What is the opposite of -15?

Solution:

15

■ 2. What is the opposite of $2/3$?

Solution:

$$-\frac{2}{3}$$

■ 3. Opposites are numbers that are equal distance from ____________.

Solution:

0 or the origin

■ 4. What is the only number that is its own opposite?

Solution:

0

■ 5. When looking at a number line, the negative numbers are to the ____________ of 0 and the positive numbers are to the ____________ of 0.

Solution:

left, right

■ 6. We know 5 and -5 are opposite numbers because they are both ____________ units away from 0.

Solution:

5

ABSOLUTE VALUE

■ 1. Simplify the expression.

$$|-4|$$

Solution:

The absolute value bars change a negative value inside them into a positive value, so $|-4|$ becomes 4.

■ 2. Simplify the expression.

$$|75|$$

Solution:

The value inside the absolute value bars is already positive, so $|75|$ becomes 75.

■ 3. Write the numbers from least to greatest.

$$|-4|, \ |1|, \ |0|, \ |-8|, \ |9|$$

Solution:

The absolute value bars change a negative value inside them into a positive value, so $|-4|$ becomes 4, and $|-8|$ becomes 8.

Some values inside the absolute value bars are already positive, so $|1|$ becomes 1, $|9|$ becomes 9, and $|0|$ becomes 0.

Putting these in order from least to greatest, we get

$$|0|,\ |1|,\ |-4|,\ |-8|,\ |9|$$

■ 4. Write the values from greatest to least.

$$|7|,\ |-3|,\ |0|,\ |-9|,\ |5|$$

Solution:

The absolute value bars change a negative value inside them into a positive value, so $|-3|$ becomes 3, and $|-9|$ becomes 9.

Some values inside the absolute value bars are already positive, so $|7|$ becomes 7, $|5|$ becomes 5, and $|0|$ becomes 0.

Putting these in order from greatest to least, we get

$$|-9|,\ |7|,\ |5|,\ |-3|,\ |0|$$

■ 5. Absolute values make positive numbers ____________ and negative numbers ____________.

Solution:

positive, positive

■ 6. Simplify the expression.

$$|-3|$$

Solution:

The absolute value bars change a negative value inside them into a positive value, so $|-3|$ becomes 3.

ADDING AND SUBTRACTING SIGNED NUMBERS

■ 1. Simplify the expression.

$$-4+2$$

Solution:

Here, the negative number is -4 and the positive number is 2. 4 is larger than 2, so the absolute value of the negative number is larger, which means the answer will be negative. So we subtract 2 from 4 to get 2. The sign needs to be negative, so we get -2.

■ 2. Simplify the expression.

$$-11-8$$

Solution:

Add the numbers as if they were both positive, but make the sign negative.

$$-11-8=-19$$

■ 3. When we add two negative numbers, we'll always get a ____________ number.

Solution:

negative

■ 4. Simplify the expression.

$$-19-26$$

Solution:

Add the numbers as if they were both positive, but make the sign negative.

$$-19-26=-45$$

■ 5. Simplify the expression.

$$5-8$$

Solution:

Here, the first number is 5 and the second number is 8. Since 5 is less than 8, the second number is larger so the result is negative.

$$5 - 8 = -3$$

■ 6. Simplify the expression.

$$3 - (-6)$$

Solution:

When we subtract a negative number from a positive number, the result will always be positive, because of the fact that the negative signs will cancel, leaving just the addition of two positive numbers.

$$3 + 6 = 9$$

MULTIPLYING SIGNED NUMBERS

■ 1. Multiplying two negative numbers will always result in a ___________ number.

Solution:

positive

■ 2. Multiplying a negative and a positive number will always result in a ___________ number.

Solution:

negative

■ 3. Multiplying two positive numbers will always result in a ___________ number.

Solution:

positive

■ 4. Simplify the expression.

$$12 \cdot -5$$

Solution:

Multiplying 12 by 5 gives 60. Because we're multiplying one positive number by one negative number, the result must be negative, so

$$12 \cdot -5 = -60$$

■ 5. Simplify the expression.

$$-8 \cdot -6$$

Solution:

Multiplying 8 by 6 gives 48. Because we're multiplying one negative number by another negative number, the result must be positive, so

$$-8 \cdot -6 = 48$$

■ 6. Simplify the expression.

$$25 \cdot 3$$

Solution:

Multiplying 25 by 3 gives 75. Because we're multiplying one positive number by another positive number, the result must be positive, so

$$25 \cdot 3 = 75$$

DIVIDING SIGNED NUMBERS

■ 1. Dividing a negative number by a negative number will always result in a ____________ number.

Solution:

positive

■ 2. Dividing a positive number by a negative number will always result in a ____________ number.

Solution:

negative

■ 3. Simplify the expression.

$$-12 \div 2$$

Solution:

-6

■ 4. Simplify the expression.

$$0 \div -8$$

Solution:

0

■ 5. Simplify the expression.

$$24 \div -6$$

Solution:

-4

■ 6. Simplify the expression.

$$-144 \div -12$$

Solution:

12

ABSOLUTE VALUE OF AN EXPRESSION

■ 1. Simplify the expression.

$$|-6|+|-5\cdot 2|$$

Solution:

First simplify the multiplication inside the absolute value bars.

$$|-6|+|-10|$$

The absolute value bars change a negative value inside them into a positive value, so $|-6|$ becomes 6 and $|-10|$ becomes 10.

$$6+10$$

$$16$$

■ 2. Simplify the expression.

$$|-6|-|7|$$

Solution:

First simplify each set of absolute value bars separately. The absolute value bars change a negative value inside them into a positive value, so $|-6|$ becomes 6, but 7 is already positive so $|7|$ becomes 7.

$$|-6|-|7|$$

$$6-7$$

Now finish the subtraction.

$$-1$$

■ 3. Simplify the expression.

$$|-5\cdot 4|$$

Solution:

First simplify the multiplication inside the absolute value bars. Multiplying -5 by 4 gives -20, so

$$|-5\cdot 4|$$

$$|-20|$$

The absolute value bars change a negative value inside them into a positive value, so $|-20|$ becomes 20.

■ 4. Simplify the expression.

$$| -5 \cdot -4 \cdot 2 |$$

Solution:

First simplify the multiplication inside absolute value bars.

$$|20 \cdot 2|$$

$$|40|$$

The absolute value bars change a negative value inside them into a positive value, so $|40|$ becomes 40.

■ 5. Simplify the expression.

$$| -11 + 3 | \cdot | -9 |$$

Solution:

First simplify the addition inside the first set of absolute value bars.

$$| -8 | \cdot | -9 |$$

Then simplify each set of absolute value bars separately. The absolute value bars change a negative value inside them into a positive value, so $| -8 |$ becomes 8, and $| -9 |$ becomes 9.

$$8 \cdot 9$$

$$72$$

■ 6. Simplify the expression.

$$|-8-2| \cdot |-4-3|$$

Solution:

First simplify the subtraction inside both sets of absolute value bars.

$$|-10| \cdot |-7|$$

Then simplify each set of absolute value bars separately. The absolute value bars change a negative value inside them into a positive value, so $|-10|$ becomes 10, and $|-7|$ becomes 7.

$$10 \cdot 7$$

$$70$$

DIVISIBILITY

■ 1. Is 369 divisible by 3?

Solution:

The sum of the digits is $3 + 6 + 9 = 18$. Since 18 is divisible by 3, 369 is also divisible by 3.

■ 2. How can we determine if a number is divisible by 5?

Solution:

If a number ends in “0” or “5,” it’s divisible by 5.

■ 3. “Divisibility” of whole numbers means we’re looking at numbers that divide into a number ____________.

Solution:

evenly

■ 4. Is 245 divisible by 7?

Solution:

The last digit is 5. Multiply it by 5 to get $5 \times 5 = 25$.

Add this product to remaining number to get $24 + 25 = 49$.

Since 49 is divisible by 7, the number 245 is divisible by 7.

■ 5. What is the smallest whole number larger than 20 that's divisible by 2 and 4?

Solution:

If a number ends in 0, 2, 4, 6, 8, it's divisible by 2, so our first set of candidates is 22, 24, 26, ...

If the last two digits are divisible by 4, it's divisible by 4, so the candidates for this list are 24, 28, 32, ...

The first matching number from both lists is 24, so the smallest whole number larger than 20 that's divisible by 2 and 4 is 24.

■ 6. What is the smallest whole number larger than 50 that's divisible by both 3 and 5?

Solution:

If a number ends in 0 or 5, it's divisible by 5, so our first set of candidates is $55, 60, 65, \ldots$

If the sum of the digits is divisible by 3, it's divisible by 3, so the candidates for this list are $51, 54, 57, 60, \ldots$

The first matching number from both lists is 60, so the smallest whole number larger than 50 that's divisible by both 3 and 5 is 60.

MULTIPLES

■ 1. List the first four multiples of 8.

Solution:

$8, 16, 24, 32$

■ 2. List the first five multiples of 40.

Solution:

$40, 80, 120, 160, 200$

■ 3. Is 8 a multiple of 8? Why or why not?

Solution:

Yes, because $8 \cdot 1 = 8$.

■ 4. What are two common multiples of 2 and 3?

Solution:

6 and 12

■ 5. What are two common multiples of 5 and 10?

Solution:

10 and 20

■ 6. The concept of multiples is related to the concept of ___________.

Solution:

divisibility

PRIME AND COMPOSITE

■ 1. ____________ numbers are numbers that are divisible by numbers other than 1 and themselves.

Solution:

Composite

■ 2. Is 7 a prime or composite number?

Solution:

Prime, because 7 is divisible by only 1 and itself.

■ 3. Is 15 a prime or composite number?

Solution:

Composite, because 15 is divisible by 1 and itself, but also by 3 and 5.

■ 4. 35 is a composite number because it's divisible by which numbers?

Solution:

1, 5, 7, and 35

■ 5. 98 is a composite number because it's divisible by which numbers?

Solution:

$1, 2, 7, 14, 49, 98$

■ 6. By how many numbers will a prime number be divisible?

Solution:

A prime number will only be divisible by two numbers: 1 and the number itself.

PRIME FACTORIZATION AND PRODUCT OF PRIMES

■ 1. What is the prime factorization of 75?

Solution:

75

$3 \cdot 25$

$3 \cdot 5 \cdot 5$ or $3 \cdot 5^2$

■ 2. What is the prime factorization of 55?

Solution:

55

$5 \cdot 11$

■ 3. What is the prime factorization of 148?

Solution:

148

$2 \cdot 74$

$2 \cdot 2 \cdot 37$ or $2^2 \cdot 37$

■ 4. The prime factorization of 156 is $2 \cdot 2 \cdot 3 \cdot$ ___________.

Solution:

The product $2 \cdot 2 \cdot 3$ is equal to 12, and $156 \div 12 = 13$, so the missing value is 13.

■ 5. The prime factorization of 63 is $3 \cdot 3 \cdot$ ___________.

Solution:

The product $3 \cdot 3$ is equal to 9, and $63 \div 9 = 7$, so the missing value is 7.

■ 6. Prime factorization is when we break down a composite number into its factors until every factor is a ___________ number.

Solution:

prime

LEAST COMMON MULTIPLE

■ 1. Find the least common multiple of 3 and 15.

Solution:

The multiples of 3 are $3, 6, 9, 12, 15, 18$, etc., and the multiples of 15 are $15, 30, 45, 60$, etc. The smallest matching multiple in these lists is 15, so 15 is the least common multiple of 3 and 15.

■ 2. Find the least common multiple of 16 and 40.

Solution:

The multiples of 16 are $16, 32, 48, 64, 80$, etc., and the multiples of 40 are $40, 80, 120$, etc. The smallest matching multiple in these lists is 80, so 80 is the least common multiple of 16 and 40.

■ 3. Find the least common multiple of the set $\{36, 84\}$.

Solution:

The multiples of 36 are 36, 72, 108, 144, 180, 216, 252, etc., and the multiples of 84 are 84, 168, 252, etc. The smallest matching multiple in these lists is 252, so 252 is the least common multiple of 36 and 84.

■ 4. Find the least common multiple of 12 and 20.

Solution:

The multiples of 12 are 12, 24, 36, 48, 60, etc., and the multiples of 20 are 20, 40, 60, etc. The smallest matching multiple in these lists is 60, so 60 is the least common multiple of 12 and 20.

■ 5. If the prime factorization of one number is $2 \cdot 3 \cdot 5^2$, and the prime factorization of another is $2^3 \cdot 3$, what's the least common multiple of the two numbers?

Solution:

Take the largest number of factors for each prime number in the prime factorizations. The first number has one factor of 2, while the second number has three factors of 2, so we'll take three factors of 2.

$$2^3$$

The first number has one factor of 3, while the second number has one factor of 3, so we'll take one factor of 3.

$$2^3 \cdot 3$$

The first number has two factors of 5, while the second number has zero factors of 5, so we'll take two factors of 5.

$$2^3 \cdot 3 \cdot 5^2$$

If we multiply this out, we'll find the least common multiple.

$$8 \cdot 3 \cdot 25$$

$$24 \cdot 25$$

$$600$$

■ 6. Is there only one possible pair of two numbers that can have a LCM of 20? Give examples to support the answer.

