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1. $y = x\sin(z)e^{-x}$

$$\frac{dy}{dx} = \sin(z)e^{-x} - x\sin(z)e^{-x}$$

$$= \sin(z)e^{-x}\cdot(1-x)$$

2. a) $y^T z = (3\times3)+(2\times1) = 11$

b) $Xy = \begin{bmatrix} 2\times1+4\times3 \\ 1\times1+3\times3 \end{bmatrix} = \begin{bmatrix} 14 \\ 10 \end{bmatrix}$

c) Yes.

$$X^* = \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$$

$$X^{-1} = \frac{1}{|X|}\cdot X^* = \frac{1}{2\times3-4\times1}\begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & -2 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

d) The Rank of x is 2, since $\begin{bmatrix} 2 \\ 1 \end{bmatrix} \neq k\begin{bmatrix} 4 \\ 3 \end{bmatrix}$ for $k \in \mathbb{R}$

3. a) sample mean $= \frac{1+1+0+1+0}{5} = \frac{3}{5}$

b) sample variance $= \frac{1}{5}\times\left[(1-\frac{3}{5})^2+(1-\frac{3}{5})^2+(0-\frac{3}{5})^2+(1-\frac{3}{5})^2+(0-\frac{3}{5})^2\right]$

$$= \frac{1}{5}\times\frac{30}{25}$$

$$= \frac{6}{25}$$

c). $P(S) = 0.5^5 = \frac{1}{32}$

d). let P represent the probability of $P(\pi=1)$

Then:

$$P(s) = P^{s_1} \cdot (1-P)^{(1-s_1)} \cdot P^{s_2}(1-P)^{(1-s_2)} \cdots P^{s_5}(1-P)^{(1-s_5)}$$

$$= \prod_{i=1}^{5} P^{s_i} (1-P)^{(1-s_i)}$$

$$= P^{\sum_{i=1}^{5} s_i} (1-P)^{5-\sum_{i=1}^{5} s_i}$$

Take the log of the function above:

$$f(P) = \left(\sum_{i=1}^{5} s_i\right) \log(P) + \left(5 - \sum_{i=1}^{5} s_i\right) \log(1-P).$$

Take derivative of $f(P)$ and set to 0, to find the maximum P.

$$\frac{df(P)}{dp} = \frac{1}{P} \sum_{i=1}^{5} s_i - \frac{1}{1-P}\left(5 - \sum_{i=1}^{5} s_i\right) = 0$$

$$\Rightarrow \frac{\sum_{i=1}^{5} s_i - 5P}{P(1-P)} = 0$$

$$\Rightarrow 5P = \sum_{i=1}^{5} s_i$$

$$P = \frac{1}{5} \cdot (1+1+0+1+0)$$

$$\boxed{P = \frac{3}{5}}$$

e). $P(X=T \mid Y=b) = \frac{0.1}{0.1+0.15} = \frac{0.1}{0.25} = 40\%$

4. a) False
b) True
c) False
d) False
e) True

5. a) with (v)
b) with (iv)
c) with (ii)
d) with (i)
e) with (iii)

6. a) The mean is p, variance is $p(1-p)$

b) The variance for x is : $\frac{1}{n} \cdot [(x_1-\sigma)^2+(x_2-\sigma)^2+\dots+(x_n-\sigma)^2] = \sigma^2$

For $2x$: $\frac{1}{n}[(x_1-2\sigma)^2+\dots+(x_n-2\sigma^2)] = \boxed{4\sigma^2}$

Since $(x_1^2-2x_1\sigma)+\dots+(x_n^2-2x_n\sigma)=0$

The variance for $x+2$ would remain $\boxed{\sigma^2}$

7. a) i). Both, since the function are multiply with same constant.

ii). $g(n) = O(f(n))$, since $f(n)$ grows much faster than $g(n)$ as n increasing.

iii). $g(n) = O(f(n))$, since $f(n)$ grows much faster as n increasing.

b). $f(s, e)$:

let mid = (e-s)/2

let left = array [mid]   right = array [mid + 1]

if left = 0 , right = 1, return mid

else if left = 0 , right = 0, return f(mid +1 , e)

else return f(s, mid)

The algorithm will start searching from middle positon, each time will divide the array to two parts, therefore the time complexity is $O(\log n)$.

