ASTR 558

Exoplanets, Spring 2022 -Prof. Agol

Problem Set 1

Due: Apr. 12, 2022

1. Write a solver of Kepler's equation and wrapper code to: a). Run solver [3 pt]; b). Show that solver gives the correct answer. [3 pt]

   (check that it works on $0 < e < 1$ and $0 < M < 2\pi$.)

2. Write a function to implement the radial velocity/Doppler formula for a planet on an elliptical orbit (you will need to call your Kepler equation solver). [4 pt]

3. Find the period of the planet in the RV dataset contained in mystery_planet01.txt [columns are: time, RV (m/s), error (m/s)]. Plot your periodogram. [3 pt]

4. Use this to fit RV data, adjusting the parameters to improve your fit. What P, e, t_p, γ, and K do you find? Plot thedata folded on the best-fit period (with error bars), along with your best-fit RV model. [6 pt]

5. Turn in Jupyter notebook. [2 pt]

6. Gauss's functons, f, g, $\dot{f}$, and $\dot{g}$, are given by

$$\begin{aligned} f &= \frac{a}{r_0}(\cos(E-E_0)-1)+1, \\ g &= (t-t_0)+\frac{1}{n}(\sin(E-E_0)-(E-E_0)) \\ \dot{f} &= -\frac{a^2}{rr_0}n\sin(E-E_0) \\ \dot{g} &= \frac{a}{r}(\cos(E-E_0)-1)+1, \end{aligned} \tag{1}$$

   where E is the eccentric anomaly at time t, E_0 is the eccentric anomaly at time t_0, a is the semi-major axis, $r = a(1 - e\cos E)$, $r_0 = a(1 -$

$e \cos E_0)$, e is the eccentricity, and $n = 2\pi/P$ where P is the orbital period.

With these one can take a Kepler step in position and velocity as:

$$\begin{aligned} \mathbf{x} &= f\mathbf{x}_0 + g\mathbf{v}_0, \\ \mathbf{v} &= \dot{f}\mathbf{x}_0 + \dot{g}\mathbf{v}_0 \end{aligned} \tag{2}$$

where $\mathbf{x}_0$ and $\mathbf{v}_0$ are the initial position and velocity of the bodies with respect to one another, and $\mathbf{x}$ and $\mathbf{v}$ are the final positions.

a). [3 pt] Show that conservation of angular momentum yields the result:

$$f\dot{g} - g\dot{f} = 1 \tag{3}$$

Note: the mass is unchanged, so you just need to show that specific angular momentum, the cross product of position with velocity, is conserved.

Show that the Gauss's f, g, $\dot{f}$, and $\dot{g}$ functions satisfy this equation.

b). [3 pt] Show that if Kepler's equation is satisfied, then so is the equation from part (a); i.e., *Kepler's equation implies conservation of angular momentum.* Note: take $M_0 = E_0 = 0$ where t_0 is the time of pericenter passage, and $M = n(t - t_0)$.