The Momentum-First Dirac Equation: A Unified Hamiltonian for Fermions in Stationary Spacetimes

Arne Klaveness[1,*]

[1] *Independent Researcher, Sandefjord, Norway*

(Dated: July 7, 2025)

The Momentum-First (M-First) framework, which posits that gravity modifies quantum kinematic rules, is applied to Dirac fermions in stationary spacetimes to derive a unified quantum Hamiltonian from first principles. The Hamiltonian's structure is uniquely determined by the spacetime's isometry group and the correspondence principle, containing no free parameters. In the static limit, it resolves the neutron-star shallow-heating puzzle via a gravitationally induced contextual mass. In its rotational limit, it provides the theoretical foundation for the anomalous Earth fly-by velocity shifts. The theory further predicts novel, testable phenomena, including a gravitational spin-Hall effect and a gravitational screening of charge. The ability of a coherent framework to solve existing anomalies and predict new physics marks it as a compelling paradigm for quantum-gravity interactions.

I. INTRODUCTION

A central challenge in theoretical physics is the consistent quantum description of particles in curved spacetime. The standard paradigm, Quantum Field Theory in Curved Spacetime (QFTCS) [1, 2], treats gravity's influence by promoting partial derivatives to covariant ones ($\partial \to \nabla$) and introducing spin connections, while preserving the local mass-shell condition. This paper explores the Momentum-First (M-First) framework [3], which proposes that gravity's primary effect is to alter a particle's fundamental kinematic relations and, consequently, the structure of its quantum operators.

Here we develop an M-First framework for spin-$\frac{1}{2}$ fermions in stationary spacetimes, deriving a Hamiltonian fixed by spacetime symmetries rather than by *ansatz*. By examining this Hamiltonian in different physical limits, we derive quantitative solutions to two long-standing anomalies from a common origin and predict new, testable physics.

II. THE UNIFIED M-FIRST HAMILTONIAN

The framework's logic flows from classical principles to a unique quantum operator.

A. Classical Foundation and Contextual Mass

The M-First framework is built on the primacy of momentum. Each particle possesses an intrinsic, invariant **fermic momentum**, $p_f \equiv m_0 c$, with units of momentum. In a gravitational field, its energy must conform to the **gravitationally influenced core momentum**, M_g, which is identified with the time-translation generator of the metric (see Appendix A). In a static potential, this defines a position-dependent **contextual mass**:

$$m_c(\boldsymbol{x}) \equiv m_0 \sqrt{-g_{00}(\boldsymbol{x})}. \tag{1}$$

B. The Unified Momentum Generator from M-First Principles

The quantum operator for a particle's momentum in a rotating spacetime is not derived from the conventional Lense-Thirring effect, but is constructed from the core principles of the M-First framework, as detailed in Appendix C.

* Contact author: arne.klaveness@outlook.com

The construction follows a strict two-part logic. First, the functional form of the rotational interaction is postulated to be the simplest vector potential constructible from the background's angular velocity $\boldsymbol{\Omega}$ and the particle's position $\hat{\mathbf{x}}$, yielding an effective potential $\boldsymbol{\mathcal{A}}_{\text{eff}} \propto \boldsymbol{\Omega} \times \hat{\mathbf{x}}$. Second, the coupling strength to this potential is fixed by the M-First EFT hierarchy [4]. This procedure dictates that after the static potential (g_{00}) renormalizes the bare mass m_0 to the contextual mass m_c, this m_c becomes the only operative mass "charge" available for subsequent couplings.

This logic uniquely determines the form of the **unified momentum generator**:

$$\hat{\boldsymbol{\Pi}} = \hat{\boldsymbol{p}} - m_c(\hat{\boldsymbol{x}})(\boldsymbol{\Omega} \times \hat{\boldsymbol{x}}). \tag{2}$$

This operator predicts a gravito-magnetic interaction corresponding to a uniform effective field, which is distinct from and significantly stronger than the dipole field of standard General Relativity. The fly-by anomaly therefore serves as a clean experimental test to distinguish the M-First prediction from that of GR [5].

C. The Unified Hamiltonian

With the generator uniquely determined, the quantum Hamiltonian is constructed. It takes the form of a Dirac Hamiltonian where both mass and momentum are contextualized:

$$\hat{H} = \beta\, m_c(\hat{\boldsymbol{x}})c^2 + c\,\boldsymbol{\alpha} \cdot \hat{\boldsymbol{\Pi}}. \tag{3}$$

We verify its closure under Poincaré generators and Hermiticity in Appendices E, F, and G.

III. ANOMALY RESOLUTION FROM A UNIFIED ORIGIN

This single Hamiltonian provides mechanisms for two well-known anomalies.

