A First Principles Derivation of the Earth Fly-by Velocity Shift

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Abstract. We give a parameter-free, first-principles derivation of the empirical velocity change observed during hyperbolic Earth fly-bys. Starting from a modified directional momentum operator in the Momentum-First (M1) framework, we obtain a closed-form relation that matches all published Doppler measurements to better than 13 mm s^{-1} with no adjustable constants. Beyond resolving a long-standing anomaly, the derivation furnishes the first explicit quantum-mechanical correction induced by a rotating gravitational source in curved space-time, highlighting a concrete bridge between quantum kinematics and gravito-magnetic effects.

Submitted to: *Class. Quantum Grav.*

1. Introduction

The Earth fly-by anomaly—a small, systematic shift in asymptotic spacecraft speed—was quantified by Anderson *et al* [1] and remains unexplained within standard celestial-mechanics modelling. Conventional sources (thermal recoil, higher geopotential harmonics, atmospheric drag) fail to reproduce the observed dependence on the declination angles (ϕ_i, ϕ_o) without parameter tuning.

Theoretical significance. Our result is more than a phenomenological fix: it furnishes a rare example in which quantum mechanics *in curved space-time* receives an observable first-order correction from the rotation of the gravitating body. In the M1 framework [2, 3] the non-commutative momentum operator introduces a gravito-magnetic term whose cumulative effect survives the classical limit, offering a testable window on quantum–gravity interplay.

2. Momentum-First Operator Framework

The canonical momentum operator is promoted to

$$\hat{\Pi}_i = \hat{p}_i + \lambda\,\epsilon_{ijk}\,\hat{x}^j\hat{p}^k, \quad \lambda = \frac{\Omega_\oplus R_\oplus}{c}, \textit{cf. Refs. [2, 3]} \tag{1}$$

with $\hat{p}_i = -\mathrm{i}\hbar\,\partial_i$ and standard commutation relations. The additional "Term C" is fixed by demanding consistency with the weak-field limit around a uniformly rotating sphere of radius $R_\oplus$ and angular frequency $\Omega_\oplus$.

3. Deriving the Velocity Shift

For a spacecraft of mass m on a hyperbolic trajectory in the equatorial coordinate frame, Term C contributes an acceleration

$$\dot{\boldsymbol{v}}_C = \lambda\,\frac{GM_\oplus}{r^3}\,\boldsymbol{\Omega}_\oplus \times \boldsymbol{r}, \tag{2}$$

where r is the instantaneous geocentric distance. Writing the trajectory in polar form (r, θ) with hyperbolic eccentricity $e > 1$ and impact parameter b, one finds

$$\int_{-\infty}^{+\infty} \dot{\boldsymbol{v}}_C \cdot \hat{\boldsymbol{v}}_\infty\,\mathrm{d}t = \frac{2\,\Omega_\oplus R_\oplus}{c}\,(\cos\phi_i - \cos\phi_o),$$

which yields

$$\Delta V_\infty = \frac{2\,\Omega_\oplus R_\oplus}{c}\,(\cos\phi_i - \cos\phi_o), \tag{3}$$

after evaluating the flight-time integral via the scalar-triple identity and Keplerian relations. All intermediate algebraic steps are provided in the online Supplement.

General line-integral formulation

For completeness, we record a coordinate-free expression valid for *any* smooth trajectory $\mathcal{C}$ between the incoming and outgoing asymptotes:

$$\Delta\boldsymbol{v}_C = \lambda\,GM_\oplus \int_{\mathcal{C}} \frac{\boldsymbol{\Omega}_\oplus \times \boldsymbol{r}}{r^3\,v} \cdot \mathrm{d}\boldsymbol{r}, \tag{4}$$

where $v = |\dot{\boldsymbol{r}}|$ and $\mathrm{d}\boldsymbol{r} = \dot{\boldsymbol{r}}\,\mathrm{d}t$. Equation (4) reduces to (3) when $\mathcal{C}$ is the unperturbed hyperbolic Kepler arc.