By this method, it would iterate until only two numbers in the array in the worse case, so it is guarenteed to have the correct answer is there is one.

8. a). By definition of $E[X]$:

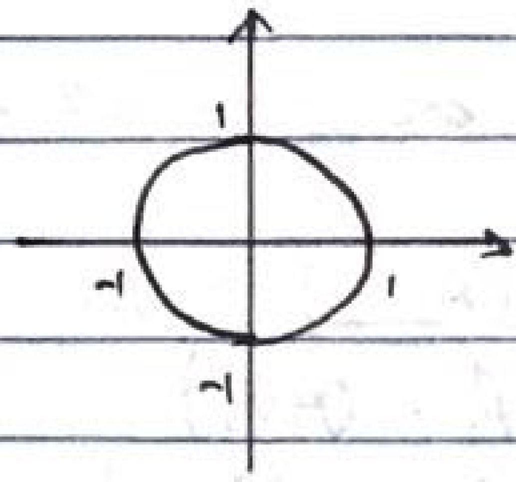
$$E[XY] = \int xy\, P(x,y)\, dx\, dy = \int xP(x)\, yP(y)\, dx\, dy$$

$$= \int xP(x)dx \int yP(y)dy = E[X]E[Y]$$

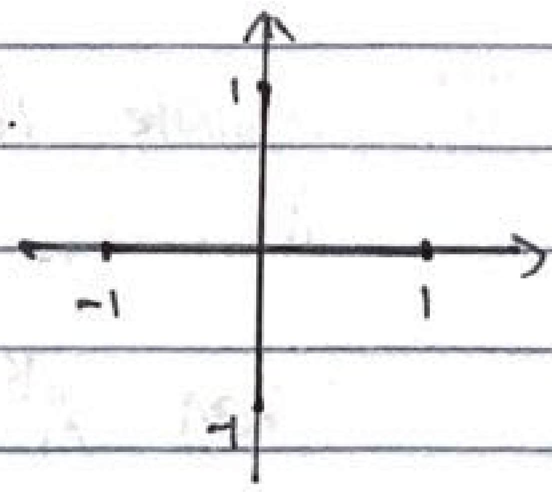
b). i). Since there is only one die, the probability of get a 3 is $\frac{1}{6}$, thus the number of times 3 shows up is close to $6000 \times \frac{1}{6} = 1000$.

ii).

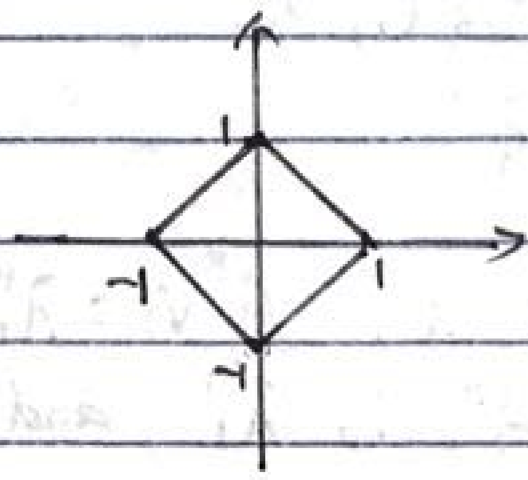
9. a). i).



