Least common multiple

A common multiple of two positive whole numbers is a number that's divisible by both of them. Their least common multiple (sometimes abbreviated LCM) is the smallest number that's divisible by both of them. For example,

4 is the least common multiple of 2 and 4

6 is the least common multiple of 2 and 3

10 is the least common multiple of 2 and 5

When we're dealing with small numbers, the easiest way to find the least common multiple is to start with the larger number, and test each of its positive multiples, one at a time starting with the smallest one, until we find one that's divisible by the other number.

Remember, the positive multiples of a positive whole number are the values we get when we multiply it by 1, 2, 3, 4, etc. For example, here's how we get the first twelve positive multiples of 3:

$3 \cdot 1 = 3$	$3 \cdot 4 = 12$	$3 \cdot 7 = 21$	$3 \cdot 10 = 30$
$3 \cdot 2 = 6$	$3 \cdot 5 = 15$	$3 \cdot 8 = 24$	$3 \cdot 11 = 33$
$3 \cdot 3 = 9$	$3 \cdot 6 = 18$	$3 \cdot 9 = 27$	$3 \cdot 12 = 36$

Let's try an example with small numbers.

Example

Find the least common multiple of 3 and 4.

Since 4 is the larger number, we'll use its positive multiples ($4 \cdot 1$, $4 \cdot 2$, etc.) to find the smallest one that 3 divides into evenly.

$4 \cdot 1 = 4$ 3 doesn't go evenly into 4, so we have to keep going.

$4 \cdot 2 = 8$ 3 doesn't go evenly into 8, so we have to keep going.

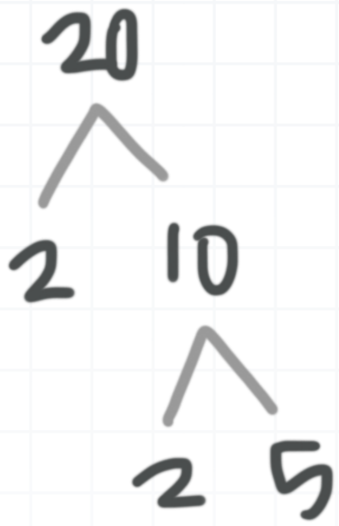
$4 \cdot 3 = 12$ 3 goes evenly into 12, so 12 is the least common multiple of 3 and 4.

Sometimes we need to find the least common multiple of larger numbers, where using this method might be more difficult. In this case, we can use a different method for finding the least common multiple: finding the prime factorization of each number, and then using the prime factors to build the least common multiple.

Example

Find the least common multiple of 20 and 75.

We need to reduce each of these numbers to its prime factors.



The prime factor 2 appears twice, and the prime factor 5 appears once, so the prime factorization of 20 is:

$$2^2 \cdot 5$$

The prime factor 3 appears once, and the prime factor 5 appears twice, so the prime factorization of 75 is:

$$3 \cdot 5^2$$

We found prime factors of 2, 3, and 5. The number of times each of these prime factors appears in the least common multiple of 20 and 75 will be the larger of the number of times it appears in the prime factorization of 20 and the number of times it appears in the prime factorization of 75.

There are two factors of 2 in 20, and none in 75, so there will be two factors of 2 in their least common multiple.

There are no factors of 3 in 20, and one in 75, so there will be one factor of 3 in their least common multiple.

There is one factor of 5 in 20, and there are two in 75, so there will be two factors of 5 in their least common multiple.

Therefore, the least common multiple of 20 and 75 is $2^2 \cdot 3 \cdot 5^2$. Now we'll multiply this out to express the least common multiple as a single number:

$$2^2 \cdot 3 \cdot 5^2$$

$$4 \cdot 3 \cdot 25$$

$$12 \cdot 25$$

$$300$$

This tells us that 300 is the least common multiple of 20 and 75.

Remember, we can always double-check our answer by dividing the least common multiple by the original numbers. Using the previous example, we said that 300 was the least common multiple of 20 and 75. If that's true, then 20 and 75 must both go evenly into 300.

$$300 \div 20 = 15$$

$$300 \div 75 = 4$$

Because we got whole numbers for answers, we know that 300 is a common multiple of 20 and 75. In fact, 300 is the **least** common multiple of 20 and 75: When we divide 300 by 20 and 75, we get 15 and 4, respectively, and the only number by which both 15 and 4 are divisible is 1.