ASTR 531 - Stellar Interiors and Evolution

Problem Set 3

Tom Wagg

May 7, 2022

12.2 - Early Radii and Timescales

Part a - Radii Estimations

Let's use a couple of different relations from the textbook to get the radii at different times. A protostar becomes ionised and stars the Hayashi concentration phase when it's radius is on the order of (Eq. 12.13)

$$R_{\rm Hayashi,start} \approx 100\,\mathrm{R}_\odot \left(\frac{M}{\mathrm{M}_\odot}\right) \tag{1}$$

We find the radius of the protostar once the Hayashi concentration phase comes to an end is approximately a factor of 50 lower (based on assumptions of the temperature and opacity) such that (page 12-8)

$$R_{\rm Hayashi,end} \approx 2\,\mathrm{R}_\odot \left(\frac{M}{\mathrm{M}_\odot}\right) \tag{2}$$

The radius at the start of the PMS phase will be the same as the end of the Hayashi concentration phase.

$$R_{\rm PMS,start} = R_{\rm Hayashi,end} \tag{3}$$

Finally, the radius at the end of the PMS phase is the same as the radius at ZAMS and so we can write that (Eq. 12.16)

$$R_{\rm PMS,end} = R_{\rm ZAMS} = \mathrm{R}_\odot \left(\frac{M}{\mathrm{M}_\odot}\right)^{0.7} \tag{4}$$

So now we can plug in numbers for the different masses of stars that we considered

$M/\,\mathrm{M}_\odot$	$R_{\rm Hayashi,start}/\,\mathrm{R}_\odot$	$R_{\rm Hayashi,end}/\,\mathrm{R}_\odot$	$R_{\rm PMS,start}/\,\mathrm{R}_\odot$	$R_{\rm PMS,end}/\,\mathrm{R}_\odot$
0.3	30	0.6	0.6	0.43
3	300	6	6	2.16
30	3000	60	60	10.8

Part b - Timescale estimations

The duration of the Hayashi concentration phase is given in Eq. 12.15 but we also showed that the timescale scales as $1/M$ such that

$$\tau_{\rm Hayashi} \approx 10^6\,\mathrm{yr}\left(\frac{M_\odot}{M}\right) \tag{5}$$

The duration of the PMS phase is given by Eq. 12.17 so we have that

$$\tau_{\rm PMS} \approx 6\times 10^7\,\mathrm{yr}\left(\frac{M}{\mathrm{M}_\odot}\right)^{-2.5} \tag{6}$$

So now we can plug in numbers for the different masses of stars that we considered

$M/\,\mathrm{M}_\odot$	$\tau_{\mathrm{Hayashi}}/\,\mathrm{yr}$	$\tau_{\mathrm{PMS}}/\,\mathrm{yr}$
0.3	3.33×10^6	1.22×10^9
3	3.33×10^5	3.85×10^6
30	3.33×10^4	1.22×10^4

15.4 - Metallicity and Mass Loss Rates

TODO: Need to check code/method with Emily

16.1 - RGB Radii

From inspection of Figure 16.1 we can find values for L and T_{eff} at the start and end of the RGB phase. This phase starts at C and ends at F. We can then use the fact that

$$R = \sqrt{\frac{L}{4\pi\sigma T_{\mathrm{eff}}^4}} \tag{7}$$

to get the radii. Since I'm in a mood for tables today, let's make another!

Stage	$\log(L/\,\mathrm{L}_\odot)$	$\log(T_{\mathrm{eff}}/\,\mathrm{K})$	$R/\,\mathrm{R}_\odot$
C (Start of RGB)	0.4	3.7	2.1
F (End of RGB)	3.4	3.48	183.1

So the radius is increasing by nearly two orders of magnitude!

TODO: Ask Emily about metallicity

17.1 - Helium Flash Duration

First we need to calculate the potential energy of a uniform sphere. We know that gravitational potential is given by

$$\Phi = -\frac{GM}{r} \tag{8}$$

So we just need to integrate this over a series of teeny tiny shells over the core of the star. Note we can assume a uniform density so $\rho(r) = \rho$. Let's do it!

$$\begin{aligned}
U &= -\int_0^R \frac{GM(r)}{r}\rho(r)\,\mathrm{d}^3 r &(9)\\
&= -\int_0^R \frac{G(\rho(r)\frac{4}{3}\pi r^3)}{r}\rho(r) 4\pi r^2\,\mathrm{d}r &(10)\\
&= -\frac{16G\pi^2\rho^2}{3}\int_0^R r^4\,\mathrm{d}r &(11)\\
U &= -\frac{16G\pi^2\rho^2 R^5}{15} &(12)
\end{aligned}$$

We can rearrange this to get rid of the radius and just write it as a function of density and mass.

$$\begin{aligned}
R &= \left(\frac{3M}{4\pi\rho}\right)^{1/3} &(13)\\
U &= -\frac{16G\pi^2\rho^2}{15}\left(\frac{3M}{4\pi\rho}\right)^{5/3} &(14)\\
U &= -\frac{G(36\pi M^5\rho)^{1/3}}{5} &(15)
\end{aligned}$$

Nice, what a...clean expression?[1]. Now let's use this to find the difference in potential energy for a $0.5\,\mathrm{M}_\odot$ core expanding from a density of $\rho = 10^6\,\mathrm{g\,cm^{-3}}$ to $\rho = 10^4\,\mathrm{g\,cm^{-3}}$.

$$\Delta U = -\frac{G(36\pi(0.5\,\mathrm{M}_\odot)^5)^{1/3}}{5}\left[(10^4\,\mathrm{g\,cm^{-3}})^{1/3} - (10^6\,\mathrm{g\,cm^{-3}})^{1/3}\right] = 5\times 10^{42}\,\mathrm{J} \tag{16}$$

Now we can calculate the duration of the Helium flash given the energy production and efficiency

$$\tau_{\text{flash}} = \frac{\Delta U}{L\cdot\phi} = \frac{5\times 10^{49}\,\mathrm{J}}{10^{10}\,\mathrm{L}_\odot\cdot 0.2} = 2.5\times 10^{40}\,\mathrm{erg\,L_\odot^{-1}} \tag{17}$$

Now because I'm an "astronomer" I can 100% (totally) understand ergs and $L_\odot$ intuitively, but just in case some readers don't live and breathe cgs in the same way this translates to

$$\boxed{\tau_{\text{flash}} = 0.21\,\mathrm{yr} \approx 76\,\mathrm{days}} \tag{18}$$

Quite the 'flash' indeed!

[1] To be fair, it looks much better if we did it in terms of M and R: $U = -\frac{3}{5}GM^2/R$