Ratio and proportion

Ratio and proportion is an application of fractions. A ratio is basically just a fraction, but one that emphasizes the relationship of its numerator to its denominator. For example, we say that

$$\frac{2}{3}$$

is "the ratio of 2 to 3." We sometimes write a ratio as $2 : 3$.

A proportion is an equation of two ratios, such as

$$\frac{2}{3} = \frac{8}{12}$$

Because the two ratios in a proportion are equal to each other, a proportion is an equation that expresses the equivalence of two fractions. We could read this proportion as follows:

"2 is to 3 as 8 is to 12"

What this says is that the relationship of 2 to 3 is the same as the relationship of 8 to 12. We sometimes write a proportion as $2 : 3 :: 8 : 12$, with a single colon within each ratio and a sequence of two colons between the two ratios.

Sometimes we're given a proportion such as

$$\frac{4}{5} = \frac{2x}{40}$$

and we're asked to solve for x. The x (which we call "a variable" or "an unknown") stands for a particular number (namely, the number that makes this equation true, which we can also refer to as "the number that satisfies this equation"). To "solve for x" means to find that number.

The expression $2x$ means "2 multiplied by x," so the numerator of the fraction on the right side of the equation is the product of 2 and x. By the way, we can also refer to the left and right sides (of an equation) as "the left-hand side" and "the right-hand side," respectively.

To solve for x, we have to take steps that will allow us to ultimately get an equation in which x is all by itself on one side. When we get that equation, the value of x (the number that makes our original equation true) will be on the other side of it.

Example

Solve for the unknown.

$$\frac{4}{5} = \frac{2x}{40}$$

To solve for the unknown, we simply need to remember the following rule:

> "If we make some change to the expression on one side of an equation, make the same change to the expression on the other side."

In the case of a proportion, the easiest way to do this is to cross multiply, which means we'll multiply both sides by both denominators.

First, we'll multiply both sides by 40, the denominator from the right side. This will cancel out the 40 in the denominator on the right side.

$$40\left(\frac{4}{5}\right) = 40\left(\frac{2x}{40}\right)$$

$$40\left(\frac{4}{5}\right) = 2x$$

$$\frac{160}{5} = 2x$$

$$5\left(\frac{160}{5}\right) = 5\,(2x)$$

$$160 = 10x$$

Now divide both sides by 10, which will cancel the 10 on the right side.

$$\frac{160}{10} = \frac{10x}{10}$$

$$16 = x$$

$$x = 16$$

Let's revisit that example, and see what we actually ended up doing when we cross multiplied.

Since each of the denominators canceled out when we multiplied both sides of the equation

$$\frac{4}{5} = \frac{2x}{40}$$

by it, what we eventually got on the left side was the product of the denominator from the right side (40) and the numerator from the left side (4), and what we eventually got on the right side was the product of the denominator from the left side (5) and the numerator from the right side ($2x$).

The arrows (from 40 to 4, and from 5 to $2x$) show which numerator each of the denominator ended up being multiplied by.

$$40(4) = 5(2x)$$

$$160 = 10x$$

From there, the only thing we had to do (to get the value of x) was to divide both sides of this equation by 10, which gave us $16 = x$.

We can think of the following as a shortcut way to cross multiply: Write the given proportion, and then (immediately below that) write the equation in which the expression on the left side is the product of the denominator from the right side (of the given proportion) and the numerator from the left side (of the given proportion), and the expression on the right side is the product of the denominator from the left side (of

the given proportion) and the numerator from the right side (of the given proportion).

If we ever forget this shortcut or feel uncomfortable using it, we can always multiply both sides of a proportion by both denominators.

We can solve for the variable, no matter where it appears in the equation. It can be on the left side or the right side, and it can be in the numerator or the denominator. Let's try an example where the variable appears in the denominator on the left side.

Example

Solve for the variable.

$$\frac{1}{6x} = \frac{3}{20}$$

This time, we'll use the shortcut way to cross multiply (which is the method that most people actually think of as cross multiplying). Our first step in solving for the variable will consist of writing an equation as follows: On the left-hand side, we'll write the product of the denominator from the right-hand side (20) and the numerator from the left-hand side (1). On the right-hand side, we'll write the product of the denominator from the left-hand side ($6x$) and the numerator from the right-hand side (3).

$$20(1) = 6x(3)$$

$$20 = 18x$$

Next, we'll divide both sides of this equation by 18, to get the x all by itself.

$$\frac{20}{18} = \frac{18x}{18}$$

$$\frac{20}{18} = x$$

Finally, we'll reduce the fraction to lowest terms.

$$\frac{10}{9} = x$$