A. The Static Limit: Neutron Star Shallow Heating

In a static, non-rotating spacetime ($\boldsymbol{\Omega} = 0$), the momentum generator reduces to $\hat{\boldsymbol{\Pi}} \to \hat{\boldsymbol{p}}$. The entire gravitational modification is carried by the contextual mass, implying an effective rest mass $m_0^{\text{contextual}}(\vec{r}) = \sqrt{-g_{00}}m_0$. This reduction enhances quantum tunneling rates by scaling the Gamow exponent G (Appendix D):

$$G_{\text{M1}} = G_{\text{Std}} \times (-g_{00}(\vec{r}))^{1/4}. \tag{4}$$

This provides a direct physical mechanism that can resolve the "shallow heating" puzzle in neutron stars [3, 6].

B. The Rotational Limit: The Fly-by Anomaly

In a rotating frame, new dynamics arise from the rotational term in $\hat{\boldsymbol{\Pi}}$. As shown via a Foldy-Wouthuysen transformation in Appendix E, the classical limit of the dynamics includes a Coriolis-like force. Integrating this force's effect yields a net change in asymptotic velocity that precisely matches the anomalous shifts (at the mm/s level) observed in spacecraft fly-bys [5, 7].

IV. NOVEL PHYSICAL PREDICTIONS

Beyond resolving known puzzles, our Hamiltonian predicts two novel effects.

A. Gravitational Spin Dynamics

As derived in Appendix E, a particle's helicity is not conserved. Its time evolution is driven by a "gravitational spin-Hall effect," coupling spin to the potential gradient, and a "frame-dragging precession." For the spin-Hall effect, an order-of-magnitude estimate for a 10 GeV electron in a gravitoelectric gradient equivalent to 1 T/m yields a spin precession of $\sim 10^{-19}$ rad/s. While extremely small, this points toward a potential target for future high-precision experiments.

B. Gravitationally Screened Currents

As derived from a U(1) gauge principle (minimal substitution $\partial_\mu \to \partial_\mu + iqA_\mu$ in the M-First action) in Appendix G, the conserved electromagnetic current acquires a gravitational correction. This implies a particle's effective charge is screened or anti-screened by the local potential, a key testable difference from standard minimally coupled theories.

V. DISCUSSION

The M-First framework offers a coherent quantum mechanical picture for fermions in stationary gravity, shifting from a "metric in the derivative" paradigm (QFTCS) to a "metric in the operators" paradigm. A profound consequence, derived in Appendix F, is the modification of the theory's fundamental Casimir invariant: the square of the Pauli-Lubanski vector becomes local (see Eq. (F4) in Appendix F). This implies that gravity is imprinted on a particle's most fundamental property—its intrinsic spin classification—by making it a local quantity. Mass and coupling parameters are expected to receive calculable, gravitationally dependent renormalization corrections in a full M-First QFT, with further work needed to explore extensions to non-stationary spacetimes.

VI. CONCLUSION

We constructed a Hamiltonian for Dirac fermions in stationary spacetimes based on Momentum-First principles. By deriving its form from fundamental symmetries, we have shown that the framework, from a common origin, resolves the neutron star shallow heating and spacecraft fly-by anomalies. Furthermore, it predicts novel, testable physics, including new forms of spin-gravity coupling and a gravitational screening of charge. The emergence of these solutions from a single theoretical structure demonstrates the potential of M-First as a robust and predictive theory of quantum-gravity interactions.

ACKNOWLEDGMENTS

The author thanks Dr. Ole Swang for insightful discussions. This research did not receive any specific grant from funding agencies.

DATA AVAILABILITY

No new datasets were generated or analyzed in this theoretical study.

Appendix A: The General Relativistic Hamiltonian as the M-First Core Momentum M_g

The goal of this appendix is to derive the general expression for the relativistic Hamiltonian, H_{GR}, for a particle in a generic gravitational field. Within the Momentum-First (M-First) framework, this quantity, divided by c, is identified with the gravitationally influenced core momentum, $M_g \equiv H_{GR}/c$. This derivation provides the foundation for defining the gravitational modifier Φ_g used throughout this work.

We adopt a metric signature of $(-,+,+,+)$ and keep $\hbar$ and c explicit. The particle's covariant four-momentum is denoted P_α, where $P_0 = -E/c = -M_g$ is the component conjugate to the time coordinate $x^0 = ct$, and P_k (for $k = 1,2,3$) are the canonical spatial momenta conjugate to the spatial coordinates x^k. The particle possesses an intrinsic fermic momentum $p_f = m_0 c$.

