Scientific notation

Scientific notation is a tool we use to write numbers that would otherwise be really tedious to express. Kind of like the way exponents allow us to express

$$x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$$

as

$$x^9$$

which is a lot easier, scientific notation allows us to express

$$964{,}000{,}000{,}000{,}000{,}000{,}000{,}000$$

as

$$9.64 \times 10^{23}$$

which is also a lot easier. Notice how the number we started with was a huge, huge number. It's actually hard for us to know what that number is, because we have to spend time counting the 0's just to figure out its actual value.

But with scientific notation, we're able to express that number without all the 0's and write a simpler number that quickly tells us how big the number is.

So scientific notation can be used to express really, really big numbers like that one, but it can also be used to express really, really small numbers like

$$0.00000000000000000000000000782$$

A number in scientific notation always has two parts: a decimal number, which comes first, and a power of 10 written in exponential form, which comes second. There are some requirements that these two parts must satisfy.

The decimal number can be positive or negative, and it has to have exactly one digit to the left of the decimal point. Also, in proper scientific notation, the digit to the left of the decimal point cannot be 0.

The exponent (in the power of 10 written in exponential form) can be positive, negative, or 0, but the base has to be 10.

Here are some examples of numbers that are in proper scientific notation:

$$7.1805 \times 10^{6}$$

$$4.32 \times 10^{-8}$$

$$-6.9 \times 10^{7}$$

$$-5.777 \times 10^{-9}$$

And here are some examples of numbers that aren't in proper scientific notation:

21.4×10^{7} (more than one digit to the left of the decimal point)

0.382×10^{-4} (the digit to the left of the decimal point is 0)

2.698×8^{3} (the base in the exponential expression is 8, not 10)

Let's use an example to talk about how to express a really big number or really small number in scientific notation.

Example

Express the number in scientific notation.

$$0.000000000000000000000000782$$

As written, this number is really difficult to understand, because it's hard to know how many 0's are included after the decimal point without spending time to carefully count them. So this is a great number to express in scientific notation.

When we take a number in decimal form and express it in proper scientific notation, the digits we include in the decimal number part are called the "**significant figures**."

The significant figures in the number in this example are the 7, the 8, and the 2. We say that 7 is the first significant figure, 8 is the second significant figure, and 2 is the third significant figure. We want to move the decimal point until it's just to the right of the first significant figure. Which means we want it to be just to the right of the 7.

Now the question is, "How many places do we need to move the decimal point in order to get it just to the right of the 7?" If we count all the 0's to the right of the decimal point, and the 7, in

$$0.000000000000000000000000782$$

we can see that we need to move the decimal point 25 places to the right (one for each of the twenty-four 0's to the right of the decimal point, and one more for the 7). This means that in scientific notation the decimal number part of this number will be 7.82, but then because we moved the decimal point 25 places to the right, we have to multiply the 7.82 by 10^{-25}, so we get

$$7.82 \times 10^{-25}$$

Now we might wonder how we know that the exponent has to be negative (that it's -25, and not 25). Well, here's the general rule for the sign of the exponent (in the power of 10 written in exponential form) when we take a number in decimal form and express it in scientific notation:

If we have to move the decimal point to the right (to get exactly one nonzero digit to the left of the decimal point), the exponent is negative.

If we have to move the decimal point to the left, the exponent is positive.

If we don't have to move the decimal point at all (if there is already exactly one digit to the left of the decimal place, and that digit isn't 0), the exponent is 0.

Since we moved the decimal point 25 places to the right in this example, the exponent has to be -25.

$$7.82 \times 10^{-25}$$