Parallel Circuits

Prepared by

Dr. Mohammad Abdul Mannan

Professor, Department of EEE

American International University – Bangladesh (AIUB)

Admittance

The reciprocal of impedance is called **admittance**.

The unit of admittance is **Siemens** [S].

$$\boldsymbol{Y} = \frac{1}{\boldsymbol{Z}} = \frac{\boldsymbol{I}}{\boldsymbol{V}} = Y\angle\theta_y = G + jB = g + jb = (\textbf{Conducatcne}) + j(\textbf{Susceptance}) \ \ [S]$$

$$\textbf{Conducatcne}: G = g = Y\cos\theta_y \ \ [S] \qquad \textbf{Susceptance}: B = b = Y\sin\theta_y \ \ [S]$$

G or g **is called conductance**. The unit of conductance is **mho** or **Siemens**.

Conductance (*G* or *g*) is the reciprocal of resistance (*R*).

B or b **is called susceptance**. The unit of susceptance is **Siemens**.

Susceptance (*B* or *b*) is the reciprocal of reactance (*X*).

$$G = g = \frac{1}{R} \ \ [S] \qquad B = b = \frac{1}{X} \ \ [S] \qquad B_L = b_L = \frac{1}{X_L} \ \ [S] \qquad B_C = b_C = \frac{1}{X_C} \ \ [S]$$

B_L *or* b_L **is called inductive susceptance**.

B_C *or* b_C **is called capacitive susceptance**.

Admittance of a Resistance, Inductance and Capacitance

Admittance of a resistance: $$\boldsymbol{Y}_R = \frac{1}{\boldsymbol{Z}_R} = \frac{1}{R\angle 0°} = G\angle 0° \quad [S]$$

Admittance of an inductance: $$\boldsymbol{Y}_L = \frac{1}{\boldsymbol{Z}_L} = \frac{1}{X_L\angle 90°} = B_L\angle -90° \quad [S]$$

Admittance of a capacitance: $$\boldsymbol{Y}_C = \frac{1}{\boldsymbol{Z}_C} = \frac{1}{X_C\angle -90°} = B_C\angle 90° \quad [S]$$

Admittance of a *RL* series branch: $$\boldsymbol{Y}_{RL} = \frac{1}{\boldsymbol{Z}_{RL}} = \frac{1}{R\angle 0° + X_L\angle 90°} = \frac{1}{R + jX_L} \quad [S]$$

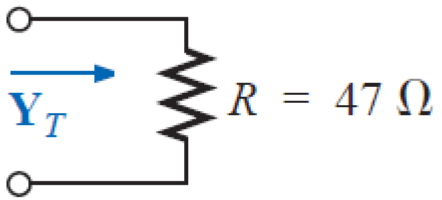
Admittance of a *RC* series branch: $$\boldsymbol{Y}_{RC} = \frac{1}{\boldsymbol{Z}_{RC}} = \frac{1}{R\angle 0° + X_C\angle -90°} = \frac{1}{R - jX_C} \quad [S]$$

Admittance of a *RLC* series branch:

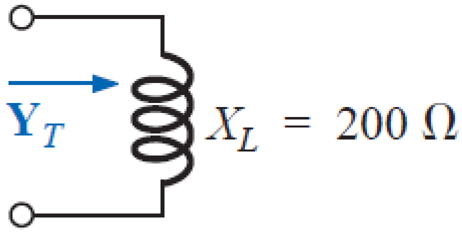
$$\boldsymbol{Y}_{RLC} = \frac{1}{\boldsymbol{Z}_{RLC}} = \frac{1}{R\angle 0° + X_L\angle 90° + X_C\angle -90°} = \frac{1}{R + jX_L - jX_C} \quad [S]$$

Example

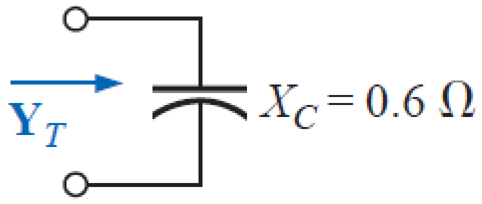
Calculate the total admittance for the following circuit.



$$\boldsymbol{Z}_T = 47\angle 0^\circ\ \Omega \qquad \boldsymbol{Y}_T = \frac{1}{47\angle 0^\circ} = 0.0213\angle 0^\circ\ \ [S]$$



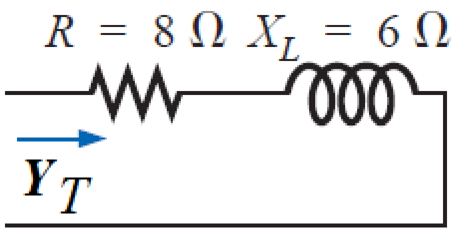
$$\boldsymbol{Z}_T = 200\angle 90^\circ\ \Omega \qquad \boldsymbol{Y}_T = \frac{1}{200\angle 90^\circ} = 0.005\angle -90^\circ\ \ [S]$$



$$\boldsymbol{Z}_T = 0.6\angle -90^\circ\ \Omega \qquad \boldsymbol{Y}_T = \frac{1}{0.6\angle -90^\circ} = 1.67\angle 90^\circ\ \ [S]$$

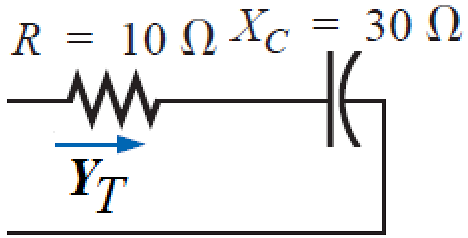
Example

Calculate the total admittance for the following circuit.



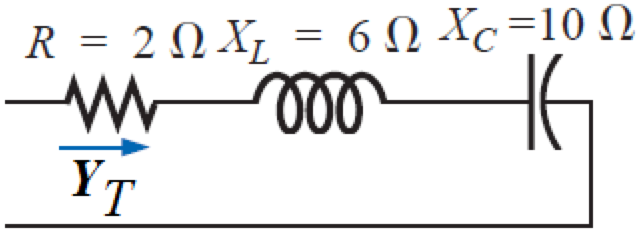
$$Z_T = 8 + j6 \quad \Omega$$

$$Y_T = \frac{1}{8 + j6} = 0.08 - j0.06 \quad [S]$$



$$Z_T = 10 - j30 \quad \Omega$$

$$Y_T = \frac{1}{10 - j30} = 0.01 + j0.03 \quad [S]$$

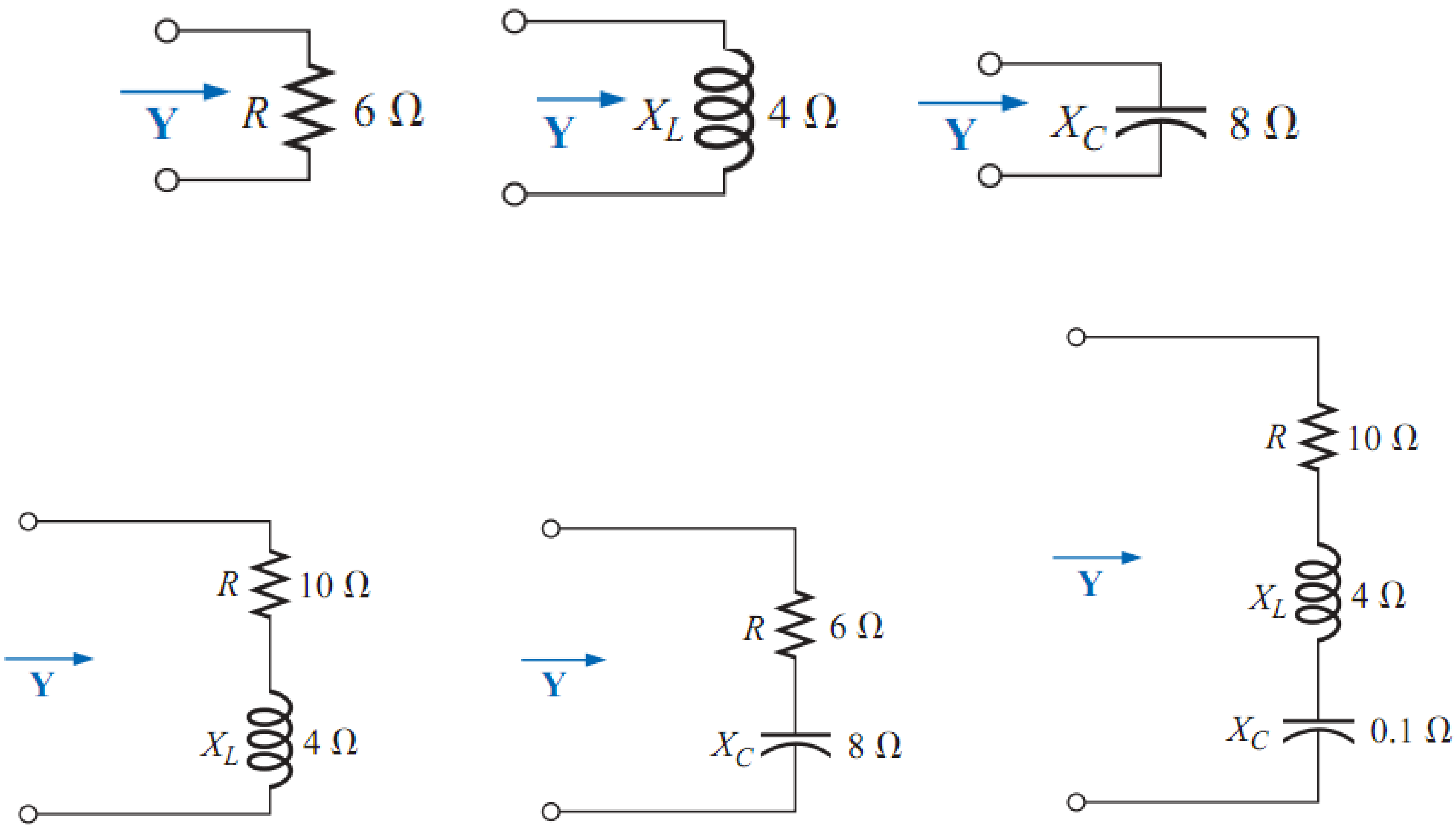


$$Z_T = 2 + j6 - j10 \quad \Omega$$

$$Y_T = \frac{1}{2 + j6 - j10} = \frac{1}{2 - j4} = 0.1 + j0.2 \quad [S]$$

Home Work 5.1

Problem 1: Calculate the total admittance for the following circuit.

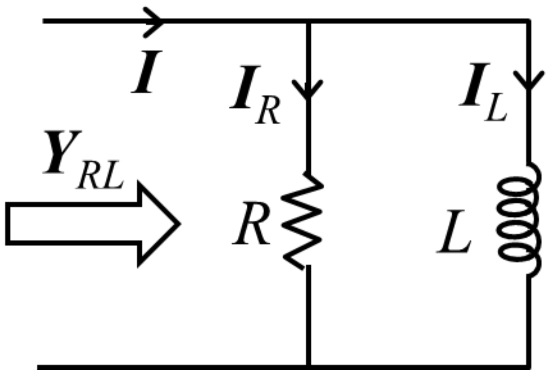


Admittance and Impedance of a *RL* Parallel Circuit

Admittance of a *RL* parallel Circuit:

$$\boldsymbol{Y} = \boldsymbol{Y}_{RL} = \boldsymbol{Y}_R + \boldsymbol{Y}_L = \frac{1}{R\angle 0^\circ} + \frac{1}{X_L\angle 90^\circ} \quad [S]$$

$$\boldsymbol{Y} = \boldsymbol{Y}_{RL} = G\angle 0^\circ + B_L\angle -90^\circ = G - jB_L \quad [S]$$



Impedance of a *RL* Parallel Circuit:

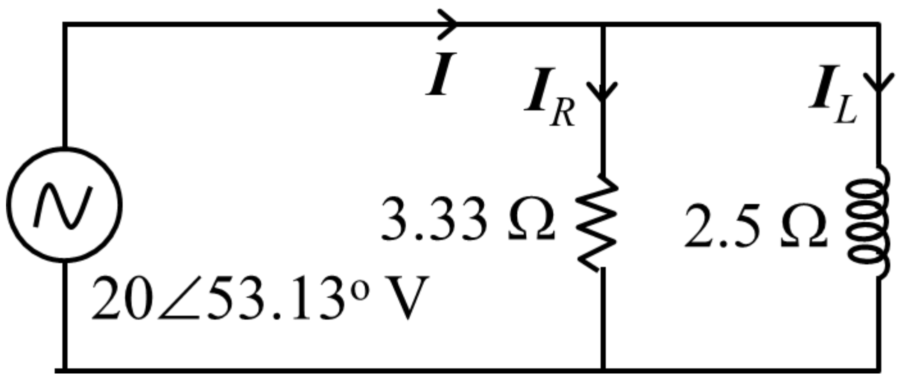
$$\boldsymbol{Z}_{RL} = \frac{1}{\boldsymbol{Y}_{RL}} = \frac{1}{\boldsymbol{Y}_R + \boldsymbol{Y}_L} = \frac{1}{\frac{1}{R\angle 0^\circ} + \frac{1}{X_L\angle 90^\circ}} = \frac{1}{\frac{1}{R} + \frac{1}{jX_L}} = \frac{(R)(jX_L)}{R + jX_L} \quad [\Omega]$$

$$\boldsymbol{Z}_{RL} = \frac{1}{\frac{1}{\boldsymbol{Z}_R} + \frac{1}{\boldsymbol{Z}_L}} = \frac{\boldsymbol{Z}_R\boldsymbol{Z}_L}{\boldsymbol{Z}_R + \boldsymbol{Z}_L} \quad [\Omega]$$

Example

Calculate the total admittance and current for the following circuit.