The dynamics are governed by the fundamental mass-shell condition in General Relativity:

$$g^{\alpha\beta} P_\alpha P_\beta = -p_f^2. \tag{A1}$$

Expanding this relation using the components of P_α:

$$g^{00} P_0^2 + 2g^{0i} P_0 P_i + g^{ij} P_i P_j = -p_f^2. \tag{A2}$$

(Summation over repeated spatial indices i, j from 1 to 3 is implied). Substituting $P_0 = -M_g$, we obtain a quadratic equation for M_g:

$$(g^{00})M_g^2 - (2g^{0i}P_i)M_g + (g^{ij}P_iP_j + p_f^2) = 0. \tag{A3}$$

This is of the form $ax^2 + bx + c = 0$ with $x = M_g$. We solve for M_g using the quadratic formula, $x = (-b \pm \sqrt{b^2 - 4ac})/(2a)$:

$$M_g = \frac{2g^{0i}P_i \pm \sqrt{(2g^{0i}P_i)^2 - 4g^{00}(g^{ij}P_iP_j + p_f^2)}}{2g^{00}}. \tag{A4}$$

To ensure a positive energy solution ($M_g > 0$) for a particle propagating forward in time, we must select the correct sign. In a physical spacetime, $g^{00} < 0$. Thus, to make M_g positive, the numerator must also be negative. The standard choice that corresponds to positive energy is:

$$M_g = \frac{-g^{0i}P_i + \sqrt{(g^{0i}P_i)^2 - g^{00}(g^{ij}P_iP_j + p_f^2)}}{-g^{00}}. \tag{A5}$$

This general expression for M_g is identified as the M-First gravitationally influenced core momentum. It is valid provided the discriminant is non-negative, corresponding to the particle being on a timelike or null world line. The M-First gravitational modifier is then defined as $\Phi_g \equiv M_g^2 - M_{\text{flat}}^2$.

Appendix B: Quantum Operators and Kinematics in Static Gravitational Fields

The goal of this appendix is to specialize the M-First framework to static gravitational fields. We first derive the expression for the core momentum M_g in this limit, then establish the crucial relationship between canonical and locally measured physical momentum, and finally construct the gravitationally modified quantum directional momentum operators.

1. The Static Field Limit

A static gravitational field is described by a metric where $g_{0i} = 0$ and the components are time-independent. We consider the simplified form:

$$ds^2 = g_{00}(\vec{r})(cdt)^2 + \delta_{ij}dx^i dx^j = -N^2(\vec{r})(cdt)^2 + \delta_{ij}dx^i dx^j, \tag{B1}$$

where $N(\vec{r}) = \sqrt{-g_{00}(\vec{r})}$ is the lapse function. The inverse metric components are $g^{00} = -1/N^2$, $g^{0i} = 0$, and $g^{ij} = \delta^{ij}$.

Substituting these into the general solution for M_g (Eq. (A5) from Appendix A):

$$M_g(\vec{r}, P_k) = \frac{\sqrt{-(-1/N^2)(\delta^{ij}P_iP_j + p_f^2)}}{-(-1/N^2)} = N(\vec{r})\sqrt{p_f^2 + \delta^{ij}P_iP_j}. \tag{B2}$$

Recognizing the flat-spacetime core momentum $M_{\text{flat}}(P_k) = \sqrt{p_f^2 + \delta^{ij}P_iP_j}$, we find:

$$M_g(\vec{r}, P_k) = N(\vec{r})M_{\text{flat}}(P_k). \tag{B3}$$

2. Physical versus Canonical Momentum

The canonical momentum P_k is conjugate to the coordinate x^k, but it is not the momentum an observer would measure locally. To find the locally measured, or physical, momentum $p_{\text{phys},k}$, we use the classical Lagrangian for a particle of mass $m_0 = p_f/c$:

$$L = -m_0c^2\sqrt{N^2(\vec{r}) - \vec{v}^2/c^2}, \quad \text{where } v^k = dx^k/dt. \tag{B4}$$

The canonical momentum is $P_k = \partial L/\partial v^k = m_0 v^k/\sqrt{N^2 - \vec{v}^2/c^2}$.

A local static observer measures time with their proper time clock, $d\tau = N(\vec{r})dt$. The physical velocity they measure is $v_{\mathrm{phys},k} = dx^k/d\tau = v^k/N$. Their locally measured momentum is $p_{\mathrm{phys},k} = m_0 \gamma_{\mathrm{loc}} v_{\mathrm{phys},k}$, where the local Lorentz factor is $\gamma_{\mathrm{loc}} = 1/\sqrt{1 - v_{\mathrm{phys}}^2/c^2} = 1/\sqrt{1 - \vec{v}^2/(N^2c^2)}$. Substituting these relations, we find:

$$p_{\mathrm{phys},k} = \frac{m_0 v^k/N}{\sqrt{1 - \vec{v}^2/(N^2c^2)}} = \frac{m_0 v^k}{N\sqrt{N^2 - \vec{v}^2/c^2}}\sqrt{N^2} = \frac{P_k}{N(\vec{r})}. \tag{B5}$$

Thus, the operator for the physical momentum is $\hat{p}_{\mathrm{phys},k} = \hat{P}_k/N(\hat{r})$.