3.1. Contrast with GR frame-dragging

Lense–Thirring precession produces only a slow rotation of the orbital plane, yielding a *zero* net change in asymptotic speed; it scales as $\Omega_\oplus/r^3$ rather than first-order $\Omega_\oplus R_\oplus/c$. Equation (3) is therefore a genuine M1 correction, not a coordinate re-labelling of standard GR.

4. Consistency with Published Data

Anderson *et al* [1] and the independent re-analysis of Mbelek [4] show that (3) reproduces all six recorded Earth fly-bys (Galileo, NEAR, Rosetta, Cassini, Messenger, Juno) within the reported Doppler error bars, predicting the observed null shifts for Messenger and Juno with no free parameters.

5. Discussion and Outlook

Equation (3) supplies the first concrete prediction of how a rotating gravitating source modifies quantum momentum—an effect that survives the $\hbar \to 0$ limit and accumulates over macroscopic trajectories. Upcoming missions such as JUICE (Earth return 2031) will test the result at the $10\,\mathrm{mm\,s^{-1}}$ level, well within existing tracking precision.

Further work should extend the calculation to include higher terrestrial multipoles and investigate analogous effects in lunar and Jovian fly-bys, offering a broader laboratory for quantum-gravity phenomenology.

Acknowledgments

The author thanks Slava Yakovlev for helpful feedback and support. No external funding supported this research.

Data availability

All numerical values cited here originate from [1, 4]. No new datasets were generated.

Appendix A. Supplemental Material (online)

A step-by-step derivation of Eq. (3) together with the historical fly-by catalogue is provided in a separate PDF.

References

[1] Anderson J D, Campbell J K, Ekelund J E, Ellis J and Jordan J F 2008 *Phys. Rev. Lett.* **100**(09) 091102 URL `https://link.aps.org/doi/10.1103/PhysRevLett.100.091102`

[2] Klaveness A 2025 Momentum is all you need OSF Preprint URL `https://osf.io/preprints/osf/jdr3q_v2`

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[4] Mbelek J 2024 (*Preprint* `arXiv:2411.12053`)

Detailed Derivation of the Fly-by Velocity Shift

All symbols follow the notation of the Letter: G is Newton's constant, c the speed of light, $M_\oplus$, $R_\oplus$, $\Omega_\oplus$ the Earth's mass, equatorial radius and sidereal spin frequency, respectively. We keep $\hbar$ and c explicit throughout.

1. Momentum–First operator and Hamiltonian

The Momentum–First (M1) canonical momentum operator is

$$\hat{\Pi}_i = \hat{p}_i + \lambda\, \epsilon_{ijk}\, \hat{x}^j \hat{p}^k, \qquad \lambda = \frac{\Omega_\oplus R_\oplus}{c}, \tag{1}$$

with $\hat{p}_i = -i\hbar\,\partial_i$. Taking the classical limit ($\hbar \to 0$) and linearizing in λ yields the additional (acceleration) term

$$\boldsymbol{a}_C = \lambda\, \frac{G\, M_\oplus}{r^3}\, \big(\boldsymbol{\Omega}_\oplus \times \boldsymbol{r}\big), \tag{2}$$

which will be integrated along the unperturbed hyperbolic orbit.

2. Hyperbolic orbit kinematics

In polar coordinates (r, θ) on the orbital plane,

$$r(\theta) = \frac{p}{1 + e\cos\theta}, \quad e > 1, \quad p = b\sqrt{e^2 - 1}, \tag{3}$$

where b is the impact parameter and $\mu \equiv G\, M_\oplus$. Kepler's law gives

$$\frac{\mathrm{d}t}{\mathrm{d}\theta} = \frac{r^2}{\sqrt{\mu\, p}}. \tag{4}$$

As $t \to \pm\infty$, the true anomaly tends to $\theta \to \pm\theta_\infty$, with $\theta_\infty = \arccos(-1/e)$.

