Exploring boundaries of privacy in e2e verifiable e-voting systems.

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Strict privacy and Privacy

In this modelling, the election system involves five types of entities, the voters $V_1, \ldots, V_n$, possibly equipped with the voting supporting device (VSD) and the auditing supporting device (ASD), the election authority (EA), the vote collector (VC), the trustee (T), and the bulletin board (BB) whose only role is to provide storage for the election transcript for the purpose of verification.

(1) BB is completely passive and is only writeable by the VC and readable by anyone.

(2) Voters submit their votes by engaging in the ballot casting protocol to the VC and they are not allowed to interact with each other.

(3) VC only role is to collect votes and write them to BB

(4) T is responsible for computing the tally and announcing the election result.

(5) EA - prepares all the election setup information and distributes the voters' ballots.

Protocols An e-voting system Π is a quintuple of algorithms and protocols **(Setup, Cast, Tally, Result, Verify)** specified as follows:

(1) The algorithm **Setup** is executed by the EA and T separately. During the setup phase EA generates Π's public parameters Pub (which include P, V, U) and the voters' secrets $s_1, \ldots, s_n$. The part of the algorithm during which EA distributes secrets among voters is defined as **Registration**. At the same time, T generates pre-election BB data and posts it on BB.

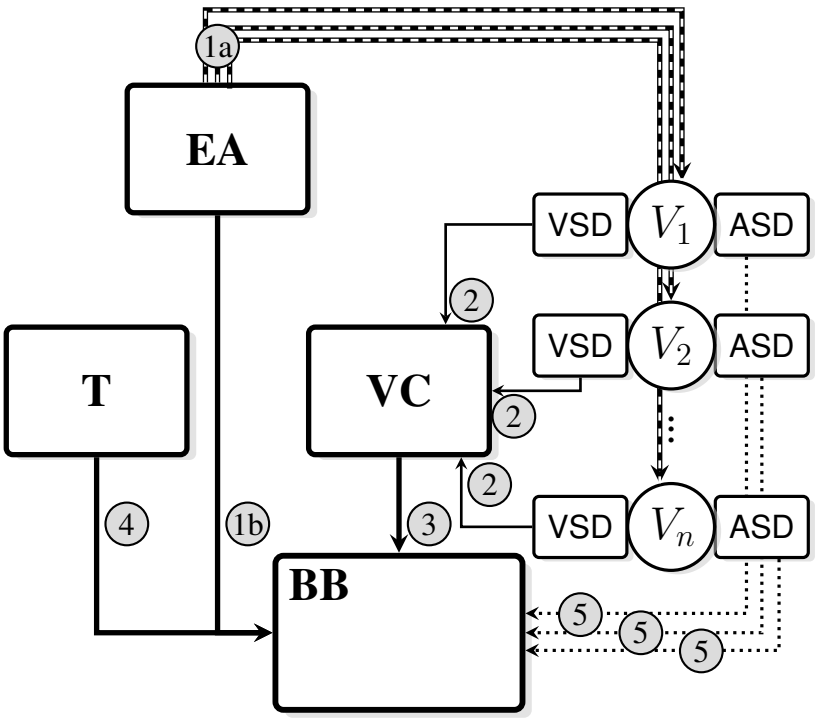


FIGURE 1: The interaction among the entities in an e-voting execution. The dotted lines denote read-only access to the BB. The dashed arrows denote channels for voters' private inputs distribution. Annotation: (1a): distribution of voter's private inputs; (1b): pre-election BB data; (2): vote casting; (3): writing votes to BB; (4) post-election BB data; (5): auditing.

(2) The interactive protocol **Cast** is executed between three parties, the voter V_l, the BB and the VC. During this interaction, the voter uses VSD, his secret s_l and an option U_l to generate the ballot b_l and sends this ballot to VC. Upon successful termination, VC posts ballot b_l to BB and the voter V_l receives a receipt α_l.

(3) The algorithm **Result** is executed by T and outputs the result τ for the election or returns $\perp$ in case such result is undefined.

(4) The algorithm **Verify** outputs a value in $\{0, 1\}$, where α is a voter receipt (that corresponds to the voter's output from the **Cast** protocol).

In some e-voting systems **Registration** part is omitted and voters are expected to receive their credentials via secure channel such as post mail, pulling place etc.

Correctness

A system Π has (perfect) correctness, if for any honest execution of any subset of not abstained voters that results in a public transcript τ, where the voters $V_1, ..., V_n$ cast votes for options $U_1, ..., U_n$, it holds that $Result(\tau) = f(U_1, ..., U_n)$, where $f(U_1, ..., U_n)$ is the m-vector whose i-th location is equal to the number of times a candidate $P_i \in \{P_1, \ldots, P_m\}$ was chosen in the candidate selections $U_1, ..., U_n$.

Privacy with respect to entities

EA only:

If EA is trusted and all other entities $(T, \mathrm{VSD}, \mathrm{VC})$ are corrupted (BB is completely passive and simply posts all information received from VC) it is impossible to preserve voter's privacy.

Proof:

Suppose there is some e-voting system Π which doesn't allow any adversary to learn anything about individual voters' intents in case when only EA is trusted. $\mathcal{A}$ strategy is to calculate tally after every vote submission and compare it with the previous tally to learn the voter's intent. Due to a correctness, Π should be able to produce meaningful election tally for any turnout, including just one voter.

Since an adversary $\mathcal{A}$ controls a trustee T, he should be able to compute results after very first cast procedure is done. This means that either $\mathcal{A}$ may learn voters' choices one by one or Π fails to produce results for any subset of abstained voters and therefore is not correct.

EA + VC
It's not enough for privacy, an adversary can perform exactly the same attack as for 'EA only case'
EA + T
Private. Example: Demos
EA + VSD
In such case Π can be private, since it's enough to have a trusted VSD for preserving voters' privacy.

VSD only:

VSD is trusted, all other entities (T, EA, VC) are controlled by an adversary. BB is completely passive and simply posts all information received from VC.

Consider the following e-voting system Π:

(1) EA generates a large random number R and commitments bases g, h. Also EA picks a random id x_i for every candidate $cand_i$. $R, g, h, \{cand_i, x_i\}$ - Π's public information.

(2) each voter submits vis VSD a perfectly hiding commitment $g^{x_i}h^{r_l}$, where r_l is a random number less than R picked by VSD.

(3) after the last voter cast his vote, VSD submits a commitment $g^{x_{abs}}h^{r_{vsd}}$, where x_{abs} corresponds to the abstain option and $r_{vsd} = R - \sum_{l=0}^{l=n} r_l$, n - number of voters.

(4) T computes the product of all hiding commitments and opens the result.

In order to break voters' privacy, an adversary has to break perfectly hiding commitments.

VC only:

Impossible. An adversary can perform exactly the same attack as for 'EA only case.

VC + T

Due to the assumption that voters are not able to collaborate for protecting their privacy and all complex math operation performed by VSD or EA, it's impossible to have privacy in this case. A voter provides VSD with an input which is either in a plain text or calculated by EA line and therefore immediately gives away his choice.

VC + VSD

In such case Π can be private, since it's enough to have a trusted VSD for preserving voters' privacy.

VC + EA

Impossible. See the EA + VC case for details.

T only:

Impossible. Same reasoning as VC + T case.