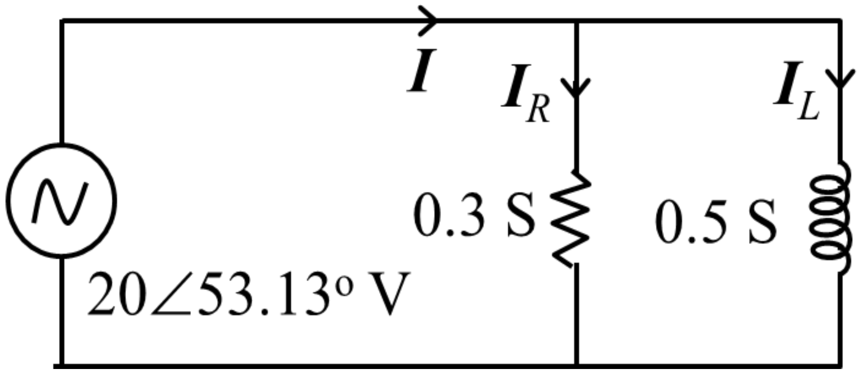
$$G=\frac{1}{3.33}=0.3\ [S] \qquad B_L=\frac{1}{2.5}=0.4\ [S]$$



$$\boldsymbol{Y}_R=0.3\angle 0^\circ=0.3\,[S] \qquad \boldsymbol{Y}_L=0.4\angle -90^\circ=-j0.4\ [S]$$

$$\boldsymbol{Y}=0.3\angle 0^\circ+0.4\angle -90^\circ=0.3-j0.4=0.5\angle -53.13^\circ\ \ [S]$$

$$\boldsymbol{Z}=\frac{1}{\boldsymbol{Y}}=\frac{1}{0.5\angle -53.13^\circ}=2\angle 53.13^\circ\ [\Omega]$$



$$\boldsymbol{I}=\frac{\boldsymbol{V}}{\boldsymbol{Z}}=\boldsymbol{YV}=(0.5\angle -53.13^\circ)(20\angle 53.13^\circ)=10\angle 0^\circ\ \ [A]$$

$$\boldsymbol{I}_R=\frac{\boldsymbol{V}}{\boldsymbol{Z}_R}=\boldsymbol{Y}_R\boldsymbol{V}=(0.3\angle 0^\circ)(20\angle 53.13^\circ)=6\angle 53.13^\circ\ \ [A]$$

$$\boldsymbol{I}_L=\frac{\boldsymbol{V}}{\boldsymbol{Z}_L}=\boldsymbol{Y}_L\boldsymbol{V}=(0.4\angle -90^\circ)(20\angle 53.13^\circ)=8\angle -36.87^\circ[A]$$

$$\theta = \theta_v - \theta_i = 53.13° - 0° = 53.13°$$

$$pf = \cos(53.13°) = 0.6 \ \text{lagging}$$

$$rf = \sin(53.13°) = 0.8$$

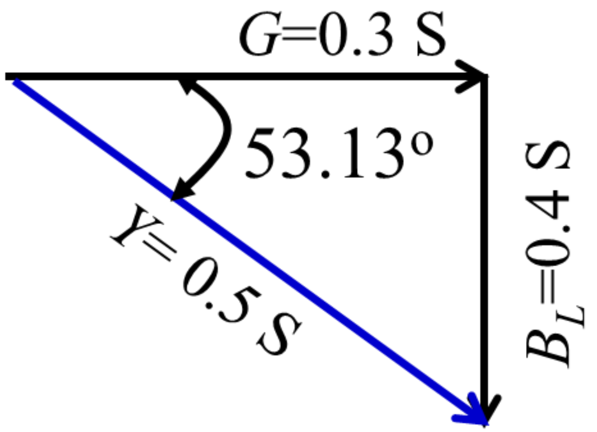
$$S = VI = 20 \times 10 = 200 \ \text{VA}$$

$$P = VI\cos\theta = 20 \times 10 \times 0.6 = 120 \ \text{W}$$

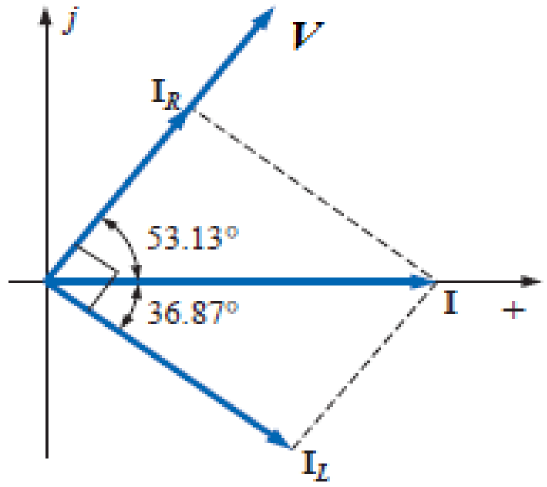
$$Q = VI\sin\theta = 20 \times 10 \times 0.8 = 160 \ \text{VAR}$$

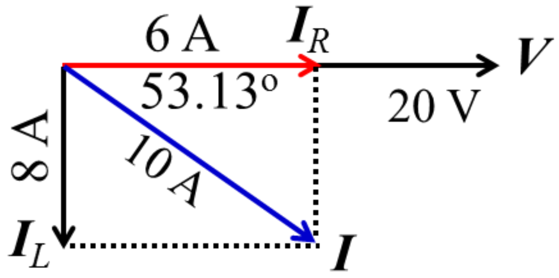
$$p(t) = 120(1 - \cos 2\omega t) + 160 \sin 2\omega t$$

$$Q = I_L^2 X_L = 8^2 \times 2.5 = 160 \ \text{VAR}$$

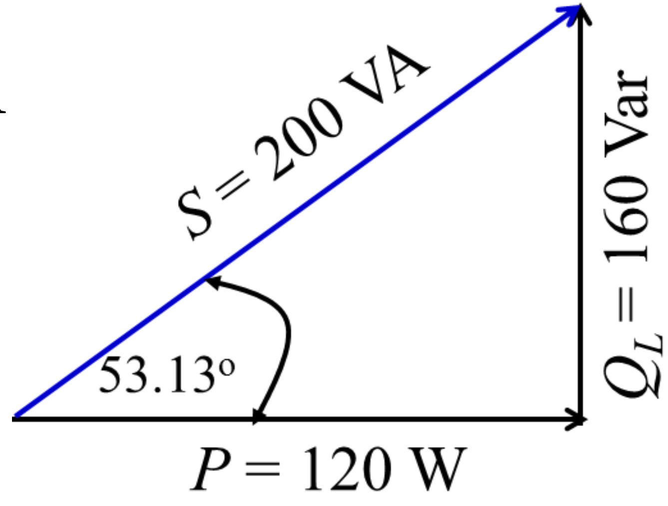


Admittance Diagram





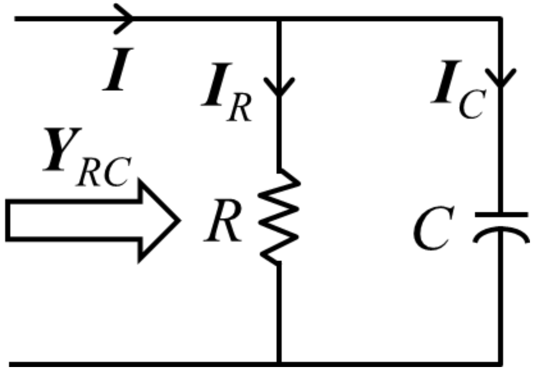
Phasor Diagram



Power Triangle

Admittance and Impedance of a *RC* Parallel Circuit

Admittance of a *RC* parallel Circuit:



$$\mathbf{Y} = \mathbf{Y}_{RC} = \mathbf{Y}_C + \mathbf{Y}_C = \frac{1}{R\angle 0^\circ} + \frac{1}{X_C\angle -90^\circ} \quad [S]$$

$$\mathbf{Y} = \mathbf{Y}_{RC} = G\angle 0^\circ + B_C\angle 90^\circ = G + jB_C \quad [S]$$

Impedance of a *RC* Parallel Circuit:

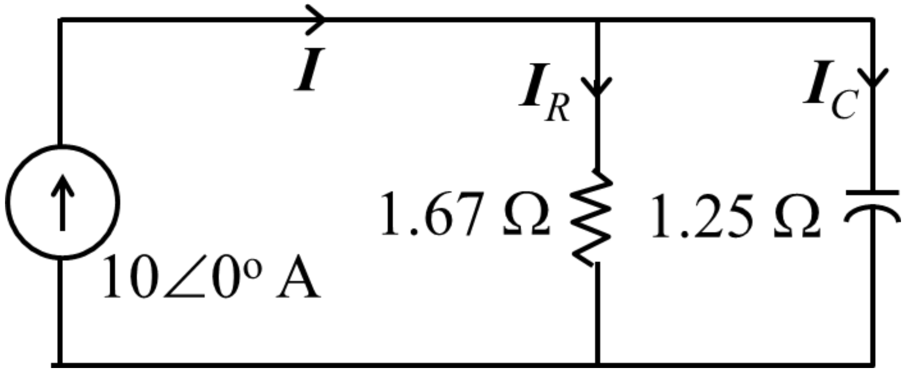
$$\mathbf{Z}_{RC} = \frac{1}{\mathbf{Y}_{RC}} = \frac{1}{\mathbf{Y}_R + \mathbf{Y}_C} = \frac{1}{\frac{1}{R\angle 0^\circ} + \frac{1}{X_C\angle -90^\circ}} = \frac{1}{\frac{1}{R} + \frac{1}{-jX_C}} = \frac{(R)(-jX_C)}{R - jX_C} \quad [\Omega]$$

$$\mathbf{Z}_{RC} = \frac{1}{\frac{1}{\mathbf{Z}_R} + \frac{1}{\mathbf{Z}_C}} = \frac{\mathbf{Z}_R\mathbf{Z}_C}{\mathbf{Z}_R + \mathbf{Z}_C} \quad [\Omega]$$

Example

Calculate the total admittance and current for the following circuit.

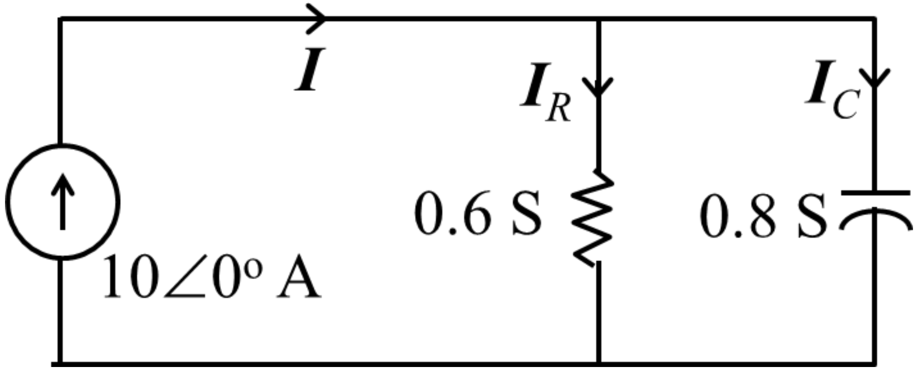
$$G = \frac{1}{1.67} = 0.6 \ [S] \qquad B_C = \frac{1}{1.25} = 0.8 \ [S]$$



$$\boldsymbol{Y}_R = 0.6\angle 0° = 0.6 \, [S] \qquad \boldsymbol{Y}_C = 0.8\angle 90° = j0.8 \ [S]$$

$$\boldsymbol{Y} = 0.6\angle 0° + 0.8\angle 90° = 0.6 + j0.8 = 1.0\angle 53.13° \ \ [S]$$

$$\boldsymbol{Z} = \frac{1}{\boldsymbol{Y}} = \frac{1}{1\angle 53.13°} = 1\angle -53.13° \ [\Omega]$$



$$\boldsymbol{V} = \boldsymbol{IZ} = \frac{\boldsymbol{I}}{\boldsymbol{Y}} = \frac{10\angle 0°}{1\angle 53.13°} = 10\angle -53.13° \ \ [\mathrm{V}]$$

$$\boldsymbol{I}_R = \boldsymbol{Y}_R \boldsymbol{V} = (0.6\angle 0°)(10\angle -53.13°) = 6\angle -53.13° \ \ [\mathrm{A}]$$

$$\boldsymbol{I}_C = \boldsymbol{Y}_C \boldsymbol{V} = (0.8\angle 90°)(10\angle -53.13°) = 8\angle 36.87° [\mathrm{A}]$$

