Exercise 1 - Trajectory of rotating ball

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1 Introduction

In this paper we aim at giving a comprehensive numerical analysis of a typical case study, the trajectory of a rotating ball under the influence of the Magnus effect. This analysis will be done using three related types of algorithms: Euler explicit, implicit and semi-implicit.

The algorithms used will be implemented through a C++ simulation and the obtained data analysed using classic python libraries. Aaaaaa yes no gravity aaaa yes with gravity and traine aerodynamique yes yes baguette. Some analytical results will also be proven in order to verify the soundness of our simulation by comparing them to computational results.

2 Analytical results

For the purpose of this study it is first needed to prove some analytical results that will later be achieved through numerical simulations.

We are considering a sphere (tennis ball) of mass m and radius R. It is rotating according to $\vec{\omega} = \omega \vec{e_z}$. It is moving in the gravity field $\vec{g} = -g\vec{e_y}$ and inside a fluid of density ρ which applies a force due to the Magnus effect:

$$\vec{F_p} = \mu R^3 \rho \vec{\omega} \times \vec{v} \tag{1}$$

We want to determine the movement of the ball knowing the initial velocity $\vec{v_0}$ and rotation $\vec{\omega}$. In an effort to simplify the problem we assume that the rotation is constant and we consider trajectories occuring only on the (x, y) plane.

2.1 System of differential equations

Question 1.1-(a)

Let us take the vector: $\mathbf{y} = (x, y, v_x, v_y)$

We are searching for f such that:

$$\frac{\mathrm{d}\mathbf{y}}{\mathrm{d}t} = \begin{pmatrix} v_x \\ v_y \\ a_x \\ a_y \end{pmatrix} = f(\mathbf{y}) \tag{2}$$

We know that $m\vec{a} = \vec{F_p} + m\vec{g}$ and from Equation 1 we have:

$$\vec{F_p} = \mu R^3 \rho \omega \begin{pmatrix} -v_y \\ v_x \\ 0 \end{pmatrix}$$

We get from this and $g_z = 0$ that $a_z = 0$ so the trajectory will indeed stay inside the (x, y) plane. Most importantly we also get:

$$\begin{pmatrix} a_x \\ a_y \end{pmatrix} = \begin{pmatrix} -\frac{\mu R^3 \rho \omega}{m} v_y \\ \frac{\mu R^3 \rho \omega}{m} v_x - g \end{pmatrix}$$

Which allows us to rewrite Equation 2 as:

$$\frac{\mathrm{d}\mathbf{y}}{\mathrm{d}t} = f(\mathbf{y}) = \begin{pmatrix} v_x \\ v_y \\ -\frac{\mu R^3 \rho \omega}{m} v_y \\ \frac{\mu R^3 \rho \omega}{m} v_x - g \end{pmatrix} \tag{3}$$

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2.2 Mechanical energy

Question 1.1-(b)

We are searching for the total mechanical energy of the system. We know that for a rigid body such as the tennis ball considered in this problem the total mechanical energy is the sum of: the translational kinetic energy, the rotational kinetic energy and the potential so:

$$\mathrm{E_{tot}} = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 + m g y$$

We get the moment of inertia of a sphere for any rotation around an axis going through the center: $I = \frac{2}{5} m R^2$ [1].

Which yields the final formula for the energy:

$$\mathrm{E_{tot}} = \frac{1}{2}mv^2 + \frac{1}{5}mR^2\omega^2 + mgy \tag{4}$$

We now want to know if this energy is conserved. We know that the gravitational force from the $\vec{g}$ field is conservative. We now consider the force $\vec{F_p}$. It is strictly orthogonal to the velocity at any point from EQUATION 1 which means that $\vec{F_p}$ does not produce any work throughout the movement. We can conclude that we have no non-conservative forces doing work so the total mechanical energy must be conserved. #

2.3 Zero-gravity situation

Question 1.1-(c)

2.4 Gravity with no initial speed situation

Question 1.1-(d)

3 Simulations

4 Analysis

5 Conclusion

References

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A Appendix