ASTR 531 - Stellar Interiors and Evolution

Problem Set 2

Tom Wagg

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8.1 - Mass Defect Fraction

The mass defects of the respective equations are as follows

Phase	Equation	$\Delta m/m$
Hydrogen	$4\,^{1}\mathrm{H} \rightarrow {}^{4}\mathrm{He}$	0.007145
Helium	$3\,^{4}\mathrm{He} \rightarrow {}^{12}\mathrm{C}$	0.000650
Carbon	${}^{12}\mathrm{C} + {}^{4}\mathrm{He} \rightarrow {}^{16}\mathrm{O}$	0.000481
Oxygen	$2\,^{16}\mathrm{O} \rightarrow {}^{28}\mathrm{Si} + {}^{4}\mathrm{He}$	0.000322
Silicon	$2\,^{28}\mathrm{Si} \rightarrow {}^{56}\mathrm{Fe}$	0.000338

You can see that the trend is that subsequent reactions have lower mass defect fractions and produce less energy per reaction (with the exception of Silicon vs. Oxygen fusion.) Since less energy is available to counter gravity in each phase, this implies that stars will spend the majority of their time in the hydrogen fusion phase and progressively less time in each fusion phase.

8.4 - Minimum Core Masses

Equation 8.24 in the textbook gives that the minimum core mass required to initiate a fusion phase with ignition temperature $T_{\rm ign}$ is

$$M_{\rm crit} \approx \left[\frac{\mathcal{R} K_1}{\mu_c \mu_e^{5/3} G^2} \cdot T_{\rm ign}\right]^{3/4} \tag{1}$$

The only variables here are μ_c, μ_e and $T_{\rm ign}$. The textbook gives us that $\mu_e = \mu_c = 2$ for all fusion phases after Helium. Thus, given that the minimum core mass for Helium fusion is $0.3\,\mathrm{M}_\odot$ (and that we know the ignition temperature is $10^8\,\mathrm{K}$ from Table 8.4), the core masses required for the subsequent phases will be

$$M_{\rm crit} \approx 0.3\,\mathrm{M}_\odot \cdot \left(\frac{T_{\rm ign}}{10^8\,\mathrm{K}}\right)^{3/4} \tag{2}$$

So we can tabulate all of this for brevity

Phase	$T_{\rm ign}/\,\mathrm{K}$	$M_{\rm crit}/\,\mathrm{M}_\odot$
Helium	10^8	0.3
Carbon	6×10^8	1.15
Neon	9×10^8	1.56
Oxygen	10^9	1.69
Silicon	3×10^9	3.85