Multiplying radicals

When we multiply two radicals with the same type of root (both square roots, both cube roots, and so on), we simply multiply the **radicands**, which are the expressions under the radical signs, and put the product under a radical sign.

Let's do an example, first with two different radicands.

Example

Find the product.

$$\sqrt{3}\sqrt{2}$$

When we see two radicals next to each other like this, it means we're supposed to multiply them.

To multiply two square roots, we just multiply the radicands and put the product under a radical sign. That is, the product of two square roots is equal to the square root of the product of the radicands.

$$\sqrt{3 \cdot 2}$$

$$\sqrt{6}$$

It’s helpful to remember that we can use this rule for multiplication of radicals to go in the opposite direction as well. In other words, if we’re given $\sqrt{6}$, we can factor the 6 as $3 \cdot 2$, then rewrite $\sqrt{6}$ as $\sqrt{3 \cdot 2}$, and finally rewrite the square root of the product (of 3 and 2) as the product of their square roots.

$$\sqrt{6}$$

$$\sqrt{3 \cdot 2}$$

$$\sqrt{3}\sqrt{2}$$

Sometimes rewriting a radical as a product of radicals can help us solve a problem we’re working on, so it’s helpful to remember that we can go both ways with this rule for multiplication of radicals.

The product of square roots theorem tells us that, if m and/or n are nonnegative real numbers, then

$$\sqrt{m}\sqrt{n} = \sqrt{mn} \qquad \text{and} \qquad \sqrt{mn} = \sqrt{m}\sqrt{n}$$

Let’s do another example where we multiply two square roots, this time with equal radicands.

Example

Find the product.

$$\sqrt{5}\sqrt{5}$$

Let's follow the same steps we did before, where we rewrite the product of the square roots as the square root of the product of the radicands.

$$\sqrt{5 \cdot 5}$$

$$\sqrt{25}$$

But now we need to realize that $\sqrt{25}$ is just 5, since 5 multiplied by itself is equal to 25. So we can write $\sqrt{25}$ as just 5.

Which brings us to the point that when we multiply two identical square roots, the result is just the same as the radicand. So

$\sqrt{5}\sqrt{5}$	is the same as	5
$\sqrt{7}\sqrt{7}$	is the same as	7
$\sqrt{12}\sqrt{12}$	is the same as	12

Similarly, when we have a product of three identical cube roots, we get the number that's equal to the radicand,

$$\sqrt[3]{-21}\sqrt[3]{-21}\sqrt[3]{-21} = -21$$

and when we have four identical fourth roots, we get the number that's equal to the radicand.

$$\sqrt[4]{13}\sqrt[4]{13}\sqrt[4]{13}\sqrt[4]{13} = 13$$

And so on for higher-numbered roots.

Let's do one more example of multiplication of square roots - this time where one of them has a coefficient other than 1.

Example

Find the product.

$$(4\sqrt{2}) \cdot \sqrt{3}$$

The fact that we have a coefficient other than 1 doesn't change anything. We can leave the coefficient in front and multiply just the square roots.

$$(4\sqrt{2}) \cdot \sqrt{3}$$

$$4(\sqrt{2} \cdot \sqrt{3})$$

$$4\sqrt{2 \cdot 3}$$

$$4\sqrt{6}$$