Rules of exponents

When it comes to dealing with exponents, we have to follow certain rules.

Addition and subtraction

When we want to find the sum or difference of two exponential expressions, they must be **like terms**, meaning that they must have the same base and the same exponent; otherwise, we can't add or subtract them.

For example, we can add or subtract $3x^2$ and x^2, because the bases are both x and the exponents are both 2. The 3 is what we call a **coefficient**; it just tells us we have three x^2's added together ($3x^2 = x^2 + x^2 + x^2$), so the sum of $3x^2$ and x^2 is found by adding one x^2 to three x^2's.

$$3x^2 + x^2$$

$$(x^2 + x^2 + x^2) + x^2$$

$$x^2 + x^2 + x^2 + x^2$$

$$4x^2$$

Now let's subtract x^2 from $3x^2$.

$$3x^2 - x^2$$

$$(x^2 + x^2 + x^2) - x^2$$

$$x^2 + x^2 + x^2 - x^2$$

Now take a look at the last two terms in the expression we just found: $x^2 - x^2$. As we might guess, when we have $x^2 - x^2$ (when we want to subtract x^2 from x^2), we get 0. That's because no matter what number the x stands for, the number $-x^2$ is the opposite of x^2.

$$x^2 + x^2 + (x^2 - x^2)$$

$$x^2 + x^2 + 0$$

$$x^2 + x^2$$

$$2x^2$$

Multiplication and division

Multiplication and division of exponential expressions is a little different. For the purposes of multiplication and division, only the bases need to be the same in order for the terms to be alike. We don't need the exponents to be the same.

For example, if we want to multiply x^4 by x^5, we can do it because the bases are the same, even though the exponents are different.

$$x^4 \cdot x^5$$

$$(xxxx) \cdot (xxxxx)$$

$$(xxxxxxxxx)$$

$$x^9$$

From this example, we realize that we're really just adding the exponents when we multiply two terms with the same base. In other words, the **product rule** for multiplication is

$$x^a \cdot x^b = x^{a+b}$$

A related exponent rule tells us that, when two terms are multiplied and the exponents are equal, we can raise the product of the bases to that exponent.

$$a^n b^n = (ab)^n$$

Similarly, if we want to divide x^5 by x^2, we can do it because the bases are the same, even though the exponents are different.

$$\frac{x^5}{x^2}$$

$$\frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x}$$

The factor that's common to the numerator and the denominator is $x \cdot x$, so we'll divide top and bottom by $x \cdot x$.

$$\frac{(x \cdot x \cdot x \cdot x \cdot x) \div (x \cdot x)}{(x \cdot x) \div (x \cdot x)}$$

$$\frac{x \cdot x \cdot x}{1}$$

$$x^3$$

From this example, we realize that we're really just subtracting the exponents when we divide two exponential expressions with the same base. In other words, the **quotient rule** for division is

$$\frac{x^a}{x^b} = x^{a-b}$$

Let's do another example.

Example

Use the quotient rule for exponents to simplify the expression.

$$\frac{x^4}{x^3}$$

The base of the expression in the numerator is x, and the base of the expression in the denominator is x, which means that the bases are the same, so we can use the quotient rule for exponents. We'll subtract the exponent in the denominator from the exponent in the numerator, keeping the base the same.

$$\frac{x^4}{x^3} = x^{4-3} = x^1 = x$$

Remember, the quotient rule works only with like bases, so

$$\frac{y^3}{x^2}$$

can't be simplified, because the bases y and x aren't the same, so we can't use the quotient rule.