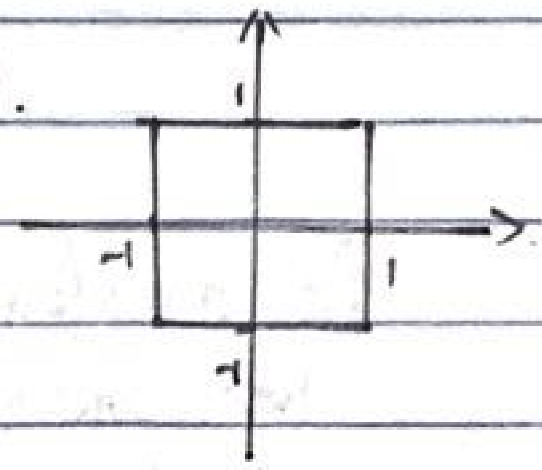
ii).



iii).



iv).



b). i). For square matrix A, $A \cdot v = \lambda \cdot v$, where v are the eigenvectors and λ are eigenvalues

ii). $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$$\det(A) = (2-\lambda)(2-\lambda) - 1 = 0$$
$$4 - 4\lambda + \lambda^2 - 1 = 0$$
$$\lambda^2 - 4\lambda + 3 = 0$$
$$(\lambda - 1)(\lambda - 3) = 0$$
$$\boxed{\lambda_1 = 1, \quad \lambda_2 = 3}$$

For $\lambda = 1$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0, \quad \boxed{\text{eigenvector} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}}$$

For $\lambda = 3$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0, \quad \boxed{\text{eigenvector} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

iii). Suppose $v_1, v_2, \cdots, v_n$ are eigenvectors for A with eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_n$

$$\text{Then } A^k v_1 = \overbrace{A \cdot A \cdot A \cdots}^{kA} (Av_1)$$
$$= \underbrace{A \cdot A \cdot A \cdots A}_{(k-1)A} (\lambda_1 v_1)$$

By transform all As to λ, $A^k v_1 = \lambda_1^k v_1$
Then one eigenvalue of A^k is λ_1^k, and one eigenvector for A^k is v_1.
Do the same to other eigenvalues and eigenvectors, we can prove that $\lambda_1^k, \lambda_2^k, \cdots, \lambda_n^k$ are eigenvalues for A^k, and same for $v_1, v_2, \cdots, v_n$.

9 c). i). $\dfrac{\partial a^T x}{\partial x} = x^T \dfrac{\partial a}{\partial x} + a^T$

ii).

9 d). i): The line include all x such that $w^T x + b = 0$.
Suppose x_1 and x_2 are such x, then $x_1 - x_2$ is a vector parallel to the line:

$w^T x_1 + b = 0$

$w^T x_2 + b = 0$

$w^T x_1 = w^T x_2$

$w^T (x_1 - x_2) = 0$

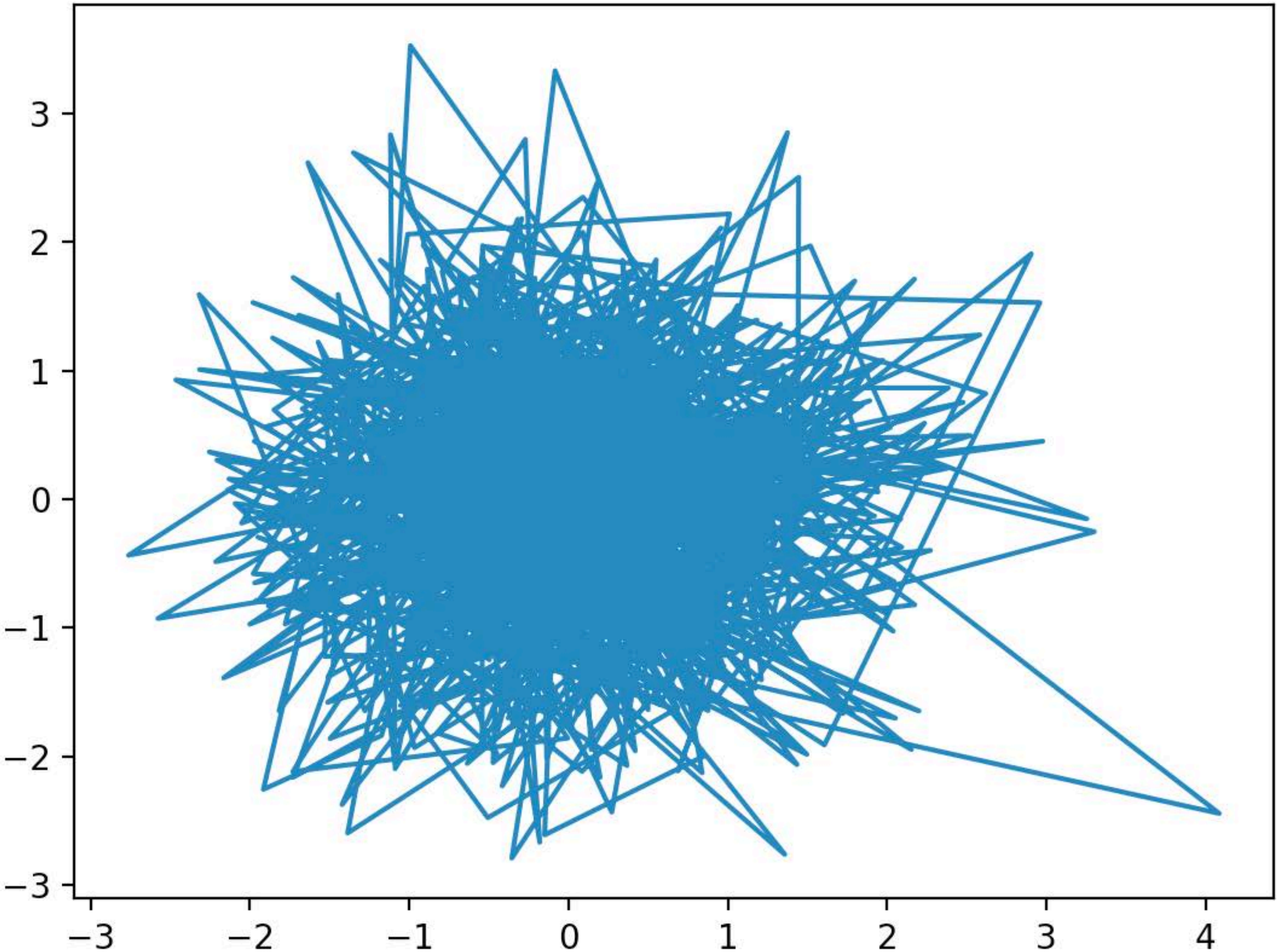
which means w^T orthogonal with the vector parallel to $w^T x + b = 0$, which means w is also orthogonal to the line $w^T x + b = 0$

ii).

12. a). SNAP Social Networks Dataset
b). https://snap.stanford.edu/data/egonets-Facebook.html
c). This dataset consists of social circles from Facebook. The dataset includes node features, circles, and ego networks.
d). There are 4039 examples
e). Each example Provide 3 features, which is profiles, circles, and ego network. These could use to analysis the social habit one Facebook users, and could also determine different kinds of social groups among Facebook users.

Figure 1

10. a)



