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HW 5
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1.

1). $P \Rightarrow \neg Q$, $Q \Rightarrow \neg P$

$= \neg P \vee \neg Q$, $\neg Q \vee \neg P$

P	Q	$\neg P \vee \neg Q$	$\neg Q \vee \neg P$
F	F	T	T
F	T	T	T
T	F	T	T
T	T	F	F

∴ These two are equivalent.

2). $P \Leftrightarrow \neg Q$ $[(P \wedge \neg Q) \vee (\neg P \wedge Q)]$

$P \Leftrightarrow \neg Q$

$= P \Rightarrow \neg Q \wedge \neg Q \Rightarrow P$.

$= (\neg P \vee \neg Q) \wedge (Q \vee P)$

P	Q	$P \Leftrightarrow \neg Q$	$P \wedge \neg Q$	$\neg P \wedge Q$	$((P \wedge \neg Q) \vee (\neg P \wedge Q))$
F	F	F	F	F	F
F	T	T	F	T	T
T	F	T	T	F	T
T	T	F	F	F	F

∴ These two are equivalent.

2. 1) (Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)

let S = Smoke, F = Fire,

(S ⇒ F) ⇒ (¬S ⇒ ¬F)

= (¬S ∨ F) ⇒ (S ∨ ¬F)

S	F	(¬S ∨ F)	(S ∨ ¬F)
F	F	T	T
F	T	T	F
T	F	F	T
T	T	T	T

Therefore it is neither

2). let H = Heat.

(S ⇒ F) ⇒ ((S ∨ H) ⇒ F)

= (¬S ∨ F) ⇒ (¬(S ∨ H) ∨ F)

= (¬S ∨ F) ⇒ ((¬S ∧ ¬H) ∨ F)

S	F	H	(¬S ∨ F)	(¬S ∧ ¬H)	((¬S ∨ ¬H) ∨ F)
F	F	F	T	T	T
F	F	T	[T]	F	[F]
F	T	F	T	T	T
F	T	T	T	F	T
T	F	F	F	F	F
T	F	T	F	F	F
T	T	F	T	F	T
T	T	T	T	F	T

The second situation in the table is neither, ~~therefore~~, this sentence is neither valid nor unsatisfiable.

3). $((S \wedge H) \Rightarrow F) \Leftrightarrow ((S \Rightarrow F) \vee (H \Rightarrow F))$

$= (\neg(S \wedge H) \vee F) \Leftrightarrow ((\neg S \vee F) \vee (\neg H \vee F))$

$= (\neg S \vee \neg H \vee F) \Leftrightarrow (\neg S \vee F \vee \neg H \vee F)$

$= (\neg S \vee \neg H \vee F) \Leftrightarrow (\neg S \vee \neg H \vee F)$

By the extention shows above, this sentence is valid.

S	H	F	$(\neg S \vee \neg H \vee F)$	$(\neg S \vee \neg H \vee F)$
F	F	F	T	T
F	F	T	T	T
F	T	F	T	T
F	T	T	T	T
T	F	F	T	T
T	F	T	T	T
T	T	F	F	F
T	T	T	T	T

Truth table also shows it is valid.

3. a) M: Mythical
I = Immortal
O = Mortal mammal
H = horned.
G = Magical

KB: $M \Rightarrow (I \wedge \neg O)$
$\neg M \Rightarrow (O \wedge \neg I)$
$(I \vee O) \Rightarrow H$
$H \Rightarrow G$

b) KB. $\neg M \vee (I \wedge \neg O) = (\neg M \vee I) \wedge (\neg M \vee \neg O)$
$M \vee (O \wedge \neg I) = (M \vee \neg I) \wedge (M \vee O)$
$\neg (I \vee O) \vee H = (\neg I \wedge \neg O) \vee H = (\neg I \vee H) \wedge (\neg O \vee H)$
$\neg H \vee G$
$\therefore$ CNF = $\{(\neg M \vee I) \wedge (\neg M \vee \neg O) \wedge (M \vee \neg I) \wedge (M \vee O) \wedge (\neg I \vee H) \wedge (\neg O \vee H) \wedge (\neg H \vee G)\}$

3). KB.
1. $\neg M \vee I$
2. $\neg M \vee \neg O$
3. $M \vee \neg I$
4. $M \vee O$
5. $(\neg I \vee H)$
6. $(\neg O \vee H)$
7. $(\neg H \vee G)$

α_1: 8. M 11. $\neg M$
α_2: 9. G 12. $\neg G$
α_3: 10. H 13. $\neg H$

Mythical	Magical	Horned
14. I (1,8)	17. $\neg H$ (7,12)	23. $\neg I$ (5, 13)
15. M (14, 3)	18. $\neg O$ (6, 17)	24. $\neg O$ (6, 13)
16. empty (11, 15)	19. M (4, 18)	25. M (4, 24)
	20. I (1, 19)	26. $\neg M$ (1, 23)
	21. H (5, 20)	27. empty (25, 26)
	22. empty (17, 21)	

*Contradiction, therefore $\alpha_{1,2,3}$ could be derived by knowledge base.

Therefore, unicorn is mythical, magical, and horned.