Astro 531; Problem Set 1

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2.1 - Stellar Properties

Part a - Radius

$$L = 4\pi R^2 \sigma T_{\text{eff}}^4 \tag{1}$$

$$R = \sqrt{\frac{L}{4\pi\sigma T_{\text{eff}}^4}} \tag{2}$$

$$\boxed{R = 2570\,\mathrm{R}_\odot} \tag{3}$$

Part b - Density

If we assume that the star is a constant density throughout then we can use the definition of density and the radius we calculated above.

$$\rho = \frac{M}{\frac{4}{3}\pi R^3} \tag{4}$$

$$\boxed{\rho = 1.66 \times 10^{-6}\,\mathrm{kg\,m^{-3}}} \tag{5}$$

Part c - Escape velocity

$$\boxed{v_{\text{esc}} = \sqrt{\frac{2GM}{R}} = 54500\,\mathrm{m\,s^{-1}}} \tag{6}$$

Part d - Comparisons to Sun

This star is clearly much larger than the sun (by a factor of 2570) but is **much** less dense (since the mean density of the sun is approximately $1400\,\mathrm{kg\,m^{-3}}$, so the density is 10^9 times lower than the sun). Finally, the escape velocity is lower than the sun (around 0.088 times lower).

3.5 - Kepler's 3rd Law from Virial Theorem

The virial theorem gives the following relations between energies

$$E_{\text{kin}} = -\frac{1}{2}E_{\text{pot}} \tag{7}$$

For the orbits of planets around the sun, their potential energy will come from the gravitational potential and so

$$E_{\text{pot}} = -\frac{Gm_1m_2}{a}, \tag{8}$$

and the kinetic energy is simply

$$E_{\text{kin}} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2. \tag{9}$$

For the next part, it will be useful to define the distance from each body to the centre of mass, a_i, in terms of the masses

$$a_i = \frac{a \cdot m_{1-i}}{m_1 + m_2} \tag{10}$$

and additionally, it's useful to write the velocity in terms of the period and masses

$$v_i = \frac{2\pi a_i}{P} \tag{11}$$

$$v_i = \frac{2\pi}{P}\frac{a \cdot m_{1-i}}{m_1 + m_2} \tag{12}$$

Now we can put this all together in the expression for the virial theorem.

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2} \cdot \frac{Gm_1m_2}{a}, \tag{13}$$

$$\frac{1}{2}m_1\left(\frac{2\pi}{P}\frac{a \cdot m_2}{m_1 + m_2}\right)^2 + \frac{1}{2}m_2\left(\frac{2\pi}{P}\frac{a \cdot m_1}{m_1 + m_2}\right)^2 = \frac{Gm_1m_2}{2a} \tag{14}$$

$$\frac{2\pi^2a^2}{P^2M^2}\left(m_1m_2^2 + m_2m_1^2\right) = \frac{Gm_1m_2}{2a} \tag{15}$$

$$\frac{a^3}{P^2} = \frac{G}{4\pi^2}\frac{M^2m_1m_2}{(m_1m_2^2 + m_2m_1^2)} \tag{16}$$

$$\frac{a^3}{P^2} = \frac{G}{4\pi^2}\frac{M^2}{(m_1 + m_2)} \tag{17}$$

$$\boxed{\frac{a^3}{P^2} = \frac{GM}{4\pi^2}} \tag{18}$$