Pre-Algebra Workbook Solutions

Factors and multiples

krista king MATH

DIVISIBILITY

■ 1. Is 369 divisible by 3?

Solution:

The sum of the digits is $3 + 6 + 9 = 18$. Since 18 is divisible by 3, 369 is also divisible by 3.

■ 2. How can we determine if a number is divisible by 5?

Solution:

If a number ends in "0" or "5," it's divisible by 5.

■ 3. "Divisibility" of whole numbers means we're looking at numbers that divide into a number ____________.

Solution:

evenly

■ 4. Is 245 divisible by 7?

Solution:

The last digit is 5. Multiply it by 5 to get $5 \times 5 = 25$.

Add this product to remaining number to get $24 + 25 = 49$.

Since 49 is divisible by 7, the number 245 is divisible by 7.

■ 5. What is the smallest whole number larger than 20 that's divisible by 2 and 4?

Solution:

If a number ends in $0, 2, 4, 6, 8$, it's divisible by 2, so our first set of candidates is $22, 24, 26, ...$

If the last two digits are divisible by 4, it's divisible by 4, so the candidates for this list are $24, 28, 32, ...$

The first matching number from both lists is 24, so the smallest whole number larger than 20 that's divisible by 2 and 4 is 24.

■ 6. What is the smallest whole number larger than 50 that's divisible by both 3 and 5?

Solution:

If a number ends in 0 or 5, it's divisible by 5, so our first set of candidates is $55, 60, 65, ...$

If the sum of the digits is divisible by 3, it's divisible by 3, so the candidates for this list are $51, 54, 57, 60, ...$

The first matching number from both lists is 60, so the smallest whole number larger than 50 that's divisible by both 3 and 5 is 60.

MULTIPLES

■ 1. List the first four multiples of 8.

Solution:

$8, 16, 24, 32$

■ 2. List the first five multiples of 40.

Solution:

$40, 80, 120, 160, 200$

■ 3. Is 8 a multiple of 8? Why or why not?

Solution:

Yes, because $8 \cdot 1 = 8$.

■ 4. What are two common multiples of 2 and 3?

Solution:

6 and 12

■ 5. What are two common multiples of 5 and 10?

Solution:

10 and 20

■ 6. The concept of multiples is related to the concept of ____________.

Solution:

divisibility

PRIME AND COMPOSITE

■ 1. ___________ numbers are numbers that are divisible by numbers other than 1 and themselves.

Solution:

Composite

■ 2. Is 7 a prime or composite number?

Solution:

Prime, because 7 is divisible by only 1 and itself.

■ 3. Is 15 a prime or composite number?

Solution:

Composite, because 15 is divisible by 1 and itself, but also by 3 and 5.

■ 4. 35 is a composite number because it's divisible by which numbers?

Solution:

1, 5, 7, and 35

■ 5. 98 is a composite number because it's divisible by which numbers?

Solution:

$1, 2, 7, 14, 49, 98$

■ 6. By how many numbers will a prime number be divisible?

Solution:

A prime number will only be divisible by two numbers: 1 and the number itself.

PRIME FACTORIZATION AND PRODUCT OF PRIMES

■ 1. What is the prime factorization of 75?

Solution:

75

$3 \cdot 25$

$3 \cdot 5 \cdot 5$ or $3 \cdot 5^2$

■ 2. What is the prime factorization of 55?

Solution:

55

$5 \cdot 11$

■ 3. What is the prime factorization of 148?

Solution:

148

$2 \cdot 74$

$2 \cdot 2 \cdot 37$ or $2^2 \cdot 37$

■ 4. The prime factorization of 156 is $2 \cdot 2 \cdot 3 \cdot$ ____________.

Solution:

The product $2 \cdot 2 \cdot 3$ is equal to 12, and $156 \div 12 = 13$, so the missing value is 13.

■ 5. The prime factorization of 63 is $3 \cdot 3 \cdot$ ____________.

Solution:

The product $3 \cdot 3$ is equal to 9, and $63 \div 9 = 7$, so the missing value is 7.

■ 6. Prime factorization is when we break down a composite number into its factors until every factor is a ____________ number.

Solution:

prime

LEAST COMMON MULTIPLE

■ 1. Find the least common multiple of 3 and 15.