Solution:

No, there are multiple pairs of numbers that have a least common multiple of 20. For instance, 10 and 20 have an LCM of 20, but so do 5 and 4.

GREATEST COMMON FACTOR

■ 1. The greatest common factor of two numbers is the ____________ number that divides evenly into both numbers.

Solution:

largest

■ 2. Find the greatest common factor of 100 and 75.

Solution:

Find the prime factorization of both numbers.

100	75
$2 \cdot 50$	$3 \cdot 25$
$2 \cdot 2 \cdot 25$	$3 \cdot 5 \cdot 5$
$2 \cdot 2 \cdot 5 \cdot 5$	

The only factors that are common to both numbers are two factors of 5, so the greatest common factor is $5 \cdot 5 = 25$.

■ 3. Find the greatest common factor of the set $\{54, 162\}$.

Solution:

Find the prime factorization of both numbers.

54	162
$2 \cdot 27$	$2 \cdot 81$
$2 \cdot 3 \cdot 9$	$2 \cdot 3 \cdot 27$
$2 \cdot 3 \cdot 3 \cdot 3$	$2 \cdot 3 \cdot 3 \cdot 9$
	$2 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

The only factors that are common to both numbers is one factor of 2 and three factors of 3, so the greatest common factor is $2 \cdot 3 \cdot 3 \cdot 3 = 54$.

■ 4. If one number has a prime factorization of $3 \cdot 5 \cdot 11$, while another has a prime factorization of $2 \cdot 3^2 \cdot 11^2$, what is their greatest common factor?

Solution:

The only factors that are common to both numbers is one factor of 3 and one factor of 11, so the greatest common factor is $3 \cdot 11 = 33$.

■ 5. If one number has a prime factorization of $2^4 \cdot 3 \cdot 11$, while another has a prime factorization of $2^3 \cdot 5$, What is their greatest common factor?

Solution:

The only factors that are common to both numbers are three factors of 2, so the greatest common factor is $2^3 = 8$.

■ 6. Is there only one possible pair of two numbers that can have a GCF of 16? Give examples to support the answer.

Solution:

No, there are multiple pairs of numbers that have a greatest common factor of 16. For instance, 16 and 32 have an GCF of 16, but so do 16 and 48.

FRACTIONS

■ 1. What is the denominator of the fraction $3/5$?

Solution:

5

■ 2. How would we write 40% as a fraction?

Solution:

Percents are always expressed with a denominator of 100, so we'd write 40% as

$$\frac{40}{100}$$

■ 3. How would we write 75% as a fraction?

Solution:

Percents are always expressed with a denominator of 100, so we'd write 75% as

$$\frac{75}{100}$$

■ 4. If a pizza is cut into 6 equal pieces and Ben eats 2 of them, what fraction of the pizza did Ben eat?

Solution:

Because Ben ate 2 of the 6 total pieces, he ate $2/6$ of the pizza.

■ 5. Hazel is cleaning out her closet. She has 8 sweaters and 2 of them are blue. What fraction of her sweaters are blue?

Solution:

Because 2 of her 8 sweaters are blue, the fraction of Hazel's sweaters that are blue is

$$\frac{2}{8}$$

■ 6. Joey cuts a pie into 10 equal slices and eats 1 slice. What fraction of the pie did he eat?

Solution:

Because Joey ate 1 of the 10 total slices, he ate $1/10$ of the pizza.

SIMPLIFYING FRACTIONS AND EQUIVALENT FRACTIONS

■ 1. Write $20/50$ as a simplified fraction.

Solution:

The greatest common factor of the numerator and denominator is 10, so we'll divide both the numerator and denominator by 10.

$$\frac{20 \div 10}{50 \div 10} = \frac{2}{5}$$

■ 2. Write the fraction $4/5$ in terms of 20ths.

Solution:

We have to multiply the denominator 5 by 4 in order to get 20, which means we'll multiply the fraction by $4/4$.

$$\frac{4}{5} \cdot \frac{4}{4} = \frac{4 \cdot 4}{5 \cdot 4} = \frac{16}{20}$$

■ 3. Write $110/154$ as a simplified fraction.

Solution:

It might be a difficult to quickly see the greatest common factor of 110 and 154, but we know both the numerator and denominator are even, so let's start by dividing both of them by 2.

$$\frac{110 \div 2}{154 \div 2} = \frac{55}{77}$$

From here, it's easier to see that the greatest common factor of 55 and 77 is 11, so we'll divide both the numerator and denominator by 11.

$$\frac{55 \div 11}{77 \div 11} = \frac{5}{7}$$

■ 4. Are the fractions $3/15$ and $6/36$ equivalent?

Solution:

No. To see if fractions are equivalent, both fractions need to be simplified. The fraction $3/15$ simplifies to $1/5$, and $6/36$ simplifies to $1/6$. Since they simplify to different fractions, the original fractions are not equivalent.

■ 5. Are the fractions $2/16$ and $4/32$ equivalent?

Solution:

Yes. To see if fractions are equivalent, both fractions need to be simplified. The fraction $2/16$ simplifies to $1/8$, and the fraction $4/32$ simplifies to $1/8$. Since they both simplify to the same fractions, the original fractions are equivalent.

■ 6. When using prime factorization to reduce fractions, we're looking for the numbers in the numerator and denominator that are the ____________ prime number.

Solution:

same

DIVISION OF ZERO

■ 1. The fraction $0/7$ means __________ divided by __________.

Solution:

0, 7

■ 2. The number __________ can never be the denominator of a fraction.

Solution:

0

■ 3. The fraction $0/8$ has a value of __________.

Solution:

0

■ 4. True or false? $5/0$ has a value of 0.

Solution:

False. It's impossible to divide by 0, so $5/0$ isn't 0, it's undefined.

■ 5. Complete the statement.

$6 \cdot 0 = 0$ and $0 \div 6 =$ ____________.

Solution:

0

■ 6. Complete the statement of why we can't divide by 0.

$7 \div 0$ means that that something times 0 has a value equal to 7. But there's nothing times 0 that will ever equal 7 because anything times 0 will always equal ____________. Therefore, it's impossible to divide by 0.

Solution:

0

ADDING AND SUBTRACTING FRACTIONS

■ 1. When we add or subtract fractions, we'll add or subtract the ____________ and the ___________ will stay the same.

Solution:

numerators, denominators

■ 2. Find the sum.

$$\frac{1}{9}+\frac{3}{9}$$

Solution:

Because the fractions have the same denominator, we can add them directly by adding the numerators, while keeping the denominator the same.

$$\frac{1}{9}+\frac{3}{9}=\frac{1+3}{9}=\frac{4}{9}$$

■ 3. Find the difference.

$$\frac{7}{12} - \frac{2}{6}$$

Solution:

We need to make the denominators equivalent, which we can do by multiplying the second fraction by $2/2$.

$$\frac{7}{12} - \frac{2}{6}\left(\frac{2}{2}\right)$$

$$\frac{7}{12} - \frac{4}{12}$$

Now that the fractions have the same denominator, we can add them directly by adding the numerators, while keeping the denominator the same.

$$\frac{7}{12} - \frac{4}{12} = \frac{7-4}{12} = \frac{3}{12}$$

Because the numerator and denominator have a common factor of 3, we'll divide both the numerator and denominator of this result by 3 in order to simplify the fraction.

$$\frac{3 \div 3}{12 \div 3} = \frac{1}{4}$$

■ 4. Find the sum.

$$\frac{1}{16}+\frac{3}{4}+\frac{5}{8}$$

Solution:

We need to make the denominators equivalent, which we can do by multiplying the second fraction by $4/4$ and the third fraction by $2/2$.

$$\frac{1}{16}+\frac{3}{4}\left(\frac{4}{4}\right)+\frac{5}{8}\left(\frac{2}{2}\right)$$

$$\frac{1}{16}+\frac{12}{16}+\frac{10}{16}$$

Now that the fractions have the same denominator, we can add them directly by adding the numerators, while keeping the denominator the same.

$$\frac{1+12+10}{16}$$

$$\frac{23}{16}$$

■ 5. Simplify the expression.

$$\frac{7}{10}-\frac{1}{10}+\frac{2}{5}$$

Solution:

We need to make the denominators equivalent, which we can do by multiplying the third fraction by $2/2$.

$$\frac{7}{10}-\frac{1}{10}+\frac{2}{5}\left(\frac{2}{2}\right)$$

$$\frac{7}{10}-\frac{1}{10}+\frac{4}{10}$$

Now that the fractions have the same denominator, we can combine them directly by combining the numerators, while keeping the denominator the same.

$$\frac{7-1+4}{10}$$

$$\frac{10}{10}$$

$$1$$

■ 6. Simplify the expression.

$$\frac{2}{15}+\frac{1}{5}-\frac{1}{30}$$

Solution:

We need to make the denominators equivalent, which we can do by multiplying the first fraction by $2/2$ and the second fraction by $6/6$.

$$\frac{2}{15}\left(\frac{2}{2}\right)+\frac{1}{5}\left(\frac{6}{6}\right)-\frac{1}{30}$$

$$\frac{4}{30}+\frac{6}{30}-\frac{1}{30}$$

Now that the fractions have the same denominator, we can combine them directly by combining the numerators, while keeping the denominator the same.

$$\frac{4+6-1}{30}$$

$$\frac{9}{30}$$

$$\frac{3}{10}$$

MULTIPLYING AND DIVIDING FRACTIONS

■ 1. When we're dividing fractions, we need to flip the ___________ fraction.

Solution:

second

■ 2. Simplify the expression.

$$\frac{4}{7}\cdot\frac{2}{9}$$

Solution:

To multiply fractions, we multiply the numerators to find the new numerator, and multiply the denominators to find the new denominator.

$$\frac{4\cdot 2}{7\cdot 9}$$

$$\frac{8}{63}$$

■ 3. Simplify the expression.

$$\frac{5}{8} \div \frac{1}{12}$$

Solution:

To divide fractions, we start by flipping the second fraction, while changing the division to multiplication.

$$\frac{5}{8} \cdot \frac{12}{1}$$

To multiply fractions, we multiply the numerators to find the new numerator, and multiply the denominators to find the new denominator.

$$\frac{5 \cdot 12}{8 \cdot 1}$$

$$\frac{60}{8}$$

$$\frac{15}{2}$$

■ 4. Simplify the expression.

$$\frac{2}{9} \div \frac{1}{15}$$

Solution:

To divide fractions, we start by flipping the second fraction, while changing the division to multiplication.

$$\frac{2}{9} \cdot \frac{15}{1}$$

To multiply fractions, we multiply the numerators to find the new numerator, and multiply the denominators to find the new denominator.

$$\frac{2 \cdot 15}{9 \cdot 1}$$

$$\frac{30}{9}$$

$$\frac{10}{3}$$

■ 5. Simplify the expression.

$$\frac{1}{10} \cdot \frac{2}{5} \div \frac{1}{4}$$

Solution:

Start by multiplying the first two fractions. To multiply fractions, we multiply the numerators to find the new numerator, and multiply the denominators to find the new denominator.

$$\frac{1 \cdot 2}{10 \cdot 5} \div \frac{1}{4}$$

$$\frac{2}{50} \div \frac{1}{4}$$

To divide fractions, we start by flipping the second fraction, while changing the division to multiplication.

$$\frac{2}{50} \cdot \frac{4}{1}$$

$$\frac{2 \cdot 4}{50 \cdot 1}$$

$$\frac{8}{50}$$

$$\frac{4}{25}$$

■ 6. Simplify the expression.

$$\frac{3}{5} \div \frac{1}{6} \cdot \frac{4}{9}$$

Solution:

Start by dividing the first two fractions. To divide fractions, we start by flipping the second fraction, while changing the division to multiplication.

$$\frac{3}{5} \cdot \frac{6}{1} \cdot \frac{4}{9}$$

To multiply fractions, we multiply the numerators to find the new numerator, and multiply the denominators to find the new denominator.

$$\frac{3 \cdot 6}{5 \cdot 1} \cdot \frac{4}{9}$$

$$\frac{18}{5} \cdot \frac{4}{9}$$

$$\frac{18 \cdot 4}{5 \cdot 9}$$

$$\frac{72}{45}$$

$$\frac{8}{5}$$

SIGNS OF FRACTIONS

■ 1. Is the statement true or false?

$-\frac{1}{6}$ is equivalent to $\frac{-6}{1}$.

Solution:

The statement is false. The fractions aren't equivalent because we switched the 6 and the 1. Regardless of the signs of the fraction, we can't swap the values in the numerator and denominator and keep the fraction's value the same, unless the values in the numerator and denominator are equal.

