

$$3. a). \eta_1 = (1, 1)^T, \quad \eta_2 = (1, 0)^T$$

$$\therefore w_1 + w_2 \geq 1, \quad -w_1 \geq 1$$

$$1 - w_1, -w_2 \leq 0, \quad 1 + w_1 \leq 0$$

$$L(w, \alpha) = \frac{1}{2} (w_1^2 + w_2^2) + \alpha_1 (1 - w_1 - w_2) + \alpha_2 (1 + w_1)$$

$$d^* = \max_{\alpha} \min_w L(w, \alpha)$$

$$\frac{\partial L}{\partial w_1} = w_1 - \alpha_1 + \alpha_2 = 0,$$

$$\frac{\partial L}{\partial w_2} = w_2 - \alpha_1 = 0$$

$$\therefore w_1 = \alpha_1 - \alpha_2$$

$$w_2 = \alpha_1$$

Sub w_1, w_2 into $L(w, \alpha)$.

$$\begin{aligned} L(w, \alpha) &= \frac{1}{2} ((\alpha_1 - \alpha_2)^2 + \alpha_1^2) + \alpha_1 (1 - \alpha_1 + \alpha_2 - \alpha_1) + \alpha_2 (1 + \alpha_1 - \alpha_2) \\ &= \frac{1}{2} (2\alpha_1^2 - 2\alpha_1\alpha_2 + \alpha_2^2) + (\alpha_1 - \alpha_1^2 + \alpha_1\alpha_2) + (\alpha_2 + \alpha_1\alpha_2 - \alpha_2^2) \end{aligned}$$

$$\frac{\partial L}{\partial \alpha_1} = 2\alpha_1 - \alpha_2 + 1 - 4\alpha_1 + \alpha_2 + \alpha_2 = -2\alpha_1 + \alpha_2 + 1 = 0$$

$$\frac{\partial L}{\partial \alpha_2} = -\alpha_1 + \alpha_2 + \alpha_1 + 1 + \alpha_1 - 2\alpha_2 = \alpha_1 - \alpha_2 + 1 = 0$$

$$\therefore \alpha_1 = \frac{\alpha_2 + 1}{2}$$

$$\alpha_2 = \alpha_1 + 1$$

$$\therefore \alpha_1 = \frac{\alpha_1 + 2}{2}$$

$$\therefore \alpha_1 = 2, \quad \alpha_2 = 3.$$

$$\text{Since } w_1 = \alpha_1 - \alpha_2$$

$$w_2 = \alpha_1$$

$$\therefore w^* = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$