

ASTR 531 - Stellar Interiors and Evolution

Problem Set 4

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20.2 - White Dwarf Luminosity

Part a - Difference in luminosity

The luminosity of a white dwarf in the slow cooling phase is given by Eq. 20.10 in the textbook

$$\frac{L}{L_{\odot}} \approx 5.2 \times 10^{10} \frac{M}{M_{\odot}} \mu_{\text{ion}}^{-7/5} \left(\frac{t}{\text{yr}} \right)^{-7.5} \quad (1)$$

Since we are comparing white dwarfs with the same cooling age, the only relevant factors are the mass and μ_{ion} when comparing a H-rich WD to He-rich and C-rich.

The values of μ_{ion} for these WDs are 1, 4, 12 respectively. This means that the relative luminosity of the WDs is

$$\frac{L}{L_{\text{H-rich}}} = 1 : 0.14 : 0.03 \quad (2)$$

respectively. This shows that H-rich WDs are the brightest for a given cooling age, following by He-rich and then C-rich WDs.

TODO: Can we assume that the mass is constant?

Part b - Reason for differences

TODO: Unsure, maybe larger ions make cooling happen faster? Why?

23.2 - Central Temperature-Density Gradient

The evolution of a star in the T_c - ρ_c diagram becomes less steep at late evolution phases. This means that for a given increase in density, the increase in temperature is not as strong.

TODO: I think this is probably something to do with the mass defect being lower and so the energy/temperature production is lower?

25.1 - Non-spherical mass loss in rotating stars

Part a - Mass loss latitude trends

The star in Figure 25.3 is very massive, for these stars the winds are driven mainly by radiation pressure. The Von Zeipel effect means that the effective temperature of the pole is higher than at the equator. This means that for most cases (the left panel) mass loss is much higher at the poles than at the equator.

In certain cases (the right panel), the decreased T_{eff} at the equator actually leads to a higher mass loss rate due to the formation of a bi-stability disk. This disk has higher mass loss because its lower effective temperature puts it on the other side of a bi-stability jump from the pole and so its winds are driven by different ions that more effectively remove mass from the star.

Part b - Terminal wind velocity latitude trends

In rapidly rotating stars, the effective surface gravity at a given radius becomes a function of latitude, with stronger gravity at the poles than the equator. The escape velocity is a direct function of this surface gravity. We have previously shown that the terminal wind velocity is approximately equal to the escape velocity (see Section 15.2.2 of the textbook). For these reasons the terminal wind velocity is higher at the poles than at the equator.

Part c - Density of bi-stability disk

A rotation-induced bi-stability disk must by definition have a lower effective temperature than the wind in the poles and it also has a lower terminal wind velocity. The lower temperature naturally leads to higher densities and the lower velocity allows more gas to remain in the disk.

TODO: check this part, particularly the velocity bit

27.1 - Limit for NS vs. BH for rotating stars