Solution:

The multiples of 3 are 3, 6, 9, 12, 15, 18, etc., and the multiples of 15 are 15, 30, 45, 60, etc. The smallest matching multiple in these lists is 15, so 15 is the least common multiple of 3 and 15.

■ 2. Find the least common multiple of 16 and 40.

Solution:

The multiples of 16 are 16, 32, 48, 64, 80, etc., and the multiples of 40 are 40, 80, 120, etc. The smallest matching multiple in these lists is 80, so 80 is the least common multiple of 16 and 40.

■ 3. Find the least common multiple of the set $\{36, 84\}$.

Solution:

The multiples of 36 are 36, 72, 108, 144, 180, 216, 252, etc., and the multiples of 84 are 84, 168, 252, etc. The smallest matching multiple in these lists is 252, so 252 is the least common multiple of 36 and 84.

■ 4. Find the least common multiple of 12 and 20.

Solution:

The multiples of 12 are 12, 24, 36, 48, 60, etc., and the multiples of 20 are 20, 40, 60, etc. The smallest matching multiple in these lists is 60, so 60 is the least common multiple of 12 and 20.

■ 5. If the prime factorization of one number is $2 \cdot 3 \cdot 5^2$, and the prime factorization of another is $2^3 \cdot 3$, what's the least common multiple of the two numbers?

Solution:

Take the largest number of factors for each prime number in the prime factorizations. The first number has one factor of 2, while the second number has three factors of 2, so we'll take three factors of 2.

$$2^3$$

The first number has one factor of 3, while the second number has one factor of 3, so we'll take one factor of 3.

$$2^3 \cdot 3$$

The first number has two factors of 5, while the second number has zero factors of 5, so we'll take two factors of 5.

$$2^3 \cdot 3 \cdot 5^2$$

If we multiply this out, we'll find the least common multiple.

$$8 \cdot 3 \cdot 25$$

$$24 \cdot 25$$

$$600$$

■ 6. Is there only one possible pair of two numbers that can have a LCM of 20? Give examples to support the answer.

Solution:

No, there are multiple pairs of numbers that have a least common multiple of 20. For instance, 10 and 20 have an LCM of 20, but so do 5 and 4.

GREATEST COMMON FACTOR

■ 1. The greatest common factor of two numbers is the ____________ number that divides evenly into both numbers.

Solution:

largest

■ 2. Find the greatest common factor of 100 and 75.

Solution:

Find the prime factorization of both numbers.

100	75
$2 \cdot 50$	$3 \cdot 25$
$2 \cdot 2 \cdot 25$	$3 \cdot 5 \cdot 5$
$2 \cdot 2 \cdot 5 \cdot 5$	

The only factors that are common to both numbers are two factors of 5, so the greatest common factor is $5 \cdot 5 = 25$.

■ 3. Find the greatest common factor of the set $\{54, 162\}$.

Solution:

Find the prime factorization of both numbers.

54	162
$2 \cdot 27$	$2 \cdot 81$
$2 \cdot 3 \cdot 9$	$2 \cdot 3 \cdot 27$
$2 \cdot 3 \cdot 3 \cdot 3$	$2 \cdot 3 \cdot 3 \cdot 9$
	$2 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

The only factors that are common to both numbers is one factor of 2 and three factors of 3, so the greatest common factor is $2 \cdot 3 \cdot 3 \cdot 3 = 54$.

■ 4. If one number has a prime factorization of $3 \cdot 5 \cdot 11$, while another has a prime factorization of $2 \cdot 3^2 \cdot 11^2$, what is their greatest common factor?

Solution:

The only factors that are common to both numbers is one factor of 3 and one factor of 11, so the greatest common factor is $3 \cdot 11 = 33$.

■ 5. If one number has a prime factorization of $2^4 \cdot 3 \cdot 11$, while another has a prime factorization of $2^3 \cdot 5$, What is their greatest common factor?

Solution:

The only factors that are common to both numbers are three factors of 2, so the greatest common factor is $2^3 = 8$.

■ 6. Is there only one possible pair of two numbers that can have a GCF of 16? Give examples to support the answer.

Solution:

No, there are multiple pairs of numbers that have a greatest common factor of 16. For instance, 16 and 32 have an GCF of 16, but so do 16 and 48.