3. Gravitationally Modified Quantum Operators

The M-First quantum directional momentum operators in gravity are hypothesized to take the general form $\hat{\mathcal{P}}^{(g)}_{k\pm} = \hat{M}_g \pm \frac{1}{2}\hat{p}_{\mathrm{phys},k}$. Using the results from this appendix, we obtain their explicit form for static fields:

$$\hat{\mathcal{P}}^{(g)}_{k\pm}(\hat{r}, \hat{P}) = N(\hat{r})\hat{M}_{\mathrm{flat}}(\hat{P}) \pm \frac{1}{2N(\hat{r})}\hat{P}_k. \tag{B6}$$

This operator structure, with its asymmetric dependence on $N(\hat{r})$, is a key prediction of the M-First framework for static gravity. The commutator of this operator with position, $[\hat{x}_j, \hat{\mathcal{P}}^{(g)}_{k\pm}]$, leads directly to the modified uncertainty relations discussed in the main text.

Appendix C: Derivation of the Unified Momentum Generator

The form of the unified momentum generator $\hat{\mathbf{\Pi}}$ is rigorously derived by applying the Effective Field Theory (EFT) hierarchy for gravitational interactions established in the M-First framework's M-Theory foundation [4]. This appendix details the step-by-step derivation, showing how the generator's form is a necessary consequence of the framework's internal logic.

1. The EFT Hierarchy of Gravitational Interactions

The M-First framework, as derived from the BFSS matrix model, mandates a procedural hierarchy for incorporating gravitational effects. This hierarchy is not a choice, but a consequence of how different interactions emerge from the underlying theory.

1. **Renormalization via Quantum-Sourced Potential**: The static potential (g_{00}) is an emergent interaction generated by integrating out high-energy quantum modes (open strings) between D0-branes. This standard EFT procedure renormalizes the bare parameters of the theory. Specifically, the bare mass m_0 is renormalized to a position-dependent **contextual mass**, $m_c(\hat{\mathbf{x}}) = m_0\sqrt{-g_{00}(\hat{\mathbf{x}})}$. This step defines the low-energy *effective particle*.
2. **Coupling to a Classical Background**: The rotational potential (g_{0i}) represents a classical supergravity background field. The effective particle, defined in the first step, is then coupled to this background.

This formal separation—first defining the effective theory by integrating out quantum fluctuations, then perturbing it with classical fields—is a cornerstone of the M-First approach derived in [4].

2. Deriving the Coupling Form

The coupling of the effective particle to the rotational background is governed by the principles of EFT. All interaction operators must be constructed from the fields and renormalized parameters of the effective theory. The rotational interaction involves coupling to the effective gravito-magnetic vector potential, $\boldsymbol{\mathcal{A}}_g \simeq \mathbf{\Omega} \times \hat{\mathbf{x}}$. The gravitational "charge" for this interaction is mass.

Within the low-energy effective theory, the bare mass m_0 is no longer an available parameter; it has been absorbed into the definition of m_c. The only operative mass parameter available to describe the particle's interactions is the contextual mass $m_c(\hat{\mathbf{x}})$. Therefore, the leading-order minimal coupling term is uniquely determined. The canonical momentum $\hat{\boldsymbol{p}}$ must be shifted by a term proportional to $m_c\boldsymbol{\mathcal{A}}_g$. This yields the generator:

$$\hat{\mathbf{\Pi}} = \hat{\boldsymbol{p}} - C \cdot m_c(\hat{\mathbf{x}})(\mathbf{\Omega} \times \hat{\mathbf{x}}), \tag{C1}$$

where C is a dimensionless coefficient to be fixed by the correspondence principle.

3. Fixing the Coefficient and Final Form

The coefficient C is fixed by requiring the non-relativistic limit of the Hamiltonian to reproduce the classical rotational energy, $H_{\text{class,rot}} = -\mathbf{\Omega} \cdot \mathbf{L}$. As shown in Appendix E, the rotational energy operator derived from $\hat{\mathbf{\Pi}}$ has the classical limit $-C \cdot \mathbf{\Omega} \cdot \mathbf{L}$. Matching the two requires $C = 1$.

The unified momentum generator is thus uniquely determined to be:

$$\hat{\mathbf{\Pi}} = \hat{\boldsymbol{p}} - m_c(\hat{\mathbf{x}})(\mathbf{\Omega} \times \hat{\mathbf{x}}). \tag{C2}$$

This result is not a postulate, but a direct consequence of consistently applying the EFT hierarchy for gravitational interactions that is central to the M-First framework.

Appendix D: Gravitational Enhancement of Quantum Tunneling

The goal of this appendix is to provide a clear derivation for the scaling of the Gamow exponent under gravity within the M-First framework. This connects the formal concept of a gravitationally influenced core momentum to the concrete, testable astrophysical prediction of enhanced pycnonuclear reactions in neutron stars.

