Pre-Algebra Workbook Solutions

Radicals

krista king
MATH

RADICALS

■ 1. Radicals are the opposite of ___________.

Solution:

exponents

■ 2. $\sqrt[4]{x}$ can also be written as ___________.

Solution:

$x^{\frac{1}{4}}$

■ 3. Find the value of $\sqrt{36}$.

Solution:

Rewrite the value under the root as a perfect square.

$$\sqrt{6 \cdot 6}$$

So the square root is

$$6$$

■ 4. $x^{\frac{1}{3}}$ can also be written as ____________.

Solution:

$$\sqrt[3]{x}$$

■ 5. Find the value of $\sqrt{300}$.

Solution:

Rewrite the value under the root as a product.

$$\sqrt{100 \cdot 3}$$

Now separate the values under the radical into separate radicals.

$$\sqrt{100}\sqrt{3}$$

Rewrite the value under the root as a perfect square.

$$\sqrt{10 \cdot 10}\sqrt{3}$$

So the square root is

$$10\sqrt{3}$$

■ 6. Find the value of $\sqrt{5{,}000}$.

Solution:

Rewrite the value under the root as a product.

$$\sqrt{100 \cdot 50}$$

$$\sqrt{100 \cdot 25 \cdot 2}$$

Now separate the values under the radical into separate radicals.

$$\sqrt{100}\sqrt{25}\sqrt{2}$$

Rewrite the value under the root as a perfect square.

$$\sqrt{10 \cdot 10}\sqrt{5 \cdot 5}\sqrt{2}$$

So the square root is

$$10(5)\sqrt{2}$$

$$50\sqrt{2}$$

ADDING AND SUBTRACTING RADICALS

■ 1. Find the value of $2\sqrt{3} + 5\sqrt{3}$.

Solution:

Because the radicals are the same, we have like-terms and we can add them directly.

$$(2+5)\sqrt{3}$$

$$7\sqrt{3}$$

■ 2. Find the value of $\sqrt{32} - \sqrt{2}$.

Solution:

Because the radicals aren't the same, we need to simplify the radicals first.

$$\sqrt{16 \cdot 2} - \sqrt{2}$$

$$\sqrt{16}\sqrt{2} - \sqrt{2}$$

$$4\sqrt{2} - \sqrt{2}$$

Now with like terms, we can find the difference.

$$(4 - 1)\sqrt{2}$$

$$3\sqrt{2}$$

■ 3. Find the value of $\sqrt{3} + \sqrt{12}$.

Solution:

Because the radicals aren't the same, we need to simplify the radicals first.

$$\sqrt{3} + \sqrt{4 \cdot 3}$$

$$\sqrt{3} + \sqrt{4}\sqrt{3}$$

$$\sqrt{3} + 2\sqrt{3}$$

Now with like terms, we can find the sum.

$$(1 + 2)\sqrt{3}$$

$$3\sqrt{3}$$

■ 4. Find the value of $\sqrt{16} + \sqrt{25}$.

Solution:

Both radicals can be evaluated directly.

$$\sqrt{16} + \sqrt{25}$$

$$4 + 5$$

$$9$$

■ 5. Find the value of $4\sqrt{3} + 2\sqrt{2} - 2\sqrt{3} - \sqrt{2}$.

Solution:

If we re-order the terms, we notice that we can combine some of the radicals.

$$(4\sqrt{3} - 2\sqrt{3}) + (2\sqrt{2} - \sqrt{2})$$

$$(4 - 2)\sqrt{3} + (2 - 1)\sqrt{2}$$

$$2\sqrt{3} + \sqrt{2}$$

The remaining roots are still different, and can't be simplified further, so this is as far as we can simplify the expression.

■ 6. Find the value of $3\sqrt{4} - 2\sqrt{9}$.

Solution:

Both radicals can be evaluated directly.

$$3(2) - 2(3)$$

$$6 - 6$$

$$0$$

MULTIPLYING RADICALS

■ 1. Find the value of $\sqrt{20} \cdot \sqrt{4}$.

Solution:

Simplify the radicals first,

$$\sqrt{4 \cdot 5} \cdot \sqrt{4}$$

$$\sqrt{4}\sqrt{5} \cdot \sqrt{4}$$

$$2\sqrt{5} \cdot 2$$

then multiply.

$$4\sqrt{5}$$

■ 2. Find the value of $\sqrt{13} \cdot \sqrt{7}$.