■ 2. Is the statement true or false?

$-\frac{3}{4}$ is equivalent to $\frac{3}{-4}$.

Solution:

The statement is true. We can change exactly two signs of the fraction, and the fraction's value will stay the same. In this case, we changed the sign of the denominator, and the sign of the fraction itself, while keeping the sign of the numerator the same. Since we changed exactly two signs, the fractions are equivalent.

■ 3. Simplify the expression.

$$\frac{2}{11} \cdot -\frac{1}{4}$$

Solution:

To multiply fractions, we multiply the numerators to find the new numerator, and we multiply the denominators to find the new denominator. The negative sign on the second fraction can remain out in front of the result.

$$-\frac{2 \cdot 1}{11 \cdot 4}$$

$$-\frac{2}{44}$$

$$-\frac{1}{22}$$

■ 4. Simplify the expression.

$$-\frac{3}{20} \cdot -\frac{2}{13}$$

Solution:

To multiply fractions, we multiply the numerators to find the new numerator, and we multiply the denominators to find the new denominator. The negative signs on the two fractions will cancel each other out.

$$\frac{3 \cdot 2}{20 \cdot 13}$$

$$\frac{6}{260}$$

$$\frac{3}{130}$$

■ 5. Simplify the expression.

$$\frac{4}{7} \div -\frac{3}{11}$$

Solution:

To divide fractions, we start by flipping the second fraction, while changing the division to multiplication.

$$\frac{4}{7} \cdot -\frac{11}{3}$$

To multiply fractions, we multiply the numerators to find the new numerator, and we multiply the denominators to find the new denominator. The negative sign on the second fraction can remain out in front of the result.

$$-\frac{4 \cdot 11}{7 \cdot 3}$$

$$-\frac{44}{21}$$

■ 6. If the numerator and the denominator are both negative, the fraction will be ____________.

Solution:

positive

RECIPROCALS

■ 1. A reciprocal is what we get when we ___________ the fraction.

Solution:

flip, or invert

■ 2. What is the reciprocal of $-1/2$?

Solution:

We change the places of the numerator and denominator to get the reciprocal, $-2/1$. The negative sign doesn't affect the value of the reciprocal. We can then simplify $-2/1$ to just -2.

■ 3. What is the reciprocal of 3?

Solution:

We remember that 3 is the same as $3/1$, and then we change the places of the numerator and denominator to get the reciprocal, $1/3$.

■ 4. What is the reciprocal of $-1/4$?

Solution:

We change the places of the numerator and denominator to get the reciprocal, $-4/1$. The negative sign doesn't affect the value of the reciprocal. We can then simplify $-4/1$ to just -4.

■ 5. The only number that does not have a reciprocal is ___________.

Solution:

0. The number 0 doesn't have a reciprocal, since we would find its reciprocal be rewriting 0 as $0/1$, and then changing the places of the numerator and denominator to get $1/0$. But we can't divide by 0, so this reciprocal would be undefined.

■ 6. When we multiply two numbers that are reciprocals of one another, the result is always ___________.

Solution:

1. For example, think about the reciprocals $3/4$ and $4/3$. If we multiply them together, we get 1.

$$\frac{3}{4} \cdot \frac{4}{3}$$

$$\frac{3 \cdot 4}{4 \cdot 3}$$

$$\frac{12}{12}$$

$$1$$

MIXED NUMBERS AND IMPROPER FRACTIONS

■ 1. Mixed numbers are a representation of what operation (addition, subtraction, multiplication, division)?

Solution:

Addition, because each mixed number is the sum of a whole number and a fraction.

■ 2. Convert $15/4$ into a mixed number.

Solution:

4 goes into 15 three times. $4 \cdot 3 = 12$, and $15 - 12 = 3$, so 4 goes into 15 three times, with a remainder of 3.

$$\frac{15}{4} = 3\frac{3}{4}$$

■ 3. Convert $34/6$ into a mixed number.

Solution:

6 goes into 34 five times. $6 \cdot 5 = 30$, and $34 - 30 = 4$, so 6 goes into 34 five times, with a remainder of 4.

$$\frac{34}{6} = 5\frac{4}{6} = 5\frac{2}{3}$$

■ **4.** Write $-114/25$ as a mixed number.

Solution:

25 goes into 114 four times. $25 \cdot 4 = 100$, and $114 - 100 = 14$, so 25 goes into 114 four times, with a remainder of 14.

$$-\left(\frac{100 + 14}{25}\right)$$

$$-\left(\frac{25 \cdot 4 + 14}{25}\right)$$

$$-\left(\frac{25 \cdot 4}{25} + \frac{14}{25}\right)$$

$$-\left(4 + \frac{14}{25}\right)$$

$$-4\frac{14}{25}$$

■ 5. Convert the mixed number into an improper fraction.

$$-2\frac{1}{6}$$

Solution:

The equivalent improper fraction will have the same denominator, 6. The numerator is the product of the denominator and the whole number, added to the numerator, $6 \cdot 2 + 1 = 12 + 1 = 13$.

$$-\left(2 + \frac{1}{6}\right)$$

$$-\left(\frac{6 \times 2}{7} + \frac{1}{6}\right)$$

$$-\left(\frac{6 \times 2 + 1}{6}\right)$$

$$-\frac{13}{6}$$

■ 6. Convert the mixed number into an improper fraction.

$$8\frac{4}{9}$$

Solution:

The equivalent improper fraction will have the same denominator, 9. The numerator is the product of the denominator and the whole number, added to the numerator, $9 \cdot 8 + 4 = 72 + 4 = 76$.

$$\frac{76}{9}$$

ADDING AND SUBTRACTING MIXED NUMBERS

■ 1. Simplify the expression.

$$5\frac{2}{3}+1\frac{1}{12}$$

Solution:

To add the mixed numbers, add the whole numbers separately from the fractions.

$$5+1+\frac{2}{3}+\frac{1}{12}$$

To add the fractions, we'll need to start by making the denominators the same, which we can do by multiplying the first fraction by $4/4$.

$$6+\frac{2}{3}\left(\frac{4}{4}\right)+\frac{1}{12}$$

$$6+\frac{2\cdot 4}{3\cdot 4}+\frac{1}{12}$$

$$6+\frac{8}{12}+\frac{1}{12}$$

Then add the numerators, while keeping the denominator the same.

$$6+\frac{8+1}{12}$$

$$6\frac{9}{12}$$

$$6\frac{3}{4}$$

■ 2. Simplify the expression.

$$8\frac{7}{8}-2\frac{1}{8}$$

Solution:

To find the difference of the mixed numbers, subtract the whole numbers separately from the fractions, but add those differences.

$$8-2+\frac{7}{8}-\frac{1}{8}$$

The fractions already have the same denominator, so we can find the difference directly, which we'll do by finding the difference of the numerators while keeping the denominator the same.

$$6+\frac{7-1}{8}$$

$$6\frac{6}{8}$$

$$6\frac{3}{4}$$

■ 3. Simplify the expression.

$$7\frac{4}{5} - 6\frac{1}{15}$$

Solution:

To find the difference of the mixed numbers, subtract the whole numbers separately from the fractions, but add those differences.

$$7 - 6 + \frac{4}{5} - \frac{1}{15}$$

To find the difference of the fractions, we'll need to start by making the denominators the same, which we can do by multiplying the first fraction by $3/3$.

$$1 + \frac{4}{5}\left(\frac{3}{3}\right) - \frac{1}{15}$$

$$1 + \frac{4 \cdot 3}{5 \cdot 3} - \frac{1}{15}$$

$$1 + \frac{12}{15} - \frac{1}{15}$$

To find the difference of the fractions, we'll find the difference of the numerators while keeping the denominator the same.

$$1+\frac{12-1}{15}$$

$$1\frac{11}{15}$$

■ 4. Simplify the expression.

$$15\frac{1}{2}-11\frac{1}{4}$$

Solution:

To find the difference of the mixed numbers, subtract the whole numbers separately from the fractions, but add those differences.

$$15-11+\frac{1}{2}-\frac{1}{4}$$

To find the difference of the fractions, we'll need to start by making the denominators the same, which we can do by multiplying the first fraction by $2/2$.

$$4+\frac{1}{2}\left(\frac{2}{2}\right)-\frac{1}{4}$$

$$4+\frac{1\cdot 2}{2\cdot 2}-\frac{1}{4}$$

$$4+\frac{2}{4}-\frac{1}{4}$$

To find the difference of the fractions, we'll find the difference of the numerators while keeping the denominator the same.

$$4 + \frac{2 - 1}{4}$$

$$4\frac{1}{4}$$

■ 5. Joey and Alex are both solving the following problem.

$$2\frac{1}{3} + 1\frac{3}{5}$$

Joey takes $2 + 1 = 3$ and then takes

$$\frac{1}{3} + \frac{3}{5} = \frac{14}{15}$$

Then he adds them together to get

$$3\frac{14}{15}$$

Alex decides to change both into improper fractions before adding. He gets

$$2\frac{1}{3} = \frac{7}{3} \text{ and } 1\frac{3}{5} = \frac{8}{5}$$

Then she finds common denominators and adds them together to get

$$\frac{59}{15}$$

Who solved this problem correctly?

Solution:

They both solved it correctly. We get Joey's answer when we convert Alex's answer to a mixed number, and we get Alex's answer when we convert Joey's answer to an improper fraction.

■ 6. Simplify the expression.

$$3\frac{2}{5}+\frac{3}{10}-2\frac{3}{5}$$

Solution:

We'll start by adding the first two mixed numbers. To add the mixed numbers, add the whole numbers separately from the fractions.

$$3+\frac{2}{5}+\frac{3}{10}-2\frac{3}{5}$$

To add the fractions, we'll need to start by making the denominators the same, which we can do by multiplying the first fraction by $2/2$.

$$3+\frac{2}{5}\left(\frac{2}{2}\right)+\frac{3}{10}-2\frac{3}{5}$$

$$3+\frac{2\cdot 2}{5\cdot 2}+\frac{3}{10}-2\frac{3}{5}$$

$$3+\frac{4}{10}+\frac{3}{10}-2\frac{3}{5}$$

Then add the numerators, while keeping the denominator the same.

$$3+\frac{4+3}{10}-2\frac{3}{5}$$

$$3\frac{7}{10}-2\frac{3}{5}$$

To find the difference of the mixed numbers, subtract the whole numbers separately from the fractions, but add those differences.

$$3-2+\frac{7}{10}-\frac{3}{5}$$

To find the difference of the fractions, we'll need to start by making the denominators the same, which we can do by multiplying the second fraction by $2/2$.

$$1+\frac{7}{10}-\frac{3}{5}\left(\frac{2}{2}\right)$$

$$1+\frac{7}{10}-\frac{3\cdot 2}{5\cdot 2}$$

$$1+\frac{7}{10}-\frac{6}{10}$$

To find the difference of the fractions, we'll find the difference of the numerators while keeping the denominator the same.

$$1+\frac{7-6}{10}$$

$$1\frac{1}{10}$$

MULTIPLYING AND DIVIDING MIXED NUMBERS

■ 1. When we multiply and divide mixed numbers, we need to change the mixed numbers into ___________ fractions before we do the multiplication or division.