T + VC

seems meaningless

T + VSD

In such case Π can be private, since it's enough to have a trusted VSD for preserving voters' privacy.

T + EA

Private. See the EA + T case for details.

Strict privacy

Only EA and T honest: $G^{\mathcal{A},Sim}_{strict,\mathrm{EA},T}(1^\lambda)$

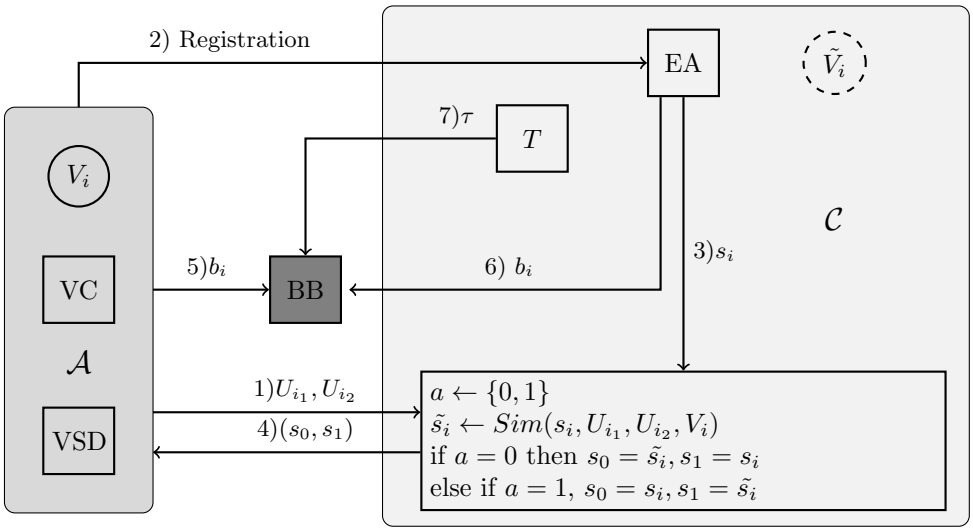


FIGURE 2: $G^{\mathcal{A},Sim}_{strict,\mathrm{EA},T}(1^\lambda)$

In the game defined above, an adversary $\mathcal{A}$ interacts with the challenger $\mathcal{C}$ on behalf of all voters, VC and VSD. $\mathcal{C}$ plays the role of EA and T. BB is completely passive and represents a publicly viewed database.

After an adversary $\mathcal{A}$ defined an election parameters, $\mathcal{C}$ doubles the voters, computes setup data and starts the election. This duplication of voters is needed because $\mathcal{C}$ would post another ballot for every adversarial's cast ballot. If $\mathcal{A}$ abstains, $\mathcal{C}$ abstains as well. During the game, we will refer to duplicated voters as fake ones.

$\mathcal{A}$ 1) picks and sends two options U_{i_1}, U_{i_2} to the challenger $\mathcal{C}$. One of the options U_{i_1}, U_{i_2} is his intent, the other – the option that the challenger would use in order to produce an indistinguishable from the intent's ballot and receipt view*. After sending options, 2) $\mathcal{A}$ schedules the **Registration** protocol with $\mathcal{C}$ on behalf of some voter V_i**. $\mathcal{C}$ creates a fake credentials $\tilde{s_i}$ and 3) generates real credentials s_i for him. 4) $\mathcal{C}$ responds $\mathcal{A}$ with a pair of credentials s_0, s_1, where one of

the credentials are real and the other were generated using the simulator Sim in a such way, that no matter what credential s_0 or s_1 $\mathcal{A}$ uses to cast a vote for U_{i_1}, the produced ballot would also correspond to the result of the **Cast** protocol for an option U_{i_2} with the other credentials and vice versa. If 5) $\mathcal{A}$ choses to post the ballot b_i to BB, 6) $\mathcal{C}$ posts exactly the same ballot to BB on behalf of the fake voter. When $\mathcal{A}$ stops the election, 7) $\mathcal{C}$ posts the tally τ ***.

Remarks:
*If $\mathcal{C}$ succeeds, $\mathcal{A}$ wouldn't be able to say whether his ballot and receipt corresponds to U_{i_1} or U_{i_2} and BB would contain both ballots (one for the option U_{i_1} ,the other for the option U_{i_2}) so the tally wouldn't reveal any information. Example: if $\mathcal{A}$ picks real credential for the voter V_i then he casts a vote for the option U_{i_1} , at the same time exactly the same ballot posted $\mathcal{C}$ on behalf of the fake voter $\tilde{V}_i$ would correspond to the fake credentials and the option U_{i_2}. In this case $\mathcal{A}$ voted for U_{i_1}. Else if $\mathcal{A}$ picks fake credentials and tries to vote for the option U_{i_1}, his ballot corresponds to the real credentials and the option U_{i_1}. At the same time, exactly the same ballot posted $\mathcal{C}$ on behalf of the fake voter $\tilde{V}_i$ would correspond to the fake credentials and the option U_{i_1}. So $\mathcal{A}$ voted for U_{i_1}. If this definition holds, $\mathcal{A}$ has no idea whom he voted for.
Registration on behalf of a voter mans that $\mathcal{A}$ is the one who runs the **Registration protocol, even if it's just picking an envelop with paper ballots.
*** The tally τ is posted only if all ballots are correctly formed: were produced with one of the provided credentials, for the candidate U_{i_1}, for every cast ballot there is a duplicate, etc.

$G^{\mathcal{A},Sim}_{strict,\mathrm{EA},T}(1^\lambda)$ defined as follows:

(1) During the game $\mathcal{C}$ plays the role of EA and T. $\mathcal{A}$ operates on behalf of the all voters, VSD and VC.

(2) $\mathcal{A}$ defines a set of voters $\mathcal{V} = \{V_1, ..., V_n\}$, a list of candidates $\mathcal{P} = \{P_1, ..., P_m\}$, a set of allowed candidates' selections $\mathcal{U}$. It provides $\mathcal{C}$ with $\mathcal{V}, \mathcal{P}, \mathcal{U}$.

(3) $\mathcal{C}$ doubles the set $\mathcal{V}$ adding fake voters $\{V'_1, ..., V'_n\}$ and starts the election on behalf of EA. Also, $\mathcal{C}$ flips a coin $a \leftarrow \{0, 1\}$ to define an order according to which real and simulated credentials would be returned to $\mathcal{A}$.

(4) The adversary $\mathcal{A}$ picks two option $U_{i_1}, U_{i_2} \in \mathcal{U}$, where U_{i_1} is an option for the real credentials and U_{i_2} is an option for the fake ones. After that, $\mathcal{A}$ and $\mathcal{C}$ engage in an interaction where $\mathcal{A}$ schedules the **Registration** protocols on behalf of all voters. For each voter $V_i \in \mathcal{V}$:

– $\mathcal{C}$ picks a fake voter $\tilde{V}_i$ and generates credentials $\tilde{s}_i$ for him using Sim*. $\mathcal{C}$ responds $\mathcal{A}$ with a pair of simulated and real credentials (s_0, s_1) in order defined by the coin a:

$$\begin{cases} \text{if } \ a = 0, \ \ (s_0, s_1) = (\tilde{s}_i, s_i) \\ \text{else } \ (s_0, s_1) = (s_i, \tilde{s}_i) \end{cases}$$

– Using one of the credentials $\mathcal{A}$ schedules the **Cast** protocol executions and sends the produced ballot to $\mathcal{C}$.