Solution:

Neither radical can be simplified initially, so simply multiply the radicals.

$$\sqrt{13 \cdot 7}$$

$$\sqrt{91}$$

The value 91 has no factors that are perfect squares, so we can't simplify it any further.

■ 3. Find the value of $8\sqrt{3} \cdot \sqrt{12}$.

Solution:

Multiply the radicals first,

$$8\sqrt{3 \cdot 12}$$

$$8\sqrt{36}$$

then simplify.

$$8(6)$$

$$48$$

■ 4. Find the value of $15\sqrt{2} \cdot \sqrt{16}$.

Solution:

Simplify the radicals first,

$$15\sqrt{2}\cdot 4$$

then multiply.

$$60\sqrt{2}$$

■ 5. Find the value of $2\sqrt{3}\cdot 5\sqrt{5}$.

Solution:

We need to use the rule that tells us that

$$\sqrt{m}\cdot\sqrt{n}=\sqrt{mn}$$

Applying this to our problem, we can rewrite it.

$$2\sqrt{3}\cdot 5\sqrt{5}$$

$$2\cdot 5\sqrt{3\cdot 5}$$

$$10\sqrt{15}$$

■ 6. Find the value of $\sqrt[3]{12}\cdot\sqrt[3]{4}$.

Solution:

We need to use the rule that tells us that

$$\sqrt{m} \cdot \sqrt{n} = \sqrt{mn}$$

Applying this to our problem, we can rewrite it.

$$\sqrt[3]{12} \cdot \sqrt[3]{4}$$

$$\sqrt[3]{12 \cdot 4}$$

$$\sqrt[3]{48}$$

Then simplify.

$$\sqrt[3]{8 \cdot 6}$$

$$\sqrt[3]{8} \cdot \sqrt[3]{6}$$

$$2\sqrt[3]{6}$$

DIVIDING RADICALS

■ 1. Simplify the expression.

$$\sqrt{\frac{36}{6}}$$

Solution:

Simplify the quotient inside the radical.

$$\sqrt{6}$$

There are no factors of 6 that are perfect squares, so we can't simplify the radical any further.

■ 2. Simplify the expression.

$$\sqrt{\frac{45}{5}}$$

Solution:

Simplify the quotient inside the radical, then simplify the radical.

$$\sqrt{9}$$

3

■ 3. Simplify the expression.

$$\frac{\sqrt{20x^5y^7}}{\sqrt{5x^3y}}$$

Solution:

When we have a fraction in which the numerator is a root and the denominator is a root, we can put the fraction under one root instead.

$$\frac{\sqrt{20x^5y^7}}{\sqrt{5x^3y}}$$

$$\sqrt{\frac{20x^5y^7}{5x^3y}}$$

$$\sqrt{4x^{5-3}y^{7-1}}$$

$$\sqrt{4x^2y^6}$$

$$2xy^3$$

■ 4. Simplify the expression.

$$\frac{\sqrt[3]{-32}}{\sqrt[3]{2}}$$

Solution:

The quotient of roots is always equal to the root of the quotient, so we can rewrite the expression as

$$\frac{\sqrt[3]{-32}}{\sqrt[3]{2}}$$

$$\sqrt[3]{\frac{-32}{2}}$$

Simplify the quotient.

$$\sqrt[3]{-16}$$

$$\sqrt[3]{-8 \cdot 2}$$

$$-2\sqrt[3]{2}$$

■ 5. Simplify the expression.

$$\frac{\sqrt{5}}{\sqrt{15}}$$

Solution:

The quotient of roots is always equal to the root of the quotient, so we can rewrite the expression as

$$\frac{\sqrt{5}}{\sqrt{15}}$$

$$\sqrt{\frac{5}{15}}$$

Simplify the quotient.

$$\sqrt{\frac{1}{3}}$$

$$\frac{\sqrt{1}}{\sqrt{3}}$$

$$\frac{1}{\sqrt{3}}$$

To rationalize the denominator, we'll multiply both the numerator and denominator by $\sqrt{3}$.

$$\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\frac{\sqrt{3}}{3}$$

6. Simplify the expression.

$$\frac{\sqrt{8}}{5\sqrt{2}}$$

Solution:

The quotient of roots is always equal to the root of the quotient, so we can rewrite the expression as

$$\frac{\sqrt{8}}{5\sqrt{2}}$$

$$\frac{1}{5} \cdot \frac{\sqrt{8}}{\sqrt{2}}$$

$$\frac{1}{5} \cdot \sqrt{\frac{8}{2}}$$

$$\frac{1}{5} \cdot \sqrt{4}$$

$$\frac{1}{5} \cdot 2$$

$$\frac{2}{5}$$

RADICAL EXPRESSIONS

■ 1. Find the value of $\sqrt{80}-\sqrt{20}$.