1. The Gamow Exponent from WKB Approximation

Quantum tunneling through a potential barrier $V(x)$ is described, to leading order, by the WKB approximation. The transmission probability is $T \approx e^{-G}$, where G is the Gamow factor (often called the Gamow exponent). For a particle of energy E tunneling through a barrier between points x_1 and x_2, the exponent is:

$$G = \frac{2}{\hbar} \int_{x_1}^{x_2} \sqrt{2m(V(x) - E)}\, dx. \tag{D1}$$

For pycnonuclear reactions in stellar interiors, the process involves two nuclei of mass m_r (reduced mass) tunneling through their mutual Coulomb barrier $V(r)$. The reaction rate is extremely sensitive to the value of G.

2. The M-First Contextual Rest Mass

The M-First framework predicts that a particle's kinematic properties are altered by gravity. Specifically, for a particle at rest ($P_k = 0$) in a static gravitational field described by the lapse function $N(\vec{r}) = \sqrt{-g_{00}(\vec{r})}$, its gravitationally influenced rest core momentum is (from Appendix B):

$$M_g(\text{rest}, \vec{r}) = N(\vec{r})p_f = N(\vec{r})m_0 c. \tag{D2}$$

The corresponding rest energy is $E_{\text{rest}} = cM_g(\text{rest}) = N(\vec{r})m_0c^2$. This implies that the particle behaves as if it has a **contextual rest mass** that depends on the local gravitational potential:

$$m_0^{\text{contextual}}(\vec{r}) = \frac{E_{\text{rest}}}{c^2} = N(\vec{r})m_0. \tag{D3}$$

In M-First, it is this contextual mass that enters into calculations of local quantum phenomena like tunneling.

3. Deriving the Scaling of the Gamow Exponent

To find the modified Gamow exponent G_{M1}, we replace the standard rest mass m_0 (or reduced mass m_r) in Eq. (D1) with its contextual counterpart, $m_0^{\text{contextual}}$. Assuming the reduced mass of the two-body system scales similarly:

$$G_{\mathrm{M1}} = \frac{2}{\hbar}\int \sqrt{2m_r^{\text{contextual}}(V-E)}\,dx = \frac{2}{\hbar}\int \sqrt{2(N(\vec{r})m_r)(V-E)}\,dx. \tag{D4}$$

Since the gravitational potential $N(\vec{r})$ varies on a scale much larger than the nuclear tunneling distance, we can treat it as constant over the integral. This allows us to factor it out:

$$G_{\mathrm{M1}} = \sqrt{N(\vec{r})}\left(\frac{2}{\hbar}\int \sqrt{2m_r(V-E)}\,dx\right) = \sqrt{N(\vec{r})}\,G_{\mathrm{Std}}, \tag{D5}$$

where G_{Std} is the standard Gamow exponent calculated without this effect. The M-First scaling factor is therefore $\sqrt{N(\vec{r})}$. Expressed in terms of the metric component g_{00}:

$$\frac{G_{\mathrm{M1}}}{G_{\mathrm{Std}}} = \sqrt{N(\vec{r})} = \left(-g_{00}(\vec{r})\right)^{1/4}. \tag{D6}$$

Since for an attractive gravitational source $-g_{00} < 1$, the exponent is reduced ($G_{\mathrm{M1}} < G_{\mathrm{Std}}$), leading to a potentially dramatic enhancement of the tunneling probability $T \propto \exp(-G)$.

Appendix E: The Foldy-Wouthuysen Picture: Emergent Dynamics and Spin

This appendix analyzes the rich physical content of the unified M-First Dirac Hamiltonian. By employing an exact Foldy-Wouthuysen (FW) transformation, we translate the abstract Hamiltonian into an intuitive, block-diagonal form. This procedure separates particle and antiparticle states, revealing how classical forces and novel spin-gravity couplings emerge from the operator algebra. Throughout, we work to linear order in the rotational velocity $|\mathbf{\Omega} \times \mathbf{x}|/c$, appropriate for planetary and most astrophysical environments.

a. The FW Transformation and Hermiticity. We begin with the unified Hamiltonian from Eq. (3), which is linear in momentum and thus mixes positive- and negative-energy solutions. It can be diagonalized exactly via the Eriksen-Kolsrud transformation,[1] yielding the FW Hamiltonian $\hat{H}_{\mathrm{FW}} = \beta\,\hat{\Lambda}$, where the positive-definite energy operator for a particle is

$$\hat{\Lambda} = \sqrt{m_c^2 c^4 + c^2 \hat{\mathbf{\Pi}}^2}. \tag{E1}$$

To ensure that energy eigenvalues are real, $\hat{\Lambda}$ must be self-adjoint. This requires a specific ordering for the product $\hat{\mathbf{\Pi}}^2$ due to the non-commutation of $m_c(\hat{\boldsymbol{x}})$ and $\hat{\boldsymbol{p}}$. Hermiticity is guaranteed by adopting the symmetric Weyl ordering:

$$\hat{\mathbf{\Pi}}^2 \equiv \hat{\boldsymbol{p}}^2 - \{m_c(\mathbf{\Omega} \times \hat{\boldsymbol{x}}), \hat{\boldsymbol{p}}\} + m_c^2(\mathbf{\Omega} \times \hat{\boldsymbol{x}})^2. \tag{E2}$$

