Pre-Algebra Workbook Solutions

Exponents

krista king
MATH

EXPONENTS

■ 1. An exponent tells us how many times to multiply the base by

____________.

Solution:

itself

■ 2. Is 2^3 the same as 3^2? Why or why not?

Solution:

No, it's not the same. 2^3 means $2 \cdot 2 \cdot 2 = 8$, whereas 3^2 means $3 \cdot 3 = 9$.

■ 3. Find the sum.

$$5^3 + 2^4$$

Solution:

Find the value of each term individually, then add the results.

$$5 \cdot 5 \cdot 5 + 2 \cdot 2 \cdot 2 \cdot 2$$

$$125 + 16$$

$$141$$

4. Write the number using exponents.

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

Solution:

Because we're multiplying nine factors of 2, we can express that as 2^9 instead.

5. Write the following number without an exponent.

$$1^6$$

Solution:

Because we're multiplying six factors of 1, we can express that as $1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1$ instead.

■ 6. Write the number without an exponent.

$$(-9)^6$$

Solution:

Because we're multiplying six factors of -9, we can express that as

$$(-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9)$$

instead.

RULES OF EXPONENTS

1. Find the sum.

$$2x^3 + x^3 + x^3 + 3x^3$$

Solution:

Because all these terms are x^3 terms, we're able to add them.

$$(2 + 1 + 1 + 3)x^3$$

$$7x^3$$

2. Find the product.

$$x^6 \cdot x^2 \cdot x^3$$

Solution:

When we multiply terms with like bases, we add the exponents.

$$x^{6+2+3}$$

$$x^{11}$$

■ 3. Simplify the expression.

$$x \cdot x \cdot x$$

Solution:

When we multiply terms with like bases, we add the exponents.

$$x^{1+1+1}$$

$$x^3$$

■ 4. Stephanie and Jimmy are trying to find a shortcut to simplify the expression below. Stephanie says that they should add the exponents $(3 + 5 = 8)$ and then raise 4 to that power. Jimmy says that since it's multiplication, they should multiply the exponents $(3 \cdot 5 = 15)$ and then raise 4 to that power. Who is correct and why?

$$4^3 \cdot 4^5$$

Solution:

Stephanie is correct. 4^3 is equivalent to $4 \cdot 4 \cdot 4$, and 4^5 is equivalent to $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$. If we're multiplying these values together, we get

$$4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$$

4^8

$65{,}536$

■ 5. Simplify the expression.

$$\frac{x^5 + x^2 \cdot x^3}{x^7}$$

Solution:

We'll use the fact that

$$x^a x^b = x^{a+b}$$

Here we have $a = 2$, and $b = 3$, so we get

$$x^2 x^3 = x^{2+3} = x^5$$

Then

$$\frac{x^5 + x^2 \cdot x^3}{x^7} = \frac{x^5 + x^5}{x^7}$$

Because all these terms in the numerator are x^5 terms, we're able to add them.

$$\frac{2x^5}{x^7}$$

Then we'll use the fact that

$$\frac{x^a}{x^b} = x^{a-b}$$

Here we have $a = 5$, and $b = 7$, so we get

$$\frac{2x^5}{x^7} = 2x^{5-7} = 2x^{-2}$$

Then, we know that $x^{-a} = 1/x^a$, so

$$2x^{-2} = \frac{2}{x^2}$$

■ 6. Simplify the expression.

$$\frac{x^{-4} \cdot x^6}{x^2}$$

Solution:

We'll use the fact that

$$x^a x^b = x^{a+b}$$

Here we have $a = -4$, and $b = 6$, so we get

$$x^{-4}x^6 = x^{-4+6} = x^2$$

Then

$$\frac{x^{-4} \cdot x^6}{x^2} = \frac{x^2}{x^2}$$

Then we'll use the fact that

$$\frac{x^a}{x^b} = x^{a-b}$$

Here we have $a = 2$, and $b = 2$, so we get

$$\frac{x^2}{x^2} = x^{2-2} = x^0 = 1$$

POWER RULE FOR EXPONENTS

■ 1. The power rule tells us that, when we raise a power to a power, we can ____________ those powers together.