$$\theta = \theta_v - \theta_i = -53.13° - 0° = -53.13°$$

$$pf = \cos(-53.13°) = 0.6 \text{ leading}$$

$$rf = \sin(-53.13°) = -0.8$$

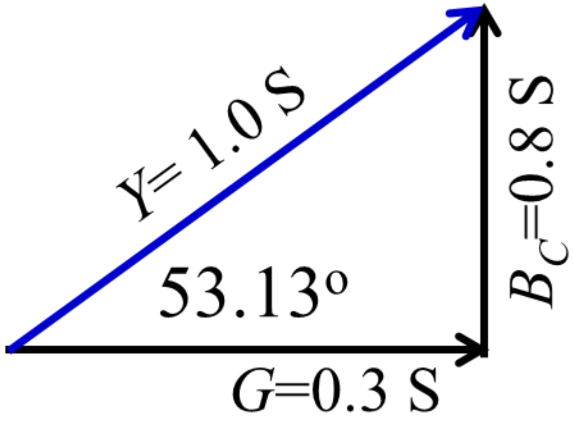
$$S = VI = 10 \times 10 = 100 \text{ VA}$$

$$P = VI\cos\theta = 10 \times 10 \times 0.6 = 60 \text{ W}$$

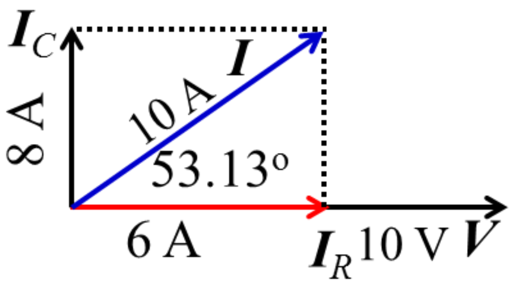
$$Q = VI\sin\theta = 10 \times 10 \times -0.8 = -80 \text{ VAR}$$

$$p(t) = 60(1 - \cos 2\omega t) - 80\sin 2\omega t$$

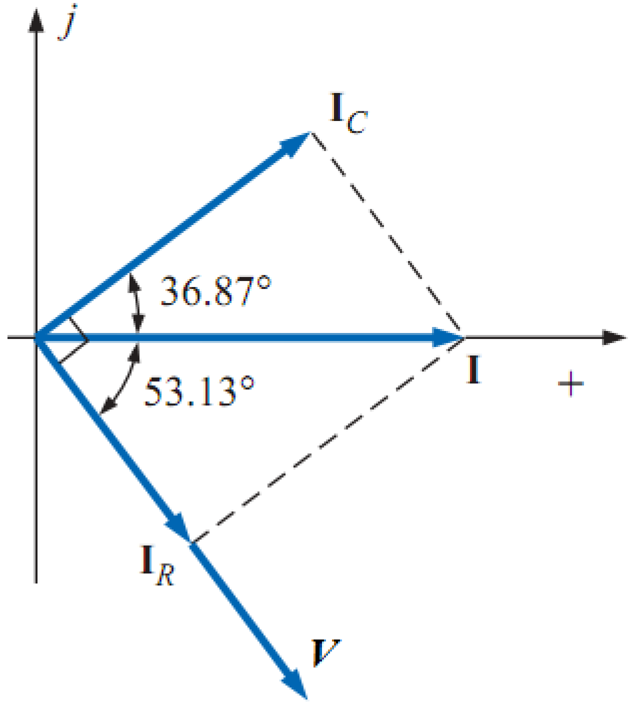
$$Q = I_C^2 X_C = 8^2 \times 1.25 = 80 \text{ VAR}$$

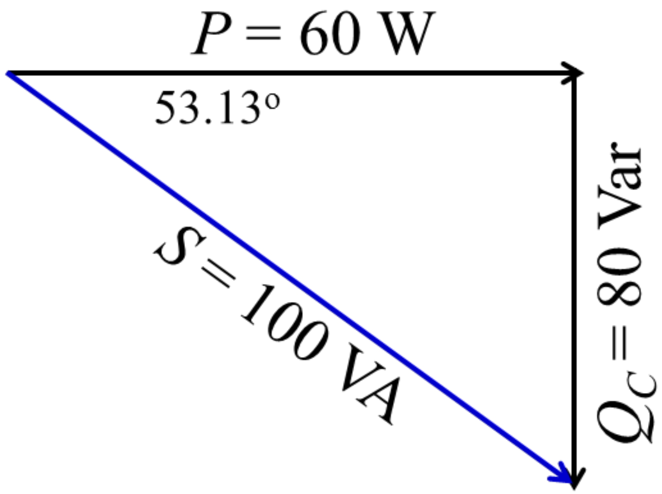


Admittance Diagram



Phasor Diagram

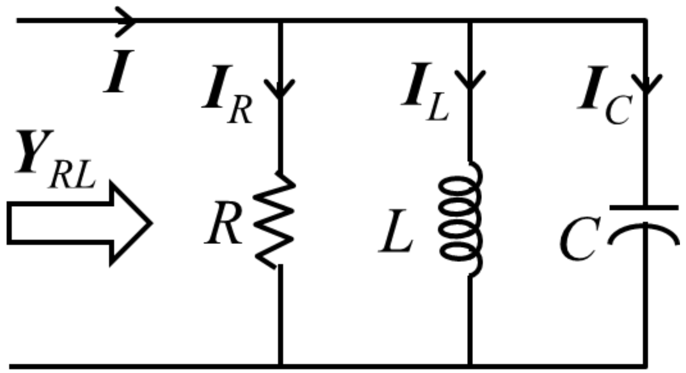




Power Triangle

Admittance and Impedance of a *RLC* Parallel Circuit

Admittance of a *RC* parallel Circuit:



$$\boldsymbol{Y} = \boldsymbol{Y}_{RLC} = \boldsymbol{Y}_R + \boldsymbol{Y}_L + \boldsymbol{Y}_C$$

$$= \frac{1}{R\angle 0^\circ} + \frac{1}{X_L\angle 90^\circ} + \frac{1}{X_C\angle -90^\circ} \quad [S]$$

$$\boldsymbol{Y} = \boldsymbol{Y}_{RLC} = G\angle 0^\circ + B_L\angle -90^\circ + B_C\angle 90^\circ = G - jB_L + jB_C \quad [S]$$

Impedance of a *RC* Parallel Circuit:

$$\boldsymbol{Z}_{RLC} = \frac{1}{\boldsymbol{Y}_{RLC}} = \frac{1}{\boldsymbol{Y}_R + \boldsymbol{Y}_L + \boldsymbol{Y}_C}$$

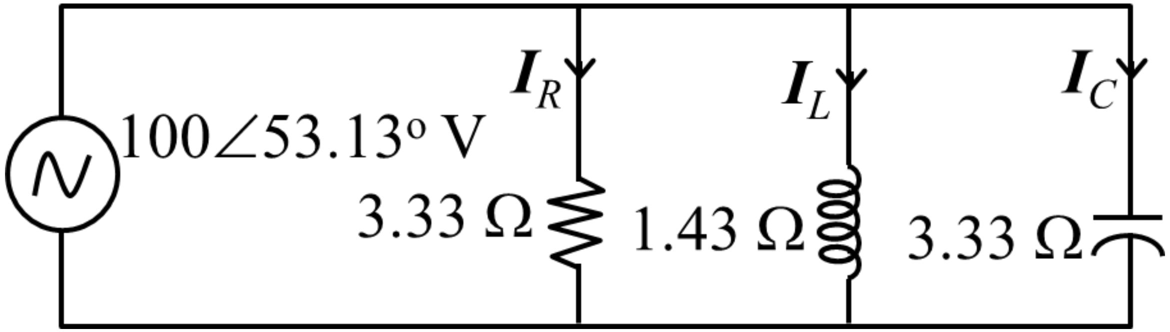
$$= \frac{1}{\frac{1}{R\angle 0^\circ} + \frac{1}{X_L\angle 90^\circ} + \frac{1}{X_C\angle -90^\circ}} = \frac{1}{\frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_C}}$$

$$\boldsymbol{Z}_{RC} = \frac{1}{\frac{1}{\boldsymbol{Z}_R} + \frac{1}{\boldsymbol{Z}_L} + \frac{1}{\boldsymbol{Z}_C}}$$

Example

Calculate the total admittance and current for the following circuit.

$$G = \frac{1}{3.33} = 0.3\ [S] \qquad B_L = \frac{1}{1.43} = 0.7\ [S]$$



$$B_C = \frac{1}{3.33} = 0.3\ [S]$$

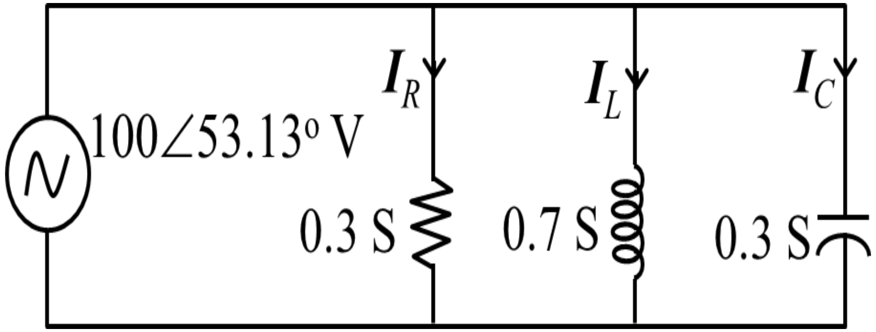
$$\boldsymbol{Y}_R = 0.3\angle 0° = 0.3\,[S]$$

$$\boldsymbol{Y}_L = 0.7\angle -90° = -j0.7\ [S]$$

$$\boldsymbol{Y}_C = 0.3\angle 90° = j0.3\ [S]$$

$$\boldsymbol{Y} = 0.3\angle 0° + 0.7\angle -90° + 0.3\angle 90°$$
$$= 0.3 - j0.4 = 0.5\angle -53.13°\ [S]$$

$$\boldsymbol{Z} = \frac{1}{\boldsymbol{Y}} = \frac{1}{0.5\angle -53.13°} = 2\angle 53.13°\ [\Omega]$$



$$\boldsymbol{I} = \boldsymbol{V}\boldsymbol{Y} = (100\angle 53.13°)(0.5\angle -53.13°) = 50\angle 0°\ [A]$$

$$\boldsymbol{I}_R = \boldsymbol{Y}_R\boldsymbol{V} = (0.3\angle 0°)(100\angle 53.13°) = 30\angle 53.13°\ [A]$$

$$\boldsymbol{I}_L = \boldsymbol{Y}_L\boldsymbol{V} = (0.7\angle -90°)(100\angle 53.13°) = 70\angle -36.87°\ [A]$$

$$\boldsymbol{I}_C = \boldsymbol{Y}_C\boldsymbol{V} = (0.3\angle 90°)(100\angle 53.13°) = 30\angle 143.13°[A]$$

$$\theta = \theta_v - \theta_i = 53.13° - 0° = 53.13°$$

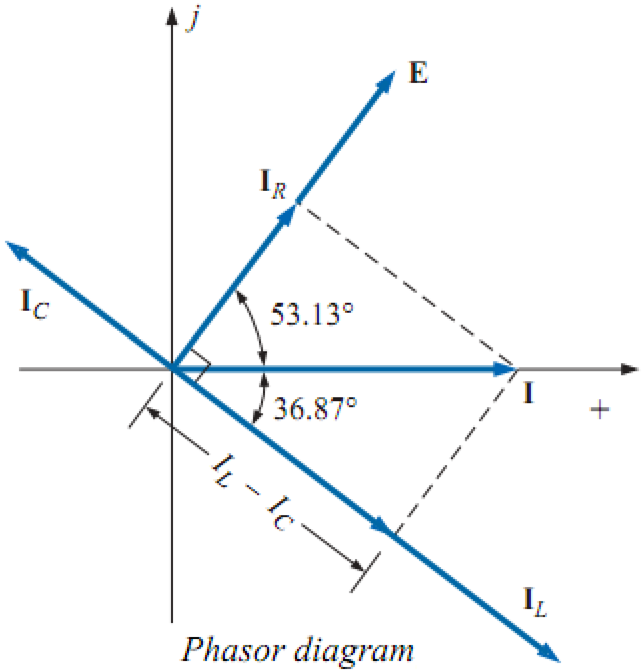
$$pf = \cos(53.13°) = 0.6 \ \text{leading}$$

