Problem 1:

a). We lose the order of words and semantic meaning of words.

b). Assume $P(Y_i=1)=\eta$ and $P(Y_i=0)=1-\eta$

$$P(D_i, y_i) = P(Y=y_i)\,P(D_i \mid Y=y_i)$$
$$= \left(P(Y=1)\,P(D_i \mid Y=1)\right)^{y_i}\left(P(Y=0)\,P(D_i \mid Y=0)\right)^{1-y_i} \dots$$

By using naive Bayes model:

$$P(D_i, y_i) = \left(\eta \frac{n!}{a_i!\,b_i!\,c_i!}\,\alpha_1^{a_i}\beta_1^{b_i}r_1^{c_i}\right)^{y_i}\left((1-\eta)\frac{n}{a_i!\,b_i!\,c_i!}\,\alpha_0^{a_i}\beta_0^{b_i}r_0^{c_i}\right)^{1-y_i}$$

The log likelihood of D_i =

$$N_i = \log P(D_i, y_i) = y_i\left[\log\eta + \log\left(\frac{n!}{a_i!\,b_i!\,c_i!}\right) + a_i\log\alpha_1 + b_i\log\beta_1 + c_i\log r_1\right] + (1-y_i)\left[\log(1-\eta) + \log\left(\frac{n!}{a_i!\,b_i!\,c_i!}\right) + a_i\log\alpha_0 + b_i\log\beta_0 + c_i\log\alpha_0\right]$$

c) Supeitute $r_1 = 1-\alpha_1-\beta_1$

$$\frac{\partial N}{\partial \alpha_1} = \sum y_i\left(\frac{a_i}{\alpha_1} + \frac{c_i}{r_1}\cdot\frac{\partial r_1}{\partial \alpha_1}\right)$$

$$= \sum y_i\left(\frac{a_i}{\alpha_1} - \frac{c_i}{r_1}\right) = \sum y_i\left(\frac{a_i r_1 - c_i\alpha_1}{\alpha_1 r_1}\right) = 0$$

$$\alpha_1 = r_1\frac{\sum y_i a_i}{\sum y_i c_i}, \quad \beta_1 = r_1\frac{\sum y_i b_i}{\sum y_i c_i}$$

Since $r_1 = 1-\alpha_1-\beta_1$

$$= 1 - r_1\frac{\sum y_i a_i}{\sum y_i c_i} - r_1\frac{\sum y_i b_i}{\sum y_i c_i}$$

$$= \frac{1}{\left(\frac{\sum y_i(a_i+b_i+c_i)}{\sum y_i c_i}\right)}$$