Solution:

improper

■ 2. Simplify the expression.

$$3\frac{3}{7} \cdot 1\frac{1}{7}$$

Solution:

Convert both mixed numbers to improper fractions.

$$\frac{7 \cdot 3 + 3}{7} \cdot \frac{7 \cdot 1 + 1}{7}$$

$$\frac{24}{7} \cdot \frac{8}{7}$$

To multiply the fractions, multiply the numerators to find the new numerator, and multiply the denominators to find the new denominator.

$$\frac{24 \cdot 8}{7 \cdot 7}$$

$$\frac{192}{49}$$

$$3\frac{45}{49}$$

■ 3. Simplify the expression.

$$5\frac{1}{5} \cdot 2\frac{2}{3}$$

Solution:

Convert both mixed numbers to improper fractions.

$$\frac{5 \cdot 5 + 1}{5} \cdot \frac{3 \cdot 2 + 2}{3}$$

$$\frac{26}{5} \cdot \frac{8}{3}$$

To multiply the fractions, multiply the numerators to find the new numerator, and multiply the denominators to find the new denominator.

$$\frac{26 \cdot 8}{5 \cdot 3}$$

$$\frac{208}{15}$$

$$13\frac{13}{15}$$

■ 4. Simplify the expression.

$$2\frac{3}{4} \div 5\frac{1}{8}$$

Solution:

Convert both mixed numbers to improper fractions.

$$\frac{4 \cdot 2 + 3}{4} \div \frac{8 \cdot 5 + 1}{8}$$

$$\frac{11}{4} \div \frac{41}{8}$$

Convert the division to multiplication by flipping the second fraction.

$$\frac{11}{4} \cdot \frac{8}{41}$$

To multiply the fractions, multiply the numerators to find the new numerator, and multiply the denominators to find the new denominator.

$$\frac{11 \cdot 8}{4 \cdot 41}$$

$$\frac{88}{164}$$

$$\frac{22}{41}$$

■ 5. Simplify the expression.

$$4\frac{5}{9} \div 2\frac{1}{4}$$

Solution:

Convert both mixed numbers to improper fractions.

$$\frac{9 \cdot 4 + 5}{9} \div \frac{4 \cdot 2 + 1}{4}$$

$$\frac{41}{9} \div \frac{9}{4}$$

Convert the division to multiplication by flipping the second fraction.

$$\frac{41}{9} \cdot \frac{4}{9}$$

To multiply the fractions, multiply the numerators to find the new numerator, and multiply the denominators to find the new denominator.

$$\frac{41 \cdot 4}{9 \cdot 9}$$

$$\frac{164}{81}$$

$$2\frac{2}{81}$$

■ 6. Simplify the expression.

$$1\frac{4}{5} \div 3\frac{3}{8}$$

Solution:

Convert both mixed numbers to improper fractions.

$$\frac{5 \cdot 1 + 4}{5} \div \frac{8 \cdot 3 + 3}{8}$$

$$\frac{9}{5} \div \frac{27}{8}$$

Convert the division to multiplication by flipping the second fraction.

$$\frac{9}{5} \cdot \frac{8}{27}$$

To multiply the fractions, multiply the numerators to find the new numerator, and multiply the denominators to find the new denominator.

$$\frac{9 \cdot 8}{5 \cdot 27}$$

$$\frac{72}{135}$$

$$\frac{8}{15}$$

RELATIONSHIPS OF NUMBERS

■ 1. Which fraction is larger?

$$\frac{1}{8} \text{ or } \frac{1}{6}$$

Solution:

When the numerators are equivalent, the larger fraction is the one with the smaller denominator, so $1/6$ is the larger fraction.

■ 2. Which fraction is smaller?

$$\frac{3}{7} \text{ or } \frac{3}{8}$$

Solution:

When the numerators are equivalent, the smaller fraction is the one with the larger denominator, so $3/8$ is the smaller fraction.

■ 3. Find a number that's $1/5$ of the way from $3/10$ to $2/5$.

Solution:

First, we'll find the distance between $3/10$ and $2/5$.

$$\frac{2}{5}-\frac{3}{10}$$

In order to do the subtraction, we have to find a common denominator.

$$\frac{2}{5}\left(\frac{2}{2}\right)-\frac{3}{10}$$

$$\frac{4}{10}-\frac{3}{10}$$

$$\frac{1}{10}$$

Now we want to find $1/5$ of this distance, which means we need to multiply it by $1/5$.

$$\frac{1}{10}\times\frac{1}{5}$$

$$\frac{1}{50}$$

This is $1/5$ the distance from $3/10$ to $2/5$, and since we want to end up exactly $1/5$ of the way from $3/10$ to $2/5$, we add $1/50$ to the smaller fraction, $3/10$.

$$\frac{3}{10}+\frac{1}{50}$$

In order to do the addition, we have to find a common denominator.

$$\frac{3}{10}\left(\frac{5}{5}\right)+\frac{1}{50}$$

$$\frac{15}{50}+\frac{1}{50}$$

$$\frac{16}{50}$$

$$\frac{8}{25}$$

So $8/25$ is the number that's $1/5$ of the way from $3/10$ to $2/5$.

■ 4. Find a number that's $2/3$ of the way from $-2/3$ to $1/4$.

Solution:

First, we'll find the distance between $-2/3$ and $1/4$.

$$\frac{1}{4}-\left(-\frac{2}{3}\right)$$

In order to do the subtraction, we have to find a common denominator.

$$\frac{1}{4}\left(\frac{3}{3}\right)-\left(-\frac{2}{3}\right)\left(\frac{4}{4}\right)$$

$$\frac{3}{12} + \frac{8}{12}$$

$$\frac{11}{12}$$

Now we want to find $2/3$ of this distance, which means we need to multiply it by $2/3$.

$$\frac{11}{12} \times \frac{2}{3}$$

$$\frac{22}{36}$$

$$\frac{11}{18}$$

This is $2/3$ of the distance from $-2/3$ to $1/4$, and since we want to end up exactly $2/3$ of the way from $-2/3$ to $1/4$, we add $11/18$ to the smaller fraction, $-2/3$.

$$-\frac{2}{3} + \frac{11}{18}$$

In order to do the addition, we have to find a common denominator.

$$-\frac{2}{3}\left(\frac{6}{6}\right) + \frac{11}{18}$$

$$-\frac{12}{18} + \frac{11}{18}$$

$$-\frac{1}{18}$$

So $-1/18$ is the number that's $2/3$ of the way from $-2/3$ to $1/4$.

■ 5. Find the fraction halfway between $1/2$ and $2/5$.

Solution:

If we find a common denominator among the fractions, $1/2$ converts to $5/10$, and $2/5$ converts to $4/10$. Now that the denominators are equivalent, we know the smaller fraction is the one with the smaller numerator, so $4/10$ is smaller than $5/10$.

We can now find the distance between these fractions by subtracting the smaller fraction from the larger fraction.

$$\frac{5}{10}-\frac{4}{10}$$

$$\frac{5-4}{10}$$

$$\frac{1}{10}$$

We need half of this total distance, so we'll divide $1/10$ by 2 to find that half of the distance is $1/20$. Now to find the fraction that's halfway between $4/10$ and $5/10$, we'll add this half distance to $4/10$.

$$\frac{4}{10}+\frac{1}{20}$$

$$\frac{4}{10}\left(\frac{2}{2}\right)+\frac{1}{20}$$

$$\frac{4\cdot 2}{10\cdot 2}+\frac{1}{20}$$

$$\frac{8}{20}+\frac{1}{20}$$

$$\frac{8+1}{20}$$

$$\frac{9}{20}$$

■ 6. Find the fraction halfway between $1/10$ and $8/13$.

Solution:

If we find a common denominator among the fractions, $1/10$ converts to $13/130$, and $8/13$ converts to $80/130$. Now that the denominators are equivalent, we know the smaller fraction is the one with the smaller numerator, so $13/130$ is smaller than $80/130$.

We can now find the distance between these fractions by subtracting the smaller fraction from the larger fraction.

$$\frac{80}{130}-\frac{13}{130}$$

$$\frac{80-13}{130}$$

$$\frac{67}{130}$$

We need half of this total distance, so we'll divide $67/130$ by 2 to find that half of the distance is $67/260$. Now to find the fraction that's halfway between $13/130$ and $80/130$, we'll add this half distance to $13/130$.

$$\frac{13}{130}+\frac{67}{260}$$

$$\frac{13}{130}\left(\frac{2}{2}\right)+\frac{67}{260}$$

$$\frac{13\cdot 2}{130\cdot 2}+\frac{67}{260}$$

$$\frac{26}{260}+\frac{67}{260}$$

$$\frac{93}{260}$$

ADDING MIXED MEASURES

■ 1. Add the mixed measures.

4 seconds, 11 minutes, 3 hours, 35 minutes, 56 minutes, 35 seconds

Solution:

Organize and combine the measures we've been given.

3 hours, $11+35+56$ minutes, $4+35$ seconds

3 hours, 102 minutes, 39 seconds

Since we have more than 60 minutes, we want to convert some of those minutes into hours.

3 hours, $60+42$ minutes, 39 seconds

3 hours, 1 hour, 42 minutes, 39 seconds

$3+1$ hours, 42 minutes, 39 seconds

4 hours, 42 minutes, 39 seconds

■ 2. Add the mixed measures.

34 inches, 2 yards, 5 feet, 8 inches, 13 feet, 1 yard

Solution:

Organize and combine the measures we've been given.

$2+1$ yards, $5+13$ feet, $34+8$ inches

3 yards, 18 feet, 42 inches

Since we have more than 12 inches, we want to convert some of those inches into feet.

3 yards, 18 feet, $36+6$ inches

3 yards, 18 feet, 3 feet, 6 inches

3 yards, $18+3$ feet, 6 inches

3 yards, 21 feet, 6 inches

Since we have more than 3 feet, we want to convert some of those feet into yards.

3 yards, 7 yards, 6 inches

10 yards, 6 inches

■ 3. Add the mixed measures.

25 seconds, 1 hour, 15 minutes, 45 seconds, 22 minutes

Solution:

Organize and combine the measures we've been given.

1 hour, $15+22$ minutes, $25+45$ seconds

1 hour, 37 minutes, 70 seconds

Since we have more than 60 seconds, we want to convert some of those seconds into minutes.

1 hour, 37 minutes, $60+10$ seconds

1 hour, 37 minutes, 1 minute, 10 seconds

1 hour, $37+1$ minutes, 10 seconds

1 hour, 38 minutes, 10 seconds

■ 4. Add the mixed measures.

13 inches, 45 feet, 35 inches, 27 feet, 9 yards

Solution:

Organize and combine the measures we've been given.

9 yards, $45+27$ feet, $13+35$ inches

9 yards, 72 feet, 48 inches

Since we have more than 12 inches, we want to convert some of those inches into feet.

9 yards, 72 feet, 4 feet

9 yards, $72+4$ feet

9 yards, 76 feet

Since we have more than 3 feet, we want to convert some of those feet into yards.

9 yards, $75+1$ feet

9 yards, 25 yards, 1 foot

34 yards, 1 foot

■ 5. Add the mixed measures.

1 foot, 38 inches

Solution:

Since we have more than 12 inches, we want to convert some of those inches into feet.

1 foot, 38 inches

1 foot, $36 + 2$ inches

1 foot, 3 feet $+2$ inches

$1 + 3$ feet $+2$ inches

4 feet, 2 inches

■ 6. Add the mixed measures.

1 hour, 85 minutes, 55 seconds, 20 minutes

Solution:

Organize and combine the measures we've been given.

1 hour, $85 + 20$ minutes, 55 seconds

1 hour, 105 minutes, 55 seconds

Since we have more than 60 minutes, we want to convert some of those minutes into hours.

1 hour, $60 + 45$ minutes, 55 seconds

1 hour, 1 hour, 45 minutes, 55 seconds

$1 + 1$ hour, 45 minutes, 55 seconds

2 hour, 45 minutes, 55 seconds

PLACE VALUE

■ 1. Identify the place value of the 2 in $4{,}562.387$.

Solution:

The 2 in $4{,}562.387$ is one place to the left of the decimal point, which means it's in the "ones" place.

■ 2. Identify the place value of the 0 in 307.119.

Solution:

The 0 in 307.119 is two places to the left of the decimal point, which means it's in the "tens" place.

■ 3. What is the number in the ten-thousandths place of $6{,}520.0019$?

Solution:

The number in the ten-thousandths place is four places to the right of the decimal point, which means it's the 9 in $6{,}520.0019$.

■ 4. What is the number in the tenths place of 0.89104?

Solution:

The number in the tenths place is one place to the right of the decimal point, which means it's the 8 in 0.89104.

■ 5. The further we move to the right of the decimal point, the ___________ (smaller or larger?) the value gets.

Solution:

smaller

■ 6. The further we move to the left of the decimal point, the ___________ (smaller or larger?) the value gets.

Solution:

larger

DECIMAL ARITHMETIC

■ 1. Find the sum.

$$4.5 + 3.75$$

Solution:

Line up the decimal points and then add the decimal numbers. There's no need to worry about the fact that one of the numbers has more digits to the right of the decimal point than the other number does.

We just pretend that there's a 0 after the 5 in 4.5 (we pretend that it's 4.50), so that the two decimal numbers we're adding have the same number of digits to the right of the decimal point.

$$\begin{array}{r} 4.50 \\ +\ \underline{3.75} \\ 8.25 \end{array}$$

■ 2. Find the difference.

$$7.87 - 4.9876$$

Solution:

Line up the decimal points and then subtract the decimal numbers. There's no need to worry about the fact that one of the numbers has more digits to the right of the decimal point than the other number does.

We just pretend that there's a 00 after the 7 in 7.87 (we pretend that it's 7.8700), so that the two decimal numbers we're subtracting have the same number of digits to the right of the decimal point.

$$\begin{array}{r} 7.8700 \\ -\ \underline{4.9876} \\ 2.8824 \end{array}$$

■ 3. Find the product.

$$1.5 \cdot 8.8$$

Solution:

Multiply normally, ignoring the decimal points.

$$\begin{array}{r} 1.5 \\ \times\ \underline{8.8} \\ 120 \end{array}$$

$$+\ \underline{1200}$$

$$1320$$

Between the two given numbers, there are two digits to the right of the decimal place, the 5 in 1.5 and the 8 in 8.8, so we'll move the decimal point two places to the left, and 1320 becomes

$$13.20$$

■ 4. Find the quotient.

$$5.65 \div 0.02$$

Solution:

To find the quotient, we'll do long division, but not until after we determine where to place the decimal point in our answer.