– If $\mathcal{A}$ posts a ballot to BB, $\mathcal{C}$ posts exactly the same ballot in the entry that corresponds to the fake voter $\tilde{V}_i$

(5) $\mathcal{C}$ executes the $Tally$ protocol.

(6) Finally, $\mathcal{A}$ using all information collected above (including the contents of the BB) outputs a bit a^*

(7) The game returns a bit which is 1 if $a = a^*$ and 0 otherwise.

***Remark:**
Sim works in a way that the fake credentials satisfy both of the following rules:

(1) The real credentials and U_{i_1} option should give ballot and receipt, which are identical* to ballot and receipt produced for the fake credentials and U_{i_2} option.

(2) The fake credentials and U_{i_1} option should give ballot and receipt, which are identical to ballot and receipt produced for the real credentials and U_{i_2} option.

* indistinguishable ?

Strict privacy: EA **and** T **are honest:**

The e-voting system Π achieves strict voter privacy in case of trusted EA and T, if there is a PPT simulator Sim such that for any PPT adversary $\mathcal{A}$:

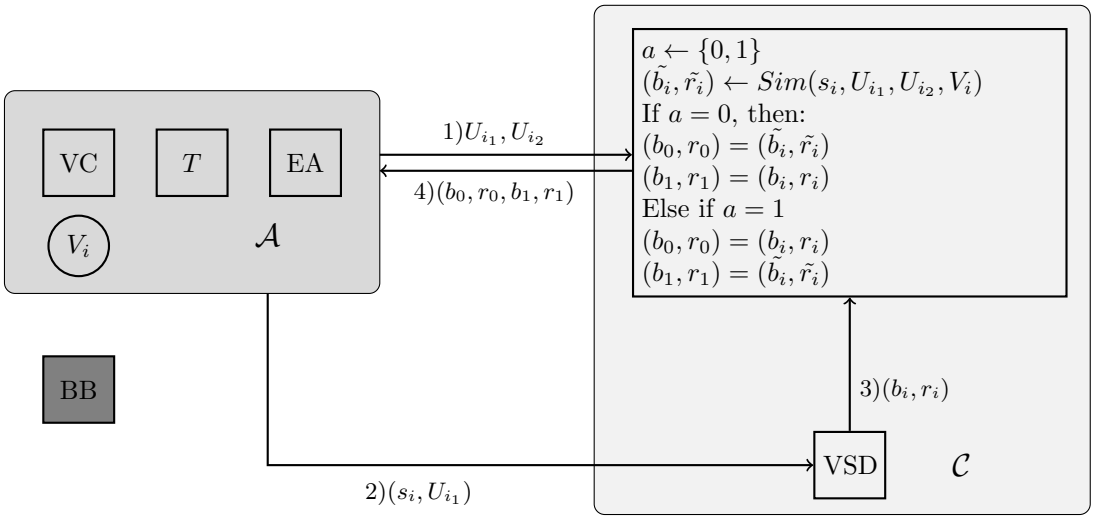


FIGURE 3: $G^{\mathcal{A},Sim}_{strict,\mathrm{VSD}}(1^\lambda)$

$|\Pr[G^{\mathcal{A},Sim}_{strict,\mathrm{EA},T}(1^\lambda) = 1] - \frac{1}{2}| = negl(\lambda)$

Only VSD is honest: $G^{\mathcal{A},Sim}_{strict,\mathrm{VSD}}(1^\lambda)$

In the game defined above, $\mathcal{A}$ operates on behalf of the all voters and all corrupted entities, such as: EA, VC and T. BB is completely passive and represents a publicly accessible database.
1) an adversary $\mathcal{A}$ picks and sends options U_{i_1}, U_{i_2} to the challenger $\mathcal{C}$. After that 2) $\mathcal{A}$ schedules the **Cast** protocol with $\mathcal{C}$ on behalf of some voter V_i. $\mathcal{C}$ 3) generates a real ballot and receipt and uses Sim to create a fake ones. At the end 4) $\mathcal{C}$ responses with a pair of ballots and receipts b_0, r_0, b_1, r_1, where one ballot and receipt corresponds to an option U_{i_1}, the other was generated using the simulator Sim for an option U_{i_2}.

The game $G^{\mathcal{A},Sim}_{strict,\mathrm{VSD}}(1^\lambda)$ is defined as follows:

(1) During the game $\mathcal{C}$ plays the role of VSD. $\mathcal{A}$ operates on behalf of the all

voters and all corrupted entities, such as: EA, VC and T.

(2) $\mathcal{A}$ defines a set of voters $\mathcal{V} = \{V_1, ..., V_n\}$, a list of candidates $\mathcal{P} = \{P_1, ..., P_m\}$, a set of allowed candidates' selections $\mathcal{U}$ and starts the election.

(3) $\mathcal{C}$ flips a coin $a \leftarrow \{0, 1\}$ to define an order according to which real and simulated ballots and receipts would be returned to $\mathcal{A}$.

(4) The adversary A and the challenger $\mathcal{C}$ engages in an interaction where $\mathcal{A}$ schedules the Cast protocols on behalf of all voters. For each voter $V_i \in \mathcal{V}$:

– $\mathcal{A}$ sends to $\mathcal{C}$ options U_{i_1}, U_{i_2} and starts with $\mathcal{C}$ the **Cast** protocol on behalf of V_i. As a result of the protocol execution $\mathcal{C}$ provides $\mathcal{A}$ with a pair of simulated and real ballot and receipt $(b_0, r_0)(b_1, r_1)$ s.t.:
$$\begin{cases} \text{if } \ a = 0, \ \ (b_0, r_0) = (\tilde{b_i}, \tilde{r_i}) \ \text{ and } \ (b_1, r_1) = (b_i, r_i) \\ \text{else } \ (b_0, r_0) = (b_i, r_i) \ \text{ and } \ (b_1, r_1) = (\tilde{b_i}, \tilde{r_i}) \end{cases}$$
where the pair (b_i, r_i) is the ballot and receipt for an adversarial option U_{i_1} and $(\tilde{b_i}, \tilde{r_i})$ is the ballot and receipt for U_{i_2} option generated via the simulator Sim.