Solution:

multiply

■ 2. Simplify the expression.

$$(x^3)^3$$

Solution:

Applying the power rule for exponents, we multiply the exponents.

$$x^{3\cdot 3}$$

$$x^9$$

■ 3. Simplify the expression.

$$(x^2)^{-4}$$

Solution:

Applying the power rule for exponents, we multiply the exponents.

$$x^{2(-4)}$$

$$x^{-8}$$

$$\frac{1}{x^8}$$

■ 4. Simplify the expression.

$$(2^m)^p$$

Solution:

Applying the power rule for exponents, we multiply the exponents.

$$2^{(m\cdot p)}$$

$$2^{mp}$$

■ 5. Simplify the expression.

$$(x^2y^2)^3$$

Solution:

Applying $a^n b^n = (ab)^n$, we can rewrite the expression as

$$((xy)^2)^3$$

Then using power rule, we get

$$(xy)^{2\cdot 3}$$

$$(xy)^6$$

$$x^6 y^6$$

6. Simplify the expression.

$$(x^{-5} \cdot x^4)^{-2}$$

Solution:

When we multiply terms with like bases, we add the exponents.

$$(x^{-5+4})^{-2}$$

$$(x^{-1})^{-2}$$

Applying the power rule for exponents, we multiply the exponents.

$$x^{(-1)\cdot(-2)}$$

x^2

NEGATIVE AND OTHER EXPONENT RULES

■ 1. Simplify the expression.

$$\frac{9a^5b^4}{3a^2b^7}$$

Solution:

Use quotient rule to simplify across the numerator and denominator.

$$\frac{9a^5b^4}{3a^2b^7}$$

$$3a^{5-2}b^{4-7}$$

$$3a^3b^{-3}$$

We know that $x^{-a} = 1/x^a$, so we get

$$\frac{3a^3}{b^3}$$

We know that

$$\frac{x^a}{y^a} = \left(\frac{x}{y}\right)^a$$

so we get

$$3\left(\frac{a}{b}\right)^3$$

■ 2. Simplify the expression.

$$\frac{2x^0y^6-(y^2)^3}{x^6}$$

Solution:

First we need to simplify the numerator. We know that $x^0=1$, so

$$\frac{2y^6-(y^2)^3}{x^6}$$

Applying the power rule for exponents, we multiply the exponents.

$$\frac{2y^6-y^6}{x^6}$$

$$\frac{y^6}{x^6}$$

We know that

$$\frac{x^a}{y^a}=\left(\frac{x}{y}\right)^a$$

so we get

$$\left(\frac{y}{x}\right)^6$$

■ 3. Simplify the expression.

$$\frac{(x^{2p})^3}{x^{3p}y^{3p}}$$

Solution:

Use power rule to simplify the exponent in the numerator.

$$\frac{x^{6p}}{x^{3p}y^{3p}}$$

Now use quotient rule to simplify across the numerator and denominator.

$$\frac{x^{6p-3p}}{y^{3p}}$$

$$\frac{x^{3p}}{y^{3p}}$$

We know that

$$\frac{x^a}{y^a} = \left(\frac{x}{y}\right)^a$$

so we get

$$\left(\frac{x}{y}\right)^{3p}$$

■ 4. Simplify the expression.

$$\frac{(x^{-3a+4})^2}{x^{-4a+8}y^{-2a}}$$

Solution:

Use power rule to simplify the exponent in the numerator.

$$\frac{x^{2(-3a+4)}}{x^{-4a+8}y^{-2a}}$$

$$\frac{x^{-6a+8}}{x^{-4a+8}y^{-2a}}$$

Now use quotient rule to simplify across the numerator and denominator.

$$\frac{x^{-6a+8+4a-8}}{y^{-2a}}$$

$$\frac{x^{-2a}}{y^{-2a}}$$

We know that

$$\frac{x^a}{y^a} = \left(\frac{x}{y}\right)^a$$

so we get

$$\left(\frac{x}{y}\right)^{-2a}$$

Applying quotient to a negative exponent

$$\left(\frac{y}{x}\right)^{2a}$$

■ 5. Simplify the expression.

$$\left(\frac{5x^{-2}}{y^{-2}}\right)^4$$

Solution:

We know that

$$\frac{x^a}{y^a} = \left(\frac{x}{y}\right)^a$$

so we get

$$\frac{(5x^{-2})^4}{(y^{-2})^4}$$

Applying the power rule for exponents, we multiply the exponents.

$$\frac{5^4x^{-8}}{y^{-8}}$$

$$625\left(\frac{x}{y}\right)^{-8}$$

Then we can change the exponent from negative to positive by taking the reciprocal of the fraction.

$$625\left(\frac{y}{x}\right)^{8}$$

■ 6. Simplify the expression.

$$\left(\frac{2x^5y^7}{y^{12}}\right)^0$$

Solution:

Any non-zero value raised to the power of 0 is 1, so the expression simplifies to just 1.

Alternately, we could use the quotient rule to simplify across the numerator and denominator.

$$\left(\frac{2x^5y^7}{y^{12}}\right)^0$$

$$(2x^5y^{7-12})^0$$

$$(2x^5y^{-5})^0$$

Then, we know that $x^{-a} = 1/x^a$, so

$$(2x^5y^{-5})^0 = \left(\frac{2x^5}{y^5}\right)^0$$

Applying the power rule for exponents, we multiply the exponents.

$$\frac{(2x^5)^0}{(y^5)^0}$$

$$\frac{2^0x^0}{y^0}$$

$$\frac{1 \cdot 1}{1}$$

$$1$$