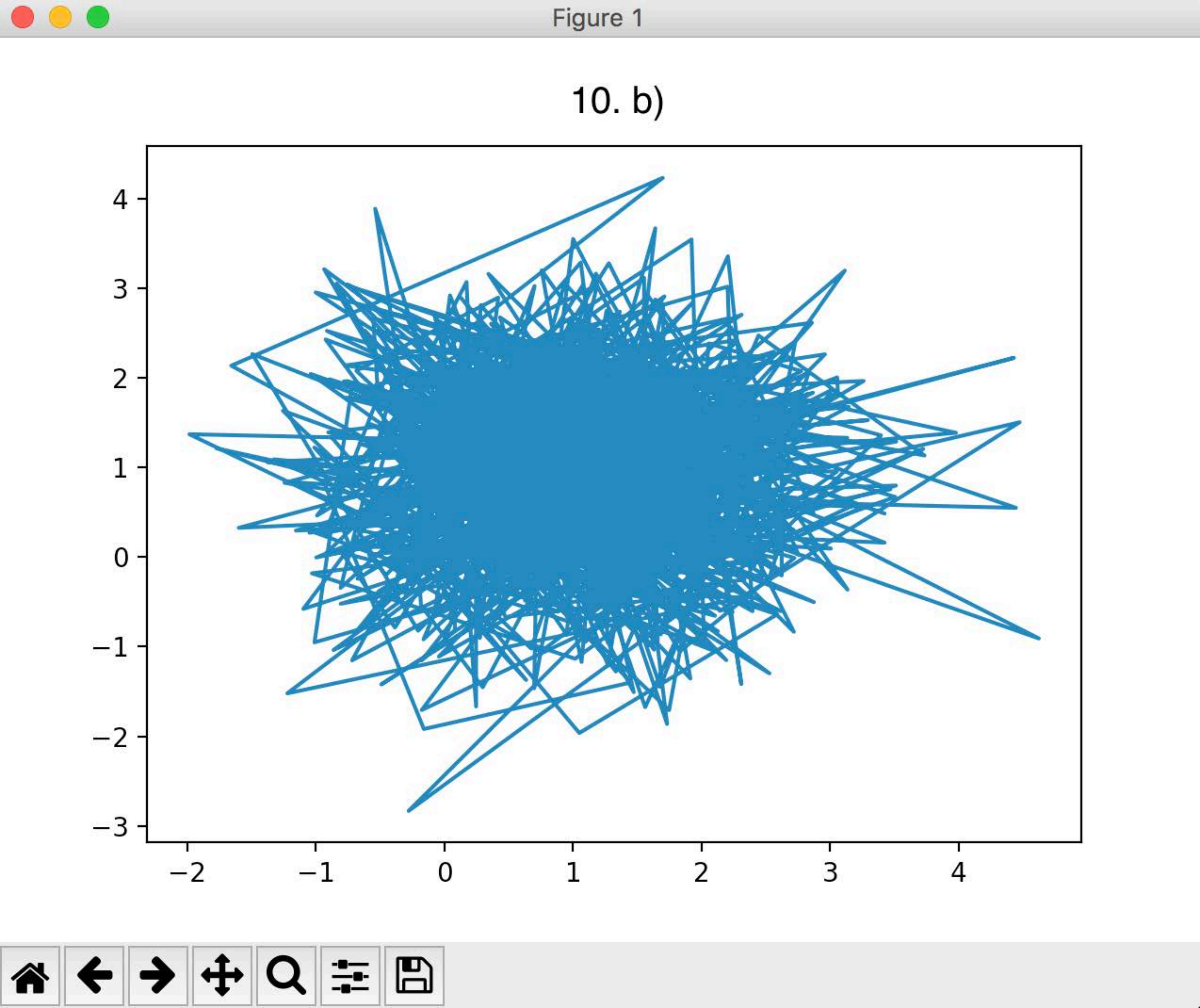


Figure 1
10. b)
4
3
