

ASTR 531 - Stellar Interiors and Evolution

Problem Set 3

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12.2 - Early Radii and Timescales

Part a - Radii Estimations

Let's use a couple of different relations from the textbook to get the radii at different times. A protostar becomes ionised and starts the Hayashi concentration phase when it's radius is on the order of (Eq. 12.13)

$$R_{\text{Hayashi,start}} \approx 100 R_{\odot} \left(\frac{M}{M_{\odot}} \right) \quad (1)$$

We find the radius of the protostar once the Hayashi concentration phase comes to an end is approximately a factor of 50 lower (based on assumptions of the temperature and opacity) such that (page 12-8)

$$R_{\text{Hayashi,end}} \approx 2 R_{\odot} \left(\frac{M}{M_{\odot}} \right) \quad (2)$$

The radius at the start of the PMS phase will be the same as the end of the Hayashi concentration phase.

$$R_{\text{PMS,start}} = R_{\text{Hayashi,end}} \quad (3)$$

Finally, the radius at the end of the PMS phase is the same as the radius at ZAMS and so we can write that (Eq. 12.16)

$$R_{\text{PMS,end}} = R_{\text{ZAMS}} = R_{\odot} \left(\frac{M}{M_{\odot}} \right)^{0.7} \quad (4)$$

So now we can plug in numbers for the different masses of stars that we considered

M/M_{\odot}	$R_{\text{Hayashi,start}}/R_{\odot}$	$R_{\text{Hayashi,end}}/R_{\odot}$	$R_{\text{PMS,start}}/R_{\odot}$	$R_{\text{PMS,end}}/R_{\odot}$
0.3	30	0.6	0.6	0.43
3	300	6	6	2.16
30	3000	60	60	10.8

Part b - Timescale estimations

The duration of the Hayashi concentration phase is given in Eq. 12.15 but we also showed that the timescale scales as $1/M$ such that

$$\tau_{\text{Hayashi}} \approx 10^6 \text{ yr} \left(\frac{M_{\odot}}{M} \right) \quad (5)$$

The duration of the PMS phase is given by Eq. 12.17 so we have that

$$\tau_{\text{PMS}} \approx 6 \times 10^7 \text{ yr} \left(\frac{M}{M_{\odot}} \right)^{-2.5} \quad (6)$$

So now we can plug in numbers for the different masses of stars that we considered

M/M_{\odot}	$\tau_{\text{Hayashi}}/\text{yr}$	$\tau_{\text{PMS}}/\text{yr}$
0.3	3.33×10^6	1.22×10^9
3	3.33×10^5	3.85×10^6
30	3.33×10^4	1.22×10^4

15.4 - Metallicity and Mass Loss Rates

TODO: Need to check code/method with Emily

16.1 - RGB Radii

From inspection of Figure 16.1 we can find values for L and T_{eff} at the start and end of the RGB phase. This phase starts at C and ends at F. We can then use the fact that

$$R = \sqrt{\frac{L}{4\pi\sigma T_{\text{eff}}^4}} \quad (7)$$

to get the radii. Since I'm in a mood for tables today, let's make another!

Stage	$\log(L/L_{\odot})$	$\log(T_{\text{eff}}/\text{K})$	R/R_{\odot}
C (Start of RGB)	0.4	3.7	2.1
F (End of RGB)	3.4	3.48	183.1

So the radius is increasing by nearly two orders of magnitude!

17.1 - Helium Flash Duration

First we need to calculate the potential energy of a uniform sphere. We know that gravitational potential is given by

$$\Phi = -\frac{GM}{r} \quad (8)$$

So we just need to integrate this over a series of teeny tiny shells over the core of the star. Note we can assume a uniform density so $\rho(r) = \rho$. Let's do it!

$$U = -\int_0^R \frac{GM(r)}{r} \rho(r) d^3r \quad (9)$$

$$= -\int_0^R \frac{G(\rho(r)\frac{4}{3}\pi r^3)}{r} \rho(r) 4\pi r^2 dr \quad (10)$$

$$= -\frac{16G\pi^2\rho^2}{3} \int_0^R r^4 dr \quad (11)$$

$$U = -\frac{16G\pi^2\rho^2 R^5}{15} \quad (12)$$

We can rearrange this to get rid of the radius and just write it as a function of density and mass.

$$R = \left(\frac{3M}{4\pi\rho}\right)^{1/3} \quad (13)$$

$$U = -\frac{16G\pi^2\rho^2}{15} \left(\frac{3M}{4\pi\rho}\right)^{5/3} \quad (14)$$

$$U = -\frac{G(36\pi M^5\rho)^{1/3}}{5} \quad (15)$$

Nice, what a...clean expression?¹. Now let's use this to find the difference in potential energy for a $0.5 M_\odot$ core expanding from a density of $\rho = 10^6 \text{ g cm}^{-3}$ to $\rho = 10^4 \text{ g cm}^{-3}$.

$$\Delta U = -\frac{G(36\pi(0.5 M_\odot)^5)^{1/3}}{5} [(10^4 \text{ g cm}^{-3})^{1/3} - (10^6 \text{ g cm}^{-3})^{1/3}] = 5 \times 10^{42} \text{ J} \quad (16)$$

Now we can calculate the duration of the Helium flash given the energy production and efficiency

$$\tau_{\text{flash}} = \frac{\Delta U}{L \cdot \phi} = \frac{5 \times 10^{49} \text{ J}}{10^{10} L_\odot \cdot 0.2} = 2.5 \times 10^{40} \text{ erg } L_\odot^{-1} \quad (17)$$

Now because I'm an "astronomer" I can 100% (totally) understand ergs and L_\odot intuitively, but just in case some readers don't live and breathe cgs in the same way this translates to

$$\boxed{\tau_{\text{flash}} = 0.21 \text{ yr} \approx 76 \text{ days}} \quad (18)$$

Quite the 'flash' indeed!

¹To be fair, it looks much better if we did it in terms of M and R : $U = -\frac{3}{5}GM^2/R$