(5) Finally, $\mathcal{A}$ executes the $Tally$ protocol and using all information collected above outputs a bit a^*

(6) The game returns a bit which is 1 if $a = a^*$ and 0 otherwise

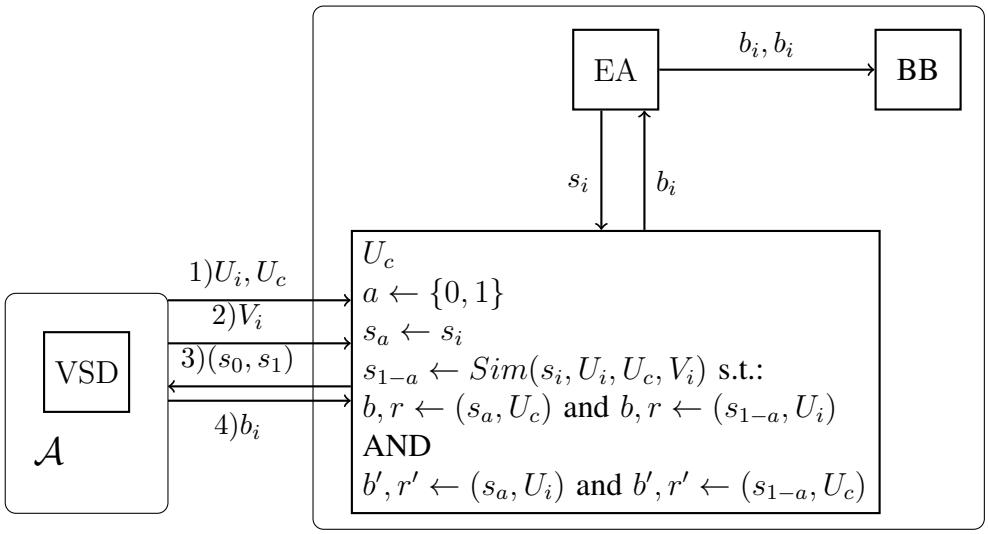
Strict privacy: VSD **is honest:**

The e-voting system Π achieves strict voter privacy in case when VSD is trusted, if there is a PPT simulator Sim such that for any PPT adversary $\mathcal{A}$:

$|\Pr[G^{\mathcal{A},Sim}_{strict,\mathrm{VSD}}(1^\lambda) = 1] - \frac{1}{2}| = negl(\lambda)$

Privacy

If EA is honest but VSD is corrupted:$G^{\mathcal{A},Sim}_{t-priv,\mathrm{EA}}(1^\lambda)$



In the game defined above, an adversary $\mathcal{A}$ picks and sends options U_i, U_c to the challenger $\mathcal{C}$. After that $\mathcal{A}$ schedules the $Registration$ protocol with $\mathcal{C}$ for some voter V_i, who at the end forwards $\mathcal{C}$'s response. $\mathcal{C}$ responses with a pair of credentials s_0, s_1, where one of the credentials belongs to the voter V_i, the other was generated for the fake voter V_i' via the simulator Sim in a such way, that no matter what credential s_0 or s_1 $\mathcal{A}$ uses to cast a vote for U_i, the produced ballot would also correspond to the result of the $Cast$ protocol for an option U_c with the other credentials.

$G^{\mathcal{A},Sim}_{t-priv,\mathrm{EA}}(1^\lambda)$defined as follows:

(1) During the game $\mathcal{C}$ plays the role of honest voters, EA and BB. $\mathcal{A}$ operates on behalf of corrupted voters and VSD.

(2) $\mathcal{A}$ defines a set of voters $\mathcal{V} = \{V_1, ..., V_n\}$, a list of candidates $\mathcal{P} = \{P_1, ..., P_m\}$, a set of allowed candidates' selections $\mathcal{U}$. It provides $\mathcal{C}$ with $\mathcal{V}, \mathcal{P}, \mathcal{U}$.

(3) $\mathcal{C}$ doubles the set $\mathcal{V}$ adding fake voters $\{V_1', ..., V_n'\}$ and starts the election on behalf of EA. Also, $\mathcal{C}$ flips a coin $a \leftarrow \{0,1\}$ to define an order according to which real and simulated credentials would be returned to $\mathcal{A}$.

(4) The adversary $\mathcal{A}$ picks two option $U_i, U_c \in \mathcal{U}$, where U_i is his intent and U_c is an option for a challenger's fake voter. After that, $\mathcal{A}$ and $\mathcal{C}$ engage in an interaction where $\mathcal{A}$ schedules the $Registration$ protocols, during which

all voters receive their credentials and forward them to $\mathcal{A}$. For each voter $V_i \in \mathcal{V}$, the adversary chooses whether V_i is corrupted:

– If V_i is corrupted, then $\mathcal{C}$ provides $\mathcal{A}$ with the real credentials s_i, and then they engage in a $Cast$ protocol where A vote on behalf of V_i and $\mathcal{C}$ plays the role of EA and BB.

– If V_i is not corrupted, then $\mathcal{C}$ provides $\mathcal{A}$ with a pair of simulated and real credentials (s_0, s_1) generated via Sim s.t.:
$$\begin{cases} \text{if } \ a = 0, \ \ (s_0, s_1) = (\tilde{s_i}, s_i) \\ \text{else } \ (s_0, s_1) = (s_i, \tilde{s_i}) \end{cases}$$
where s_i - real credentials produced by $\mathcal{C}$ on behalf of EA for the voter V_i and $\tilde{s_i}$ are simulated credentials generated via the simulator Sim for the fake voter V_i'. Fake credentials satisfy both of the following rules:

i. Simulated credentials and option U_c should give ballot and receipt, which are identical to ballot and receipt produced for real credentials and option U_i:
$b, r \leftarrow (\tilde{s_i}, U_c)$ and $b, r \leftarrow (s_i, U_i)$

ii. Ballot and receipt produced for simulated credentials and option U_i should be identical to ones produced for real credentials and an option U_c:
$b', r' \leftarrow (\tilde{s_i}, U_i)$ and $b', r' \leftarrow (s_i, U_c)$

– Honest voters forward both credentials to $\mathcal{A}$

– Using one of the credentials $\mathcal{A}$ schedules the $Cast$ protocol executions and sends the produced ballot to $\mathcal{C}$.

– $\mathcal{C}$ posts the received ballot twice: as a result of the $Cast$ protocol execution for V_i and V_i'.

(5) $\mathcal{C}$ executes the $Tally$ protocol.

(6) Finally, $\mathcal{A}$ using all information collected above (including the contents of the BB) outputs a bit a^*

(7) Denote the set of corrupted voters as $\mathcal{V}_{corr}$ and the set of honest voters as $\tilde{\mathcal{V}} = \mathcal{V} \backslash \mathcal{V}_{corr}$. The game returns a bit which is 1 if and only if the following hold true:

(a) $a = a^*$

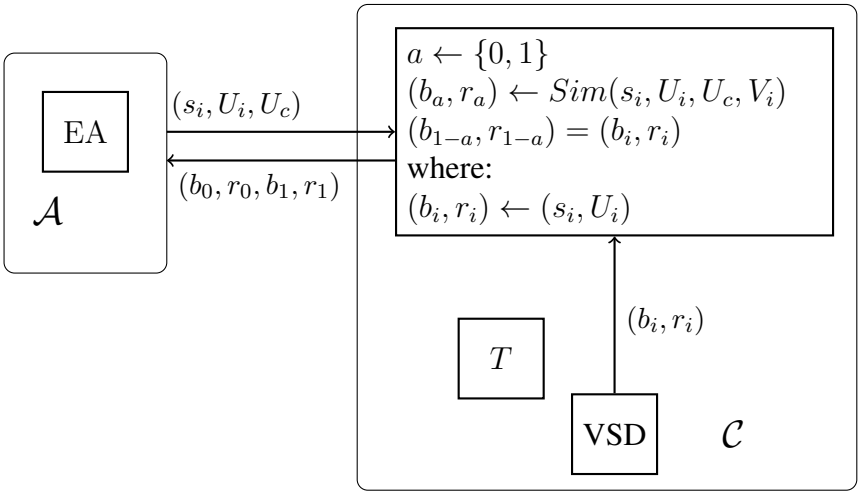
(b) $|\mathcal{V}_{corr}| \leq t$ (i.e., the number of corrupted voters is bounded by t).