$$rf = \sin(53.13°) = 0.8$$

$$S = VI = 100 \times 50 = 5000 \ \ \text{VA}$$

$$P = VI\cos\theta = 100 \times 50 \times 0.6 = 3000 \ \ \text{W}$$

$$Q = VI\sin\theta = 100 \times 50 \times 0.8 = 4000 \ \ \text{VAR}$$



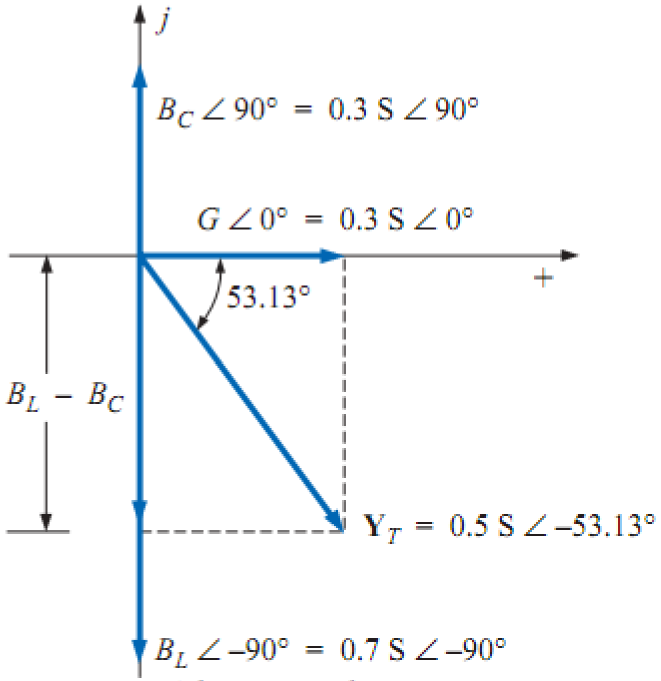
Phasor diagram

$$p(t) = 3000(1 - \cos 2\omega t) + 4000\sin 2\omega t$$

$$Q_L = I_L^2 X_L = 70^2 \times 1.43 = 7007 \ \ \text{VAR}$$

$$Q_C = I_C^2 X_C = 30^2 \times 3.33 = 2997 \ \ \text{VAR}$$

$$Q = Q_C - Q_C = 7007 - 2997 = 4010 \ \ \text{VAR}$$



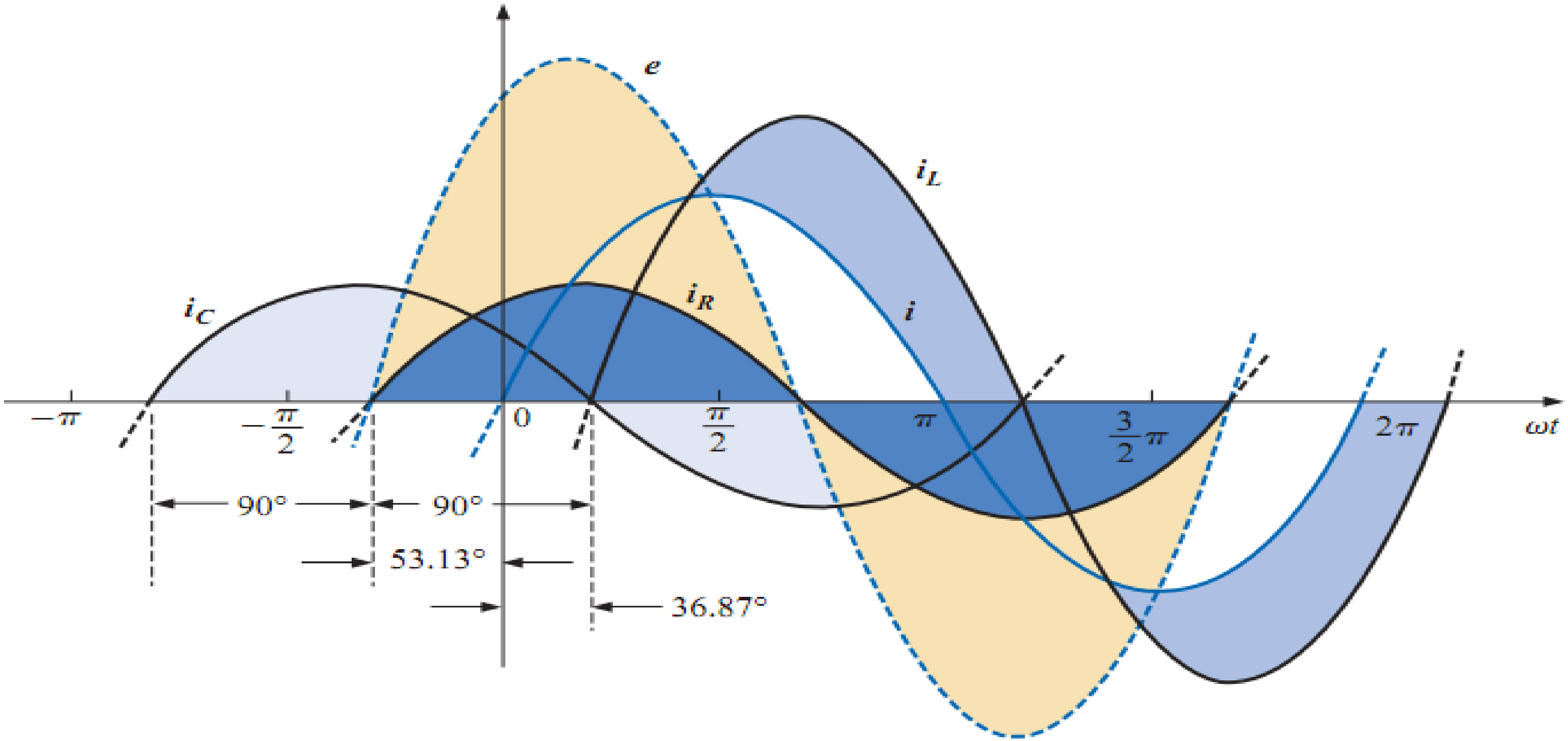
Admittance diagram

$$\boldsymbol{I} = 50\angle 0° \;\; [\mathrm{A}] \qquad i(t) = \sqrt{2}(50)\sin\omega t = 70.7\sin\omega t \;\; [\mathrm{A}]$$

$$\boldsymbol{I}_R = 30\angle 53.13° \;\; [\mathrm{A}] \qquad i_R(t) = \sqrt{2}(30)\sin(\omega t + 53.13°) = 42.42\sin(\omega t + 53.13°) \;\; [\mathrm{A}]$$

$$\boldsymbol{I}_L = 70\angle -36.87° \;\; [\mathrm{A}] \qquad i_L(t) = \sqrt{2}(70)\sin(\omega t - 36.87°) = 98.98\sin(\omega t - 36.87°) \;\; [\mathrm{A}]$$

$$\boldsymbol{I}_C = 30\angle 143.13° [\mathrm{A}] \qquad i_C(t) = \sqrt{2}(30)\sin(\omega t + 143.13°) = 42.42\sin(\omega t + 143.13°) \;\; [\mathrm{A}]$$



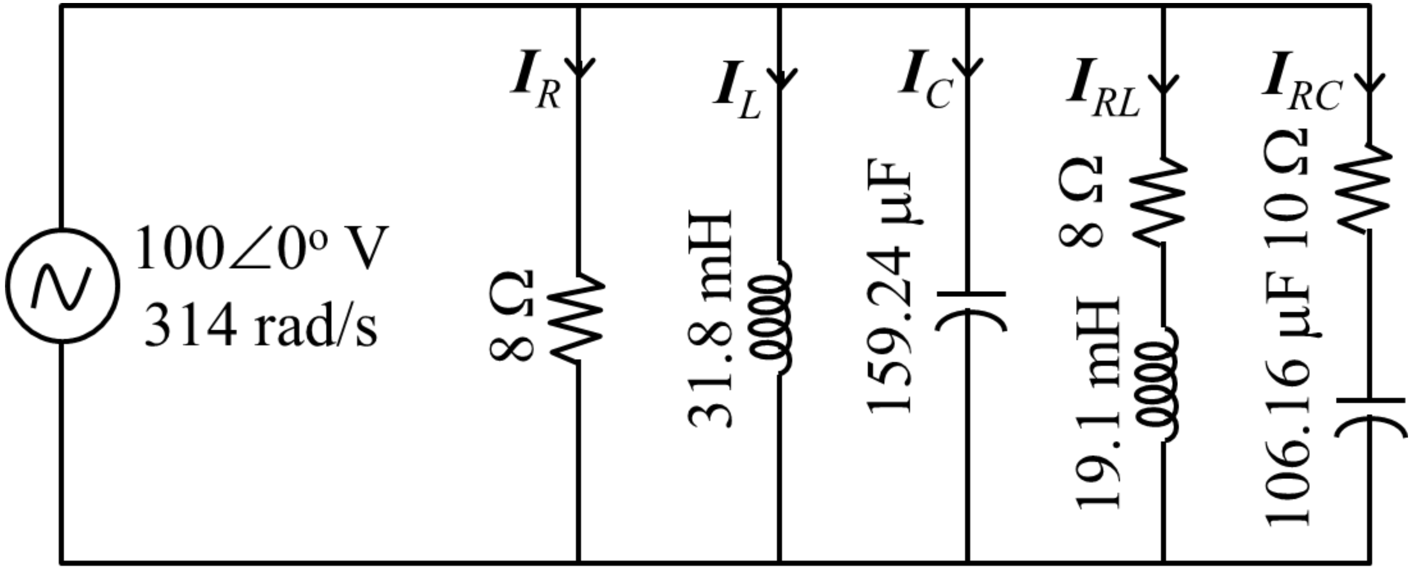
Admittance of Parallel Impedances

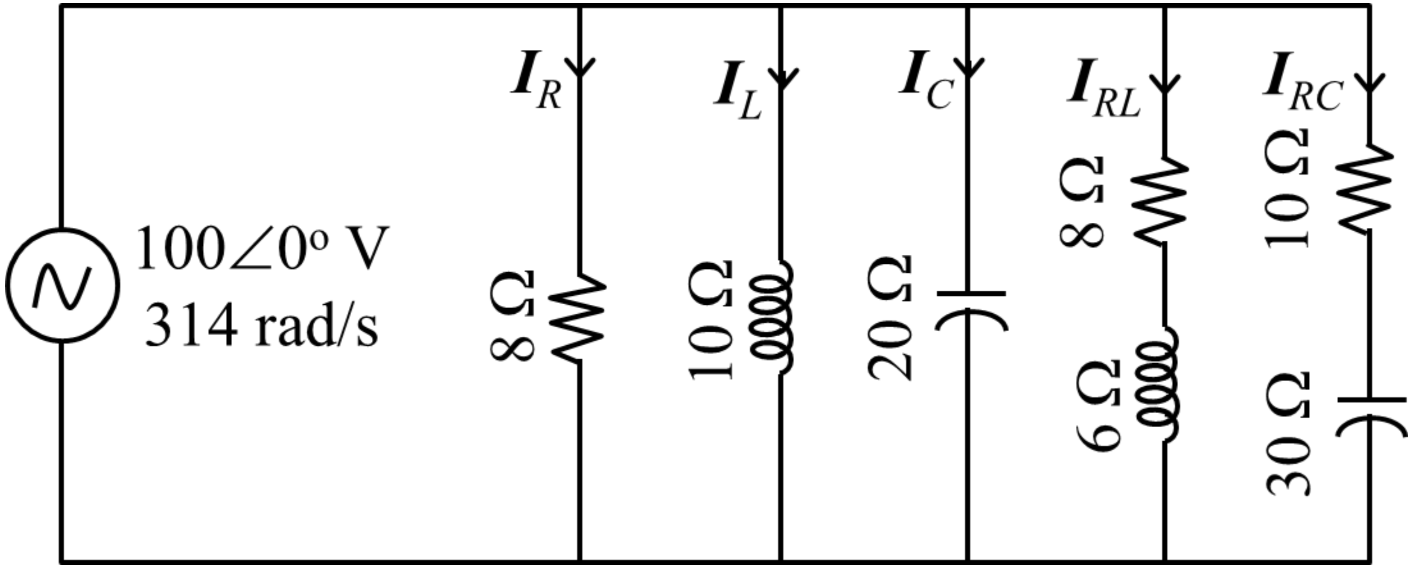
$$X_{L1} = 314 \times 31.8 \times 10^{-3} = 10\ \Omega$$

$$X_{L2} = 314 \times 19.1 \times 10^{-3} = 8\ \Omega$$

$$X_{C1} = \frac{1}{314 \times 159.24 \times 10^{-9}} = 20\ \Omega$$

$$X_{C2} = \frac{1}{314 \times 106.16 \times 10^{-9}} = 30\ \Omega$$





$$Z_R = 8\Omega$$

$$Y_R = \frac{1}{Z_R} = \frac{1}{8} = 0.125\ S$$

$$Z_L = j10\ \Omega$$

$$Y_L = \frac{1}{Z_L} = \frac{1}{j10} = -j0.1\ S$$

$$Z_C = -j20\ \Omega$$

$$Y_C = \frac{1}{Z_C} = \frac{1}{-j20} = j0.05\ S$$

$$Z_{RL} = 8 + j6\Omega$$

$$Y_{RL} = \frac{1}{Z_{RL}} = \frac{1}{8 + j6} = 0.08 - j0.06\ S$$

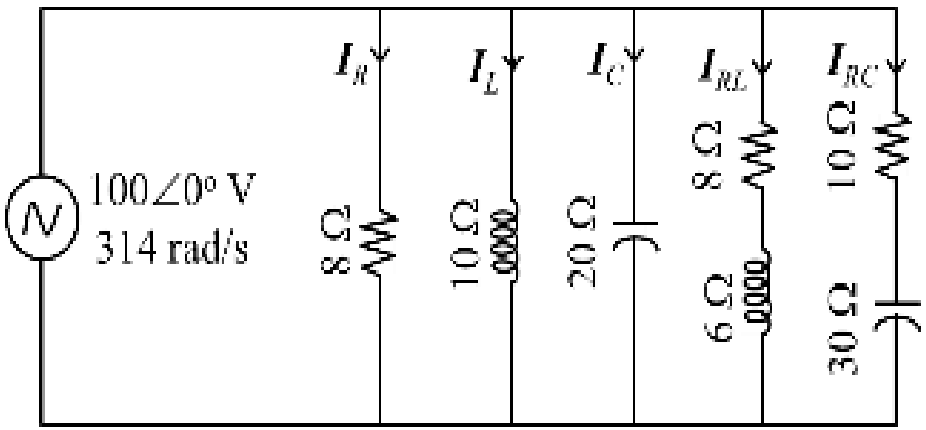
$$Z_{RC} = 10 - j30\Omega$$

$$Y_{RC} = \frac{1}{Z_{Rc}} = \frac{1}{10 - j30} = 0.01 + j0.03\ S$$

$$Y_T = Y_R + Y_L + Y_C + Y_{RL} + Y_{RC}$$

$$Y_T = 0.125 - j0.1 + j0.05 + 0.08 - j0.06 + 0.01 + j0.03$$
$$= 0.215 - j0.08\ S$$

