

$$b). \sum 2w_n (\theta_0 + \theta_1 x_{n,1} - y_n) = 0 \dots \textcircled{1}$$

$$\sum 2w_n (\theta_0 + \theta_1 x_{n,1} - y_n) \cdot x_{n,1} = 0 \dots \textcircled{2}$$

From ①:

$$\sum w_n \theta_0 = \sum (w_n y_n - w_n \theta_1 x_{n,1}) \dots \textcircled{3}$$

From ②:

$$\sum w_n \theta_0 x_{n,1} = \sum (w_n y_n x_{n,1} - w_n \theta_1 x_{n,1}^2) \dots \textcircled{4}$$

From ③:

$$\theta_0 = \frac{\sum (w_n y_n - w_n \theta_1 x_{n,1})}{\sum w_n} \dots \textcircled{5}$$

Substitute ⑤ into ④:

$$\frac{\sum (w_n y_n - w_n \theta_1 x_{n,1})}{\sum w_n} \cdot \sum w_n x_{n,1} = \sum (w_n y_n x_{n,1} - w_n \theta_1 x_{n,1}^2)$$

$$\sum (w_n y_n) \sum (w_n x_{n,1}) - \sum (w_n \theta_1 x_{n,1}) \sum (w_n x_{n,1}) = \sum w_n \cdot \sum (w_n y_n x_{n,1} - w_n \theta_1 x_{n,1}^2)$$

$$\sum w_n \sum w_n \theta_1 x_{n,1}^2 - \sum (w_n \theta_1 x_{n,1}) \sum w_n x_{n,1} = \sum w_n \sum w_n y_n x_{n,1} - \sum w_n y_n \sum w_n x_{n,1}$$

$$\theta_1 (\sum w_n \sum w_n x_{n,1}^2 - (\sum w_n x_{n,1})^2) = \sum w_n \sum w_n y_n x_{n,1} - \sum w_n y_n \sum w_n x_{n,1}$$

$$\theta_1 = \frac{\sum_{n=1}^N w_n \sum_{n=1}^N w_n y_n x_{n,1} - \sum_{n=1}^N w_n y_n \cdot \sum_{n=1}^N w_n x_{n,1}}{\sum_{n=1}^N w_n \sum_{n=1}^N w_n x_{n,1}^2 - \left( \sum_{n=1}^N w_n x_{n,1} \right)^2}$$

$$\theta_0 = \frac{\sum_{n=1}^N (w_n y_n - w_n \theta_1 x_{n,1})}{\sum_{n=1}^N w_n} \quad \text{Substitute } \theta_1 \text{ with}$$