Privacy: VSD **is corrupted:**

The e-voting system Π achieves voter privacy in case of corrupted VSD for at most t corrupted voters if there is a PPT simulator Sim such that for any PPT adversary $\mathcal{A}$:

$$|\Pr[G^{\mathcal{A},Sim}_{t-priv,\mathrm{EA}}(1^\lambda) = 1] - \tfrac{1}{2}| = negl(\lambda)$$

If EA is corrupted but VSD is honest: $G^{\mathcal{A},Sim}_{t-priv,\mathrm{VSD},T}(1^\lambda)$



This case similar to the strict privacy with respect to honest VSD and T is meaningful for e-voting, only if there exists some entity, let us call it a trustee T, which must participate in the $Tally$ protocol for an election transcript to be produced. If there is no such trusted entity, than an adversary, who corrupts EA, has all necessary information for breaking everyone's privacy. It happens due to the following e-voting assumption: voters are not expected to collaborate in order to preserve privacy or integrity of an election.

The following definition of strict privacy applies only to cases where EA is corrupted, VSD is honest and there is at lest one trusted entity, which must participate in the $Tally$ protocol for producing an election tally.

In the game defined above, an adversary $\mathcal{A}$ picks and sends an option U_c to the challenger $\mathcal{C}$. After that $\mathcal{A}$ schedules the $Cast$ protocol with $\mathcal{C}$ for some voter V_i, who at the end forwards $\mathcal{C}$'s response. $\mathcal{C}$ responses with a pair of ballots and receipts b_0, r_0, b_1, r_1, where one ballot and receipt corresponds to an option U_i, the other was generated using the simulator Sim for an option U_c.

The game $G^{\mathcal{A},Sim}_{t-priv,\mathrm{VSD},T}(1^\lambda)$ is defined as follows:

(1) During the game $\mathcal{C}$ plays the role of the honest voters, T and VSD. $\mathcal{A}$ operates on behalf of corrupted voters, EA and BB.

(2) $\mathcal{A}$ defines a set of voters $\mathcal{V} = \{V_1, ..., V_n\}$, a list of candidates $\mathcal{P} = \{P_1, ..., P_m\}$, a set of allowed candidates' selections $\mathcal{U}$.

(3) After $\mathcal{A}$ starts the election, $\mathcal{C}$ flips a coin $a \leftarrow \{0, 1\}$ to define an order according to which real and simulated ballots and receipts would be returned to $\mathcal{A}$.

(4) $\mathcal{A}$ picks and sends to $\mathcal{C}$ an option U_c . After that $\mathcal{A}$ and $\mathcal{C}$ engage in an interaction where $\mathcal{A}$ schedules the $Cast$ protocols of all voters which may run concurrently. For each voter $V_i \in \mathcal{V}$, the adversary chooses whether $V_i \in \mathcal{V}$ is corrupted:

– If V_i is corrupted, then $\mathcal{C}$ provides $\mathcal{A}$ with the real ballot and receipt (b_i, r_i).

– If V_i is not corrupted, $\mathcal{C}$ provides $\mathcal{A}$ with a pair of simulated and real ballot and receipt $(b_0, r_0)(b_1, r_1)$ s.t.:

$$\begin{cases} \text{if } \ a = 0, \ \ (b_0, r_0) = (\tilde{b_i}, \tilde{r_i}) \ \text{ and } \ (b_1, r_1) = (b_i, r_i) \\ \text{else } \ (b_0, r_0) = (b_i, r_i) \ \text{ and } \ (b_1, r_1) = (\tilde{b_i}, \tilde{r_i}) \end{cases}$$

where the pair (b_i, r_i) is the ballot and receipt for an adversarial option and $(\tilde{b_i}, \tilde{r_i})$ is the ballot and receipt for an U_c option generated via the simulator Sim.

(5) $\mathcal{A}$ interacts with $\mathcal{C}$ to execute the $Tally$ protocol. During the execution, $\mathcal{C}$ plays the role of the trustee T. $\mathcal{C}$ aborts the protocol if BB doesn't contain **all** generated by $\mathcal{C}$ ballots (including fake ballots).

(6) Finally, $\mathcal{A}$ using all information collected above outputs a bit a^*

(7) Denote the set of corrupted voters as $\mathcal{V}_{corr}$ and the set of honest voters as $\tilde{\mathcal{V}} = \mathcal{V} \backslash \mathcal{V}_{corr}$. The game returns a bit which is 1 if and only if the following hold true:

   (a) $a = a^*$

   (b) $|\mathcal{V}_{corr}| \leq t$ (i.e., the number of corrupted voters is bounded by t).

Privacy: EA **is corrupted:**

The e-voting system Π achieves voter privacy in case of corrupted EA , but trusted T and VSD, for at most t corrupted voters , if there is a PPT simulator Sim such that for any PPT adversary $\mathcal{A}$:

$$|\Pr[G^{\mathcal{A},Sim}_{t-priv,\mathrm{VSD},T}(1^\lambda) = 1] - \tfrac{1}{2}| = negl(\lambda)$$

Strict privacy vs E2E Verifiability

Any system Π that satisfies the definition of strict privacy, is "receipt free" but not E2E Verifiable.
The Real and Ideal executions in this case are shown in the following diagrams:
Security requires that the above two systems are indistinguishable for some simulator program $\mathcal{S}$. Unfortunately, as we are about to show, no such simulator exists.

Proposition:
For any system Π that satisfies the definition of strict privacy, there is an adversary $\mathcal{A}$ such that for any simulator $\mathcal{S}$, there is an environment $\mathcal{Z}$ that distinguishes ideal and real executions $\mathrm{EXEC}^{\mathcal{F},\mathcal{G}_{BB}}_{\mathcal{Z},\mathcal{S}} \not\approx \mathrm{EXEC}^{\Pi,\mathcal{G}_{BB}}_{\mathcal{Z},\mathcal{A}}$

Proof:
PART 1.
Suppose we have a system Π, which is strictly private in case "EA is corrupted, but VSD is honest".
Consider the following attack against E2E Verifiability:

$\mathcal{A}$:
– corrupts EA and VSDs but doesn't corrupt ASDs.
– corrupts t voters $(t \leq n)$, where n is the total number of voters.
– creates fake voters $\{V'_0, V'_1, \ldots, V'_n\}$ and using its power substitutes some part γ of honest voters with fake ones for the U_a option.
– fake voters vote for adversarial options according to a vote-casting procedure.
– substituted honest voters receive receipts generated for fake voters.

let $\mathcal{Z}$ be the environment that works as follows:

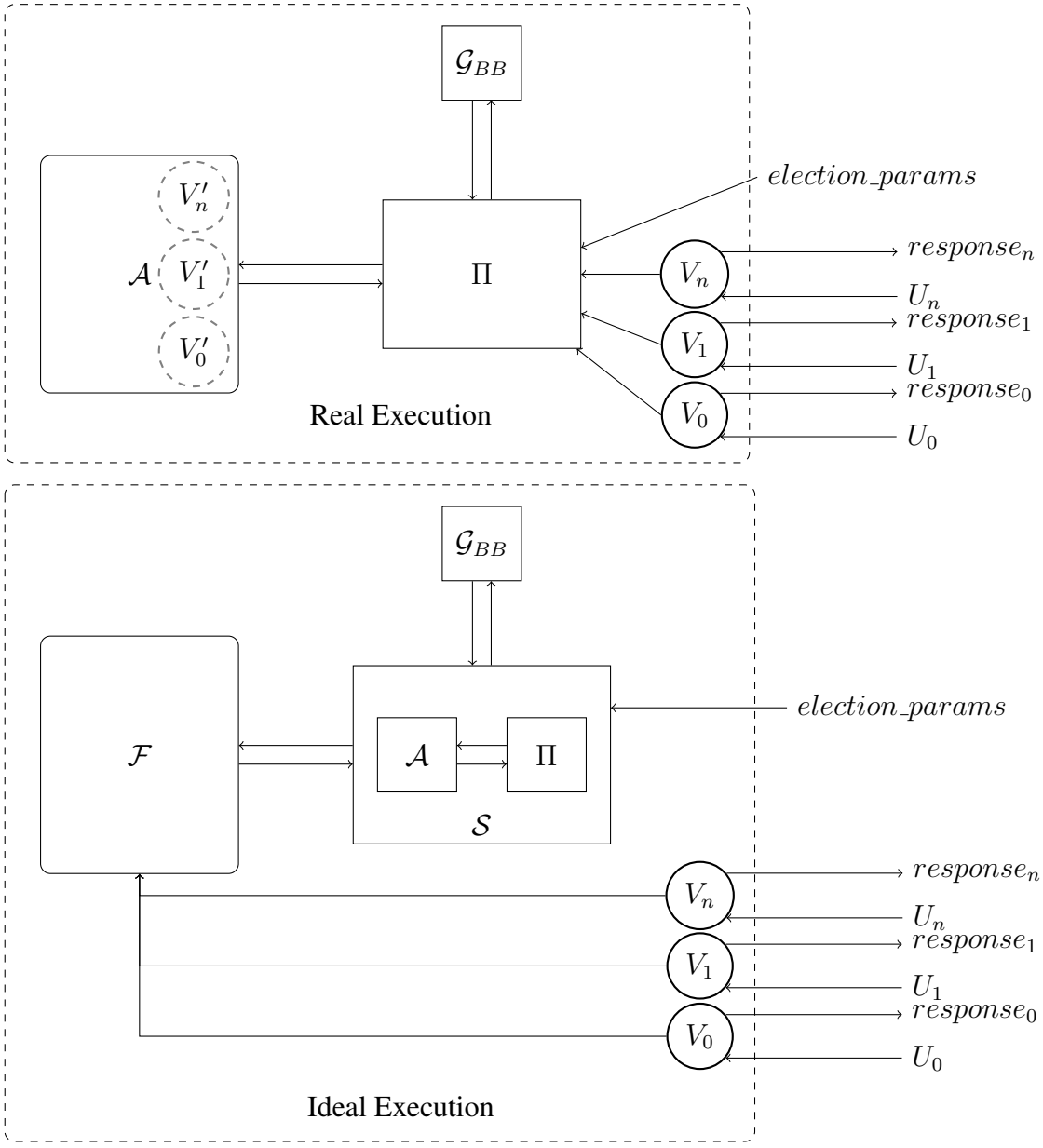


FIGURE 4: Real and Ideal Execution

$\mathcal{Z}$:
– defines an election setup information:
$election\_params = (\mathcal{C}, \mathcal{V}, \mathcal{U}, params)$, where $\mathcal{C}$ - list of candidates, $\mathcal{V}$ - list of voters, $\mathcal{U}$ - list of allowed candidates' selections, $params$ - other required information.
– instructs each voter V_i to vote for the blank '$no\ one$' option.
– stops the vote-casting phase.
– asks $\mathcal{G}_{BB}$ for the election result τ
– asks every voter V_i to verify his choice and return the result of verification - $response_i$, where $response_i$ equal to $(sid, verify\_responce, \tau')$ in case of successful verification and $\perp$ otherwise.
– If the number of successful verification responses equal to the number of voters and in all responses provided by a voter tally τ' is equal to the $\mathcal{G}_{BB}$'s tally τ return 1. Otherwise return 0.

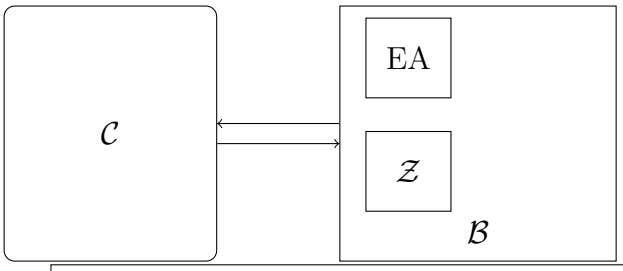
Since Π is strictly private, probability, that an adversary $\mathcal{D}$ engaging in the **Cast** protocol would distinguish real and simulated view and win the attack against the strict privacy $Pr[G^{\mathcal{D},Sim}_{strict,\mathrm{VSD},T}(1^\lambda) = 1] = \frac{1}{2} + \alpha$ where α is negligible. In case of corrupted EA, $\mathcal{C}$ returns a real ballot and receipt pair and a simulated via Sim pair. This means that in a strictly private system an adversary has a negligible chance to distinguish the ballot and receipt for his option and for some U_c option.

During the Real Execution a voter can either accept or reject the cast ballot. Rejection is possible only if he detects that the receipt and ballot are faked. The probability that a voter distinguish the receipt and ballot for his option from for the 'no-one' option and rejects is $\Pr[V_i\ rejects] \leq \alpha$, where α is negligible.

By nature of the attack $\mathcal{A}$, fake receipt is the perfectly valid receipt for the U_a option and the cast ballot would always be successfully verified. Therefore in the Real Execution $\mathcal{Z}$ would get a successful verification response from all n voters and output 1 with the probability $\Pr[\mathrm{EXEC}^{\Pi,\mathcal{G}_{BB}}_{\mathcal{Z},\mathcal{A}} = 1] = \Pr[all\ V_i\ accept] = \prod_{i=0}^{n}(1 - \Pr[V_i\ rejects]) \geq (1-\alpha)^n = 1 - n\alpha = 1 - \beta$, where $\beta = n\alpha$ is negligible. Thus, $\Pr[\mathrm{EXEC}^{\Pi,\mathcal{G}_{BB}}_{\mathcal{Z},\mathcal{A}} = 0] = 1 - \Pr[\mathrm{EXEC}^{\Pi,\mathcal{G}_{BB}}_{\mathcal{Z},\mathcal{A}} = 1] \leq \beta$, where β is negligible.

Suppose for the sake of contradiction that $\Pr[\mathrm{EXEC}^{\Pi,\mathcal{G}_{BB}}_{\mathcal{Z},\mathcal{A}} = 0] \geq \beta$, where β is non-negligible. This means that at least one voter rejects the receipt. I will show that this contradicts the definition of strict privacy. Consider an attacker $\mathcal{B}$

against strict privacy which exploits the environment $\mathcal{Z}$.