Solution:

Notice that $80=4\cdot 20$, and use that to simplify the first radical.

$$\sqrt{4\cdot 20}-\sqrt{20}$$

$$\sqrt{4}\sqrt{20}-\sqrt{20}$$

$$2\sqrt{20}-\sqrt{20}$$

Now that the radicals are the same, combine like terms.

$$(2-1)\sqrt{20}$$

$$1\sqrt{20}$$

$$\sqrt{20}$$

Now simplify the remaining radical.

$$\sqrt{4\cdot 5}$$

$$\sqrt{4}\sqrt{5}$$

$$2\sqrt{5}$$

■ 2. Find the value of $5\sqrt{24} \cdot \sqrt{15}$.

Solution:

First simplify the radicals individually.

$$5\sqrt{4 \cdot 6} \cdot \sqrt{15}$$

$$5\sqrt{4}\sqrt{6} \cdot \sqrt{15}$$

$$5(2)\sqrt{6} \cdot \sqrt{15}$$

$$10\sqrt{6} \cdot \sqrt{15}$$

Now factor the values inside both radicals, then split the radicals apart, group together like terms, and simplify.

$$10\sqrt{2 \cdot 3} \cdot \sqrt{3 \cdot 5}$$

$$10\sqrt{2}\sqrt{3} \cdot \sqrt{3}\sqrt{5}$$

$$10(\sqrt{3}\sqrt{3})\sqrt{2}\sqrt{5}$$

$$10(3)\sqrt{2}\sqrt{5}$$

$$30\sqrt{10}$$

■ 3. The square root of a number multiplied by the square root of the same number is equal to ___________.

Solution:

The square root of a number multiplied by the square root of the same number is equal to the number itself. For example,

$$\sqrt{5}\sqrt{5} = 5$$

■ 4. Find the value of $\sqrt{2} + \sqrt{32} - \sqrt{50}$.

Solution:

First simplify the radicals individually.

$$\sqrt{2} + \sqrt{16 \cdot 2} - \sqrt{25 \cdot 2}$$

$$\sqrt{2} + \sqrt{16}\sqrt{2} - \sqrt{25}\sqrt{2}$$

$$\sqrt{2} + 4\sqrt{2} - 5\sqrt{2}$$

Now that all the radicals are the same, combine like terms.

$$(1 + 4 - 5)\sqrt{2}$$

$$(0)\sqrt{2}$$

$$0$$

■ 5. To be able to add or subtract radicals, the roots must be ____________ when they are simplified.

Solution:

To be able to add or subtract radicals, the roots must be identical when they are simplified. In other words, we can could simplify an expression like

$$4\sqrt{2}+3\sqrt{2}$$

because the radicals are identical, but we couldn't simplify

$$3\sqrt{2}-4\sqrt{5}$$

because the radicals are different and they can't be rewritten in a way that would make them identical.

■ 6. Roberta is trying to simplify the following radical expression,

$$\sqrt{4}+\sqrt{20}-2\sqrt{5}+\sqrt{25}$$

and her work is shown below.

Step 1: $2+\sqrt{20}-2\sqrt{5}+5$

Step 2: $2 + \sqrt{4 \cdot 5} - 2\sqrt{5} + 5$

Step 3: $2 + 4\sqrt{5} - 2\sqrt{5} + 5$

Step 4: $7 + 2\sqrt{5}$

In which step did she make a mistake? What should she have done differently, and what is the correct answer?

Solution:

Roberta made her mistake in Step 3. Instead of writing $4\sqrt{5}$, she should have written $2\sqrt{5}$, because she should have simplified $\sqrt{4}$ to 2, instead of simplifying it to 4. Starting from Step 2, her work should have looked like this:

$$2 + \sqrt{4 \cdot 5} - 2\sqrt{5} + 5$$

$$2 + \sqrt{4}\sqrt{5} - 2\sqrt{5} + 5$$

$$2 + 2\sqrt{5} - 2\sqrt{5} + 5$$

$$2 + 5$$

$$7$$

W W W . K R I S T A K I N G M A T H . C O M