ASTR 531 - Stellar Interiors and Evolution

Problem Set 4 Tom Wagg

May 16, 2022

20.2 - White Dwarf Luminosity

Part a - Difference in luminosity

The luminosity of a white dwarf in the slow cooling phase is given by Eq. 20.10 in the textbook

$$\frac{L}{\mathrm{L}_\odot} \approx 5.2 \times 10^{10} \frac{M}{\mathrm{M}_\odot} \mu_{\mathrm{ion}}^{-7/5} \left(\frac{t}{\mathrm{yr}}\right)^{-7.5} \tag{1}$$

Since we are comparing white dwarfs with the same cooling age, the only relevant factors are the mass and μ_{ion} when comparing a H-rich WD to He-rich and C-rich.

The values of μ_{ion} for these WDs are $1, 4, 12$ respectively. This means that the relative luminosity of the WDs is

$$\frac{L}{L_{\mathrm{H-rich}}} = 1 : 0.14 : 0.03 \tag{2}$$

respectively. This shows that H-rich WDs are the brightest for a given cooling age, following my He-rich and then C-rich WDs.

Part b - Reason for differences

The cooling of a WD is due to the loss of the thermal energy of the ions. The specific heat of a substance gives a measure for how much energy is required to change its temperature by a given amount. Therefore, a smaller specific heat means that cooling an object is easier and will happen more quickly. The specific heat of the ion is given by

$$c_V = \frac{3k}{2m_{\mathrm{ion}}} \tag{3}$$

This means that for helium and carbon, the increased m_{ion} results in a smaller specific heat and thus faster cooling rate.

23.2 - Central Temperature-Density Gradient

For most of its life the star follows

$$T_c \sim \rho_c^{1/3}, \tag{4}$$

however this trend becomes less steep over time. This is because the full expression is actually

$$T_c \sim M_c^{2/3} \rho_c^{1/3}, \tag{5}$$

For the later fusion phases, the core mass becomes much lower (see Eq. 14.10) and so we can't assume it to be constant. Therefore, we see that for late fusion phases the increase in temperature for a given density is lower, thus the evolution in the plot is less steep.

25.1 - Non-spherical mass loss in rotating stars

Part a - Mass loss latitude trends

The star in Figure 25.3 is very massive, for these stars the winds are driven mainly by radiation pressure. The Von Zeipel effect means that the effective temperature of the pole is higher than at the equator. This means that for most cases (the left panel) mass loss is much higher at the poles than at the equator.

In certain cases (the right panel), the decreased T_{eff} at the equator actually leads to a higher mass loss rate due to the formation of a bi-stability disk. This disk has higher mass loss because its lower effective temperature puts it on the other side of a bi-stability jump from the pole and so its winds are driven by different ions that more effectively remove mass from the star.

Part b - Terminal wind velocity latitude trends

In rapidly rotating stars, the effective surface gravity at a given radius becomes a function of latitude, with stronger gravity at the poles than the equation. The escape velocity is a direct function of this surface gravity. We have previously shown that the terminal wind velocity is approximately equal to the escape velocity (see Section 15.2.2 of the textbook). For these reasons the terminal wind velocity is higher at the poles than at the equator.

Part c - Density of bi-stability disk

A rotation-induced bi-stability disk must by definition have a lower effective temperature than the wind in the poles and it also has a lower terminal wind velocity. The lower temperature naturally leads to higher densities and the lower velocity allows more gas to remain in the disk.

TODO: check this part, particularly the velocity bit

27.1 - Limit for NS vs. BH for rotating stars

The limit of stars with initial masses resulting in NSs or BHs could change due to rotation in the following ways.

Higher mass limit: The mass needed to create a BH would need to increase due to the increased mass loss due to rotation. This means that less mass is available to go into the creation of a remnant and so stars that would have resulted in BHs instead result in NSs.

Lower mass limit: However in addition to the overall mass loss, the mass of the helium core instead increases for rapidly-rotating stars due to the induced mixing. A larger core mass can also lead to the larger remnant mass and thus more easily create a BH with a given initial mass.

Overall, I think that…

TODO: Check with Emily what's more important