Simplifying fractions and equivalent fractions

We already understand that a fraction just represents “part of a whole.” If a baseball player gets three “at bats” in a game (gets an opportunity as a hitter three times), and out of those three chances they get a hit two times, then their success rate for that game is

$$\frac{2 \text{ hits}}{3 \text{ chances}} = \frac{2}{3}$$

Or if I borrow five books from the library but read only two of them, then I’ve read two out of five books, or

$$\frac{2 \text{ books read}}{5 \text{ books total}} = \frac{2}{5}$$

What we want to be able to do now is learn to simplify fractions. For example, we know that 50 is half of 100, so if we see the fraction

$$\frac{50}{100}$$

then we want to be able to rewrite that as

$$\frac{1}{2}$$

Fractions as relationships

Because remember, a fraction is really just a relationship between the numerator and the denominator. If I’m driving 100 miles to visit my family, and I’ve already driven 50 miles, then I know that I’m halfway there. It would be simpler for me to express my progress as $1/2$ than as $50/100$, so we need to know how to change $50/100$ into $1/2$.

The reason we want to reduce fractions to lowest terms is that even though a fraction like

$$\frac{630}{945}$$

is actually the same as

$$\frac{2}{3},$$

that isn’t obvious to us when we look at it, because the numbers 630 and 945 are so large. But if we simplify that fraction to $2/3$, we’ll be able to easily tell that we have “2 out of 3 parts.”

Example

Simplify the fraction to lowest terms.

$$\frac{630}{945}$$

There are a couple ways to tackle this, but here’s a reliable way to go about simplifying a fraction to lowest terms. We can first find the prime factorizations of the numerator and the denominator.

630 $\qquad\qquad\qquad\qquad$ 945

$$3 \cdot 210 \qquad\qquad 3 \cdot 315$$

$$3 \cdot 3 \cdot 70 \qquad\qquad 3 \cdot 3 \cdot 105$$

$$3 \cdot 3 \cdot 5 \cdot 14 \qquad\qquad 3 \cdot 3 \cdot 5 \cdot 21$$

$$3 \cdot 3 \cdot 5 \cdot 7 \cdot 2 \qquad\qquad 3 \cdot 3 \cdot 5 \cdot 7 \cdot 3$$

What we get is

$$\frac{630}{945} = \frac{3 \cdot 3 \cdot 5 \cdot 7 \cdot 2}{3 \cdot 3 \cdot 5 \cdot 7 \cdot 3}$$

Now we'll group together the factors that are common to the numerator and denominator, so our fraction can be expressed as

$$\frac{630}{945} = \frac{(3 \cdot 3 \cdot 5 \cdot 7) \cdot 2}{(3 \cdot 3 \cdot 5 \cdot 7) \cdot 3} = \frac{2}{3}$$

What we want to remember at this point is that, whenever a factor is common to the numerator and denominator of a fraction, that factor in the numerator "cancels against" the matching factor in the denominator. It's as if that factor just disappears from both places. So the $(3 \cdot 3 \cdot 5 \cdot 7)$ cancels in the numerator and denominator, leaving us with just $2/3$.

When the numerator and denominator are equal

However, if the numerator is equal to the denominator, the fraction doesn't disappear (we have to have something there), so the fraction simplifies to 1. For example,

$$\frac{3}{3} = 1$$

$$\frac{10}{10} = 1$$

$$\frac{67}{67} = 1$$

Equivalent fractions

We already know how to simplify a fraction to lowest terms. We just pull out the common factors from the numerator and denominator, cancel those out, and what's left is the fraction simplified to lowest terms, resulting in a fraction that's equivalent to the original. For example, given the fraction

$$\frac{30}{45}$$

we first find the prime factorizations of the numerator and denominator,

$$\frac{3 \cdot 5 \cdot 2}{3 \cdot 5 \cdot 3}$$

then we cancel the 3 in the numerator with one of the 3's in the denominator, and the 5 in the numerator with the 5 in the denominator. The result is $2/3$.

$$\frac{\cancel{3}\cdot\cancel{5}\cdot 2}{\cancel{3}\cdot\cancel{5}\cdot 3}=\frac{2}{3}$$

So we can say that the two fractions $30/45$ and $2/3$ are **equivalent fractions**, which just means that they're equal to each other. They represent the same proportion of the whole.

$$\frac{30}{45}=\frac{2}{3}$$

Now we want to turn this process around, and learn how to express something like $2/3$ in terms of 12ths in the denominator, instead of 3rds in the denominator.

Let's do an example.

Example

Express $3/5$ as an equivalent fraction, but with 10 in the denominator instead of 5.

If we start with the fraction $3/5$, we can say that we have "3 out of 5 parts." Since we're being asked to express this as an equivalent fraction with 10 in the denominator, we're being asked the question "If we have 3 out of every 5 pieces, how many pieces would we have if there were 10 total pieces?"

Mathematically, we can represent this as

$$\frac{3}{5} = \frac{?}{10}$$

Now the question becomes, how did we get from 5 to 10 in the denominator? Well, we had to multiply 5 by 2 in order to get to 10. So in order to figure out what numerator we'll get, we need to multiply the numerator 3 by 2 as well.

$$\frac{3}{5} = \frac{3 \cdot 2}{5 \cdot 2} = \frac{6}{10}$$

This means that $3/5$ and $6/10$ are equivalent fractions.

We can double-check our answer by simplifying $6/10$ to show that we get back to $3/5$.

$$\frac{6}{10} = \frac{3 \cdot 2}{5 \cdot 2} = \frac{3 \cdot \cancel{2}}{5 \cdot \cancel{2}} = \frac{3}{5}$$