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
4

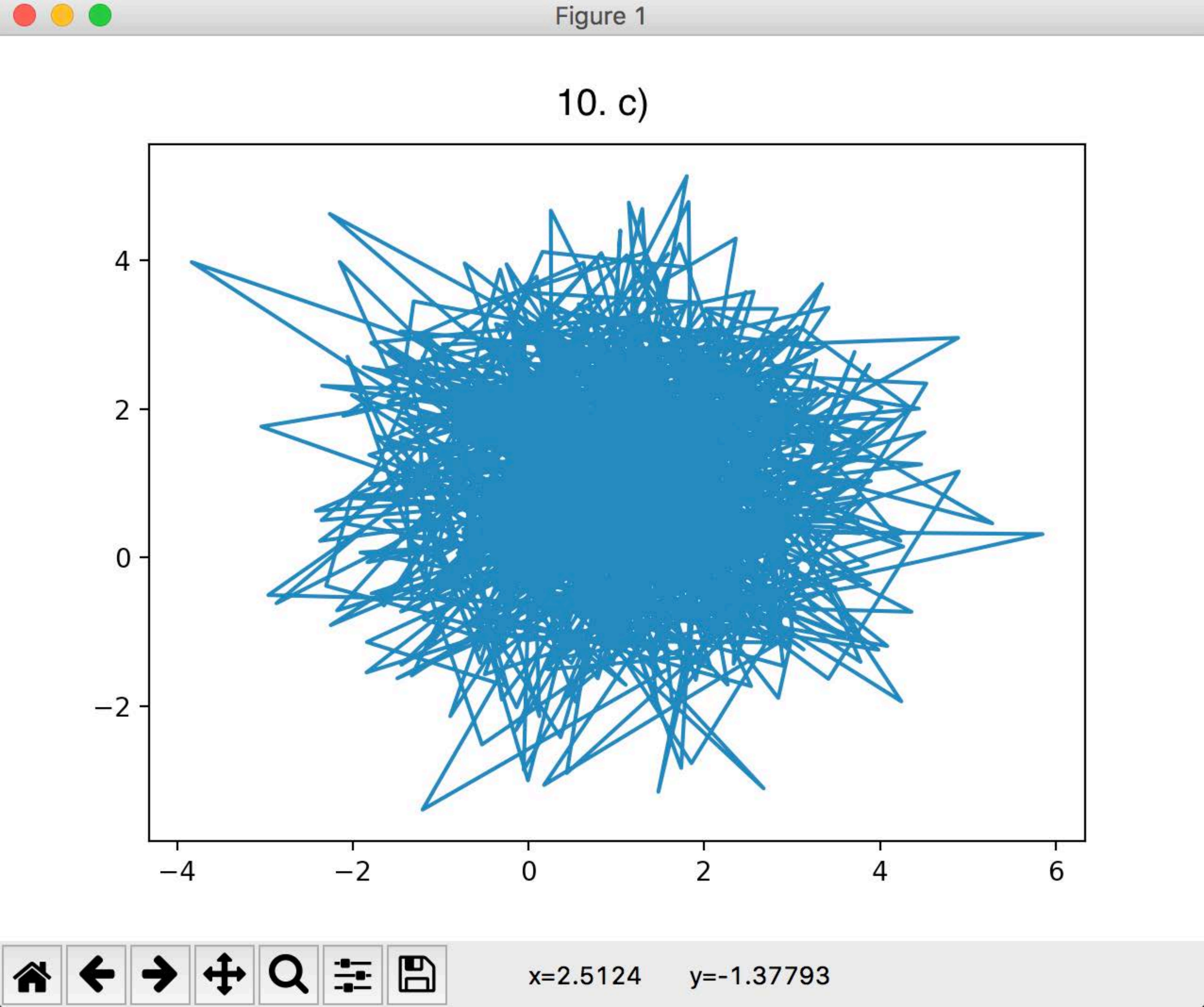
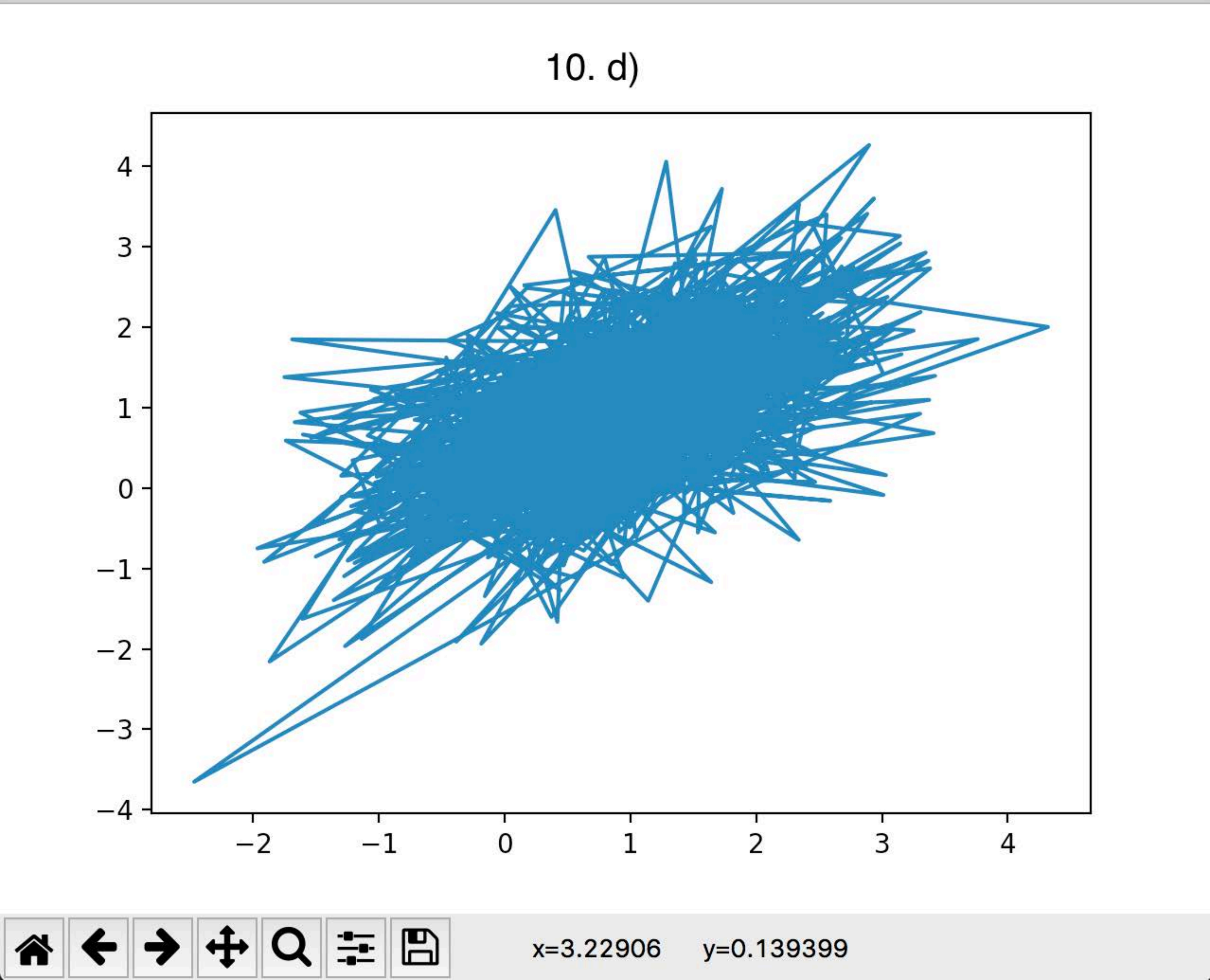