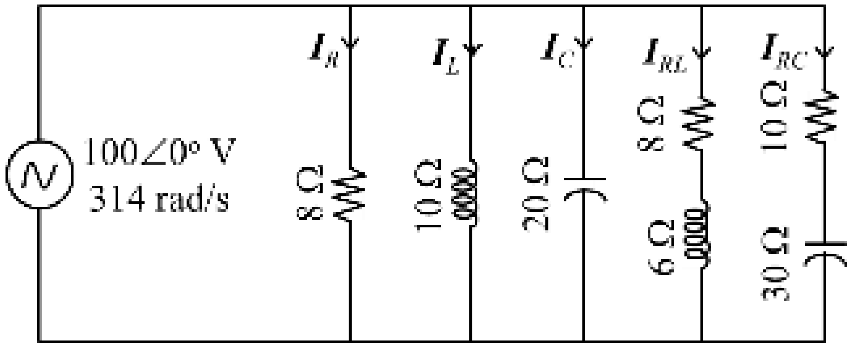
$$Z_T = \frac{1}{Y_T} = \frac{1}{0.215 - j0.08} = 4.086 + j1.52\ \Omega$$



$$\boldsymbol{I}_T = \boldsymbol{V}\boldsymbol{Y}_T = (100\angle 0^\circ)(0.215 - j0.08) = 21.5 - j8 = 22.94\angle -20.41^\circ \text{ A}$$

$$\boldsymbol{I}_R = \boldsymbol{V}\boldsymbol{Y}_R = (100\angle 0^\circ)(0.125) = 12.5 \text{ A}$$

$$\boldsymbol{I}_L = \boldsymbol{V}\boldsymbol{Y}_L = (100\angle 0^\circ)(-j0.1) = -j10 = 10\angle -90^\circ \text{ A}$$



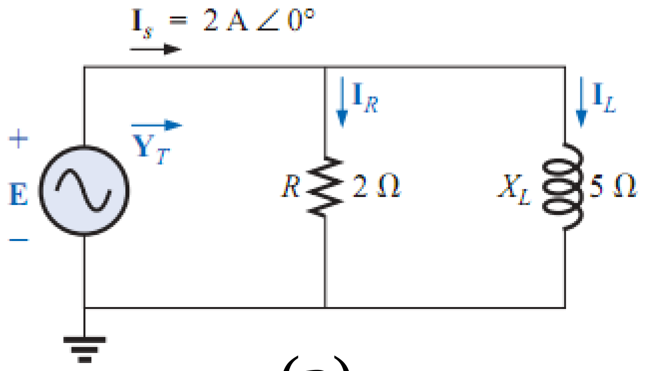
$$\boldsymbol{I}_C = \boldsymbol{V}\boldsymbol{Y}_C = (100\angle 0^\circ)(j0.05) = j5 = 5\angle 90^\circ \text{ A}$$

$$\boldsymbol{I}_{RL} = \boldsymbol{V}\boldsymbol{Y}_{RL} = (100\angle 0^\circ)(0.08 - j0.06) = 8 - j6 \text{ A}$$

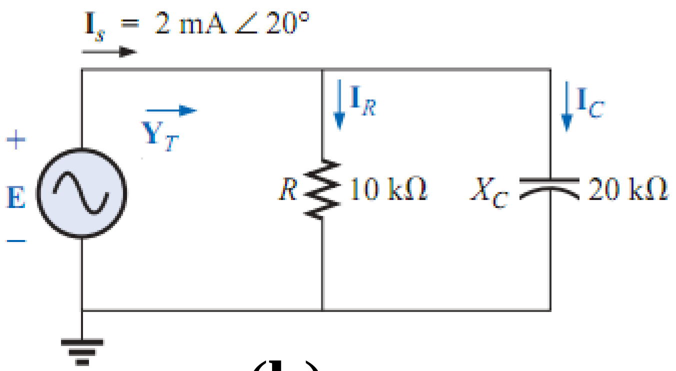
$$\boldsymbol{I}_{RC} = \boldsymbol{V}\boldsymbol{Y}_{RC} = (100\angle 0^\circ)(0.01 + j0.03) = 1 + j3 \text{ A}$$

Home Work 5.2

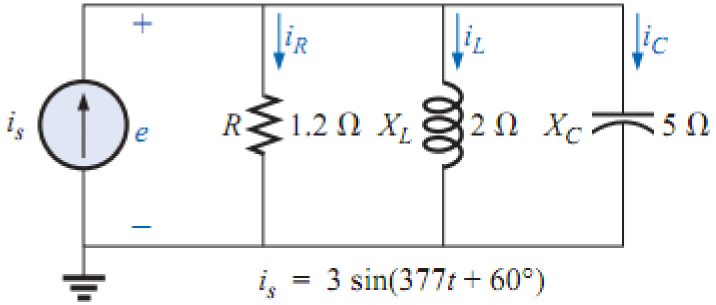
Problem 1: For the following circuits: (**i**) Find the total admittance. (**ii**) Draw the admittance diagram. (**iii**) Find the value of C in microfarads and L in henries. (**iv**) Find the source voltage (or source current. (**v**) Find the current of each branch. (**vi**) Draw the phasor diagram. (**vii**) Verify Kirchhoff's current law at one node. (**viii**) Find the average power, reactive power, apparent power, power factor, and reactive factor. (**ix**) Find the sinusoidal expressions for the currents and voltage. (**x**) Plot the waveforms for the currents and voltage on the same set of axes.



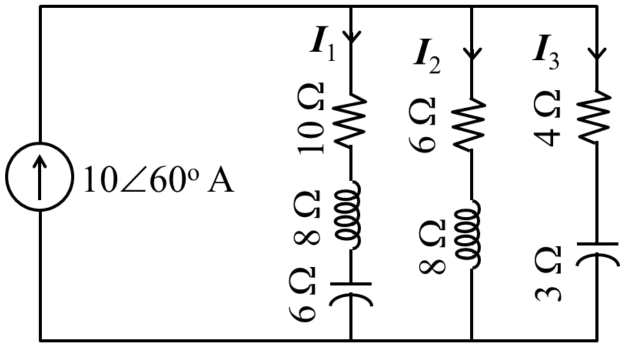
(a)



(b)



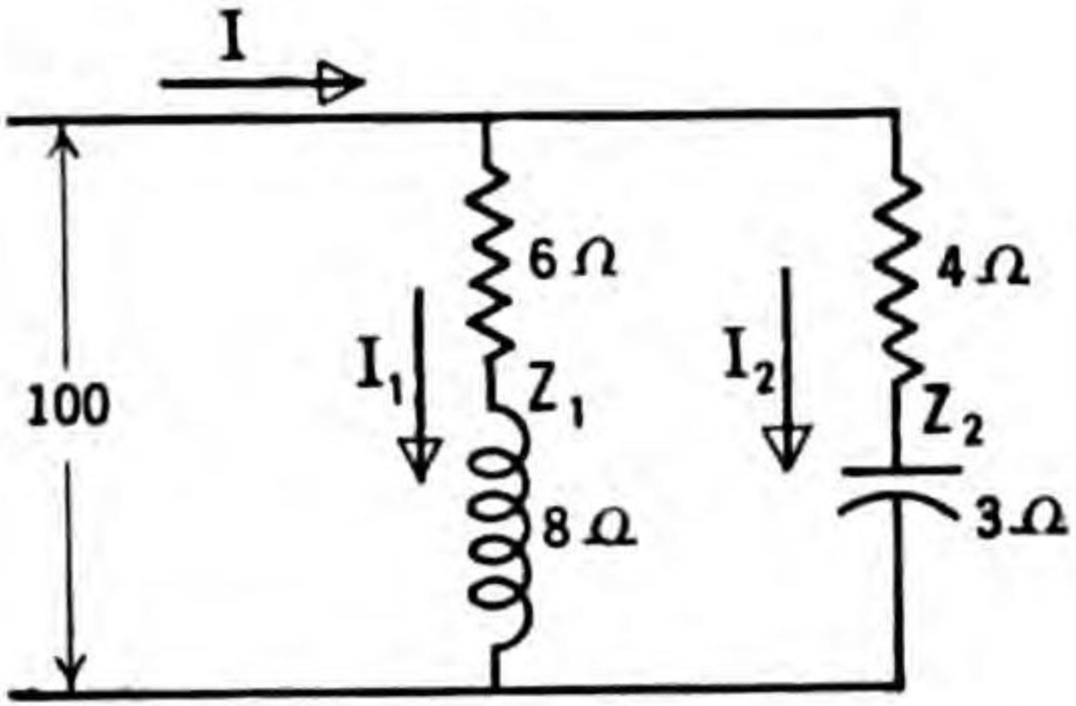
(c)



(d)

Home Work 5.3

Problem 1: For the following circuit diagram (**i**) find the conductance and susceptance of each branch. (**ii**) Find the resultant conductance and susceptance. (**iii**) Find the current I, I_1, and I_2. (**iv**) Draw the vector diagram.



Current Divider Rule

The current ($\boldsymbol{I}_x$) flows one or more elements in parallel that have total admittance $\boldsymbol{Y}_x$ or impedance $\boldsymbol{Z}_x$, can be given by:

$$\boldsymbol{I}_X = \frac{\boldsymbol{Y}_X}{\boldsymbol{Y}_T}\boldsymbol{I} = \frac{\boldsymbol{Z}_T}{\boldsymbol{Z}_X}\boldsymbol{I}$$

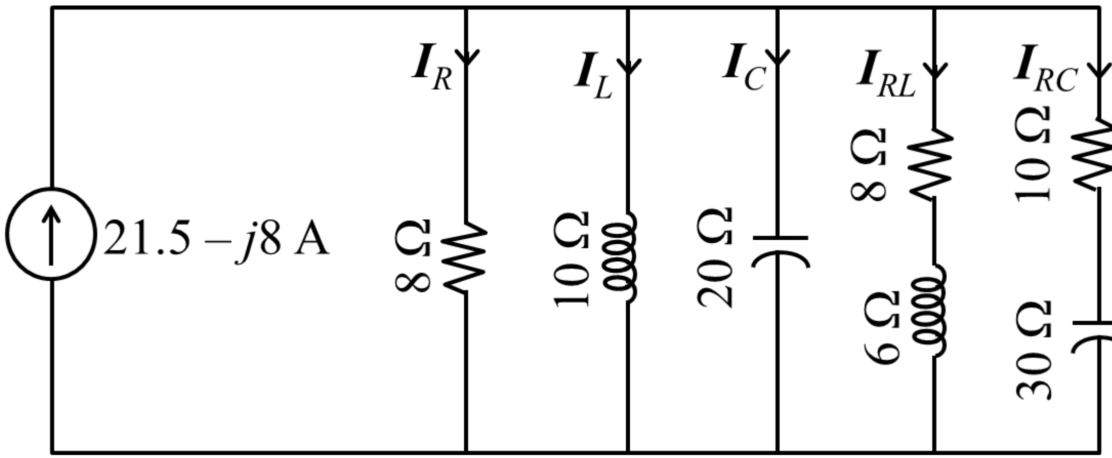
where, $\boldsymbol{I}$ is the total current flows the parallel circuit, and $\boldsymbol{Y}_T$ is the total admittance of the parallel circuit.

