ASTR 531 - Stellar Interiors and Evolution

Problem Set 3 Tom Wagg

May 7, 2022

12.2 - Early Radii and Timescales

Part a - Radii Estimations

Let's use a couple of different relations from the textbook to get the radii at different times. A protostar becomes ionised and stars the Hayashi concentration phase when it's radius is on the order of (Eq. 12.13)

$$R_{\rm Hayashi,start} \approx 100 \,\mathrm{R}_{\odot} \left(\frac{M}{\mathrm{M}_{\odot}}\right)$$
 (1)

We find the radius of the protostar once the Hayashi concentration phase comes to an end is approximately a factor of 50 lower (based on assumptions of the temperature and opacity) such that (page 12-8)

$$R_{\rm Hayashi,end} \approx 2 \,\mathrm{R}_{\odot} \left(\frac{M}{\mathrm{M}_{\odot}}\right)$$
 (2)

The radius at the start of the PMS phase will be the same as the end of the Hayashi concentration phase.

$$R_{\text{PMS,start}} = R_{\text{Hayashi,end}}$$
 (3)

Finally, the radius at the end of the PMS phase is the same as the radius at ZAMS and so we can write that (Eq. 12.16)

$$R_{\rm PMS,end} = R_{\rm ZAMS} = R_{\odot} \left(\frac{M}{\rm M_{\odot}}\right)^{0.7}$$
 (4)

So now we can plug in numbers for the different masses of stars that we considered

$M/{ m M}_{\odot}$	$R_{ m Hayashi, start}/{ m R}_{\odot}$	$R_{ m Hayashi,end}/ m R_{\odot}$	$R_{\mathrm{PMS,start}}/\mathrm{R}_{\odot}$	$R_{ m PMS,end}/ m R_{\odot}$
0.3	30	0.6	0.6	0.43
3	300	6	6	2.16
30	3000	60	60	10.8

Part b - Timescale estimations

The duration of the Hayashi concentration phase is given in Eq. 12.15 but we also showed that the timescale scales as 1/M such that

$$\tau_{\rm Hayashi} \approx 10^6 \,\mathrm{yr} \bigg(\frac{M_{\odot}}{M} \bigg)$$
(5)

The duration of the PMS phase is given by Eq. 12.17 so we have that

$$\tau_{\rm PMS} \approx 6 \times 10^7 \,\mathrm{yr} \left(\frac{M}{\mathrm{M}_{\odot}}\right)^{-2.5}$$
(6)

So now we can plug in numbers for the different masses of stars that we considered

$M/{ m M}_{\odot}$	$\tau_{ m Hayashi}/ m yr$	$ au_{\mathrm{PMS}}/\mathrm{yr}$
0.3	3.33×10^{6}	1.22×10^{9}
3	3.33×10^{5}	3.85×10^{6}
30	3.33×10^{4}	1.22×10^{4}

15.4 - Metallicity and Mass Loss Rates

TODO: Need to check code/method with Emily

16.1 - RGB Radii

From inspection of Figure 16.1 we can find values for L and $T_{\rm eff}$ at the start and end of the RGB phase. This phase starts at C and ends at F. We can then use the fact that

$$R = \sqrt{\frac{L}{4\pi\sigma T_{\text{eff}}^4}} \tag{7}$$

to get the radii. Since I'm in a mood for tables today, let's make another!

Stage	$\log(L/{ m L}_{\odot})$	$\log(T_{\rm eff}/{ m K})$	$R/{ m R}_{\odot}$
C (Start of RGB)	0.4	3.7	2.1
F (End of RGB)	3.4	3.48	183.1

17.1 - Helium Flash Duration