To figure out where it should go, we need to change both numbers into whole numbers. So 5.65 becomes 565 and 0.02 becomes 2. Then we can do the long division as if we were doing division with whole numbers, instead of decimal numbers.

$$282.5$$

$$2\,|\overline{565.0}$$

$$-\ \underline{4\quad\ }$$

$$\begin{array}{r} 16 \\ -\underline{16} \\ 5 \\ -\underline{4} \\ 10 \\ -\underline{10} \\ 0 \end{array}$$

■ 5. Simplify the expression.

$$2.5783 + 5.789 - 3.25$$

Solution:

We'll start by adding $2.5783 + 5.789$. We just pretend that there's a 0 after the 9 in 5.789 (we pretend that it's 5.7890), so that the two decimal numbers we're adding have the same number of digits to the right of the decimal point.

$$\begin{array}{r} 2.5783 \\ +\ \underline{5.7890} \\ 8.3673 \end{array}$$

Now to do the subtraction, line up the decimal points and then subtract.

$$\begin{array}{r} 8.3673 \\ -\ \underline{3.2500} \\ 5.1173 \end{array}$$

■ 6. Simplify the expression.

$$1.24 \cdot 2.61$$

Solution:

Multiply normally, ignoring the decimals.

$$\begin{array}{r} 1.24 \\ \times\ \ \underline{2.61} \\ 124 \\ +\ \ 7440 \\ +\ \underline{24800} \\ 32364 \end{array}$$

Between the two given numbers, there are four digits to the right of the decimal point, so we'll move the decimal point four places to the left to get

3.2364

REPEATING DECIMALS

■ 1. A finite decimal number is a number that ____________.

Solution:

ends

■ 2. Rewrite 0.888888 as a repeating decimal.

Solution:

$0.\overline{8}$

■ 3. Rewrite 0.1818181818 as a repeating decimal.

Solution:

$0.\overline{18}$

■ 4. What is the next digit in $3.\overline{142857}$?

Solution:

1

5. What is the next digit in $0.41\overline{6}$?

Solution:

6

6. Name an example of a decimal number that does not end, but does not repeat.

Solution:

π is an example of a non-repeating, non-finite decimal number.

ROUNDING

■ 1. If a number is ___________ or greater, we round up.

Solution:

5

■ 2. Round 11.451 to the nearest tenth.

Solution:

The 4 is the digit in the tenths place, so we'll use the 5 that follows the 4 to round up to 11.5.

■ 3. Round 691.014 to the tens place.

Solution:

The 9 is the digit in the tens place, so we'll use the 1 that follows the 9 to round down to 690.

■ 4. Round $11.\overline{6}$ to the nearest thousandth.

Solution:

If we extend the decimal to one digit past the thousandths place, we can write it as 11.6666. The third 6 is in the thousandths place, so we'll use the fourth 6 that follows it to round up to 11.667.

■ 5. When we round a number to the tenths place, we look at the digit in the ___________ place in order to determine which way to round the number.

Solution:

hundredths

■ 6. Judith types $2 \div 3$ into the calculator and gets the answer 0.66666666667. Judith tells her friend Andy that this is not a repeating decimal because there is a 7 at the end. Andy disagrees and says the calculator rounds the number and that is why there is a 7. Who is correct? Why?

Solution:

Andy is correct because calculators cannot show repeating decimals going on and on. So, it must round the number based on the number of digits that can fit on the screen. When the calculator rounds, 6 is higher than 5, so the number gets rounded up from a 6 to a 7.

RATIO AND PROPORTION

■ 1. Solve for the variable.

$$\frac{6}{10} = \frac{m}{15}$$

Solution:

Cross multiply to get rid of the fractions.

$$6(15) = 10(m)$$

$$90 = 10m$$

Divide both sides by 10 in order to solve for m.

$$\frac{90}{10} = \frac{10m}{10}$$

$$9 = 1m$$

$$m = 9$$

■ 2. Solve for the variable.

$$\frac{d}{7} = \frac{14}{49}$$

Solution:

Cross multiply to get rid of the fractions.

$$49(d) = 7(14)$$

$$49d = 98$$

Divide both sides by 49 in order to solve for d.

$$\frac{49d}{49} = \frac{98}{49}$$

$$1d = 2$$

$$d = 2$$

■ 3. Solve for the variable.

$$\frac{5}{v} = \frac{25}{40}$$

Solution:

Cross multiply to get rid of the fractions.

$$5(40) = 25(v)$$

$$200 = 25v$$

Divide both sides by 25 in order to solve for v.

$$\frac{200}{25}=\frac{25v}{25}$$

$$8=1v$$

$$v=8$$

■ 4. Solve for the variable.

$$\frac{22}{30}=\frac{33}{t}$$

Solution:

Cross multiply to get rid of the fractions.

$$22(t)=30(33)$$

$$22t=990$$

Divide both sides by 22 in order to solve for t.

$$\frac{22t}{22}=\frac{990}{22}$$

$$1t=45$$

$$t=45$$

■ 5. Solve for the variable.

$$\frac{8}{12}=\frac{20}{x}$$

Solution:

Cross multiply to get rid of the fractions.

$$8(x)=12(20)$$

$$8x=240$$

Divide both sides by 8 in order to solve for x.

$$\frac{8x}{8}=\frac{240}{8}$$

$$1x=30$$

$$x=30$$

■ 6. The reason we multiply the left and right side by the same number when we cross multiply is because, when we're solving equations, we must keep both sides ____________.

Solution:

balanced

UNIT PRICE

■ 1. If we can purchase 2 oranges for $\$0.20$, how many oranges can we purchase for $\$2.00$?

Solution:

Write an equation that expresses this proportion. On the left, the ratio we'll use will be "two oranges for $\$0.20$," and we'll equate that to a ratio on the right that says "x oranges for $\$2.00$."

$$\frac{2}{0.20}=\frac{x}{2.00}$$

Cross multiply to get rid of the fractions.

$$2(2.00)=0.20(x)$$

$$4.00=0.20x$$

Divide both sides by 0.20 in order to solve for x.

$$\frac{4.00}{0.20}=\frac{0.20x}{0.20}$$

$$20=1x$$

$$x=20$$

■ 2. If we purchase 2 oranges for $\$0.20$, how much will it cost to purchase 5 oranges?

Solution:

Write an equation that expresses this proportion. On the left, the ratio we'll use will be "two oranges for $\$0.20$," and we'll equate that to a ratio on the right that says "five oranges for x."

$$\frac{2}{0.20} = \frac{5}{x}$$

Cross multiply to get rid of the fractions.

$$2(x) = 0.20(5)$$

$$2x = 1$$

Divide both sides by 2 in order to solve for x.

$$\frac{2x}{2} = \frac{1}{2}$$

$$x = \frac{1}{2}$$

$$x = \$0.50$$

■ 3. Sally went to the candy store and bought 40 jelly beans for $\$0.50$. How much would 60 jelly beans cost her?

Solution:

Write an equation that expresses this proportion. On the left, the ratio we'll use will be "40 jelly beans for $\$0.50$," and we'll equate that to a ratio on the right that says "60 jelly beans for x."

$$\frac{40}{0.5} = \frac{60}{x}$$

Cross multiply to get rid of the fractions.

$$40(x) = 60(0.5)$$

$$40x = 30$$

Divide both sides by 40 in order to solve for x.

$$\frac{40x}{40} = \frac{30}{40}$$

$$x = \$0.75$$

■ 4. While Steven is at the grocery store, he's trying to determine which bag of popcorn is the better deal. The first bag is a 10-ounce bag of popcorn for $\$1.59$. The second bag is a 15-ounce bag of popcorn for $\$1.89$. Which bag is the better deal? Why?

Solution:

The better deal will be the bag with the better price per ounce. The price per ounce for the first bag is $\$1.59/10 \approx \$0.16$, while the price per ounce for the second bag is $\$1.89/15 \approx \$0.13$. Because the second bag has a lower price per ounce, $\$0.13 < \$0.16$, the second bag is a better deal.

■ 5. We can purchase 15 pencils for $\$4$. If we want to find the price per pencil, we would divide ____________ by ____________.

Solution:

price, number of pencils

■ 6. We can purchase 15 pencils for $\$4$. If we want to find the number of pencils we can buy per dollar, we would divide ____________ by ____________.

Solution:

number of pencils, number of dollars (price)

UNIT MULTIPLIERS

■ 1. When we're setting up a unit multiplier, the units we want to keep need to be placed in the ___________ of the fraction.

Solution:

numerator

■ 2. Convert 8 yards to inches.

Solution:

There are 3 feet in 1 yard. The units we want to keep are feet, and the units we want to get rid of are yards, so we'll set up a unit multiplier with feet on top and yards on bottom.

$$8 \text{ yards} \cdot \frac{3 \text{ feet}}{1 \text{ yard}}$$

This will convert yards into feet, but then we'll need to convert those feet into inches. There are 12 inches in 1 foot. The units we want to keep are inches, and the units we want to get rid of are feet, so we'll set up a unit multiplier with inches on top and feet on bottom.

$$8 \text{ yards} \cdot \frac{3 \text{ feet}}{1 \text{ yard}} \cdot \frac{12 \text{ inches}}{1 \text{ foot}}$$

$$8(3) \text{ feet} \cdot \frac{12 \text{ inches}}{1 \text{ foot}}$$

$$8(3)(12) \text{ inches}$$

$$288 \text{ inches}$$

■ 3. Convert 4 square feet to square inches.

Solution:

There are 144 square inches in 1 square foot. The units we want to keep are square inches, and the units we want to get rid of are square feet, so we'll set up a unit multiplier with square inches on top and square feet on bottom.

$$4 \text{ square feet} \cdot \frac{144 \text{ square inches}}{1 \text{ square foot}}$$

$$4(144) \text{ square inches}$$

$$576 \text{ square inches}$$

■ 4. Convert 144 square inches to square feet.

Solution:

There are 144 square inches in 1 square foot. The units we want to keep are square feet, and the units we want to get rid of are square inches, so we'll set up a unit multiplier with square feet on top and square inches on bottom.

$$144 \text{ square inches} \cdot \frac{1 \text{ square foot}}{144 \text{ square inches}}$$

$$1 \text{ square foot}$$

■ 5. Convert $4{,}320$ cubic inches to cubic feet.

Solution:

There are 12 inches in 1 foot. The units we want to keep are feet, and the units we want to get rid of are inches, so we'll set up a unit multiplier with feet on top and inches on bottom.

$$4{,}320 \text{ inches}^3 \cdot \frac{1 \text{ foot}}{12 \text{ inches}} \cdot \frac{1 \text{ foot}}{12 \text{ inches}} \cdot \frac{1 \text{ foot}}{12 \text{ inches}}$$

$$\frac{4{,}320(1)(1)(1)}{(12)(12)(12)} \text{ feet}^3$$

$$2.5 \text{ cubic feet}$$

6. Jason is converting $4{,}536$ cubic feet to cubic yards. His work is shown below. Did he solve the problem correctly? Why or why not?

$$4{,}536 \text{ cubic feet} \cdot \frac{3 \text{ feet}}{1 \text{ yard}} \cdot \frac{3 \text{ feet}}{1 \text{ yard}} \cdot \frac{3 \text{ feet}}{1 \text{ yard}} = 122{,}472 \text{ cubic yards}$$

Solution:

He solved it incorrectly. He did not place the yards on the top of the fraction and the feet on the bottom of the fraction so that the feet would cancel out and leave just cubic yards as units. He should have set it up as

$$4{,}536 \text{ cubic feet} \cdot \frac{1 \text{ yard}}{3 \text{ feet}} \cdot \frac{1 \text{ yard}}{3 \text{ feet}} \cdot \frac{1 \text{ yard}}{3 \text{ feet}} = 168 \text{ cubic yards}$$

EXPONENTS

■ 1. An exponent tells us how many times to multiply the base by ___________.

Solution:

itself

■ 2. Is 2^3 the same as 3^2? Why or why not?

Solution:

No, it's not the same. 2^3 means $2 \cdot 2 \cdot 2 = 8$, whereas 3^2 means $3 \cdot 3 = 9$.

■ 3. Find the sum.

$$5^3 + 2^4$$

Solution:

Find the value of each term individually, then add the results.

$$5 \cdot 5 \cdot 5 + 2 \cdot 2 \cdot 2 \cdot 2$$

$$125 + 16$$

$$141$$

■ 4. Write the number using exponents.

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

Solution:

Because we're multiplying nine factors of 2, we can express that as 2^9 instead.

■ 5. Write the following number without an exponent.

$$1^6$$

Solution:

Because we're multiplying six factors of 1, we can express that as $1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1$ instead.

■ 6. Write the number without an exponent.

$$(-9)^6$$

Solution:

Because we're multiplying six factors of -9, we can express that as

$$(-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9)$$

instead.