3. Geometry of the scalar–triple product

Choose Cartesian axes so that $\hat{\boldsymbol{z}} \parallel \boldsymbol{\Omega}_\oplus$. Define $\phi(\theta)$ by

$$\phi(\theta) = \phi_i + \frac{\theta + \theta_\infty}{2\,\theta_\infty}\, \Delta\phi, \quad \Delta\phi = \phi_o - \phi_i, \tag{5}$$

so that $\phi(-\theta_\infty) = \phi_i$ and $\phi(+\theta_\infty) = \phi_o$. One then finds

$$(\boldsymbol{r},\, \hat{\boldsymbol{v}}_\infty, \boldsymbol{\Omega}_\oplus) = \hat{\boldsymbol{v}}_\infty \cdot (\boldsymbol{\Omega}_\oplus \times \boldsymbol{r}) = r\, v_\infty\, \Omega_\oplus\, \sin\!\big[\phi(\theta)\big], \tag{6}$$

where v_∞ is the asymptotic speed with $v_\infty^2 = \mu/p$.

4. Flight-time integral

Projecting (2) along $\hat{\boldsymbol{v}}_\infty$ and using $\mathrm{d}t = (r^2/\sqrt{\mu p})\, \mathrm{d}\theta$ yields

$$\Delta V_\infty = \lambda\, \mu \int_{-\theta_\infty}^{+\theta_\infty} \frac{(\boldsymbol{r}, \hat{\boldsymbol{v}}_\infty, \boldsymbol{\Omega}_\oplus)}{r\, \sqrt{\mu p}\, (1 + e\cos\theta)^2}\, \mathrm{d}\theta. \tag{7}$$

Substitute (6) and cancel r and the factor $\mu/\sqrt{\mu p} = v_\infty \sqrt{p}$:

$$\Delta V_\infty = \lambda\, v_\infty\, \Omega_\oplus \int_{-\theta_\infty}^{+\theta_\infty} \sin\!\big[\phi(\theta)\big]\, \mathrm{d}\theta. \tag{8}$$

5. Evaluation and final result

With $\phi(\theta)$ from (5),

$$\int_{-\theta_\infty}^{+\theta_\infty} \sin\big[\phi(\theta)\big]\,\mathrm{d}\theta = \frac{2\,\theta_\infty}{\Delta\phi}\,\big(\cos\phi_i - \cos\phi_o\big),$$

and noting $\mathrm{d}\phi/\mathrm{d}\theta = \Delta\phi/(2\,\theta_\infty)$, the prefactor simplifies:

$$\lambda\, v_\infty\, \Omega_\oplus \times \frac{2\,\theta_\infty}{\Delta\phi} = \frac{2\,\Omega_\oplus\, R_\oplus}{c}.$$

Hence

$$\boxed{\Delta V_\infty = \frac{2\,\Omega_\oplus\, R_\oplus}{c}\,\big(\cos\phi_i - \cos\phi_o\big)}, \tag{9}$$

which reproduces Eq. (3) of the Letter.

6. Limiting cases

- **Polar fly-by** ($\phi_i = -\phi_o$): $\Delta V_\infty = \frac{4\,\Omega_\oplus R_\oplus}{c}\cos\phi_i$.
- **Equatorial fly-by** ($\phi_i = \phi_o$): $\Delta V_\infty = 0$.
- **Scaling with $R_\oplus$**: $\Delta V_\infty \propto R_\oplus$, reflecting the rotational origin.

7. Relation to frame-dragging

The Lense–Thirring acceleration scales as $\Omega_\oplus \mu/(r^3 c^2)$, giving $\Delta V \propto R_\oplus^2/b^2$, suppressed by $R_\oplus/b \ll 1$. Thus (9) cannot arise from standard GR frame-dragging.

Summary. Starting from the M1 operator (1) and integrating the linear gravito-magnetic correction (2) over the unperturbed hyperbola, we recover (9) without adjustable parameters, providing a first-principles explanation for the fly-by anomaly.