Relationships of numbers

This topic can be a little challenging, so let's walk through it one step at a time. We're talking about relationships between numbers.

The first thing we'll deal with is how to determine which of two fractions is greater than the other. If two fractions have the same denominator, the fraction with the greater numerator is the greater one.

For example, the denominators of $2/7$ and $5/7$ are equal, and the numerator 5 is greater than the numerator 2, so $5/7$ is greater than $2/7$.

When the numerators are equivalent, the larger fraction is the one with the smaller denominator.

If the denominators of two fractions are different, we can't compare them directly; we first have to find a common denominator. For example, consider $5/8$ and $2/3$. For a common denominator, we'll use the least common multiple of the denominators 8 and 3, which is 24. So

$$\frac{5}{8} = \frac{5}{8}\left(\frac{3}{3}\right) = \frac{5 \times 3}{8 \times 3} = \frac{15}{24}$$

$$\frac{2}{3} = \frac{2}{3}\left(\frac{8}{8}\right) = \frac{2 \times 8}{3 \times 8} = \frac{16}{24}$$

The fractions $15/24$ and $16/24$ are equivalent to $5/8$ and $2/3$, respectively. Since they have the same denominator 24, the fraction with the greater numerator is greater than the fraction with the lesser numerator.

Therefore, $16/24$ is greater than $15/24$, which means that $2/3$ is greater than $5/8$, or we could say $5/8$ is less than $2/3$.

Now let's talk about the relationship between two integers - in particular, how we can find the number that's a fraction of the distance (along the number line) from the smaller integer to the larger one. Let's say we're thinking about the integers 3 and 8. We know that 3 is five units to the left of 8, or that 8 is five units to the right of 3. In other words, they're five units apart, since $8 - 3 = 5$.

Now what if we're asked for the number that's two-fifths of the way from 3 to 8? In other words, "If we divide the distance between 3 and 8 into five equal pieces, and then we start from 3 and move toward 8 by two of those five equal pieces, where do we end up?"

Here's how we figure that out. First we find the distance between 3 and 8 by subtracting the smaller number from the bigger number.

$$8 - 3 = 5$$

The distance between 3 and 8 is 5. Now, since we're looking to go two-fifths of that distance, we want to first divide the distance 5 into five equal pieces (each of which is one-fifth of that distance), which we do by dividing 5 by 5.

$$\frac{5}{5} = 1$$

So one-fifth of the 5 units of distance between 3 and 8 is 1 unit. Since I want two-fifths of the distance between 3 and 8, I need to multiply 1 unit by 2, and I get 2 units, so two-fifths of the distance between 3 and 8 is 2.

This means that if we want to go two-fifths of the way from 3 to 8, we start at 3, and add 2, and we end up at

$$3 + 2 = 5$$

So the number that's two-fifths of the way from 3 to 8 is 5.

In general, when we have some fraction "of" another number, it means we need to multiply the fraction by the other number. For example, $2/3$ of 6 is

$$\frac{2}{3} \cdot 6 = \frac{2}{3} \cdot \frac{6}{1} = \frac{2 \cdot 6}{3 \cdot 1} = \frac{12}{3} = 4$$

Therefore, we say that $2/3$ of 6 is 4, or that two-thirds of 6 is 4.

Now we can also do this with fractions. The process is exactly the same; we're just dealing with fractions instead of integers.

Example

Find a number that's $1/2$ of the way from $1/7$ to $6/11$.

First, we'll find the distance between $1/7$ and $6/11$.

$$\frac{6}{11} - \frac{1}{7}$$

In order to do the subtraction, we have to find a common denominator.

$$\frac{6}{11}\left(\frac{7}{7}\right) - \frac{1}{7}\left(\frac{11}{11}\right)$$

$$\frac{42}{77} - \frac{11}{77}$$

$$\frac{31}{77}$$

Now we want to find $1/2$ of this distance, which means we need to multiply it by $1/2$.

$$\frac{31}{77} \times \frac{1}{2}$$

$$\frac{31}{154}$$

This is half the distance from $1/7$ to $6/11$, and since we want to end up exactly one-half of the way from $1/7$ to $6/11$, we simply add $31/154$ to the smaller fraction, $1/7$.

$$\frac{1}{7} + \frac{31}{154}$$

In order to do the addition, we have to find a common denominator.

$$\frac{1}{7}\left(\frac{22}{22}\right) + \frac{31}{154}$$

$$\frac{22}{154} + \frac{31}{154}$$

$$\frac{53}{154}$$

So $53/154$ is the number that's $1/2$ of the way from $1/7$ to $6/11$.