Estimating scientific notation

Most of the time we'll be finding exact values with scientific notation, but sometimes it's nice just to get a quick estimate of the value of an expression.

This is a simple process where we round the decimal numbers, and then do our multiplication and/or division. Obviously we don't get a result that's quite as accurate, but the process is a little quicker, so it's a trade-off.

Example

Estimate the value of the expression.

$$\frac{(2.3 \times 10^{-4})(6.4 \times 10^{12})}{4.2 \times 10^{10}}$$

First, we'll round each of the decimal numbers to the nearest whole number.

$$\frac{(2 \times 10^{-4})(6 \times 10^{12})}{4 \times 10^{10}}$$

Then we'll express the fraction as the product of two fractions, one for the whole numbers and the other for the powers of 10.

$$\frac{2 \times 6}{4} \cdot \frac{10^{-4} \times 10^{12}}{10^{10}}$$

Now we'll simplify.

$$\frac{12}{4} \cdot \frac{10^{-4+12}}{10^{10}}$$

$$3 \cdot \frac{10^{8}}{10^{10}}$$

$$3 \cdot 10^{8-10}$$

$$3 \times 10^{-2}$$

If we were to compute the exact value of this expression with a calculator and then round the answer to the nearest tenth (to one decimal place), we would get 3.5×10^{-2}. Not only was our estimate not too far off, but we were able to arrive at the estimate faster than we could have if we'd computed the exact value.

We can also use scientific notation to estimate a product of several numbers. Let's look at an example of that.

Example

Find the product.

$$(13)(476)(52{,}450)(975)(143)$$

Instead of computing the exact value, we can use scientific notation to get an estimate of it. We'll do that by first expressing all the numbers in scientific notation.

$$(1.3 \times 10^1)(4.76 \times 10^2)(5.245 \times 10^4)(9.75 \times 10^2)(1.43 \times 10^2)$$

Then we'll multiply the decimal numbers and the powers of 10 separately.

$$(1.3 \times 4.76 \times 5.245 \times 9.75 \times 1.43)(10^1 \times 10^2 \times 10^4 \times 10^2 \times 10^2)$$

$$(1.3 \times 4.76 \times 5.245 \times 9.75 \times 1.43)(10^{1+2+4+2+2})$$

$$(1.3 \times 4.76 \times 5.245 \times 9.75 \times 1.43)(10^{11})$$

Next, we'll use a calculator to multiply all the decimal numbers, and we'll round our answer to the nearest ten-thousandth (to four decimal places).

$$1.3 \times 4.76 \times 5.245 \times 9.75 \times 1.43 \approx 452.5186$$

Therefore, we have

$$(452.5186)(10^{11})$$

Now we'll express 452.5186 in proper scientific notation. To do that, we need to move the decimal point 2 places to the left, so the exponent will be 2.

$$452.5186 = 4.525186 \times 10^2$$

Next, we'll multiply that by 10^{11}.

$$(4.525186 \times 10^2) \times 10^{11}$$

$$4.525186 \times (10^2 \times 10^{11})$$

$$4.525186 \times 10^{2+11}$$

$$4.525186 \times 10^{13}$$

Finally, we'll round the answer to three significant figures.

$$4.53 \times 10^{13}$$

This isn't an exact value for the product we were asked to find, but it's a pretty quick way to get a decent estimate.