1. Prove:

Generalized product rule: Pr(A, B | K) = Pr(A | B, K)Pr(B | K):

RHS =

Pr(A | B, K)Pr(B | K) = (Pr(A, B, K)/Pr(B, K))(Pr(B, K)/Pr(K))

= Pr(A, B, K)/Pr(K)

= Pr(A, B | K)

= LHS

Generalized Bayes’ rule: Pr(A | B, K) = Pr(B | A, K)Pr(A | K)/Pr(B | K):

RHS =

Pr(B | A, K)Pr(A | K)/Pr(B | K) = (Pr(A, B, K)/Pr(A,K))\*(Pr(A, K)/Pr(K)) / (Pr(B, K)/Pr(K))

= (Pr(A, B, K)/Pr(K)) / (Pr(B, K)/Pr(K))

= (Pr(A, B, K)/Pr(K)) \* (Pr(K)/Pr(B, K))

= Pr(A, B, K) / Pr(B, K)

= Pr(A | B, K)

= LHS

2. We have a bag of three biased coins a, b, and c with probabilities of coming up heads of 20%, 60%, and 80%, respectively. One coin is drawn randomly from the bag (with equal likelihood of drawing each of the three coins), and then the coin is flipped three times to generate the outcomes X1, X2, and X3.

Draw the Bayesian network corresponding to this setup and define the necessary CPTs (Conditional Probability Table).

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Pr(a) | Pr(b) | Pr(c) | Pr(~a) | Pr(~b) | Pr(~c) |
| 1/3 | 1/3 | 1/3 | 2/3 | 2/3 | 2/3 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Pr(head|a) | Pr(head|b) | Pr(head|c) | Pr(tail|a) | Pr(tail|b) | Pr(tail|c) |
| 0.2 | 0.6 | 0.8 | 0.8 | 0.4 | 0.2 |

3. Mr. Y picked up an object at random from the above set. We want to compute the probabilities

of the following events:

α1: the object is black;

α2: the object is square;

α3: if the object is one or black, then it is also square

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| World | Black | Square | One | Probability |
| 1 | T | T | T | 2/13 |
| 2 | T | T | F | 4/13 |
| 3 | T | F | T | 1/13 |
| 4 | T | F | F | 2/13 |
| 5 | F | T | T | 1/13 |
| 6 | F | T | F | 1/13 |
| 7 | F | F | T | 1/13 |
| 8 | F | F | F | 1/13 |

α1 is true for world 1, 2, 3, 4:

Pr(α1) = (2 + 4 + 1 + 2)/13 = 9/13

α2 is true for world 1, 2, 5, 6:

Pr(α2) = (2 + 4 + 1 + 1)/13 = 8/13

α3 is true for world 1, 3, 5, 7:

Pr(α3) = (2 + 1 + 1 + 1)/13 = 5/13

Sets of sentences:

1. Alpha = One

Beta = Square

Gama = White

One is independent of Square given White.

Pr(One | White) = 2/4 = 1/2

Pr(One | Square, White) = 1/2

1. Alpha = Two

Beta = Square

Gama = White

Two is independent of Square given White.

Pr(Two | White) = 2/4 = 1/2

Pr(Two | Square, White) = 1/2

Consider the DAG in Figure 1:

(a) List the Markovian assumptions asserted by the DAG.

I(A, ∅, {B, E})

I(B, ∅, {A, C})

I(C, A, {B, D, E})

I(D, {A, B}, {C, E})

I(E, B, {A, C, D, F, G})

I(F, {C, D}, {A, B, E})

I(G, F, {A, B, C, D, E, H})

I(H, {F, E}, {A, B, C, D, G})

(b) True or false? Why?

d separated(A, BH, E)

False, since path ACFHE is not blocked.

d separated(G, D, E)

True, since D blocked FDB and H blocked FHE.

d separated(AB, F, GH)

False, since path BEH is not blocked.

(c) Express Pr(a, b, c, d, e, f, g, h) in factored form using the chain rule for Bayesian networks.

Pr(a, b, c, d, e, f, g, h) = Pr(a)\*

Pr(b)\*

Pr(c | a)\*

Pr(d | a, b)\*

Pr(e | b)\*

Pr(f | c, d)\*

Pr(g | f)\*

Pr(h | e, f)

(d) Compute Pr(A = 0, B = 0) and Pr(E = 1 | A = 1). Justify your answers.

Pr(A = 0, B = 0)

= Pr(A = 0)\*Pr(B = 0) = 0.8 \* 0.3 = 0.24

Pr(E = 1)

= Pr(E = 1| B = 0)\*Pr(B = 0) + Pr(E = 1 | B = 1)\*Pr(B = 1)

= 0.9\*0.3 + 0.1\*0.7

= 0.27 + 0.07

= 0.34

Pr(A = 1) = 0.2

Since A and E are independent, we choose the one has higher probability.

Pr(E = 1 | A = 1) = Pr(E = 1) = 0.34