Example

Calculate the current of each branch for the circuit of following figure in phasor form using the current divider rule.

$$\mathbf{Y}_R = \frac{1}{8} = 0.125 \ S$$

$$\mathbf{Y}_L = \frac{1}{j10} = -j0.1 \ S$$



$$\mathbf{Y}_C = \frac{1}{-j20} = j0.05 \ S$$

$$\mathbf{Y}_{RL} = \frac{1}{8+j6} = 0.08 - j0.06 \ S$$

$$\mathbf{Y}_{RC} = \frac{1}{10-j30} = 0.01 + j0.03 \ S$$

$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.215 - j0.08} = 4.086 + j1.52 \ \Omega$$

$$\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L + \mathbf{Y}_C + \mathbf{Y}_{RL} + \mathbf{Y}_{RC}$$

$$\mathbf{Y}_T = 0.125 - j0.1 + j0.05 + 0.08 - j0.06 + 0.01 + j0.03 = 0.215 - j0.08 \ S$$

$$\boldsymbol{I}_R = \frac{\boldsymbol{Z}_T}{\boldsymbol{Z}_R}\boldsymbol{I} = \frac{\boldsymbol{Y}_R}{\boldsymbol{Y}_T}\boldsymbol{I} = \frac{0.125}{0.215 - j0.08}(21.1 - j8) = 12.5 \text{ A}$$

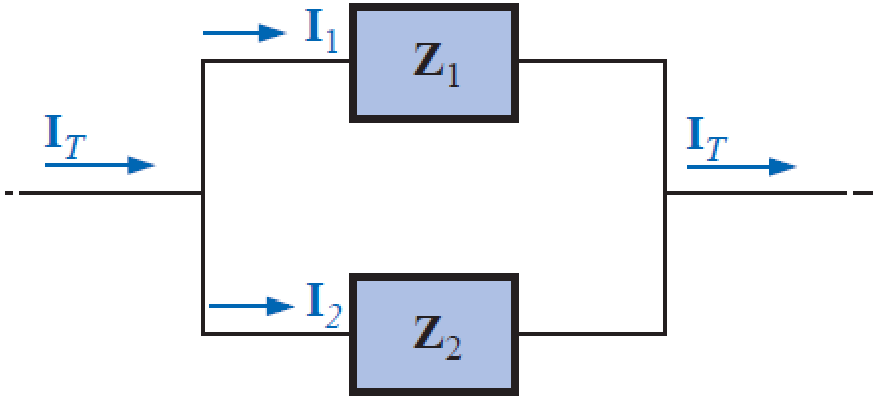
$$\boldsymbol{I}_L = \frac{\boldsymbol{Z}_T}{\boldsymbol{Z}_L}\boldsymbol{I} = \frac{\boldsymbol{Y}_L}{\boldsymbol{Y}_T}\boldsymbol{I} = \frac{-j0.1}{0.215 - j0.08}(21.1 - j8) = -j10 \text{ A}$$

$$\boldsymbol{I}_C = \frac{\boldsymbol{Z}_T}{\boldsymbol{Z}_L}\boldsymbol{I} = \frac{\boldsymbol{Y}_C}{\boldsymbol{Y}_T}\boldsymbol{I} = \frac{j0.05}{0.215 - j0.08}(21.1 - j8) = j5 \text{ A}$$

$$\boldsymbol{I}_{RL} = \frac{\boldsymbol{Z}_T}{\boldsymbol{Z}_{RL}}\boldsymbol{I} = \frac{\boldsymbol{Y}_{RL}}{\boldsymbol{Y}_T}\boldsymbol{I} = \frac{0.08 - j0.06}{0.215 - j0.08}(21.1 - j8) = 8 - j6 \text{ A}$$

$$\boldsymbol{I}_{RC} = \frac{\boldsymbol{Z}_T}{\boldsymbol{Z}_{RC}}\boldsymbol{I} = \frac{\boldsymbol{Y}_{RC}}{\boldsymbol{Y}_T}\boldsymbol{I} = \frac{0.01 + j0.03}{0.215 - j0.08}(21.1 - j8) = 1 + j3 \text{ A}$$

Current Divider Rule Only for Two Branch Circuit



$$I_1 = \frac{Y_1}{Y_T} I_T = \frac{Z_T}{Z_1} I_T = \frac{Z_2}{Z_1 + Z_2} I_T$$

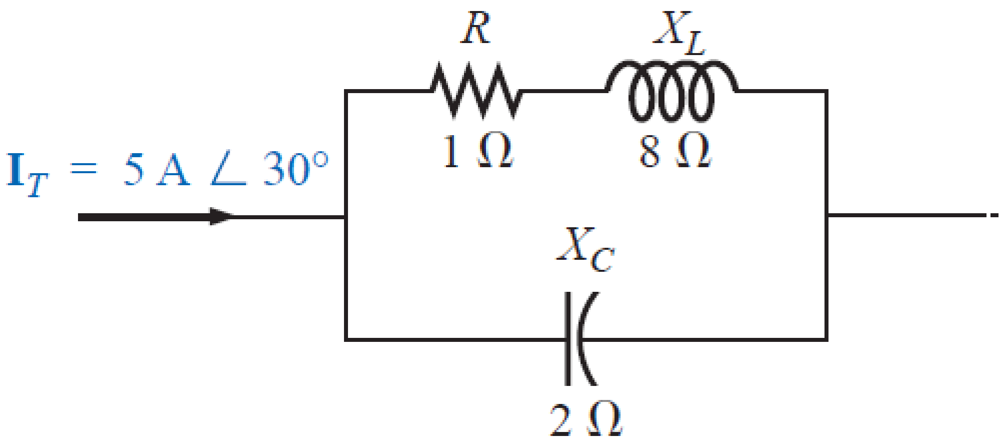
$$I_2 = \frac{Y_2}{Y_T} I_T = \frac{Z_T}{Z_2} I_T = \frac{Z_1}{Z_1 + Z_2} I_T$$

Example

Using the current divider rule, find the current through each impedance of following figure.

$$\mathbf{Z}_{RL} = 1 + j8\ \Omega$$

$$\mathbf{Z}_C = -j2 = 2\angle -90^\circ\ \Omega$$

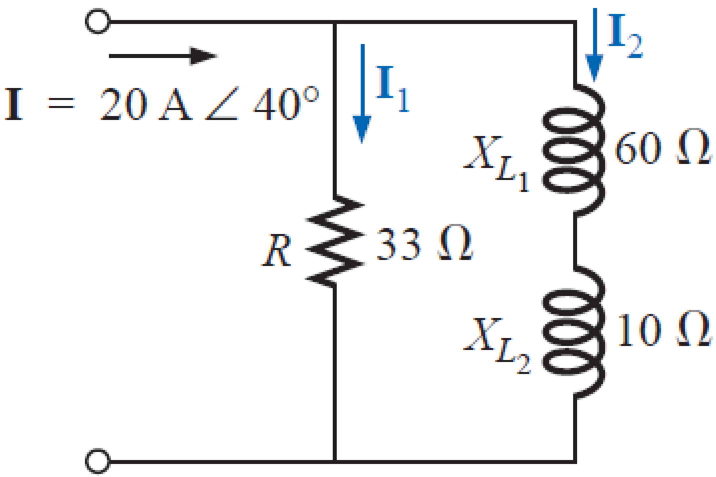


$$\mathbf{I}_{RL} = \frac{\mathbf{Z}_C}{\mathbf{Z}_{RL} + \mathbf{Z}_C}\mathbf{I}_T = \frac{-j2}{1 + j8 - j2}(5\angle 30^\circ) = 1.644\angle -140.54^\circ\ \text{A}$$

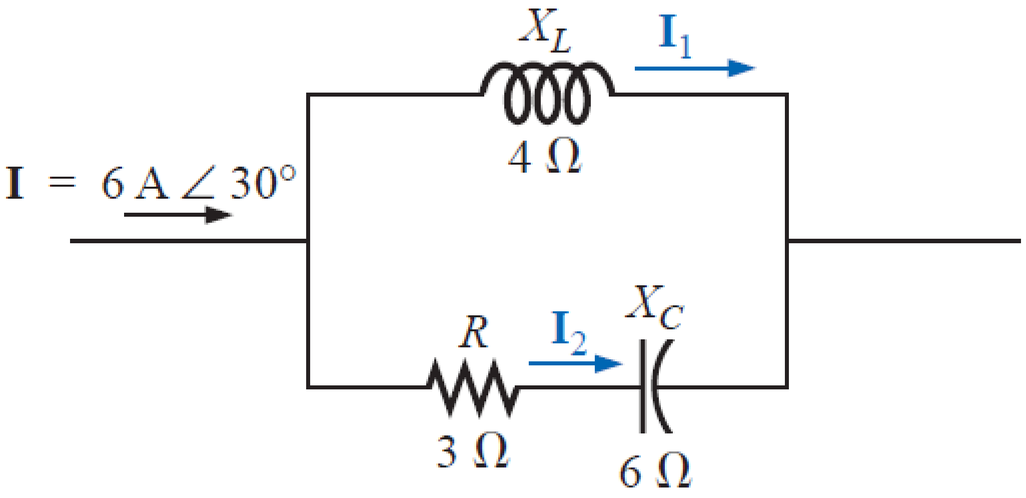
$$\mathbf{I}_C = \frac{\mathbf{Z}_{RL}}{\mathbf{Z}_{RL} + \mathbf{Z}_C}\mathbf{I}_T = \frac{1 + j8}{1 + j8 - j2}(5\angle 30^\circ) = 6.625\angle 32.33^\circ\ \text{A}$$

Home Work 5.4

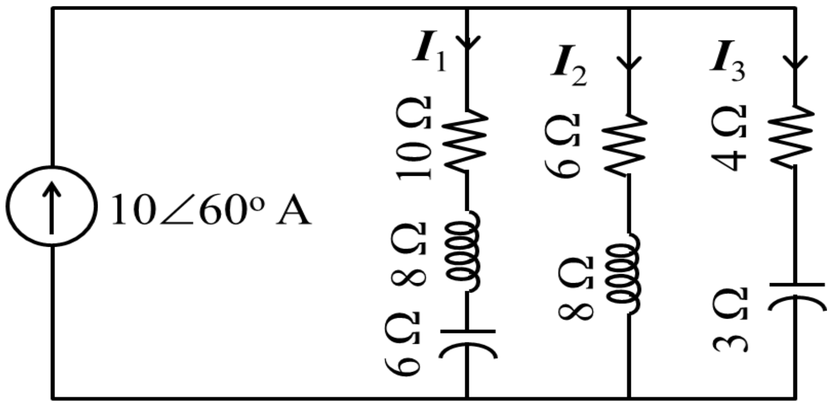
Problem 1: Calculate the current of each branch for the following figure using current divider rule.



(a)

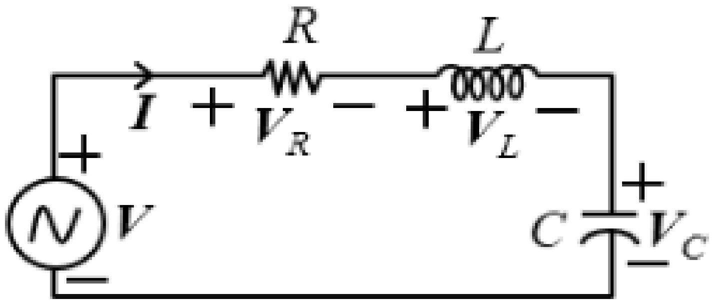
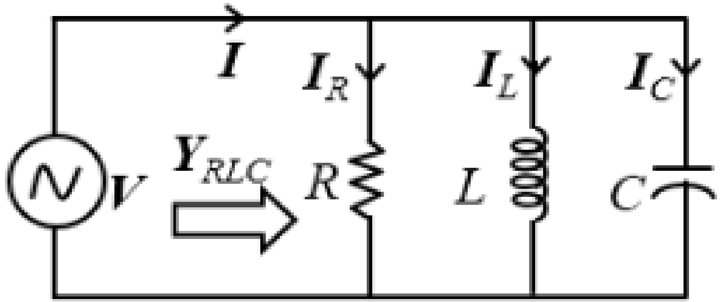


(b)



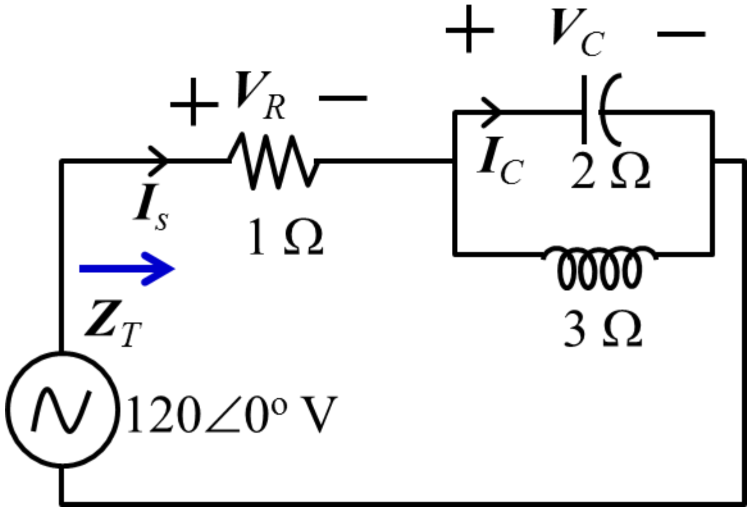
(c)