RULES OF EXPONENTS

■ 1. Find the sum.

$$2x^3 + x^3 + x^3 + 3x^3$$

Solution:

Because all these terms are x^3 terms, we're able to add them.

$$(2 + 1 + 1 + 3)x^3$$

$$7x^3$$

■ 2. Find the product.

$$x^6 \cdot x^2 \cdot x^3$$

Solution:

When we multiply terms with like bases, we add the exponents.

$$x^{6+2+3}$$

$$x^{11}$$

■ 3. Simplify the expression.

$$x \cdot x \cdot x$$

Solution:

When we multiply terms with like bases, we add the exponents.

$$x^{1+1+1}$$

$$x^3$$

■ 4. Stephanie and Jimmy are trying to find a shortcut to simplify the expression below. Stephanie says that they should add the exponents $(3 + 5 = 8)$ and then raise 4 to that power. Jimmy says that since it's multiplication, they should multiply the exponents $(3 \cdot 5 = 15)$ and then raise 4 to that power. Who is correct and why?

$$4^3 \cdot 4^5$$

Solution:

Stephanie is correct. 4^3 is equivalent to $4 \cdot 4 \cdot 4$, and 4^5 is equivalent to $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$. If we're multiplying these values together, we get

$$4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$$

$$4^8$$

$$65{,}536$$

■ 5. Simplify the expression.

$$\frac{x^5 + x^2 \cdot x^3}{x^7}$$

Solution:

We'll use the fact that

$$x^a x^b = x^{a+b}$$

Here we have $a = 2$, and $b = 3$, so we get

$$x^2 x^3 = x^{2+3} = x^5$$

Then

$$\frac{x^5 + x^2 \cdot x^3}{x^7} = \frac{x^5 + x^5}{x^7}$$

Because all these terms in the numerator are x^5 terms, we're able to add them.

$$\frac{2x^5}{x^7}$$

Then we'll use the fact that

$$\frac{x^a}{x^b} = x^{a-b}$$

Here we have $a = 5$, and $b = 7$, so we get

$$\frac{2x^5}{x^7} = 2x^{5-7} = 2x^{-2}$$

Then, we know that $x^{-a} = 1/x^a$, so

$$2x^{-2} = \frac{2}{x^2}$$

■ 6. Simplify the expression.

$$\frac{x^{-4} \cdot x^6}{x^2}$$

Solution:

We'll use the fact that

$$x^a x^b = x^{a+b}$$

Here we have $a = -4$, and $b = 6$, so we get

$$x^{-4}x^6 = x^{-4+6} = x^2$$

Then

$$\frac{x^{-4} \cdot x^6}{x^2} = \frac{x^2}{x^2}$$

Then we'll use the fact that

$$\frac{x^a}{x^b} = x^{a-b}$$

Here we have $a = 2$, and $b = 2$, so we get

$$\frac{x^2}{x^2} = x^{2-2} = x^0 = 1$$

POWER RULE FOR EXPONENTS

■ 1. The power rule tells us that, when we raise a power to a power, we can ____________ those powers together.

Solution:

multiply

■ 2. Simplify the expression.

$$(x^3)^3$$

Solution:

Applying the power rule for exponents, we multiply the exponents.

$$x^{3\cdot 3}$$

$$x^9$$

■ 3. Simplify the expression.

$$(x^2)^{-4}$$

Solution:

Applying the power rule for exponents, we multiply the exponents.

$$x^{2(-4)}$$

$$x^{-8}$$

$$\frac{1}{x^8}$$

■ 4. Simplify the expression.

$$(2^m)^p$$

Solution:

Applying the power rule for exponents, we multiply the exponents.

$$2^{(m \cdot p)}$$

$$2^{mp}$$

■ 5. Simplify the expression.

$$(x^2y^2)^3$$

Solution:

Applying $a^n b^n = (ab)^n$, we can rewrite the expression as

$$((xy)^2)^3$$

Then using power rule, we get

$$(xy)^{2\cdot 3}$$

$$(xy)^6$$

$$x^6 y^6$$

■ 6. Simplify the expression.

$$(x^{-5} \cdot x^4)^{-2}$$

Solution:

When we multiply terms with like bases, we add the exponents.

$$(x^{-5+4})^{-2}$$

$$(x^{-1})^{-2}$$

Applying the power rule for exponents, we multiply the exponents.

$$x^{(-1)\cdot(-2)}$$

x^2

NEGATIVE AND OTHER EXPONENT RULES

■ 1. Simplify the expression.

$$\frac{9a^5b^4}{3a^2b^7}$$

Solution:

Use quotient rule to simplify across the numerator and denominator.

$$\frac{9a^5b^4}{3a^2b^7}$$

$$3a^{5-2}b^{4-7}$$

$$3a^3b^{-3}$$

We know that $x^{-a} = 1/x^a$, so we get

$$\frac{3a^3}{b^3}$$

We know that

$$\frac{x^a}{y^a} = \left(\frac{x}{y}\right)^a$$

so we get

$$3\left(\frac{a}{b}\right)^3$$

■ 2. Simplify the expression.

$$\frac{2x^0y^6-(y^2)^3}{x^6}$$

Solution:

First we need to simplify the numerator. We know that $x^0=1$, so

$$\frac{2y^6-(y^2)^3}{x^6}$$

Applying the power rule for exponents, we multiply the exponents.

$$\frac{2y^6-y^6}{x^6}$$

$$\frac{y^6}{x^6}$$

We know that

$$\frac{x^a}{y^a}=\left(\frac{x}{y}\right)^a$$

so we get

$$\left(\frac{y}{x}\right)^6$$

■ 3. Simplify the expression.

$$\frac{(x^{2p})^3}{x^{3p}y^{3p}}$$

Solution:

Use power rule to simplify the exponent in the numerator.

$$\frac{x^{6p}}{x^{3p}y^{3p}}$$

Now use quotient rule to simplify across the numerator and denominator.

$$\frac{x^{6p-3p}}{y^{3p}}$$

$$\frac{x^{3p}}{y^{3p}}$$

We know that

$$\frac{x^a}{y^a} = \left(\frac{x}{y}\right)^a$$

so we get

$$\left(\frac{x}{y}\right)^{3p}$$

■ 4. Simplify the expression.

$$\frac{(x^{-3a+4})^2}{x^{-4a+8}y^{-2a}}$$

Solution:

Use power rule to simplify the exponent in the numerator.

$$\frac{x^{2(-3a+4)}}{x^{-4a+8}y^{-2a}}$$

$$\frac{x^{-6a+8}}{x^{-4a+8}y^{-2a}}$$

Now use quotient rule to simplify across the numerator and denominator.

$$\frac{x^{-6a+8+4a-8}}{y^{-2a}}$$

$$\frac{x^{-2a}}{y^{-2a}}$$

We know that

$$\frac{x^a}{y^a} = \left(\frac{x}{y}\right)^a$$

so we get

$$\left(\frac{x}{y}\right)^{-2a}$$

Applying quotient to a negative exponent

$$\left(\frac{y}{x}\right)^{2a}$$

■ 5. Simplify the expression.

$$\left(\frac{5x^{-2}}{y^{-2}}\right)^4$$

Solution:

We know that

$$\frac{x^a}{y^a} = \left(\frac{x}{y}\right)^a$$

so we get

$$\frac{(5x^{-2})^4}{(y^{-2})^4}$$

Applying the power rule for exponents, we multiply the exponents.

$$\frac{5^4x^{-8}}{y^{-8}}$$

$$625\left(\frac{x}{y}\right)^{-8}$$

Then we can change the exponent from negative to positive by taking the reciprocal of the fraction.

$$625\left(\frac{y}{x}\right)^{8}$$

■ 6. Simplify the expression.

$$\left(\frac{2x^5y^7}{y^{12}}\right)^0$$

Solution:

Any non-zero value raised to the power of 0 is 1, so the expression simplifies to just 1.

Alternately, we could use the quotient rule to simplify across the numerator and denominator.

$$\left(\frac{2x^5y^7}{y^{12}}\right)^0$$

$$(2x^5y^{7-12})^0$$

$$(2x^5y^{-5})^0$$

Then, we know that $x^{-a} = 1/x^a$, so

$$(2x^5y^{-5})^0 = \left(\frac{2x^5}{y^5}\right)^0$$

Applying the power rule for exponents, we multiply the exponents.

$$\frac{(2x^5)^0}{(y^5)^0}$$

$$\frac{2^0x^0}{y^0}$$

$$\frac{1 \cdot 1}{1}$$

$$1$$

RADICALS

■ 1. Radicals are the opposite of ____________.

Solution:

exponents

■ 2. $\sqrt[4]{x}$ can also be written as ____________.

Solution:

$x^{\frac{1}{4}}$

■ 3. Find the value of $\sqrt{36}$.

Solution:

Rewrite the value under the root as a perfect square.

$$\sqrt{6 \cdot 6}$$

So the square root is

$$6$$

■ 4. $x^{\frac{1}{3}}$ can also be written as ___________.

Solution:

$\sqrt[3]{x}$

■ 5. Find the value of $\sqrt{300}$.

Solution:

Rewrite the value under the root as a product.

$$\sqrt{100 \cdot 3}$$

Now separate the values under the radical into separate radicals.

$$\sqrt{100}\sqrt{3}$$

Rewrite the value under the root as a perfect square.

$$\sqrt{10 \cdot 10}\sqrt{3}$$

So the square root is

$$10\sqrt{3}$$

■ 6. Find the value of $\sqrt{5{,}000}$.

Solution:

Rewrite the value under the root as a product.

$$\sqrt{100 \cdot 50}$$

$$\sqrt{100 \cdot 25 \cdot 2}$$

Now separate the values under the radical into separate radicals.

$$\sqrt{100}\sqrt{25}\sqrt{2}$$

Rewrite the value under the root as a perfect square.

$$\sqrt{10 \cdot 10}\sqrt{5 \cdot 5}\sqrt{2}$$

So the square root is

$$10(5)\sqrt{2}$$

$$50\sqrt{2}$$

ADDING AND SUBTRACTING RADICALS

■ 1. Find the value of $2\sqrt{3}+5\sqrt{3}$.

Solution:

Because the radicals are the same, we have like-terms and we can add them directly.

$$(2+5)\sqrt{3}$$

$$7\sqrt{3}$$

■ 2. Find the value of $\sqrt{32}-\sqrt{2}$.

Solution:

Because the radicals aren't the same, we need to simplify the radicals first.

$$\sqrt{16\cdot 2}-\sqrt{2}$$

$$\sqrt{16}\sqrt{2}-\sqrt{2}$$

$$4\sqrt{2}-\sqrt{2}$$

Now with like terms, we can find the difference.

$$(4-1)\sqrt{2}$$

$$3\sqrt{2}$$

■ 3. Find the value of $\sqrt{3}+\sqrt{12}$.

Solution:

Because the radicals aren't the same, we need to simplify the radicals first.

$$\sqrt{3}+\sqrt{4\cdot 3}$$

$$\sqrt{3}+\sqrt{4}\sqrt{3}$$

$$\sqrt{3}+2\sqrt{3}$$

Now with like terms, we can find the sum.

$$(1+2)\sqrt{3}$$

$$3\sqrt{3}$$

■ 4. Find the value of $\sqrt{16}+\sqrt{25}$.

Solution:

Both radicals can be evaluated directly.

$$\sqrt{16}+\sqrt{25}$$

$$4+5$$

$$9$$

■ 5. Find the value of $4\sqrt{3}+2\sqrt{2}-2\sqrt{3}-\sqrt{2}$.

Solution:

If we re-order the terms, we notice that we can combine some of the radicals.

$$(4\sqrt{3}-2\sqrt{3})+(2\sqrt{2}-\sqrt{2})$$

$$(4-2)\sqrt{3}+(2-1)\sqrt{2}$$

$$2\sqrt{3}+\sqrt{2}$$

The remaining roots are still different, and can't be simplified further, so this is as far as we can simplify the expression.

■ 6. Find the value of $3\sqrt{4}-2\sqrt{9}$.

Solution:

Both radicals can be evaluated directly.

$$3(2) - 2(3)$$

$$6 - 6$$

$$0$$

MULTIPLYING RADICALS

■ 1. Find the value of $\sqrt{20} \cdot \sqrt{4}$.

Solution:

Simplify the radicals first,

$$\sqrt{4 \cdot 5} \cdot \sqrt{4}$$

$$\sqrt{4}\sqrt{5} \cdot \sqrt{4}$$

$$2\sqrt{5} \cdot 2$$

then multiply.

$$4\sqrt{5}$$

■ 2. Find the value of $\sqrt{13} \cdot \sqrt{7}$.

Solution:

Neither radical can be simplified initially, so simply multiply the radicals.

$$\sqrt{13 \cdot 7}$$

$$\sqrt{91}$$

The value 91 has no factors that are perfect squares, so we can't simplify it any further.

■ 3. Find the value of $8\sqrt{3} \cdot \sqrt{12}$.

