

#1 a). $\pi_i (i \geq 4)$ don't effect value of Y , only π_1, π_2, π_3 effect value of Y .

There are 8 combinations of π_1, π_2, π_3 , and there is one mistake among these 8 options.

Therefore, over 2^n training examples, there should be $\frac{2^n}{8}$ mistakes.

b). There is no such split that reduces the number of mistakes by at least one.

Split on $\pi_i, i \geq 4$ makes no different since it's always $\frac{1}{8}$ mistake.

Split on π_1, π_2 , or π_3 would be:



$$c). H[Y] = -\frac{1}{8} \log\left(\frac{1}{8}\right) - \frac{7}{8} \log\left(\frac{7}{8}\right) = 0.543$$

d). Do the split in b) on π_1, π_2, π_3 , the resulting entropy would be decreased:

$$H[Y|\pi_i] = \left(-\frac{1}{4} \log\left(\frac{1}{4}\right) - \frac{3}{4} \log\left(\frac{3}{4}\right) - 0\right) \times \frac{1}{2}$$

$$= 0.406$$