Prime factorization and product of primes

Remember that we learned previously that a prime number is a whole number greater than 1 which is divisible by only 1 and itself. In contrast, a composite number is a whole number greater than 1 which is divisible by 1 and itself, but also by at least one other number.

This lesson is all about prime factorization and product of primes, so we'll define what these mean. Before we do that, however, we need to talk about factors.

Factors and factorization

Factors are just things that get multiplied by one another. In math, when we multiply numbers or expressions together, we call each piece a "factor." On the other hand, when we add numbers or expressions together, we call each piece a "term."

For now, when we talk about factors, we'll think about positive whole numbers. So for now, when we talk about "factorization" of a whole number, we're talking about coming up with the whole numbers that multiply together to give us that original number.

For example, the factors of 12 are 1, 2, 3, 4, 6, and 12, because they're the positive whole numbers that divide evenly into 12.

Product of primes

A “product of primes” is a product in which every factor is a prime number. Of all the factorizations of 12 we could come up with, the only one that's a product of primes is

$$2 \times 2 \times 3$$

A “prime factorization" of a composite number is an expression of that number as a product of primes.

The factors in a prime factorization can appear in any order, but we usually list them from smallest to largest, and we group factors together that are the same. For example, we already know that the prime factorization of 12 is $2 \times 2 \times 3$, but we’d actually write this more compactly as

$$2^2 \times 3$$

where the little 2 indicates that there are two factors of 2. Taking another example, if in the prime factorization of some other number, the factor 3 occurs twice, the factor 5 occurs four times, and the factor 13 appears once, we could write its prime factorization as

$$3 \times 3 \times 5 \times 5 \times 5 \times 5 \times 13$$

or more compactly as

$$3^2 \times 5^4 \times 13$$

These little numbers are called “exponents” and we’ll learn more about them later on when we get to the exponents section of the course.

Example

Find the prime factorization of 45.

The goal in finding the prime factorization of a composite number is to keep breaking that number down into smaller and smaller factors until all the factors are prime numbers. There are multiple ways that we could do this.

We know that 45 is the product of 5 and 9, so we could say

$$45 = 5 \times 9$$

5 is a prime number, so we can't break that down any further. But 9 can be expressed as the product 3×3, so we could break down the 9 into 3×3 and write 45 as

$$45 = 5 \times 3 \times 3$$

Now we're done because 5, 3, and 3 are all prime numbers, so they can't be broken down any further. We can also write the prime factorization in exponential form as $3^2 \times 5$.