Adding and subtracting radicals

In this section we'll talk about how to add and subtract terms containing radicals.

When we have two terms that contain the same type of root (the radical in both terms is a square root, the radical in both terms is a cube root, etc.) and identical radicands (the expressions under the radical signs in the two terms are the same), they are like terms, and adding and subtracting is really simple.

Example

Simplify the expression.

$$\sqrt{3} + 4\sqrt{3}$$

Here, the two terms contain the same type of root (a square root), and the radicands are the same, which means they're like terms, and we can just do the addition. The coefficient of $\sqrt{3}$ in the first term is understood to be 1, so we can rewrite the expression as

$$1\sqrt{3} + 4\sqrt{3}$$

Now we can think of this as follows: "If we have one square root of 3, and we add to that four square roots of 3, then how many square roots of 3 (total) do we have?" Well, one of them plus four of them is five of them, total. So we get

$$5\sqrt{3}$$

If the radicals aren't the same, then they aren't like terms, and we can't combine them. So if we have $\sqrt{2}$ and $\sqrt{3}$, we can't add them or subtract one of them from the other. The radicands are different (one is 2 and the other is 3), so they aren't like terms and we can't combine them.

Example

Find the difference.

$$\sqrt{5} - \sqrt{3}$$

Because the radicands are different, $\sqrt{5}$ and $\sqrt{3}$ aren't like terms, so we can't add them or subtract one of them from the other. Therefore, we can't simplify this expression at all.

It isn't always true that terms with the same type of root but different radicands can't be added or subtracted. Sometimes we can simplify a radical within itself, and end up with like terms.

Example

Find the sum.

$$\sqrt{2}+\sqrt{8}$$

At first it looks as if we can't combine these terms, since the radicands are different, and therefore they're not like terms. But $\sqrt{8}$ can be simplified.

$$\sqrt{8}$$

$$\sqrt{4\cdot 2}$$

When a radicand can be factored, the radical can be expressed as a product of radicals with the individual factors as the radicands, so here we get

$$\sqrt{4}\cdot\sqrt{2}$$

$$2\sqrt{2}$$

Which means that the sum $\sqrt{2}+\sqrt{8}$ can be rewritten as

$$\sqrt{2}+2\sqrt{2}$$

And now we have

$$1\sqrt{2}+2\sqrt{2}$$

$$3\sqrt{2}$$

So when it comes to adding and subtracting radicals, we want to remember that only like terms can be combined, which means that those

terms have to contain the same type of root and their radicands have to be the same.

But even when the radicands are different, sometimes one or both of the radicals can be rewritten in a way that will actually make the radicands the same. So watch out for opportunities to rewrite the radicals in one or both of the terms before concluding that they're not like terms and can't be combined.