$\mathcal{B}$ interacts with the challenger in the strict privacy attack $\mathcal{C}$ as follows:
– $\mathcal{Z}$ defines an election parameters $election\_params$, assign every voter to vote for the blank option $\{V_i, `no\ one'\}$ and send this information to the $\mathcal{B}$.
– $\mathcal{B}$ forwards received $election\_params$ to the EA.
– $\mathcal{B}$ corrupts the EA.
– EA starts the election.
– $\mathcal{B}$ interacts with the challenger $\mathcal{C}$ provides $(V_i, `no\ one', U_{\mathcal{Z}_i})$ as the input.
– $\mathcal{C}$ sends back to the $\mathcal{B}$ real and simulated view $(b_0, r_0), (b_1, r_1)$ in the order defined by the challenger's coin a.
– $\mathcal{B}$ votes on behalf of all voters
– $\mathcal{Z}$ stops the vote-casting phase.
– B computes the election's tally and proof of the tally's correctness.
– $\mathcal{Z}$ asks $\mathcal{G}_{BB}$ for the election result τ
– $\mathcal{Z}$ requests every voter to verify his ballot correctness.
– $\mathcal{B}$ will run the verification on behalf of all voters using ballots and receipts from $\{b_0, r_0\}$.
– $\mathcal{Z}$ outputs 1 or 0 depending on the voters' verdict.
–$\mathcal{B}$ outputs whatever the $\mathcal{Z}$ outputs.

The challenger $\mathcal{C}$ outputs simulated ballot and receipt as (b_0, r_0), when the coin $a = 0$ and as (b_1, r_1) otherwise.

In case when $a = 0, \mathcal{B}$'s behaviour is identical to the $\mathcal{A}$'s strategy and $\mathcal{B}$ wins if $\mathcal{Z}$ outputs 0, which happens if at least one voter rejects the simulated receipt. By assumption $\Pr[\text{EXEC}^{\Pi,\mathcal{G}_{BB}}_{\mathcal{Z},\mathcal{A}} = 0] \geq \beta$, where β is non-negligible. Therefore if (b_0, r_0) is the set of simulated ballot and receipt, $\mathcal{B}$ wins with the probability $Pr[\mathcal{B} \to 0 | a = 0] = \Pr[\text{EXEC}^{\Pi,\mathcal{G}_{BB}}_{\mathcal{Z},\mathcal{A}} = 0] \geq \beta$, where β is non-negligible.

On the other hand, when $a = 1$, $\mathcal{B}$ plays honestly and the probability of $\mathcal{Z}$ out-

putting 1 is equal to probability that all voters successfully verify their votes in the honest execution, which is happens with overwhelming probability $1 - negl(\lambda)$.

The probability of $\mathcal{B}$ winning the attack against the strict privacy is $\Pr[G^{\mathcal{B}}_{strict}(1^\lambda) = 1] = \Pr[\mathcal{B} \to 0 | a = 0] \Pr[a = 0] + \Pr[\mathcal{B} \to 1 | a = 1] \Pr[a = 1] = \Pr[\mathrm{EXEC}^{\Pi, \mathcal{G}_{BB}}_{\mathcal{Z}, \mathcal{A}} = 0] \Pr[a = 0] + \Pr[\mathrm{EXEC}^{\Pi, \mathcal{G}_{BB}}_{\mathcal{Z}, honest} = 1] \Pr[a = 1] \geq \frac{1}{2}\beta + \frac{1}{2} - negl(\lambda)$, where β is not negligible. This implies that $\mathcal{B}$ wins the attack against strict privacy with the probability more than $\frac{1}{2} + negl(\lambda)$, which contradicts the assumption that the Π is strictly private.

In the Ideal Execution any simulator S can either:
1) post in $\mathcal{G}_{BB}$ the tally τ' generated by $\mathcal{A}$ or any other tally $\tau' \neq \tau$
or
2) ignore the $\mathcal{A}$'s tally and post the actual tally τ.

In the first case, the ideal functionality for E2E verifiability $\mathcal{F}$ would always detect the tally deviation caused by $\mathcal{A}$ if such exists. And since $\mathcal{A}$ doesn't corrupt ASDs, for all honest voters the ideal functionality $\mathcal{F}$ would block verification responses. This implies that in the Ideal Execution $\mathrm{EXEC}^{\mathcal{F}, \mathcal{G}_{BB}}_{\mathcal{Z}, \mathcal{S}}$ $\mathcal{Z}$ would get no response from honest voters. The total number of successful verifications would be equal to the number of corrupted voters, which is less (if not voters are corrupted) than the total number of voters – $\mathcal{Z}$ outputs 0.

In the second case, there exists a class of simulators which ignore $\mathcal{A}'$ actions and post the actual tally. For those simulators consider an modified environment $\tilde{\mathcal{Z}}$ that works as follows:

$\tilde{\mathcal{Z}}$:
Outputs a bit according to the following rules:
$\begin{cases} \text{if } \mathcal{Z} \text{ outputs 1 and the number of non-blank votes greater or equal } \gamma \text{– output 1} \\ \text{else output 0} \end{cases}$.

$\tilde{\mathcal{Z}}$ would still output 1 in case of the real execution since the number of non-blank votes would be at least γ due to successful attack $\mathcal{A}$. However $\tilde{\mathcal{Z}}$ would not find at least γ non-blank votes and output 0 in the ideal execution.

Thus, there is the attacker $\mathcal{A}$ such that for any simulator $\mathcal{S}$ there is the environment $\tilde{\mathcal{Z}}$ or $\mathcal{Z}$ which can always distinguish real and ideal executions.

PART 2.
Suppose we have a system Π', which is strictly private in case "EA is honest, but VSD is corrupted".
Consider the following attack against E2E Verifiability:

$\mathcal{A}'$:
– corrupts EA and VSDs but doesn't corrupt ASDs.
– chooses an option $U_a \in \mathcal{U}$
– corrupts t voters $(t \leq n)$, where n is the total number of voters.
– provides every honest voter V_i with fake credentials s'_i s.t. the ballot and receipt produced using $(s'_i, `no\ one')$ are identical to the ballot and receipt produced using real credentials for an option $U_a$: $b, r \leftarrow (s'_i, `no\ one')$ AND $b, r \leftarrow (s_i, U_a)$
– creates fake voters $\{V'_0, V'_1, \ldots, V'_n\}$ and provides them with real credentials s_i
– using its power substitutes some part γ of honest voters' cast protocols with the fake voters' protocols for the U_a option.

let $\mathcal{Z}'$ be the environment that works as follows:

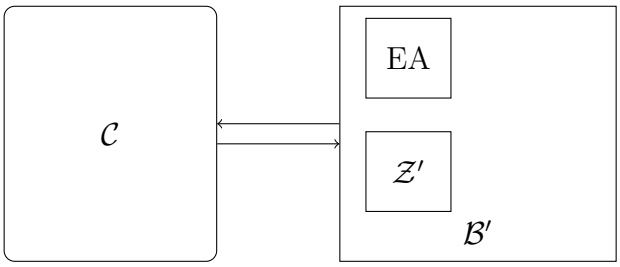
$\mathcal{Z}'$:
– defines an election setup information:
$election\_params = (\mathcal{C}, \mathcal{V}, \mathcal{U}, params)$, where $\mathcal{C}$ - list of candidates, $\mathcal{V}$ - list of voters, $\mathcal{U}$ - list of allowed candidates' selections, $params$ - other required information.
– instructs each voter V_i to vote for the blank option ('no one').
– stops the vote-casting phase.
– asks $\mathcal{G}_{BB}$ for the election result τ
– asks every voter V_i to verify his choice and return the result of verification - $responce_i$, where $responce_i$ equal to $(sid, verify\_responce, \tau')$ in case of successful verification and $\perp$ otherwise.
– If the number of successful verification responses equal to the number of voters and in all responses provided by a voter tally τ' is equal to the $\mathcal{G}_{BB}$'s tally τ return 1. Otherwise return 0.