Solution:

Multiply the radicals first,

$$8\sqrt{3 \cdot 12}$$

$$8\sqrt{36}$$

then simplify.

$$8(6)$$

$$48$$

■ 4. Find the value of $15\sqrt{2} \cdot \sqrt{16}$.

Solution:

Simplify the radicals first,

$$15\sqrt{2} \cdot 4$$

then multiply.

$$60\sqrt{2}$$

■ 5. Find the value of $2\sqrt{3} \cdot 5\sqrt{5}$.

Solution:

We need to use the rule that tells us that

$$\sqrt{m} \cdot \sqrt{n} = \sqrt{mn}$$

Applying this to our problem, we can rewrite it.

$$2\sqrt{3} \cdot 5\sqrt{5}$$

$$2 \cdot 5\sqrt{3 \cdot 5}$$

$$10\sqrt{15}$$

■ 6. Find the value of $\sqrt[3]{12} \cdot \sqrt[3]{4}$.

Solution:

We need to use the rule that tells us that

$$\sqrt{m} \cdot \sqrt{n} = \sqrt{mn}$$

Applying this to our problem, we can rewrite it.

$$\sqrt[3]{12} \cdot \sqrt[3]{4}$$

$$\sqrt[3]{12 \cdot 4}$$

$$\sqrt[3]{48}$$

Then simplify.

$$\sqrt[3]{8 \cdot 6}$$

$$\sqrt[3]{8} \cdot \sqrt[3]{6}$$

$$2\sqrt[3]{6}$$

DIVIDING RADICALS

■ 1. Simplify the expression.

$$\sqrt{\frac{36}{6}}$$

Solution:

Simplify the quotient inside the radical.

$$\sqrt{6}$$

There are no factors of 6 that are perfect squares, so we can't simplify the radical any further.

■ 2. Simplify the expression.

$$\sqrt{\frac{45}{5}}$$

Solution:

Simplify the quotient inside the radical, then simplify the radical.

$$\sqrt{9}$$

3

■ 3. Simplify the expression.

$$\frac{\sqrt{20x^5y^7}}{\sqrt{5x^3y}}$$

Solution:

When we have a fraction in which the numerator is a root and the denominator is a root, we can put the fraction under one root instead.

$$\frac{\sqrt{20x^5y^7}}{\sqrt{5x^3y}}$$

$$\sqrt{\frac{20x^5y^7}{5x^3y}}$$

$$\sqrt{4x^{5-3}y^{7-1}}$$

$$\sqrt{4x^2y^6}$$

$$2xy^3$$

■ 4. Simplify the expression.

$$\frac{\sqrt[3]{-32}}{\sqrt[3]{2}}$$

Solution:

The quotient of roots is always equal to the root of the quotient, so we can rewrite the expression as

$$\frac{\sqrt[3]{-32}}{\sqrt[3]{2}}$$

$$\sqrt[3]{\frac{-32}{2}}$$

Simplify the quotient.

$$\sqrt[3]{-16}$$

$$\sqrt[3]{-8 \cdot 2}$$

$$-2\sqrt[3]{2}$$

■ 5. Simplify the expression.

$$\frac{\sqrt{5}}{\sqrt{15}}$$

Solution:

The quotient of roots is always equal to the root of the quotient, so we can rewrite the expression as

$$\frac{\sqrt{5}}{\sqrt{15}}$$

$$\sqrt{\frac{5}{15}}$$

Simplify the quotient.

$$\sqrt{\frac{1}{3}}$$

$$\frac{\sqrt{1}}{\sqrt{3}}$$

$$\frac{1}{\sqrt{3}}$$

To rationalize the denominator, we'll multiply both the numerator and denominator by $\sqrt{3}$.

$$\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\frac{\sqrt{3}}{3}$$

■ 6. Simplify the expression.

$$\frac{\sqrt{8}}{5\sqrt{2}}$$

Solution:

The quotient of roots is always equal to the root of the quotient, so we can rewrite the expression as

$$\frac{\sqrt{8}}{5\sqrt{2}}$$

$$\frac{1}{5}\cdot\frac{\sqrt{8}}{\sqrt{2}}$$

$$\frac{1}{5}\cdot\sqrt{\frac{8}{2}}$$

$$\frac{1}{5}\cdot\sqrt{4}$$

$$\frac{1}{5}\cdot 2$$

$$\frac{2}{5}$$

RADICAL EXPRESSIONS

■ 1. Find the value of $\sqrt{80}-\sqrt{20}$.

Solution:

Notice that $80=4\cdot 20$, and use that to simplify the first radical.

$$\sqrt{4\cdot 20}-\sqrt{20}$$

$$\sqrt{4}\sqrt{20}-\sqrt{20}$$

$$2\sqrt{20}-\sqrt{20}$$

Now that the radicals are the same, combine like terms.

$$(2-1)\sqrt{20}$$

$$1\sqrt{20}$$

$$\sqrt{20}$$

Now simplify the remaining radical.

$$\sqrt{4\cdot 5}$$

$$\sqrt{4}\sqrt{5}$$

$$2\sqrt{5}$$

■ 2. Find the value of $5\sqrt{24} \cdot \sqrt{15}$.

Solution:

First simplify the radicals individually.

$$5\sqrt{4 \cdot 6} \cdot \sqrt{15}$$

$$5\sqrt{4}\sqrt{6} \cdot \sqrt{15}$$

$$5(2)\sqrt{6} \cdot \sqrt{15}$$

$$10\sqrt{6} \cdot \sqrt{15}$$

Now factor the values inside both radicals, then split the radicals apart, group together like terms, and simplify.

$$10\sqrt{2 \cdot 3} \cdot \sqrt{3 \cdot 5}$$

$$10\sqrt{2}\sqrt{3} \cdot \sqrt{3}\sqrt{5}$$

$$10(\sqrt{3}\sqrt{3})\sqrt{2}\sqrt{5}$$

$$10(3)\sqrt{2}\sqrt{5}$$

$$30\sqrt{10}$$

■ 3. The square root of a number multiplied by the square root of the same number is equal to ___________.

Solution:

The square root of a number multiplied by the square root of the same number is equal to the number itself. For example,

$$\sqrt{5}\sqrt{5} = 5$$

■ 4. Find the value of $\sqrt{2} + \sqrt{32} - \sqrt{50}$.

Solution:

First simplify the radicals individually.

$$\sqrt{2} + \sqrt{16 \cdot 2} - \sqrt{25 \cdot 2}$$

$$\sqrt{2} + \sqrt{16}\sqrt{2} - \sqrt{25}\sqrt{2}$$

$$\sqrt{2} + 4\sqrt{2} - 5\sqrt{2}$$

Now that all the radicals are the same, combine like terms.

$$(1 + 4 - 5)\sqrt{2}$$

$$(0)\sqrt{2}$$

$$0$$

■ 5. To be able to add or subtract radicals, the roots must be ____________ when they are simplified.

Solution:

To be able to add or subtract radicals, the roots must be identical when they are simplified. In other words, we can could simplify an expression like

$$4\sqrt{2} + 3\sqrt{2}$$

because the radicals are identical, but we couldn't simplify

$$3\sqrt{2} - 4\sqrt{5}$$

because the radicals are different and they can't be rewritten in a way that would make them identical.

■ 6. Roberta is trying to simplify the following radical expression,

$$\sqrt{4} + \sqrt{20} - 2\sqrt{5} + \sqrt{25}$$

and her work is shown below.

Step 1: $\quad 2 + \sqrt{20} - 2\sqrt{5} + 5$

Step 2: $2+\sqrt{4\cdot 5}-2\sqrt{5}+5$

Step 3: $2+4\sqrt{5}-2\sqrt{5}+5$

Step 4: $7+2\sqrt{5}$

In which step did she make a mistake? What should she have done differently, and what is the correct answer?

Solution:

Roberta made her mistake in Step 3. Instead of writing $4\sqrt{5}$, she should have written $2\sqrt{5}$, because she should have simplified $\sqrt{4}$ to 2, instead of simplifying it to 4. Starting from Step 2, her work should have looked like this:

$$2+\sqrt{4\cdot 5}-2\sqrt{5}+5$$

$$2+\sqrt{4}\sqrt{5}-2\sqrt{5}+5$$

$$2+2\sqrt{5}-2\sqrt{5}+5$$

$$2+5$$

$$7$$

POWERS OF 10

■ 1. If we multiply a number by a power of 10, e can count the number of zeroes to know how many spaces to move the ____________ to the ____________.

Solution:

decimal point, right

■ 2. Find the product.

$$450 \cdot 10^0$$

Solution:

$450 \cdot 1$

450

■ 3. Find the quotient.

$$6.4 \div 100$$

Solution:

0.064

4. Find the product.

$$3.5 \times 10^4$$

Solution:

35,000

5. Find the product.

$$1.8 \times 10^{-2}$$

Solution:

0.018

6. Find the quotient.

$$420 \div 10^3$$

Solution:

0.42

SCIENTIFIC NOTATION

■ 1. Scientific notation has 2 parts. The first part is the decimal number and the second part is the ____________.

Solution:

power of 10

■ 2. The decimal number of a number written in scientific notation must be greater than or equal to 1 and less than ____________.

Solution:

10

■ 3. Write the number in scientific notation.

$$0.000000056$$

Solution:

5.6×10^{-8}

■ 4. Write the number in scientific notation.

0.00000000000012

Solution:

1.2×10^{-13}

■ 5. Write number in expanded form.

7.2×10^{12}

Solution:

7,200,000,000,000

■ 6. Write number in expanded form.

4.9×10^{-7}

Solution:

0.00000049

MULTIPLYING SCIENTIFIC NOTATION

■ 1. Write the product $(3.1 \times 10^5)(5.5 \times 10^{-7})$ in scientific notation.

Solution:

Multiply the decimal numbers together, separately from the powers of 10.

$$(3.1 \times 5.5)(10^5 \times 10^{-7})$$

$$17.05 \times (10^5 \times 10^{-7})$$

To multiply the powers of 10, add the exponents.

$$17.05 \times 10^{5+(-7)}$$

$$17.05 \times 10^{5-7}$$

$$17.05 \times 10^{-2}$$

Proper scientific notation requires just one non-zero digit in front of the decimal point, which means we need to move the decimal point one place to the left. Therefore, we rewrite the expression as

$$1.705 \times 10^1 \times 10^{-2}$$

Now simplify the powers of 10 again.

$$1.705 \times 10^{1+(-2)}$$

$$1.705 \times 10^{1-2}$$

$$1.705 \times 10^{-1}$$

■ 2. Write the product $(1.8 \times 10^4)(5.9 \times 10^6)$ in scientific notation.

Solution:

Multiply the decimal numbers together, separately from the powers of 10.

$$(1.8 \times 5.9)(10^4 \times 10^6)$$

$$10.62 \times (10^4 \times 10^6)$$

To multiply the powers of 10, add the exponents

$$10.62 \times 10^{4+6}$$

$$10.62 \times 10^{10}$$

Proper scientific notation requires just one non-zero digit in front of the decimal point, which means we need to move the decimal point one place to the left. Therefore, we rewrite the expression as

$$1.062 \times 10^1 \times 10^{10}$$

Now simplify the powers of 10 again.

$$1.062 \times 10^{1+10}$$

$$1.062 \times 10^{11}$$

■ 3. Write the product $(8.8 \times 10^{-2})(7.85 \times 10^{-5})$ in scientific notation.

Solution:

Multiply the decimal numbers together, separately from the powers of 10.

$$(8.8 \times 7.85)(10^{-2} \times 10^{-5})$$

$$69.08 \times (10^{-2} \times 10^{-5})$$

To multiply the powers of 10, add the exponents.

$$69.08 \times 10^{-2+(-5)}$$

$$69.08 \times 10^{-2-5}$$

$$69.08 \times 10^{-7}$$

Proper scientific notation requires just one non-zero digit in front of the decimal point, which means we need to move the decimal point one place to the left. Therefore, we rewrite the expression as

$$6.908 \times 10^{1} \times 10^{-7}$$

Now simplify the powers of 10 again.

$$6.908 \times 10^{1+(-7)}$$

$$6.908 \times 10^{1-7}$$

$$6.908 \times 10^{-6}$$

■ 4. Write the product $(1.3 \times 10^3)(2.6 \times 10^{-4})$ in scientific notation.

Solution:

Multiply the decimal numbers together, separately from the powers of 10.

$$(1.3 \times 2.6)(10^3 \times 10^{-4})$$

$$3.38 \times (10^3 \times 10^{-4})$$

To multiply the powers of 10, add the exponents.

$$3.38 \times 10^{3+(-4)}$$

$$3.38 \times 10^{3-4}$$

$$3.38 \times 10^{-1}$$

■ 5. If we're given 3.6×10^{-2} in scientific notation, will we get a smaller or larger number when we multiply it by a positive power of 10?

