Powers of 10

We want to start getting comfortable with powers of 10, since we'll be using them all the time for scientific notation. When we talk about powers of 10, we mean the result of raising 10 to some power:

$$10^0 = 1$$

$$10^1 = 10$$

$$10^2 = 100$$

$$10^3 = 1{,}000$$

$$10^4 = 10{,}000$$

etc.

Notice how the power (exponent) on the 10 is the same as the number of 0's in the power of 10 (in the number to the right of the equals sign). For example, the exponent in 10^4 is 4, and there are four 0's in $10{,}000$.

The key thing to remember then is that, when we multiply a number by a power of 10, all we do is count the 0's in the power of 10, and then move the decimal point that many places to the right in the other number, and that gives us the product.

Example

Find the product.

$$67 \times 1{,}000$$

Since we're multiplying by a power of 10, we need to count the 0's in the power of 10. There are three 0's in $1{,}000$, so we need to move the decimal point in 67 three places to the right. Since 67 has no decimal point (and so it looks as if there's no way to move a decimal point to the right), we have to first put a decimal point to the right of the 7 (which gives $67.$), and then (so that we'll be able to move the decimal point three places to the right) we put three 0's to the right of the decimal point.

At that stage in the process, we have 67.000, so when we move the decimal point three places to the right, we get

$$67{,}000$$

We can divide by powers of 10 just as easily. When we multiply by a power of 10, we move the decimal point to the right, but when we divide by a power of 10 (which is equivalent to multiplying by the number we would get if we raised 10 to the corresponding negative power), we move the decimal point to the left.

Notice the pattern in powers of 10:

$$10^0 = 1$$

$$10^{-1} = 0.1$$

$$10^{-2} = 0.01$$

$$10^{-3} = 0.001$$

etc.

Let's do an example.

Example

Do the division.

$$4.3 \div 100$$

There are two zeros in 100, and since we're dividing, we need to move the decimal point two places to the left. Since 4.3 has only one digit to the left of the decimal point (and so it looks as if there's no way to move the decimal point two places to the left), we have to first put a 0 to the left of the 4 (to give us a total of two digits to the left of the decimal point).

At that stage, we have 04.3, so when we move the decimal point two places to the left, we get $.043$. In any decimal number, however, we always want to have at least one digit to the left of the decimal point. Since there is no such digit in $.043$, the part that's to the left of the decimal point is understood to be 0, so we put a 0 to the left of the decimal point and get

$$0.043$$

Let's do another example.

Example

Simplify the expression.

$$510.75 \times 10^{-2}$$

When the power of 10 is negative, we move the decimal point to the left by the number of places indicated by the exponent. Since the exponent is negative 2, we move the decimal point to the left two places, and we get

$$510.75 \times 10^{-2}$$

$$5.1075$$

So, regardless of the number we start with, if we multiply that number by a power of 10, then we move the decimal point to the right (by the number of places equal to the number of 0's in that power of 10). If we divide a number by a power of 10, then we move the decimal point to the left (by the number of places equal to the number of 0's in that power of 10).