Since Π' is strictly private, probability, that an adversary engaging in the **Registration** protocol would distinguish real and simulated view and win the attack against the strict privacy $|\Pr[G^{\mathcal{A},Sim}_{strict,\mathrm{EA}}(1^\lambda) = 1] - \frac{1}{2}| = \alpha$ where α is negligible. In case "EA is honest, but VSD is corrupted", simulated view is credentials $\tilde{s}_i$

generated via the simulator Sim.

In the Real Execution a voter V_i starts the Cast protocol using fake credentials s_i' and an option '$no\ one'$. However, $\mathcal{A}'$ substitutes his ballot and receipt with the ballot and receipt generated for a fake voter with real credentials and U_a option. By nature of the attack, the returned receipt generated for real credentials and the U_a option is identical to the receipt produced for a fake credentials and '$no\ one'$ option. Therefore in the Real Execution $\mathcal{Z}'$ would output 1 if non of the voters would detect that he was given a receipt for fake credentials. The probability of this event is $\Pr[\text{EXEC}^{\Pi',\mathcal{G}_{BB}}_{\mathcal{Z}',\mathcal{A}'} = 1] = \Pr[all\ V_i\ accept] = \prod_{i=0}^{n}(1 - \Pr[V_i\ rejects])$. Suppose, the probability of rejection by V_i is $\Pr[V_i\ rejects] = \zeta$. Thus, $\Pr[\text{EXEC}^{\Pi',\mathcal{G}_{BB}}_{\mathcal{Z}',\mathcal{A}'} = 1] = (1-\zeta)^n = 1 - n\zeta = 1 - \zeta'$.

Suppose that ζ' is not negligible. This means that $\Pr[\text{EXEC}^{\Pi',\mathcal{G}_{BB}}_{\mathcal{Z}',\mathcal{A}'} = 0] = 1 - \Pr[\text{EXEC}^{\Pi',\mathcal{G}_{BB}}_{\mathcal{Z}',\mathcal{A}'} = 1] = \zeta'$, where ζ' is not negligible. This means that at least one voter rejects the receipt with a non-negligible probability. We will show that this contradicts the definition of strict privacy. Consider an attacker $\mathcal{B}'$ against strict privacy which exploits the environment $\mathcal{Z}'$.



$\mathcal{B}'$ interacts with the challenger in the strict privacy attack $\mathcal{C}$ as follows:
– $\mathcal{Z}$ defines an election parameters $election\_params$, assign each voter the blank option $\{V_i, `no\ one'\}$ and send this information to the $\mathcal{B}$.
– $\mathcal{B}'$ forwards received $election\_params$ to the $\mathcal{C}$.
– $\mathcal{B}'$ corrupts the VSD.
– $\mathcal{C}$ starts the election.
– $\mathcal{B}'$ sends to $\mathcal{C}$ $(V_i, `no\ one', U_a)$.
– $\mathcal{C}$ sends back s_0, s_1.
– $\mathcal{B}'$ provides voters with credentials s_0 and uses $(s_0, `no\ one')$ to produce the ballot and receipt and posts the result on BB.
– $\mathcal{Z}'$ stops the vote-casting phase.
– $\mathcal{C}$ executes the $Tally$ protocol.
– $\mathcal{Z}'$ asks $\mathcal{G}_{BB}$ for the election result τ
– $\mathcal{Z}'$ requests every voter to verify his ballot correctness.
– $\mathcal{B}'$ will run the verification on behalf of all voters.
– $\mathcal{Z}'$ outputs 1 or 0 depending on the voters' verdict.
– $\mathcal{B}'$ outputs whatever the $\mathcal{Z}'$ outputs.

In case when the simulated credentials are chosen ($a = 0$), $\mathcal{B}'$ provides a voter with fake credentials and generated ballot and receipt for the blank option using the fake credentials, which means that real credentials correspond to the U_a option. $\mathcal{B}'$'s behaviour is identical to the $\mathcal{A}'$'s strategy. $\mathcal{B}'$ wins if outputs 0, which happens if $\mathcal{Z}'$ outputs 0. $\Pr[\mathcal{B}' \to 0 | a = 0] = \Pr[\mathrm{EXEC}^{\Pi', \mathcal{G}_{BB}}_{\mathcal{Z}', \mathcal{A}'} = 0] = \zeta'$

Else if $a = 1$, $\mathcal{B}'$ provides a voter with real credentials and uses the real credentials to vote for the blank option, which is honest behaviour. $\Pr[B' \to 1 | a = 1] = \Pr[\mathrm{EXEC}^{\Pi', \mathcal{G}_{BB}}_{\mathcal{Z}', honest} = 1] = 1$

Thus, the probability of $\mathcal{B}'$ winning the attack against the strict privacy is $\Pr[G^{\mathcal{B}, Sim}_{strict, \mathrm{EA}}(1^\lambda) = 1] = \frac{1}{2}(\Pr[B' \to 0 | a = 0] + \Pr[B' \to 1 | a = 1]) = \frac{1}{2}(\zeta' + 1) = \frac{1}{2} + \frac{1}{2}\zeta'$, where ζ' is not negligible. This implies that $\mathcal{B}'$ wins the attack against strict privacy with the probability more than $\frac{1}{2} + negl(\lambda)$, which contradicts the assumption that the Π is strictly private.

In the Ideal Execution any simulator S can either:
1) post in $\mathcal{G}_{BB}$ the tally τ' generated by $\mathcal{A}'$ or any other tally $\tau' \neq \tau$
or

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In the first case, the ideal functionality for E2E verifiability $\mathcal{F}$ would always detect the tally deviation caused by $\mathcal{A}'$ if such exists. And since $\mathcal{A}'$ doesn't corrupt ASDs, for all honest voters the ideal functionality $\mathcal{F}$ would block verification responses. This implies that in the Ideal Execution $\mathrm{EXEC}^{\mathcal{F},\mathcal{G}_{BB}}_{\mathcal{Z}',\mathcal{S}}\ \mathcal{Z}'$ would get no response from honest voters. The total number of successful verifications would be equal to the number of corrupted voters, which is less (if not voters are corrupted) than the total number of voters – $\mathcal{Z}'$ outputs 0.

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$\tilde{\mathcal{Z}}'$ would still output 1 in case of the real execution since the number of non-blank votes would be at least γ due to successful attack $\mathcal{A}$. However $\tilde{\mathcal{Z}}'$ would not find at least γ non-blank votes and output 0 in the ideal execution.

Thus, there is the attacker $\mathcal{A}'$ such that for any simulator $\mathcal{S}$ there is the environment $\tilde{\mathcal{Z}}'$ or $\mathcal{Z}'$ which can always distinguish real and ideal executions.