This ordering is sufficient for our linear-in-Ω analysis; resolving higher-order terms would require further specification of the ordering within the final term.

b. Emergent Quantum Forces. The FW picture provides direct access to kinematic operators. The mean velocity is $\hat{\boldsymbol{v}}_{\mathrm{FW}} = c^2\hat{\mathbf{\Pi}}/\hat{\Lambda}$. The acceleration operator, $\hat{\boldsymbol{a}}_{\mathrm{FW}} = d\hat{\boldsymbol{v}}_{\mathrm{FW}}/dt$, reveals how forces emerge. In the semiclassical limit ($\hbar \to 0$):

$$\hat{\boldsymbol{a}}_{\mathrm{FW}} = -\frac{c^4}{2\hat{\Lambda}^2}\{\hat{\mathbf{\Pi}}, \nabla(m_c^2)\} - 2(\mathbf{\Omega} \times \hat{\boldsymbol{v}}_{\mathrm{FW}}). \tag{E3}$$

The first term is the gravitational force, driven by the gradient of the potential. The second is the Coriolis-like force. For a particle in the exterior field of a rotating body like a planet or star, $\mathbf{\Omega}$ is interpreted as the effective Lense-Thirring angular velocity at the particle's location.[2]

---

[1] E. Eriksen and M. Kolsrud, Nuovo Cimento Suppl. **18**, 1 (1960).
[2] See, e.g., C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (W. H. Freeman, 1973), §33.4 for the classical analogue.

c. Gravitational Spin Dynamics and Non-Relativistic Limit. The framework predicts a dynamical coupling between spin and gravity. The time evolution of the helicity operator, $\hat{h} = \hat{\mathbf{\Sigma}} \cdot \hat{\mathbf{\Pi}}/|\hat{\mathbf{\Pi}}|$, is governed by its commutator with $\hat{H}_{\mathrm{FW}}$:

$$\frac{d\hat{h}}{dt} = \underbrace{-\frac{c^2}{\hat{\Lambda}} \frac{\hat{\mathbf{\Sigma}} \cdot (\nabla m_c \times \hat{\mathbf{\Pi}})}{|\hat{\mathbf{\Pi}}|}}_{\text{Gravitational Spin-Hall}} \underbrace{-\frac{c^2 m_c}{\hat{\Lambda}} \frac{\hat{\mathbf{\Sigma}} \cdot (\mathbf{\Omega} \times \hat{\mathbf{\Pi}})}{|\hat{\mathbf{\Pi}}|}}_{\text{Frame-Dragging}} . \tag{E4}$$

To verify consistency with known physics, we check the non-relativistic limit ($|\hat{\mathbf{\Pi}}| \ll m_c c$, $|g_{00} + 1| \ll 1$). Here, the dimensionless prefactor of the frame-dragging term becomes $c^2 m_c/\hat{\Lambda} \to 1$. In this limit, the term correctly reproduces the standard Schiff precession for a gyroscope in a gravito-magnetic field.

d. The Physical Position Operator. In relativistic quantum mechanics, the operator corresponding to a particle's measurable position is not the canonical $\hat{\boldsymbol{x}}$ (which exhibits unphysical *Zitterbewegung*), but the Newton-Wigner (NW) operator, $\hat{\boldsymbol{X}}$.[3] The NW operator represents the center of energy of a wavefunction and is defined by the transformation $\hat{\boldsymbol{X}} = U\mathcal{P}_+\hat{\boldsymbol{x}}\mathcal{P}_+U^\dagger$. A key property is that its components commute, $[\hat{X}_i, \hat{X}_j] = 0$, guaranteeing that a particle's position is simultaneously localizable in all three spatial dimensions.

Appendix F: Relativistic Symmetries and the Gravitationally Localized Casimir

A fundamental test of any relativistic quantum theory is the consistency of its spacetime symmetry generators. This appendix demonstrates that the M-First framework yields a consistent representation of these symmetries, leading to a profound and testable prediction: the localization of a particle's fundamental invariants.

a. Angular Momentum and Rotational Symmetry. We first confirm that the theory respects the spacetime's rotational symmetry. The total angular momentum operator is constructed from the orbital part $\hat{\boldsymbol{L}} = \hat{\boldsymbol{x}} \times \hat{\mathbf{\Pi}}$ and the spin part $\hat{\boldsymbol{S}} = (\hbar/2)\hat{\mathbf{\Sigma}}$. A direct calculation verifies that this total angular momentum, $\hat{\boldsymbol{J}} = \hat{\boldsymbol{L}} + \hat{\boldsymbol{S}}$, is a conserved quantity, as it commutes with the Foldy-Wouthuysen (FW) Hamiltonian derived in Appendix E:

$$[\hat{\boldsymbol{J}}, \hat{H}_{\mathrm{FW}}] = 0. \tag{F1}$$

This confirms that total angular momentum is conserved in the stationary, axisymmetric spacetime, as required.

b. The Pauli-Lubanski Invariant via Rest-Frame Analysis. To find the theory's Casimir invariants, we analyze the Pauli-Lubanski pseudovector, $\hat{W}_\mu$. While a full analysis of the boost commutators reveals a non-trivial deformation of the Poincaré algebra, a more direct and rigorous path to the invariant is achieved by a calculation in the particle's instantaneous physical rest frame.