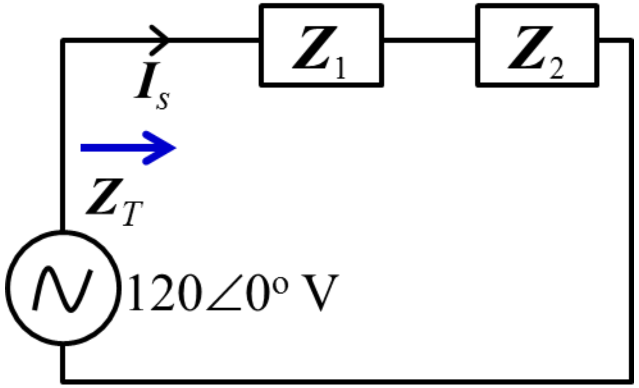
Series – Parallel Circuit

RLC Series Circuit	RLC Parallel Circuit
	
$X_L = \omega L$; $X_C = \frac{1}{\omega C}$	$G = \frac{1}{R}$; $B_L = \frac{1}{X_L} = \frac{1}{\omega L}$; $B_C = \frac{1}{X_C} = \omega C$
$\mathbf{Z}_R = R$; $\mathbf{Z}_L = jX_L$; $\mathbf{Z}_C = -jX_C$	$\mathbf{Y}_R = G$; $\mathbf{Y}_L = -jB_L$; $\mathbf{Y}_C = jB_C$
$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \mathbf{Z}_R + \mathbf{Z}_L + \mathbf{Z}_C = R + jX_L - jX_C$	$\mathbf{Y} = \frac{\mathbf{I}}{\mathbf{V}} = \mathbf{Y}_R + \mathbf{Y}_L + \mathbf{Y}_C = G + jB_C - jB_L$
$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}}$; $\mathbf{V} = \mathbf{IZ}$	$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \mathbf{VY}$; $\mathbf{V} = \mathbf{IZ} = \frac{\mathbf{I}}{\mathbf{Y}}$
$\mathbf{V}_R = \mathbf{IZ}_R = \frac{\mathbf{Z}_R}{\mathbf{Z}}\mathbf{V}$; $\mathbf{V}_L = \mathbf{IZ}_L = \frac{\mathbf{Z}_L}{\mathbf{Z}}\mathbf{V}$ $\mathbf{V}_C = \mathbf{IZ}_C = \frac{\mathbf{Z}_C}{\mathbf{Z}}\mathbf{V}$	$\mathbf{I}_R = \mathbf{VY}_R = \frac{\mathbf{Y}_R}{\mathbf{Y}}\mathbf{I}$; $\mathbf{I}_L = \mathbf{VY}_L = \frac{\mathbf{Y}_L}{\mathbf{Y}}\mathbf{I}$ $\mathbf{I}_C = \mathbf{VY}_C = \frac{\mathbf{Y}_C}{\mathbf{Y}}\mathbf{I}$
$P = I_{rms}^2 R$; $Q_L = I_{rms}^2 X_L$; $Q_C = I_{rms}^2 X_C$	$P = I_R^2 R$; $Q_L = I_L^2 X_L$; $Q_C = I_C^2 X_C$

Example

For the following circuits: (**i**) Calculate the total impedance, (**ii**) Calculate the source current, $\boldsymbol{I_s}$, and current $\boldsymbol{I_C}$.(**iii**) Calculate $\boldsymbol{V_R}$ and $\boldsymbol{V_C}$. (iv) Compute the power delivered. (**v**) Find the power factor.





$$\boldsymbol{I_S} = \frac{\boldsymbol{V}}{\boldsymbol{Z_T}} = \frac{120\angle 0^{\circ}}{6.08\angle -80.54^{\circ}}$$

$$= 19.74\angle 80.54^{\circ} \text{ A}$$

$$\boldsymbol{V_R} = \boldsymbol{I_S}\boldsymbol{Z_1} = (19.74\angle 80.54^{\circ})(1)$$

$$= 19.74\angle 80.54^{\circ} \text{ V}$$

$$\boldsymbol{Z_1} = 3\ \Omega$$

$$\boldsymbol{Z_2} = \frac{(-j2)(j3)}{-j2 + j3} = -j6\ \Omega$$

$$\boldsymbol{Z_T} = \boldsymbol{Z_1} + \boldsymbol{Z_2} = 3 - j6$$

$$= 6.08\angle -80.54^{\circ}\ \Omega$$

$$\boldsymbol{V_C} = \boldsymbol{I_S}\boldsymbol{Z_2} = (19.74\angle 80.54^{\circ})(-j6) = 118.44\angle -9.46^{\circ} \text{ V}$$

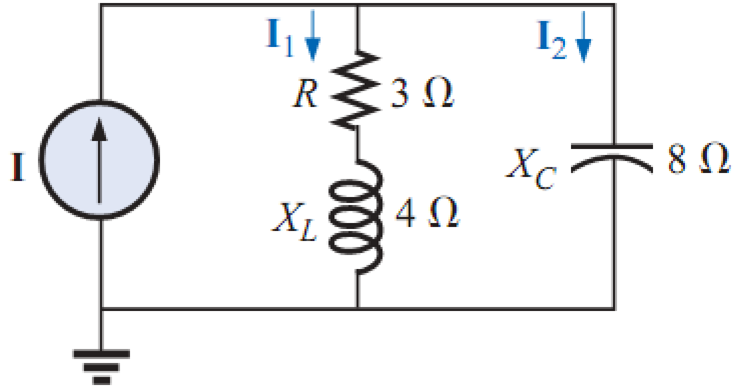
$$\boldsymbol{I_C} = \frac{\boldsymbol{V_C}}{\boldsymbol{Z_C}} = \frac{118.44\angle -9.46^{\circ}}{-j2} = 59.22\angle 80.54^{\circ} \text{ A}$$

$$P = \boldsymbol{I_S}^2 R = 19.74^2 \times 1 = 389.67 \text{ W}$$

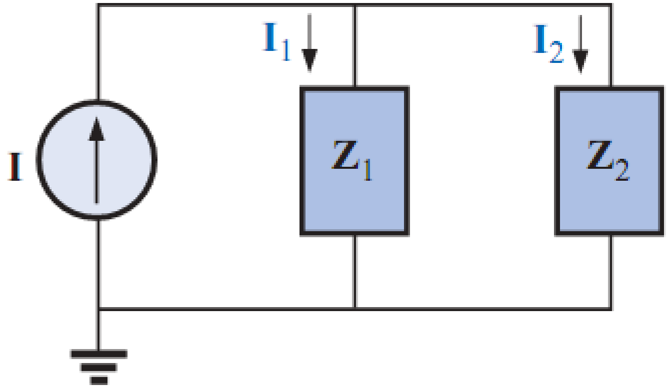
$$pf = \cos\theta = \cos(80.54^{\circ}) = 0.164 \text{ leading}$$

EXAMPLE 16.2 For the network of Fig. 16.3:

a. If $\mathbf{I}$ is 50 A $\angle 30°$, calculate $\mathbf{I}_1$ using the current divider rule.
b. Repeat part (a) for $\mathbf{I}_2$.
c. Verify Kirchhoff's current law at one node.



$$\mathbf{Z}_1 = R + j\,X_L = 3\ \Omega + j\,4\ \Omega = 5\ \Omega \angle 53.13°$$
$$\mathbf{Z}_2 = -j\,X_C = -j\,8\ \Omega = 8\ \Omega \angle -90°$$



Using the current divider rule yields

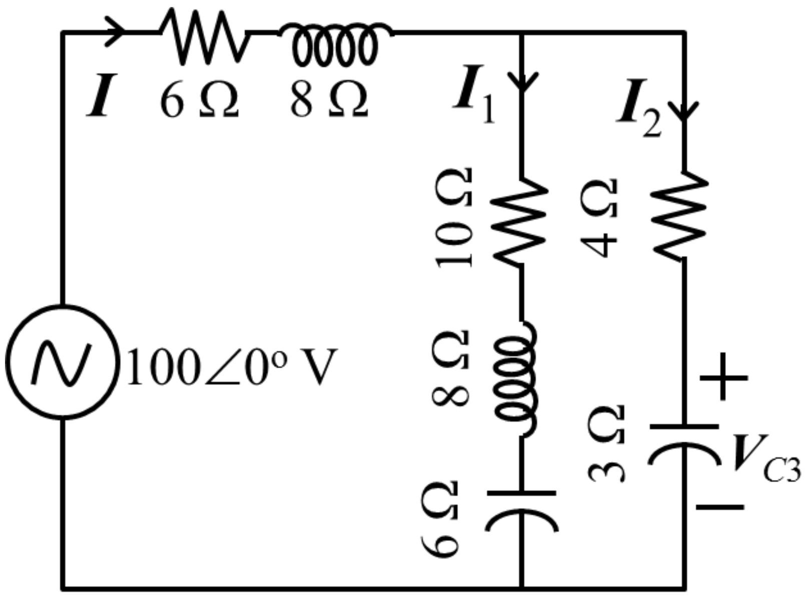
$$\mathbf{I}_1 = \frac{\mathbf{Z}_2\mathbf{I}}{\mathbf{Z}_2 + \mathbf{Z}_1} = \frac{(8\ \Omega \angle -90°)(50\ \text{A} \angle 30°)}{(-j\,8\ \Omega) + (3\ \Omega + j\,4\ \Omega)} = \frac{400 \angle -60°}{3 - j\,4}$$
$$= \frac{400 \angle -60°}{5 \angle -53.13°} = \mathbf{80\ A \angle -6.87°}$$

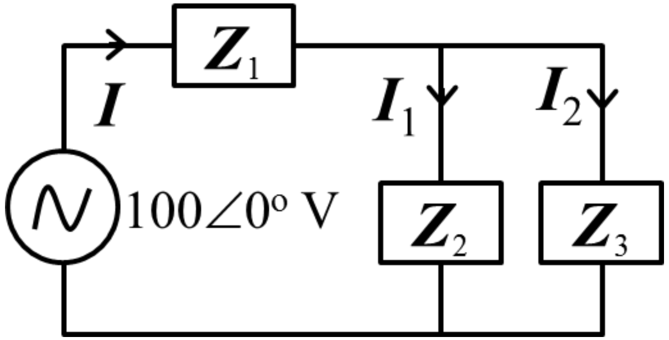
$$\mathbf{I}_2 = \frac{\mathbf{Z}_1\mathbf{I}}{\mathbf{Z}_2 + \mathbf{Z}_1} = \frac{(5\ \Omega \angle 53.13°)(50\ \text{A} \angle 30°)}{5\ \Omega \angle -53.13°} = \frac{250 \angle 83.13°}{5 \angle -53.13°}$$
$$= \mathbf{50\ A \angle 136.26°}$$

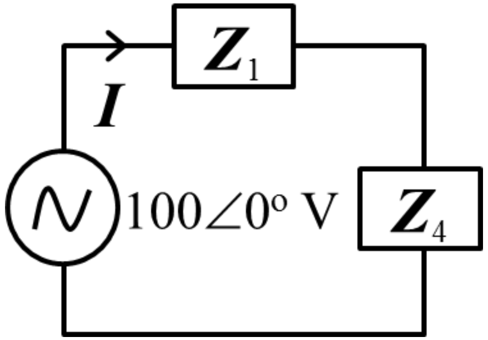
$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2$$
$$50\ \text{A} \angle 30° = 80\ \text{A} \angle -6.87° + 50\ \text{A} \angle 136.26°$$
$$= (79.43 - j\,9.57) + (-36.12 + j\,34.57)$$
$$= 43.31 + j\,25.0$$
$$50\ \text{A} \angle 30° = 50\ \text{A} \angle 30° \quad \text{(checks)}$$

Example

For the following circuits: Calculate the total impedance, $\boldsymbol{I}$, $\boldsymbol{I}_1$, $\boldsymbol{I}_2$, $\boldsymbol{V}_{C3}$.







$$\boldsymbol{Z}_1 = 6 + j8\ \Omega$$

$$\boldsymbol{Z}_2 = 10 + j8 - j6 = 10 + j2\ \Omega$$

$$\boldsymbol{Z}_3 = 4 - j3\ \Omega$$

$$\boldsymbol{Z}_4 = \frac{\boldsymbol{Z}_2\boldsymbol{Z}_3}{\boldsymbol{Z}_2 + \boldsymbol{Z}_3} = \frac{(10 + j2)(4 - j3)}{10 + j2 + 4 - j3} = 3.38 - j1.33\ \Omega$$

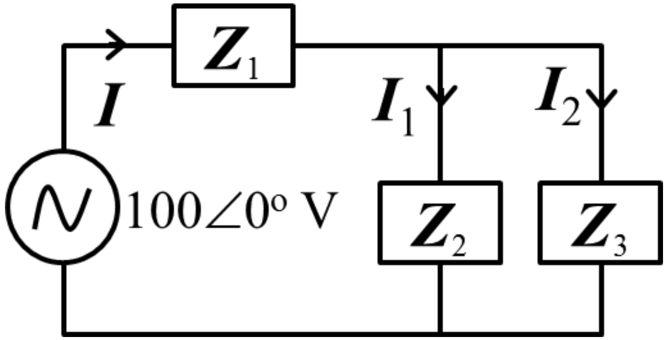
$$\boldsymbol{Z}_T = \boldsymbol{Z}_1 + \boldsymbol{Z}_4 = 6 + j8 + 3.38 - j1.33 = 9.38 + j6.67\ \Omega$$

