ASTR 541 - Interstellar Medium

Problem Set 2

Tom Wagg

October 10, 2022

1. Maxwellians

The minimum velocity needed to ionise a hydrogen atom can be calculated by setting the kinetic energy equal to the ionisation energy

$$\frac{1}{2}m_e v_{\text{ionise}}^2 = E_{\text{ionise}} \tag{1}$$

$$v_{\text{ionise}} = \sqrt{\frac{2E_{\text{ionise}}}{m_e}} \tag{2}$$

$$\boxed{v_{\text{ionise}} = 2187\,\text{km}\,\text{s}^{-1}} \tag{3}$$

We now need to integrate the Maxwellian (yay?). Now I don't intend to do that analytically because I am not sadistic and therefore will be numerically integrating it will `scipy`'s quadrature function. Additionally, it doesn't play nicely with infinity so we're going to be sneaky and flip it around

$$f_{\text{ionise}} = \int_{v_{\text{ionise}}}^{\infty} 4\pi \left(\frac{m_e}{2\pi k_B T}\right)^{\frac{3}{2}} \exp\left(-\frac{mv^2}{2k_B T}\right) v^2\,\text{d}v \tag{4}$$

$$f_{\text{ionise}} = 1 - \int_{0}^{v_{\text{ionise}}} 4\pi \left(\frac{m_e}{2\pi k_B T}\right)^{\frac{3}{2}} \exp\left(-\frac{mv^2}{2k_B T}\right) v^2\,\text{d}v \tag{5}$$

Now we can evaluate this for both temperatures given

$$\boxed{f_{\text{ionise,HII}} = 6.5 \times 10^{-7}, \quad f_{\text{ionise,halo}} = 0.96} \tag{6}$$

From this we can conclude that collisional ionisation is extremely unlikely to occur in an HII region, but it will be dominant for hot halo gas.

2. The Dust Collector