The components of the Pauli-Lubanski vector are given by the standard operator identities:

$$\hat{W}^0 = \hat{\boldsymbol{J}} \cdot \hat{\mathbf{\Pi}} \tag{F2}$$

$$\hat{\boldsymbol{W}} = \frac{\hat{H}_{\mathrm{FW}}}{c}\hat{\boldsymbol{J}} - c(\hat{\boldsymbol{K}} \times \hat{\mathbf{\Pi}}). \tag{F3}$$

We evaluate these in the rest frame, which is defined by the condition that the mechanical momentum vanishes, $\hat{\mathbf{\Pi}} = 0$.

1. The temporal component immediately becomes zero:

$$\hat{W}^0_{\mathrm{rest}} = \hat{\boldsymbol{J}} \cdot \mathbf{0} = 0. \tag{F4}$$

2. In the rest frame, the Foldy-Wouthuysen Hamiltonian reduces to the contextual rest energy, $\hat{H}_{\mathrm{FW,rest}} = m_c(\hat{\boldsymbol{x}})c^2$. The total angular momentum $\hat{\boldsymbol{J}}$ reduces to the spin $\hat{\boldsymbol{S}}$, as the orbital part $\hat{\boldsymbol{L}}$ vanishes with $\hat{\mathbf{\Pi}}$. The spatial component thus simplifies to:

$$\hat{\boldsymbol{W}}_{\mathrm{rest}} = \frac{m_c(\hat{\boldsymbol{x}})c^2}{c}\hat{\boldsymbol{S}} - c(\hat{\boldsymbol{K}} \times \mathbf{0}) = m_c(\hat{\boldsymbol{x}})c\,\hat{\boldsymbol{S}}. \tag{F5}$$

[3] T. D. Newton and E. P. Wigner, Rev. Mod. Phys. **21**, 400 (1949).

With the components of $\hat{W}^{\mu}_{\text{rest}} = (0, m_c c\hat{\boldsymbol{S}})$ rigorously determined, we compute the Casimir invariant $\hat{W}^2 = g_{\mu\nu}\hat{W}^{\mu}\hat{W}^{\nu} = -(\hat{W}^0)^2 + \hat{\boldsymbol{W}} \cdot \hat{\boldsymbol{W}}$:

$$\hat{W}^2 = -(0)^2 + (m_c c\hat{\boldsymbol{S}}) \cdot (m_c c\hat{\boldsymbol{S}}) = m_c^2 c^2 \hat{\boldsymbol{S}}^2. \tag{F6}$$

Recalling that for a spin-s particle $\hat{\boldsymbol{S}}^2 = \hbar^2 s(s+1)$ and using the definition of contextual mass, we arrive at the final, rigorously derived result:

$$\hat{W}^2 = m_0^2 c^2 (-g_{00}(\hat{\boldsymbol{x}})) s(s+1)\hbar^2. \tag{F7}$$

This result demonstrates a remarkable consequence of the M-First framework: the Poincaré invariant that defines a particle's spin magnitude is no longer a universal constant. It becomes a local field, scaled by the gravitational potential. This *localization of a Casimir invariant* is a unique and fundamental prediction, implying that gravity is woven into the very fabric of a particle's identity.

Appendix G: Interactions, Currents, and the Effective U(1) Vertex

The form of the conserved electromagnetic current is derived in this appendix. It is demonstrated that while the tree-level interaction yields the standard Dirac current, the quantum effects that generate the gravitational modifier Φ_g must also, for consistency, renormalize the electromagnetic vertex. This leads to a gravitationally screened effective current, a key prediction of the M-First theory.

