ASTR 531 - Stellar Interiors and Evolution

Problem Set 2 Tom Wagg

April 13, 2022

8.1 - Mass Defect Fraction

The mass defects of the respective equations are as follows

Phase	Equation	$\Delta m/m$
Hydrogen	$4^1\mathrm{H} \rightarrow {}^4\mathrm{He}$	0.007145
Helium	$3^4\mathrm{He} \rightarrow {}^{12}\mathrm{C}$	0.000650
Carbon	${}^{12}\mathrm{C} + {}^4\mathrm{He} \rightarrow {}^{16}\mathrm{O}$	0.000481
Oxygen	$2^{16}\mathrm{O} \rightarrow {}^{28}\mathrm{Si} + {}^4\mathrm{He}$	0.000322
Silicon	$2^{28}\mathrm{Si} \rightarrow {}^{56}\mathrm{Fe}$	0.000338

You can see that the trend is that subsequent reactions have lower mass defect fractions and produce less energy per reaction (with the exception of Silicon vs. Oxygen fusion.) Since less energy is available to counter gravity in each phase, this implies that stars will spend the majority of their time in the hydrogen fusion phase and progressively less time in each fusion phase.

8.4 - Minimum Core Masses

Equation 8.24 in the textbook gives that the minimum core mass required to initiate a fusion phase with ignition temperature $T_{\rm ign}$ is

$$M_{\rm crit} \approx \left[\frac{\mathcal{R} K_1}{\mu_c \mu_e^{5/3} G^2} \cdot T_{\rm ign}\right]^{3/4} \tag{1}$$

The only variables here are μ_c, μ_e and $T_{\rm ign}$. The textbook gives us that $\mu_e = \mu_c = 2$ for all fusion phases after helium. Thus, given that the minimum core mass for helium fusion is $0.3\,\mathrm{M}_\odot$ (and that we know the ignition temperature is $10^8\,\mathrm{K}$ from Table 8.4), the core masses required for the subsequent phases will be

$$M_{\rm crit} \approx 0.3\,\mathrm{M}_\odot \cdot \left(\frac{T_{\rm ign}}{10^8\,\mathrm{K}}\right)^{3/4} \tag{2}$$

So we can tabulate all of this for brevity

Phase	$T_{\rm ign}/\,\mathrm{K}$	$M_{\rm crit}/\,\mathrm{M}_\odot$
Helium	10^8	0.3
Carbon	6×10^8	1.15
Neon	9×10^8	1.56
Oxygen	10^9	1.69
Silicon	3×10^9	3.85

9.1 - Typical Timescales

For calculating the timescales based on M, R, L, I use the following equations from the textbook (specifically, 9.3, 9.4, 9.5d)

$$\tau_{\text{dyn}} = \sqrt{\frac{1}{G\bar{\rho}}}, \qquad \tau_{\text{KH}} = \frac{GM^2}{RL}, \qquad \tau_{\text{nuc}} \approx \frac{M/\,\mathrm{M}_\odot}{L/\,\mathrm{L}_\odot} 10^{10}\,\mathrm{yr} \tag{3}$$

Specifically, this gives the following values

Star	$\tau_{\text{dyn}}/\,\mathrm{s}$	$\tau_{\text{KH}}/\,\mathrm{s}$	$\tau_{\text{nuc}}/\,\mathrm{s}$	Ratios
$1\,\mathrm{M}_\odot$ MS	3.3×10^{03}	9.9×10^{14}	3.2×10^{17}	$1 : 3.0 \times 10^{11} : 9.7 \times 10^{13}$
$60\,\mathrm{M}_\odot$ MS	2.4×10^{04}	3.0×10^{11}	2.4×10^{13}	$1 : 1.2 \times 10^{07} : 9.7 \times 10^{08}$
$15\,\mathrm{M}_\odot$ RSG	1.6×10^{08}	1.5×10^{08}	1.1×10^{13}	$1.1 : 1 : 7.0 \times 10^{04}$
$0.6\,\mathrm{M}_\odot$ WD	5.5×10^{00}	3.0×10^{19}	N/A	$1 : 5.4 \times 10^{18}$

There's a couple of things to note here. First, in the general case of a lower mass main sequence star, the dynamical timescale is far shorter than both other timescales, meaning that these stars are always in quasi-hydrostatic equilibrium. The thermal timescale is also shorter than the nuclear timescale and so the star is in thermal equilibrium for most of its life.

For higher mass main sequence stars, the nuclear timescale is significantly reduced (by a factor of 10,000), thus higher mass stars will run out of fuel much more quickly. The thermal timescale is also relatively closer to this nuclear timescale so we'd expect this star to spend more of its time out of thermal equilibrium.

For RSGs, an interesting thing to note if that the dynamical timescale is actually slightly longer than the thermal timescale! For WDs the dynamical timescale is incredibly short (5 seconds!), whilst the thermal timescale is much longer than for any of the other stars.

9.2 - Hey, where'd the sun go!?

Well it depends what we are measuring. If we happen to have a fancy neutrino detector then the answer would be the photon travel time since the neutrinos would immediately stop appearing - though I think people would probably assume some sort of instrument error at first, rather than the sun kicking the hypothetical bucket ;)

Assuming we're just using good old EM detectors, then I think the answer is instead the **dynamical timescale**. We can immediately rule out the nuclear timescale since that's only relevant with nuclear fuel running out (rather than being arbitrarily cut off). One may argue for the thermal timescale, since this is how long it would actually take for the Sun's luminosity to reflect its lack of nuclear energy production. However, upon the removal of the central energy source, the sun would start to contract on the dynamical timescale and thus we could measure this contraction and notice the sun has broken in this time (about an hour for the sun).

11.1 - Density structure

A polytrope is defined to obey the relation $P \sim \rho^\gamma$, or equivalently $\rho \sim P^{1/\gamma}$. Therefore, a star with $\gamma = 4/3$ has a more concentrated density structure than a star with $\gamma = 5/3$. We can see this is true because ρ follows a steeper power law with pressure when $\gamma = 4/3$ and thus more of the mass will be concentrated towards the centre and decrease quickly towards the surface when pressure declines.