Radicals

We can think about radicals (also called "roots") as the opposite of exponents.

We already know that the expression x^2 with the exponent of 2 means "multiply x by itself two times". The opposite operation would be "what do we have to multiply by itself two times in order to get x?" That's where radicals come in. If we see

$$\sqrt{x}$$

it means "the number we have to multiply by itself to get x." The symbol that contains the x is the **radical sign**, and the expression inside the symbol - in this case x - is the **radicand**.

Instead of saying that an expression is *inside* the radical sign, however, we usually say that it's *under* the radical sign. When we see $\sqrt{x}$, we can call it "the square root of x." But there are other kinds of roots of x too (which are indicated by little numbers tucked into the left side of the radical sign):

$\sqrt[3]{x}$ "the cube root of x"

$\sqrt[4]{x}$ "the fourth root of x"

$\sqrt[5]{x}$ "the fifth root of x"

"The cube root of x" means the number that's multiplied by itself three times in order to get x; "the fourth root of x" means the number that's multiplied by itself four times in order to get x, and so on.

Since roots are the opposite operation of exponents, we can convert between roots and exponents. For example, taking the square root of x is the same as raising x to the $1/2$ power. To see this, apply the power rule for exponents:

$$(x^{\frac{1}{2}})^2 = x^{(\frac{1}{2} \cdot 2)} = x^1 = x$$

Here's how to convert between roots and exponents.

$\sqrt{x}$	is the same as	$x^{\frac{1}{2}}$
$\sqrt[3]{x}$	is the same as	$x^{\frac{1}{3}}$
$\sqrt[4]{x}$	is the same as	$x^{\frac{1}{4}}$
$\sqrt[5]{x}$	is the same as	$x^{\frac{1}{5}}$

Now let's do an example where we simplify a radical.

Example

Simplify the radical expression.

$$\sqrt{9}$$

We're taking the square root of 9, which means we need to figure out what number we have to multiply by itself in order to get 9.

If we multiply 3 by itself, we get 9, which means that the square root of 9 is 3. So we can say

$$\sqrt{9} = 3$$

If we're given a number that x stands for, we want $\sqrt{x}$ to represent only one number - and a number that everyone will agree on. Notice, however, that we can get 9 not only by multiplying 3 by itself, but also by multiplying -3 by itself:

$$(+3)(+3) = 9 = (-3)(-3)$$

The way we get around this is that everyone agrees that both $\sqrt{9}$ and $9^{\frac{1}{2}}$ mean the positive number that we can multiply by itself in order to get 9:

$$\sqrt{9} = 3 = 9^{\frac{1}{2}}$$

Also

$$\sqrt{0} = 0 = 0^{\frac{1}{2}}$$

because 0 is the only number that when multiplied by itself gives 0. Now notice that there is no negative number that when multiplied by itself gives a negative number. (A positive number multiplied by itself is positive, 0 multiplied by itself is 0, and a negative number multiplied by itself is positive.) So $\sqrt{x}$ and $x^{\frac{1}{2}}$ are undefined if x is negative.

If x stands for any positive number, there's one and only one positive number that when multiplied by itself gives x. So $\sqrt{x}$ and $x^{\frac{1}{2}}$ are defined, and they represent that "one and only one positive number."

Sometimes the argument of a radical is not a perfect square, although it may contain a perfect square within its factors. When we're trying to simplify roots, we need to factor the radicand and take out any factor that's a perfect square. The rule we apply is

$$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$$

We'll factor the radicand, then separate the perfect square factor into its own radical, and then simplify that radical individually.

Let's do an example.

Example

Simplify the radical.

$$\sqrt{40}$$

The radicand 40 is not itself a perfect square, but we can factor 40 as $4 \cdot 10$.

$$\sqrt{4 \cdot 10}$$

Now we can separate each factor into its own radical,

$$\sqrt{4}\sqrt{10}$$

and then $\sqrt{4} = 2$, so $\sqrt{40}$ simplifies to

$$2\sqrt{10}$$

We know the simplification is done, because the remaining radicand 10 is not a perfect square, nor does it contain any factors that are perfect squares.

In later sections, we'll look at the specific rules we use to handle radical expressions.