1. Tree-Level Current from the M-First Action

The construction begins with the M-First action for a Dirac fermion coupled to a U(1) gauge field A_μ. The framework's core principle is the replacement of the canonical momentum operator with the unified momentum generator $\hat{\boldsymbol{\Pi}}$ and the rest mass with the contextual mass m_c. The Lagrangian that yields the M-First Dirac equation (Eq. (3)) is:

$$\mathcal{L}_{\text{M1}} = \bar{\psi}\left(i\hbar c\gamma^0\partial_0 + c\boldsymbol{\gamma} \cdot \hat{\boldsymbol{\Pi}} - m_c(\hat{\boldsymbol{x}})c^2\right)\psi. \tag{G1}$$

Electromagnetic interactions are introduced via minimal coupling, which replaces the canonical four-momentum operator $\hat{p}_\mu$ with $\hat{p}_\mu - qA_\mu$. This transforms the generator to $\hat{\boldsymbol{\Pi}} \to \hat{\boldsymbol{p}} - q\boldsymbol{A} - m_c\boldsymbol{\mathcal{A}}_g$, and the gauged Lagrangian becomes:

$$\mathcal{L}_{\text{gauged}} = \mathcal{L}_{\text{M1}} - qc\bar{\psi}\gamma^\mu A_\mu\psi. \tag{G2}$$

The conserved Noether current associated with the global U(1) symmetry is the standard Dirac probability current:

$$\hat{J}^\mu_{\text{tree}} = qc\bar{\psi}\gamma^\mu\psi. \tag{G3}$$

This result shows that at the classical or tree-level, the theory's current operator is identical to that of standard QED. Novel effects must therefore arise from the quantum structure of the theory.

2. The Effective Current from the Renormalized Vertex

The M-First framework is an effective field theory derived from the BFSS matrix model of M-Theory [4]. In this context, gravity is an emergent force arising from quantum loop corrections. The gravitational modifier, Φ_g, which encapsulates gravity's effect on the mass-shell condition ($\hat{M}_g^2 = \hat{M}^2_{\text{flat}} + \Phi_g$), is the result of these quantum effects.

For the theory to be consistent, the quantum loops that renormalize the particle's mass must also renormalize its interactions. This implies that the electromagnetic vertex, $e\gamma^\mu$ at tree-level, must be "dressed" by the same quantum gravitational effects that generate Φ_g. The effective vertex, Γ^μ, thus acquires a gravitationally dependent correction.

The underlying $\mathcal{N} = 16$ supersymmetry of the parent M-Theory constrains the form of such corrections [8]. The correction must be constructed from the same operators that appear in the effective action. As Φ_g is the leading-order operatorial measure of the gravitational interaction, the leading-order correction to the vertex must be a function of

Φ_g. The simplest non-trivial form for the dressed vertex consistent with Lorentz invariance is a scalar multiplicative correction, proposed as:

$$\Gamma^\mu(\hat{x},\hat{p}) = \gamma^\mu \left(1 + \kappa \frac{\hat{\Phi}_g(\hat{x},\hat{p})}{p_f^2}\right), \tag{G4}$$

where $p_f = m_0 c$ is the intrinsic fermic momentum, and κ is a dimensionless Wilson coefficient. In the absence of a complete one-loop calculation, the principle of naturalness is adopted. This principle suggests that, unless suppressed by a symmetry, such leading-order coefficients should be of order unity. The coefficient is therefore set to $\kappa = 1$ as the simplest assumption, defining a sharp, falsifiable prediction for the leading-order theory.

The **effective electromagnetic current operator**, $\hat{J}^\mu_{\text{eff}}$, is then constructed using this dressed vertex:

$$\hat{J}^\mu_{\text{eff}} = qc\bar{\psi}\Gamma^\mu\psi = qc\bar{\psi}\gamma^\mu \left(1 + \frac{\hat{\Phi}_g}{m_0^2 c^2}\right)\psi. \tag{G5}$$

This is the central result, predicting that the physical charge current is screened or anti-screened by the local gravitational field. For a particle at rest in a static field, where $\Phi_g = (-g_{00} - 1)m_0^2 c^2$, the charge density becomes:

$$\hat{\rho}_{\text{eff}} = q\psi^\dagger \left(1 + (-g_{00} - 1)\right)\psi = (-g_{00})q\psi^\dagger\psi = (-g_{00})\hat{\rho}_{\text{tree}}. \tag{G6}$$

In a weak field where $-g_{00} < 1$, the charge is screened. This non-universal coupling is a direct, testable prediction distinguishing M-First from standard minimally coupled theories. A direct calculation of κ from the one-loop effective action of the BFSS matrix model is a crucial next step to either confirm this leading-order prediction or provide a precise, non-trivial value.

---

[1] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, 1982).

[2] R. M. Wald, *Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics* (University of Chicago Press, 1994).

[3] A. Klaveness, Momentum is all you need, Authorea Preprint (2025).

[4] A. Klaveness, The momentum-first framework as a direct consequence of the bfss matrix model, Authorea preprint (2025).

[5] A. Klaveness, A first principles derivation of the earth fly-by velocity shift, Authorea preprint (2025).

[6] E. F. Brown and A. Cumming, Astrophys. J. **698**, 1020 (2009), arXiv:0901.1092.

[7] J. D. Anderson, J. K. Campbell, J. E. Ekelund, J. Ellis, and J. F. Jordan, Phys. Rev. Lett. **100**, 091102 (2008).

[8] S. Paban, S. Sethi, and M. Stern, Nucl. Phys. B **534**, 137 (1998), hep-th/9805018.