Sampling flares

1 Flare energies

1.1 M & K stars

The general form of the flare frequency distribution is given by

$$\log_{10} \nu(E) = \alpha \log_{10} E + \beta \tag{1}$$

where $\nu(E)$ is the number of flares per day with at least an energy E, α and β are constants and E is the energy of the flare. This form is sort of the opposite of a CDF, and can be referred to as a top-tail distribution. In order to sample from this distribution we can use the inverse transform method. Assuming the distribution is defined between $E_{\min}$ and $E_{\max}$, the total number of flares per day (for normalisation) is given by

$$N_{\text{tot}} = \nu(E_{\min}) - \nu(E_{\max}). \tag{2}$$

Then we can use this to get the CDF pretty simply as just

$$F(E) = 1 - \frac{\nu(E)}{N_{\text{tot}}}. \tag{3}$$

Then we need to convert this to the inverse CDF:

$$E = 1 - \frac{\nu(F^{-1}(E))}{N_{\text{tot}}} \tag{4}$$

$$(1-E)N_{\text{tot}} = \nu(F^{-1}(E)) \tag{5}$$

$$(1-E)N_{\text{tot}} = 10^{\alpha \log_{10} F^{-1}(E) + \beta} \tag{6}$$

$$(1-E)N_{\text{tot}} = 10^{\beta}[F^{-1}(E)]^{\alpha} \tag{7}$$

$$\boxed{F^{-1}(E) = \left(\frac{(1-E)N_{\text{tot}}}{10^{\beta}}\right)^{1/\alpha}} \tag{8}$$

Nice! For sampling we just plug in a random uniform variable into this to get flare energies.

1.2 G stars

The form we use for G stars is similar, but needs to be converted from years to days and multiplied by the energy bin widths.

$$\nu(E) = \frac{10^{\beta}}{365} E^{1+\alpha} \tag{9}$$

This gives the same N_{tot} and $F(E)$ as before, but the inverse CDF is slightly different:

$$E = 1 - \frac{\nu(F^{-1}(E))}{N_{\text{tot}}} \tag{10}$$

$$(1-E)N_{\text{tot}} = \nu(F^{-1}(E)) \tag{11}$$

$$(1-E)N_{\text{tot}} = \frac{10^{\beta}}{365}[F^{-1}(E)]^{1+\alpha} \tag{12}$$

$$\boxed{F^{-1}(E) = \left(\frac{365(1-E)N_{\text{tot}}}{10^{\beta}}\right)^{1/(1+\alpha)}} \tag{13}$$

2 Amplitudes

We use the distributions from the Superflares paper to get a relation between the flare amplitude and energy for different stellar classes. This relation gives an amplitude A for a flare of energy E, with some additional scatter such that it is not a 1-to-1 relation.

3 FWHM

This one is simple, applying Lupita's flare model we know that

$$\text{FWHM} = E/2.0487 * A \tag{14}$$

4 Summary

So in summary we can sample a flare energy, use that to get an amplitude, and then use the energy and amplitude to get the FWHM. This is all we need to generate a flare light curve with Lupita's flare model and inject it into the light curves.