

b). With offset b be non-zero, we get:

$$\begin{aligned} w_1 + w_2 + b \geq 1 &\Rightarrow 1 - w_1 - w_2 - b \leq 0 \\ -(w_1 + b) \geq 1 &\Rightarrow 1 + w_1 + b \leq 0 \end{aligned}$$

$$\therefore L(w, \alpha, b) = \frac{1}{2}(w_1^2 + w_2^2) + \alpha_1(1 - w_1 - w_2 - b) + \alpha_2(1 + w_1 + b)$$

$$\frac{\partial L}{\partial w_1} = w_1 - \alpha_1 + \alpha_2 = 0$$

$$\frac{\partial L}{\partial w_2} = w_2 - \alpha_1 = 0$$

$$\therefore w_1 = \alpha_1 - \alpha_2$$

$$w_2 = \alpha_1$$

$$\Rightarrow L(w, \alpha, b) = \frac{1}{2}(2\alpha_1^2 - 2\alpha_1\alpha_2 + \alpha_2^2) + (\alpha_1 - 2\alpha_1^2 + \alpha_1\alpha_2 - \alpha_1 b) + (\alpha_2 + \alpha_1\alpha_2 - \alpha_2^2 + \alpha_2 b)$$

$$\frac{\partial L}{\partial \alpha_1} = -2\alpha_1 + \alpha_2 + 1 - b = 0$$

$$\frac{\partial L}{\partial \alpha_2} = -\alpha_2 + \alpha_1 + 1 + b = 0$$

$$\therefore \alpha_1 = \frac{\alpha_2 + 1 - b}{2}$$

$$\alpha_2 = \alpha_1 + 1 + b$$

$$\alpha_1 = \frac{\alpha_1 + 1 + b + 1 - b}{2}, \quad \alpha_1 = 2, \quad \alpha_2 = \alpha_1 = 2$$

$$\therefore b = -1, \quad w_1 = 0, \quad w_2 = 2$$

$$\therefore (w^*, b^*) = \left(\begin{bmatrix} 0 \\ 2 \end{bmatrix}, -1 \right), \quad r = \frac{1}{2}$$

$$\text{with } b = 0, \quad r = \frac{1}{\sqrt{2}}$$

Therefore, we could have larger margin within decision boundary with offset.