Solution:

Let's pretend we're multiplying by 10^3. When we do, we get

$$3.6 \times 10^{-2} \times 10^3$$

$$3.6 \times 10^{-2+3}$$

$$3.6 \times 10^1$$

$$3.6 \times 10$$

$$36$$

This is a larger value than 3.6×10^{-2}, so we can say that multiplying by a positive power of 10 will give a larger number.

■ 6. Yvonne is asked to find the product of two numbers written in scientific notation:

$$(2.8 \times 10^4)(4.46 \times 10^{-6})$$

She solves the problem in three steps.

Step 1	$2.8 \times 4.46 = 12.488$
Step 2	$4 + (-6) = -2$
Step 3	12.488×10^{-2}

In what step did she make her mistake? What is the correct answer?

Solution:

She made her mistake in Step 3. She forgot to change the decimal back to a number between 1 and 10. She would get the correct answer in one more step:

$$1.2488 \times 10^1 \times 10^{-2}$$

$$1.2488 \times 10^{1+(-2)}$$

$$1.2488 \times 10^{1-2}$$

$$1.2488 \times 10^{-1}$$

DIVIDING SCIENTIFIC NOTATION

■ 1. When we divide two numbers that have the same base, we
____________ the exponents.

Solution:

subtract

■ 2. Find the value of $(1.5 \times 10^8) \div (2.0 \times 10^{-3})$.

Solution:

Divide the decimal numbers, separately from the powers of 10.

$$\frac{1.5}{2.0} \times \frac{10^8}{10^{-3}}$$

$$0.75 \times \frac{10^8}{10^{-3}}$$

To divide the powers of 10, subtract the exponent in the denominator from the exponent in the numerator.

$$0.75 \times 10^{8-(-3)}$$

$$0.75 \times 10^{8+3}$$

$$0.75 \times 10^{11}$$

Proper scientific notation requires one non-zero digit in front of the decimal point, which means we need to move the decimal point one place to the right. Therefore, we rewrite the expression as

$$7.5 \times 10^{-1} \times 10^{11}$$

$$7.5 \times 10^{-1+11}$$

$$7.5 \times 10^{10}$$

■ 3. Find the value of $(6.75 \times 10^3) \div (1.5 \times 10^9)$.

Solution:

Divide the decimal numbers, separately from the powers of 10.

$$\frac{6.75}{1.5} \times \frac{10^3}{10^9}$$

$$4.5 \times \frac{10^3}{10^9}$$

To divide the powers of 10, subtract the exponent in the denominator from the exponent in the numerator.

$$4.5 \times 10^{3-9}$$

$$4.5 \times 10^{-6}$$

■ 4. Find the value of $(2.75 \times 10^{10}) \div (8.0 \times 10^8)$.

Solution:

Divide the decimal numbers, separately from the powers of 10.

$$\frac{2.75}{8.0} \times \frac{10^{10}}{10^8}$$

$$0.34375 \times \frac{10^{10}}{10^8}$$

To divide the powers of 10, subtract the exponent in the denominator from the exponent in the numerator.

$$0.34375 \times 10^{10-8}$$

$$0.34375 \times 10^2$$

Proper scientific notation requires one non-zero digit in front of the decimal point, which means we need to move the decimal point one place to the right. Therefore, we rewrite the expression as

$$3.4375 \times 10^{-1} \times 10^2$$

$$3.4375 \times 10^{-1+2}$$

$$3.4375 \times 10^1$$

■ 5. Find the value of $(7.5 \times 10^4) \div (1.5 \times 10^{-4})$.

Solution:

Divide the decimal numbers, separately from the powers of 10.

$$\frac{7.5}{1.5} \times \frac{10^4}{10^{-4}}$$

$$5 \times \frac{10^4}{10^{-4}}$$

To divide the powers of 10, subtract the exponent in the denominator from the exponent in the numerator.

$$5 \times 10^{4-(-4)}$$

$$5 \times 10^{4+4}$$

$$5 \times 10^8$$

$$5.0 \times 10^8$$

■ 6. If we're given 5.75×10^6 in scientific notation and we divide it by a negative power of 10, will we get a larger or smaller result?

Solution:

Let’s pretend we’re dividing by 10^{-2}. When we do, we get

$$5.75 \times 10^{6-(-2)}$$

$$5.75 \times 10^{6+2}$$

$$5.75 \times 10^{8}$$

This is a larger value than 5.75×10^{6}, so we can say that dividing by a negative power of 10 will give a larger number.

MULTIPLYING AND DIVIDING SCIENTIFIC NOTATION

■ 1. Simplify the expression.

$$\frac{(4.5 \times 10^3)(1.4 \times 10^{-5})}{2.8 \times 10^{-1}}$$

Solution:

Simplify the numerator first by multiplying the scientific notation expressions.

$$\frac{(4.5 \times 1.4)(10^3 \times 10^{-5})}{2.8 \times 10^{-1}}$$

$$\frac{6.3 \times 10^{3+(-5)}}{2.8 \times 10^{-1}}$$

$$\frac{6.3 \times 10^{-2}}{2.8 \times 10^{-1}}$$

Now divide the scientific notation.

$$\frac{6.3}{2.8} \times \frac{10^{-2}}{10^{-1}}$$

$$2.25 \times 10^{-2-(-1)}$$

$$2.25 \times 10^{-1}$$

■ 2. Simplify the expression.

$$\frac{(7.6 \times 10^5)(1.1 \times 10^{-7})}{5.1 \times 10^{-3}}$$

Solution:

Simplify the numerator first by multiplying the scientific notation expressions.

$$\frac{(7.6 \times 1.1)(10^5 \times 10^{-7})}{5.1 \times 10^{-3}}$$

$$\frac{8.36 \times 10^{5+(-7)}}{5.1 \times 10^{-3}}$$

$$\frac{8.36 \times 10^{-2}}{5.1 \times 10^{-3}}$$

Now divide the scientific notation.

$$\frac{8.36}{5.1} \times \frac{10^{-2}}{10^{-3}}$$

$$1.639 \times 10^{-2-(-3)}$$

$$1.639 \times 10^1$$

■ 3. Simplify the expression.

$$\frac{(1.7 \times 10^{-3})(3.4 \times 10^{-4})}{(6.3 \times 10^{-3})(7.3 \times 10^{-2})}$$

Solution:

Simplify the numerator and denominator first by multiplying the scientific notation expressions.

$$\frac{(1.7 \times 3.4)(10^{-3} \times 10^{-4})}{(6.3 \times 7.3)(10^{-3} \times 10^{-2})}$$

$$\frac{5.78 \times 10^{-3+(-4)}}{45.99 \times 10^{-3+(-2)}}$$

$$\frac{5.78 \times 10^{-7}}{45.99 \times 10^{-5}}$$

Now divide the scientific notation.

$$\frac{5.78}{45.99} \times \frac{10^{-7}}{10^{-5}}$$

$$0.126 \times 10^{-7-(-5)}$$

$$0.126 \times 10^{-2}$$

Move the decimal point one place to the right to put the result in proper scientific notation.

$$1.26 \times 10^{-1} \times 10^{-2}$$

$$1.26 \times 10^{-1+(-2)}$$

$$1.26 \times 10^{-3}$$

■ 4. Simplify the expression.

$$\frac{(4.9 \times 10^4)(6.4 \times 10^{-4})}{(8.2 \times 10^{-3})(2 \times 10^3)}$$

Solution:

Simplify the numerator and denominator first by multiplying the scientific notation expressions.

$$\frac{(4.9 \times 6.4)(10^4 \times 10^{-4})}{(8.2 \times 2)(10^{-3} \times 10^3)}$$

$$\frac{31.36 \times 10^{4+(-4)}}{16.4 \times 10^{-3+3}}$$

$$\frac{31.36 \times 10^0}{16.4 \times 10^0}$$

Any non-zero value raised to the power of 0 is 1, so

$$\frac{31.36 \times 1}{16.4 \times 1}$$

$$\frac{31.36}{16.4}$$

$$1.912$$

■ 5. Simplify the expression.

$$\frac{(6.1\times10^6)(6.8\times10^{-4})}{(1.1\times10^{-5})(1.8\times10^5)}$$

Solution:

Simplify the numerator and denominator first by multiplying the scientific notation expressions.

$$\frac{(6.1\times6.8)(10^6\times10^{-4})}{(1.1\times1.8)(10^{-5}\times10^5)}$$

$$\frac{41.48\times10^{6+(-4)}}{1.98\times10^{-5+5}}$$

$$\frac{41.48\times10^2}{1.98\times10^0}$$

Now divide the scientific notation.

$$\frac{41.48}{1.98}\times\frac{10^2}{10^0}$$

$$20.949\times10^{2-0}$$

$$20.949\times10^2$$

Move the decimal point one place to the left to put the result in proper scientific notation.

$$2.0949 \times 10^1 \times 10^2$$

$$2.0949 \times 10^{1+2}$$

$$2.0949 \times 10^3$$

■ 6. Danny and Deacon are working on finding the quotient below. Danny decides to multiply out the numerator and gets $10{,}000$ for the powers of 10 portion. Then he divides it by 0.00061 to get $16{,}393{,}442.6$ or 1.63934426×10^7. Deacon decides he wants to divide, so he divides each number by 0.00061 and gets $819{,}672{,}131$ and 32.78689852. Then he multiplies those numbers to get 2.68745×10^{10}. Why are the answers different? Who is correct?

$$\frac{(5 \times 10^5)(2 \times 10^{-2})}{6.1 \times 10^{-4}}$$

Solution:

Deacon is incorrect. He should have divided just one of the numbers and then multiplied the other number by that quotient.

ESTIMATING SCIENTIFIC NOTATION

■ 1. Estimate the value of 3.65×10^{-5}.

Solution:

We'll round 3.65 up to the nearest whole number, 4, and then multiply that by 10^{-5}, or 0.00001.

$$4 \times 0.00001$$

$$0.00004$$

■ 2. Use scientific notation to estimate the value $(5.75 \times 10^{6})(2.34 \times 10^{-1})$.

Solution:

Round 5.75 to 6 and 2.34 to 2, then rewrite the expression as

$$(6 \times 10^{6})(2 \times 10^{-1})$$

$$(6 \times 2)(10^{6} \times 10^{-1})$$

$$12 \times 10^{5}$$

Rewrite the answer in proper scientific notation by moving the decimal point one place to the left.

$$1.2 \times 10^1 \times 10^5$$

$$1.2 \times 10^6$$

■ 3. Use scientific notation to estimate the value of $(2.456 \times 10^3)(1.67 \times 10^{-7})$.

Solution:

Round 2.456 to 2 and 1.67 to 2, then rewrite the expression as

$$(2 \times 10^3)(2 \times 10^{-7})$$

$$(2 \times 2)(10^3 \times 10^{-7})$$

$$4 \times 10^{-4}$$

■ 4. Use scientific notation to estimate the value of the expression.

$$\frac{7.152 \times 10^2}{2.91 \times 10^2}$$

Solution:

Round 7.152 to 7 and 2.91 to 3, then rewrite the expression as

$$\frac{7 \times 10^2}{3 \times 10^2}$$

$$\frac{7}{3} \times \frac{10^2}{10^2}$$

$$2.333 \times 1$$

$$2.333$$

5. Use scientific notation to estimate the value of the expression.

$$\frac{(6.2 \times 10^6)(6.4 \times 10^{-3})}{(4.25 \times 10^{-2})(2.9 \times 10^{-3})}$$

Solution:

Round 6.2 to 6, 6.4 to 6, 4.25 to 4, and 2.9 to 3, then rewrite the expression as

$$\frac{(6 \times 10^6)(6 \times 10^{-3})}{(4 \times 10^{-2})(3 \times 10^{-3})}$$

$$\frac{(6 \times 6)(10^6 \times 10^{-3})}{(4 \times 3)(10^{-2} \times 10^{-3})}$$

$$\frac{36 \times 10^3}{12 \times 10^{-5}}$$

$$\frac{36}{12} \times \frac{10^3}{10^{-5}}$$

$$3 \times 10^8$$

■ 6. Use scientific notation to estimate the value of the expression.

$$\frac{(1.7 \times 10^{-5})(2.6 \times 10^2)}{(3.334 \times 10^{-3})(2.5 \times 10^{-1})}$$

Solution:

Round 1.7 to 2, 2.6 to 3, 3.334 to 3, and 2.5 to 3, then rewrite the expression as

$$\frac{(2 \times 10^{-5})(3 \times 10^2)}{(3 \times 10^{-3})(3 \times 10^{-1})}$$

$$\frac{(2 \times 3)(10^{-5} \times 10^2)}{(3 \times 3)(10^{-3} \times 10^{-1})}$$

$$\frac{6 \times 10^{-3}}{9 \times 10^{-4}}$$

$$\frac{6}{9} \times \frac{10^{-3}}{10^{-4}}$$

$$0.6667 \times 10^1$$

$$6.6667$$

W W W . K R I S T A K I N G M A T H . C O M