Problem 4:

a). Since data set D is linear separable, exist a hyperplane $\vec{V}^T\vec{x}+P$ such that:

$$\min_{(\vec{x},y)\in D,\, y=1} (\vec{V}^T\vec{x}+P) \geq 0 > \max_{(\vec{x},y)\in D,\, y=-1} (\vec{V}^T\vec{x}+P)$$

Therefore, let $\vec{x}_i$ be the positive sample which is closet to $\vec{V}^T\vec{x}+P$, let $\vec{x}_j$ be the negative sample which is closet to $\vec{V}^T\vec{x}+P$.

let $P_{pos} = \vec{V}^T\vec{x}_i + P$

$P_{neg} = \vec{V}^T\vec{x}_j + P$

From definition of linear separable, $P_{pos} \geq 0 > P_{neg}$, therefore, exist $\eta \geq 0$ s.t $P_{pos} - \eta \geq P_{neg} - \eta$.

For some value of $\eta$, we have $\vec{V}^T\vec{x}+P-\eta=0$ separate D for both $\vec{x}_i$ and $\vec{x}_j$, and the distance from $\vec{x}_i$ and $\vec{x}_j$ to $\vec{V}^T\vec{x}+P-\eta=0$ is the same. So we have:

$$\frac{|\vec{V}^T\vec{x}_i+P-\eta|}{\|\vec{V}\|} = \frac{|\vec{V}^T\vec{x}_j+P-\eta|}{\|\vec{V}\|}$$

$$P_{pos}-\eta = -(P_{neg}-\eta)$$

$$\eta = \frac{P_{pos}-P_{neg}}{2}$$

With $\min_{(\vec{x},y)\in D,\, y=1} (\vec{V}^T\vec{x}+P-\eta) = \frac{P_{pos}-P_{neg}}{2}$

$$\max_{(\vec{x},y)\in D,\, y=-1} (\vec{V}^T\vec{x}+P-\eta) = -\frac{P_{pos}-P_{neg}}{2}$$

Therefor, $y(\vec{V}^T\vec{x}+P-\eta) \geq \frac{P_{pos}-P_{neg}}{2}$ for all $(\vec{x},y)\in D$

Since $P_{pos} > P_{neg}$, $\eta = \frac{P_{pos}+P_{neg}}{2}$, it becomes:

$$y(\vec{w}^T\vec{x}+\theta) \geq 1-\delta, \ \forall(\vec{x},y)\in D$$

where $\vec{w} = \frac{\vec{V}}{\eta}$, $\theta = \frac{P-\eta}{\eta}$, and $\delta$ with optimal solution $\delta=0$.