Adding and subtracting signed numbers

Think about signed numbers just as positive and negative numbers. Positive numbers have positive signs (even though we often write positive numbers without actually putting a positive sign in front of them), whereas negative numbers have negative signs. So 3, 7, and 11 are all positive numbers, and -2, -6, and -9 are all negative numbers.

When it comes to adding and subtracting signed numbers, let's break down the three possible combinations we could face when we have two signed numbers:

1. Two positive numbers
2. Two negative numbers
3. One positive number and one negative number

Let's tackle **adding** numbers for these three combinations.

Positive + **Positive** = **Positive.** The trick is: add the numbers, keeping the sign positive.

$$3 + 4 = 7$$

$$10 + 1 = 11$$

Negative + **Negative** = **Negative.** The trick is: add the numbers as if they were both positive, but make the sign negative.

$$-3 + (-4) = -7$$

$$-10+(-1)=-11$$

Positive + Negative, Negative + Positive. When we add one positive number and one negative number, we want to start by considering their absolute values. For instance, if we're trying to add -7 and 4, we want to instead first consider $|-7|=7$ and $|4|=4$. The result will be **positive if the absolute value of** the **positive number is larger, but negative if the absolute value of** the **negative number is larger.**

So 7 is larger than 4, but 7 was originally the negative number, so the absolute value of the negative number is larger, in this case. The trick is to find the absolute value of both numbers and subtract the absolute value of the smaller number from the absolute value of the larger number. The sign of the answer will be the original sign of the number whose absolute value is larger.

$$3+(-4)=-1$$

Here, the negative number is -4 and the positive number is 3. 4 is larger than 3, so the absolute value of the negative number is larger, which means the answer will be negative. So we subtract 3 from 4 and get 1. The sign needs to be negative, so we get -1.

$$10+(-1)=9$$

Here, the negative number is -1, and the positive number is 10. 10 is larger than 1, so the absolute value of the positive number is larger, which means the answer will be

positive. So we subtract 1 from 10 and get 9. The sign needs to be positive, so we get 9.

If the positive number and the negative number are opposites, the answer is 0.

$$3 + (-3) = 0$$

$$-10 + 10 = 0$$

Let's tackle **subtracting** numbers for the same combinations we considered for addition.

Positive – Positive. When we subtract one positive number from another, the result will be **positive if the first number is larger, but negative if the second number is larger.**

$$3 - 4 = 3 + (-4) = -1$$

Here, the first number is 3 and the second number is 4. Since $4 > 3$, the second number is larger so the result is negative.

$$10 - 1 = 10 + (-1) = 9$$

Here, the first number is 10 and the second number is 1. Since $10 > 1$, the first number is larger so the sign of the result is positive.

If the two positive numbers are equal, the result is 0.

$$3 - 3 = 3 + (-3) = 0$$

Negative – Negative. When we subtract one negative number from another, the result will be **positive if the first number is larger, but negative if the second number is larger.** In this context, the "larger" number refers to the number further right on a number line. For instance, the number -2 is to the right of -6 on the number line, so -2 is the larger number.

$$-3-(-4)=-3+4=1$$

Here, the first number is -3 and the second number is -4. Since $-3>-4$, the first number is larger so the result is positive.

$$-10-(-1)=-10+1=-9$$

Here, the first number is -10 and the second number is -1. Since $-1>-10$, the second number is larger so the sign of the result is negative.

If the two negative numbers are equal, the result is 0.

$$-3-(-3)=-3+3=0$$

Notice that the effect of subtracting a negative number is that the two negative signs cancel.

Positive – Negative = Positive. When we subtract a negative number from a positive number, the result will always be positive, because of the fact that the negative signs will cancel, leaving just the addition of two positive numbers.

$$3-(-4)=3+4=7$$

$$10-(-1)=10+1=11$$

Negative − Positive = Negative. When we subtract a positive number from a negative number, the result will always be negative.

$$-3-4=-3+(-4)=-7$$

$$-10-1=-10+(-1)=-11$$

Here's a summary of our findings:

Positive + Positive	Positive
Negative + Negative	Negative
Positive + Negative	Positive if the absolute value of the positive number is larger than the absolute value of the negative number Negative if the absolute value of the negative number is larger than the absolute value of the positive number 0 if the numbers are equal
Negative + Positive	Positive if the absolute value of the positive number is larger than the absolute value of the negative number

	Negative if the absolute value of the negative number is larger than the absolute value of the positive number
	0 if the numbers are equal
Positive – Positive	Positive if the first number is larger
	Negative if the second number is larger
	0 if the numbers are equal
Negative – Negative	Positive if the first number is larger
	Negative if the second number is larger
	0 if the numbers are equal
Positive – Negative	Positive
Negative – Positive	Negative

Keep in mind that when we add signed numbers, the order of the numbers doesn't make a difference.

$$3 + 4 = 7 = 4 + 3$$

$$-10 + (-1) = -11 = -1 + (-10)$$

$$-3 + (-4) = -7 = -4 + (-3)$$

$$10 + (-1) = 9 = -1 + 10$$

$$3 + (-3) = 0 = -3 + 3$$

But if we subtract signed numbers, the order of the numbers always matters.

$3-4=3+(-4)=-1$ but $4-3=4+(-3)=1$

$-10-(-1)=-10+1=-9$ but $-1-(-10)=-1+10=9$

$-3-(-4)=-3+4=1$ but $-4-(-3)=-4+3=-1$

$10-(-1)=10+1=11$ but $-1-10=-1+(-10)=-11$