Dividing radicals

Dividing radicals is really similar to multiplying radicals. Remember that when we multiply radicals with the same type of root, we just multiply the radicands and put the product under a radical sign. So

$$\sqrt{3}\sqrt{2}$$
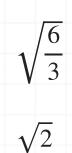
$$\sqrt{3 \cdot 2}$$

$$\sqrt{6}$$

We'll do the same when we divide radicals, because when we divide one radical by another with the same type of root, we just divide the radicands and put the quotient under a radical sign.

Example

Find the quotient.

$$\frac{\sqrt{6}}{\sqrt{3}}$$

Since we're dividing one square root by another, we can simply divide the radicands and put the quotient under a radical sign. That is, the quotient of square roots is equal to the square root of the quotient of the radicands.

$$\sqrt{\frac{6}{3}}$$

$$\sqrt{2}$$

Just as with multiplication of radicals, we can reverse this process and go the other way. So, if we wanted to, we could recognize that 2 is the same as $6/3$, and we could rewrite $\sqrt{2}$ as the square root of $6/3$, and finally rewrite that as the quotient of the square roots of 6 and 3.

$$\sqrt{2}$$

$$\sqrt{\frac{6}{3}}$$

$$\frac{\sqrt{6}}{\sqrt{3}}$$

Let's do an example where the radicands are the same.

Example

Simplify the radical expression.

$$\frac{\sqrt{5}}{\sqrt{5}}$$

We could follow the steps we did in the previous example.

$$\sqrt{\frac{5}{5}}$$

$$\sqrt{1}$$

$$1$$

This should remind us that when the roots are of the same type and the radicands are equal, the result will always be 1, because anything divided by itself is 1 (except, of course, that 0 divided by itself is undefined!).

Mathematicians don't like to end up with a radical in the denominator of a fraction. When there's a square root in the denominator, we can turn it into a rational number by multiplying the numerator and denominator of the fraction by that square root and then simplifying. That process is known as **rationalizing the denominator**, because the result has a rational number in the denominator.

Example

Rationalize the denominator.

$$\frac{7}{\sqrt{5}}$$

To get rid of the radical in the denominator, we'll multiply the numerator and denominator by $\sqrt{5}$.

$$\frac{7\sqrt{5}}{\sqrt{5}\sqrt{5}}$$

Now we have $\sqrt{5}\sqrt{5}$ in the denominator, which is equal to 5, so we get

$$\frac{7\sqrt{5}}{5}$$