Figure 1
10. c)
4
2
0
−2
−4
−2
0
2
4
6
x=2.5124
y=-1.37793

Figure 1



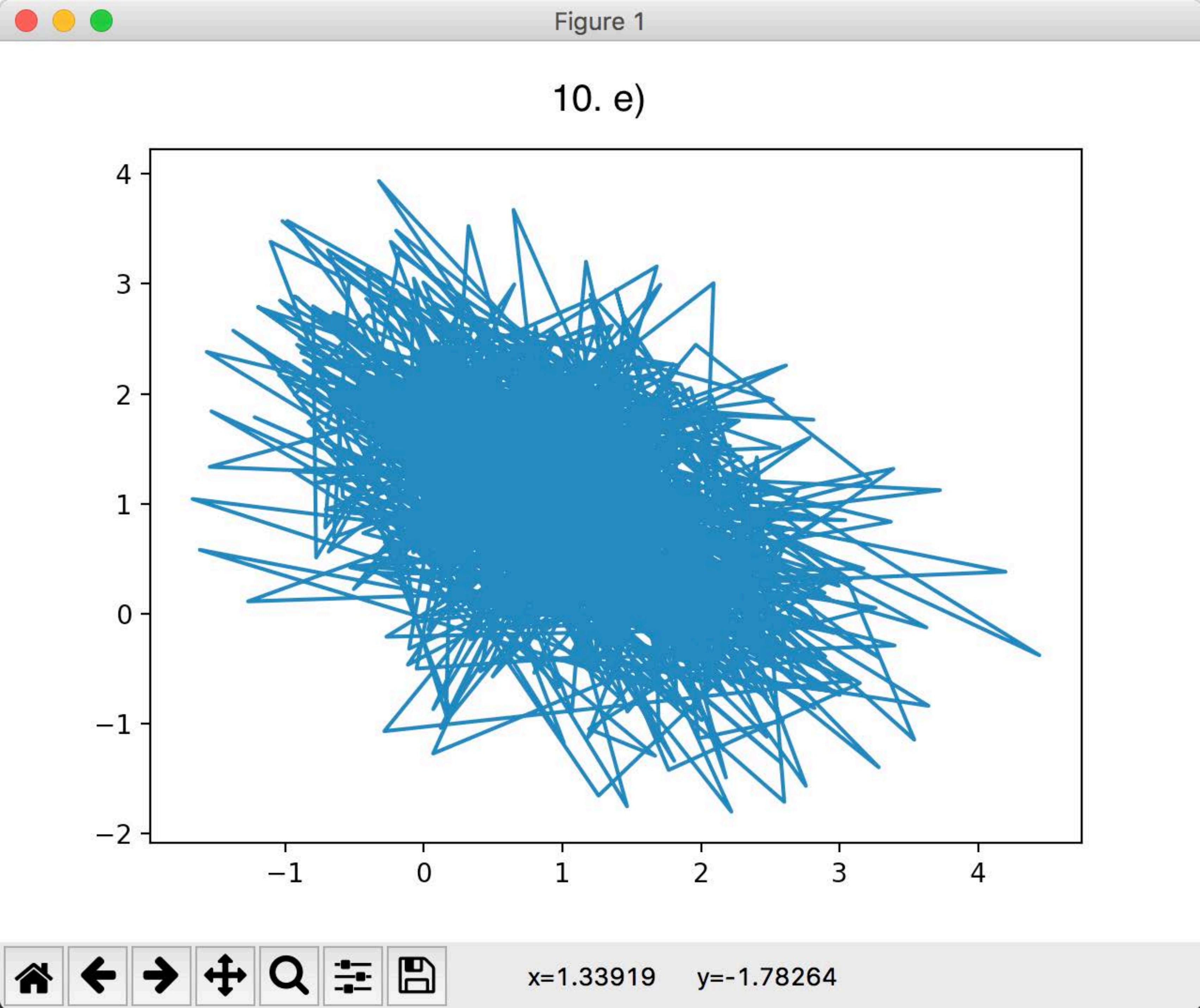


Figure 1
10. e)
4
3
2
1
0
−1
−2
−1
0
1
2
3
4
x=1.33919
y=-1.78264

Project

PS0 ~/Desktop/Study/CS_M146/PS0

- 1.jpg
- 2.jpg
- 3.jpg
- 4.jpg
- 5.jpg
- 6.jpg
- 7.jpg
- 10.a).png
- 10.b).png
- 10.c).png
- 10.d).png
- 10.e).png
- problem10.py
- problem11.py
- PS0_Solutions.pdf

External Libraries

problem11.py

```python
#Problem 11
import numpy as np

matrix = [[1, 0], [1, 3]]
e_vals, e_vecs = np.linalg.eig(matrix)
index = 0
for i in range(1, len(e_vals)):
    if e_vals[i] > e_vals[index]:
        index = i
print(e_vals[index])
print(e_vecs[index])
```

Run problem11

```
/usr/local/bin/python3 /Users/Tom/Desktop/Study/CS_M146/PS0/problem11.py
3.0
[ 0.          0.89442719]

Process finished with exit code 0
```