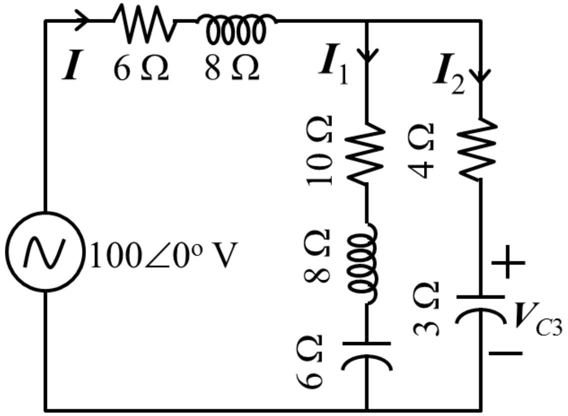
$$\boldsymbol{I} = \frac{\boldsymbol{V}}{\boldsymbol{Z}_T} = \frac{100\angle 0^\circ}{9.38 + j6.67} = 7.08 - j5.03\ \text{A}$$

$$\boldsymbol{I}_1 = \frac{\boldsymbol{Z}_3}{\boldsymbol{Z}_2 + \boldsymbol{Z}_3}\boldsymbol{I} = \frac{4 - j3}{10 + j2 + 4 - j3}(7.08 - j5.03) = 1.15 - j2.87 \text{ A}$$

$$\boldsymbol{I}_2 = \frac{\boldsymbol{Z}_2}{\boldsymbol{Z}_2 + \boldsymbol{Z}_3}\boldsymbol{I} = \frac{10 + j2}{10 + j2 + 4 - j3}(7.08 - j5.03) = 5.9 - j2.16 \text{ A}$$

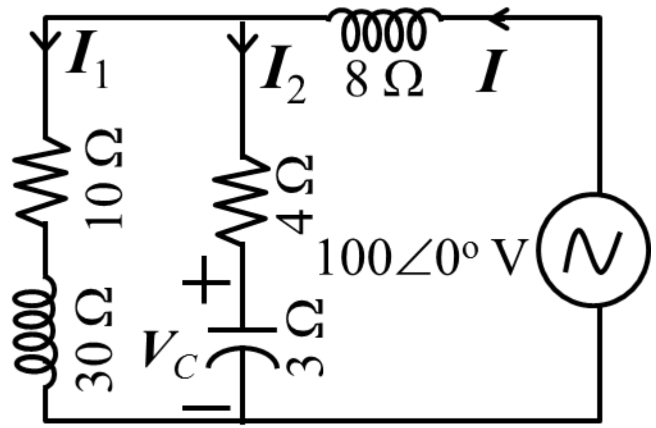
$$\boldsymbol{V}_{C3} = -j3\boldsymbol{I}_2 = -j3(5.9 - j2.16) = -6.48 - j17.8 \text{ A}$$

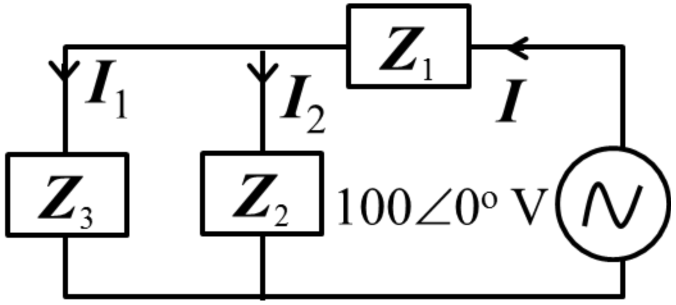


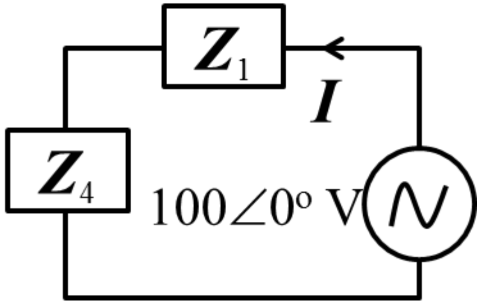


Example

For the following circuits: Calculate the total impedance, $\boldsymbol{I}$, $\boldsymbol{I}_1$, $\boldsymbol{I}_2$, $\boldsymbol{V}_C$. Calculate power consumed by load.







$$\boldsymbol{Z}_1 = j8\ \Omega$$

$$\boldsymbol{Z}_2 = 4 - j3\ \Omega$$

$$\boldsymbol{Z}_3 = 10 + j30\ \Omega$$

$$\boldsymbol{Z}_4 = \frac{\boldsymbol{Z}_2\boldsymbol{Z}_3}{\boldsymbol{Z}_2 + \boldsymbol{Z}_3} = \frac{(4 - j3)(10 + j30)}{4 - j3 + 10 + j30}$$

$$= 3.84 - j3.84\ \Omega$$

$$\boldsymbol{Z}_T = \boldsymbol{Z}_1 + \boldsymbol{Z}_4 = j8 + 3.84 - j3.84 = 3.84 + j4.15\ \Omega$$

$$\boldsymbol{I} = \frac{\boldsymbol{V}}{\boldsymbol{Z}_T} = \frac{100}{3.84 + j4.15} = 12 - j12.96 = 17.66\angle -47.1^\circ\ \text{A}$$

$$\boldsymbol{I}_1 = \frac{\boldsymbol{Z}_2}{\boldsymbol{Z}_2 + \boldsymbol{Z}_3}\boldsymbol{I} = \frac{4 - j3}{4 - j3 + 10 + j30}(12 - j12.96) = 3.04\angle -163.77^\circ \text{ A}$$

$$\boldsymbol{I}_2 = \frac{\boldsymbol{Z}_3}{\boldsymbol{Z}_2 + \boldsymbol{Z}_3}\boldsymbol{I} = \frac{10 + j30}{4 - j3 + 10 + j30}(12 - j12.96) = 19.22\angle -39.07^\circ \text{ A}$$

$$\boldsymbol{V}_C = 3\boldsymbol{I}_2 = 3(19.22\angle -39.07^\circ) = 57.66\angle -39.07^\circ \text{ V}$$

$$P = VI\cos\theta = 100 \times 17.66\cos(-47.1^\circ) = 1200 \text{ W}$$

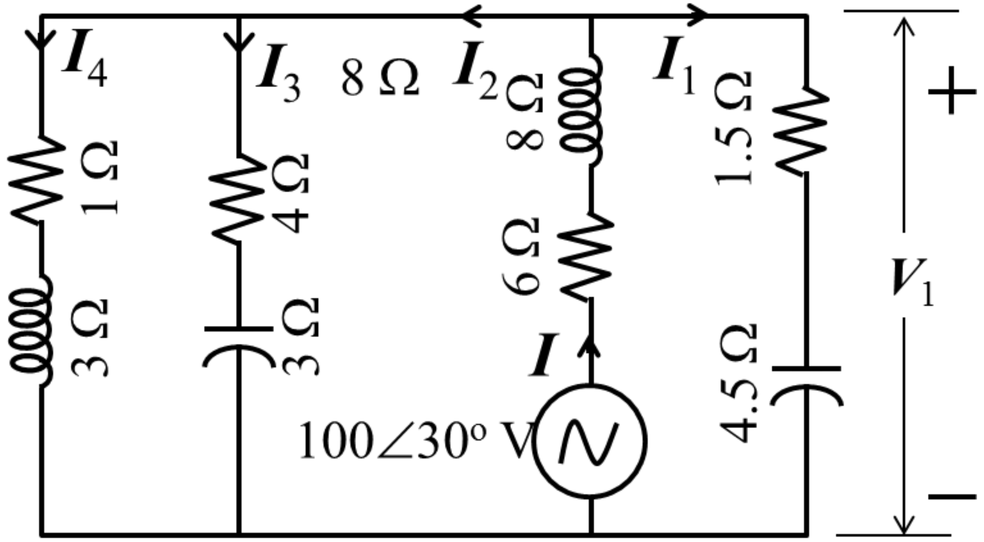
Power Consumed by $\boldsymbol{Z}_2$ load. $P_{4\Omega} = I_2{}^2 \times 4 = 19.22^2 \times 4 = 1107.8 \text{ W}$

Power Consumed by $\boldsymbol{Z}_3$ load. $P_{10\Omega} = I_1{}^2 \times 10 = 3.04^2 \times 10 = 92.32 \text{ W}$

$$P = P_{4\Omega} + P_{10\Omega} = 1200 \text{ W}$$

Example

For the following circuits: Calculate the total impedance, $\boldsymbol{I}$, $\boldsymbol{I}_1$, $\boldsymbol{I}_2$, $\boldsymbol{I}_3$, $\boldsymbol{I}_4$, $\boldsymbol{V}_1$. Calculate power consumed by load.

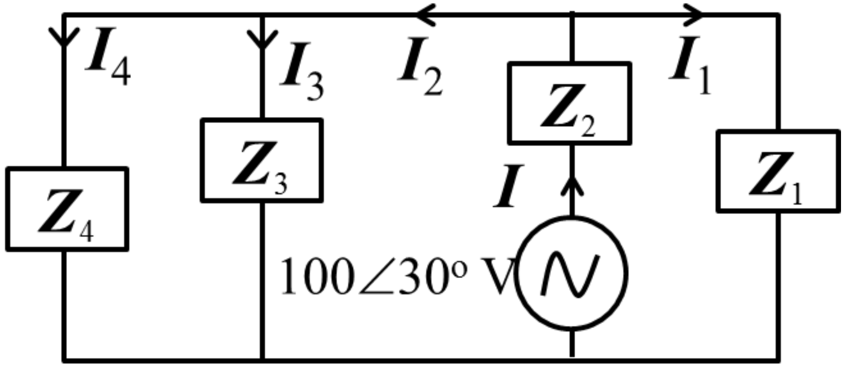


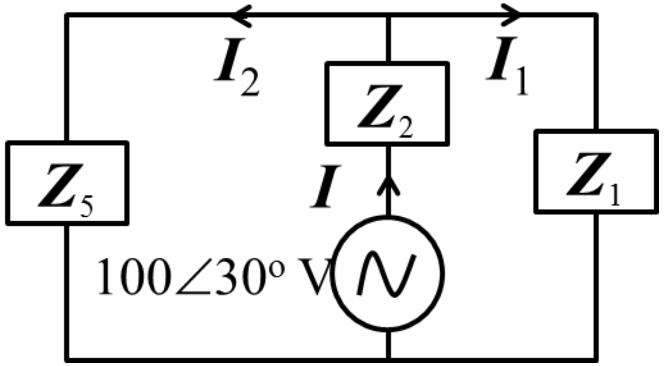
$$\mathbf{Z}_1 = 1.5 - j4.5 = 4.74\angle -71.6^\circ\ \Omega$$

$$\mathbf{Z}_2 = 6 + j8 = 10\angle 53.13^\circ\ \Omega$$

$$\mathbf{Z}_3 = 4 - j3 = 5\angle -36.87^\circ\ \Omega$$

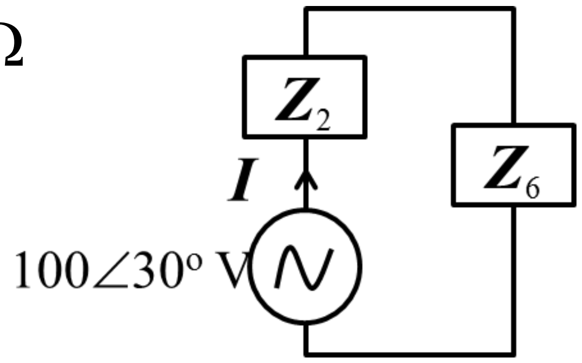
$$\mathbf{Z}_4 = 1 + j3 = 3.16\angle 71.6^\circ\ \Omega$$





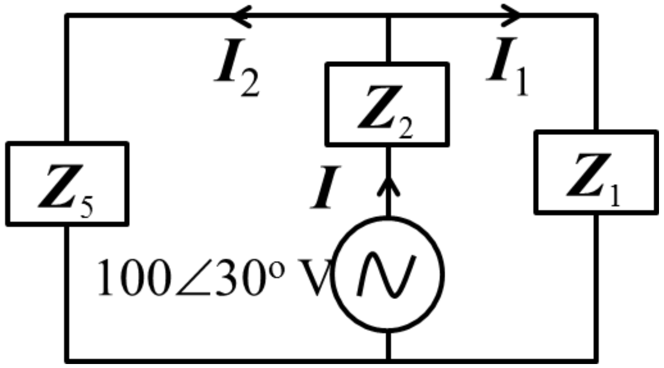
$$\mathbf{Z}_5 = \frac{\mathbf{Z}_3\mathbf{Z}_4}{\mathbf{Z}_3 + \mathbf{Z}_4} = \frac{(4 - j3)(1 + j3)}{4 - j3 + 1 + j3} = 2.6 + j1.8 = 3.2\angle 34.7^\circ\ \Omega$$

$$Z_6 = \frac{Z_5 Z_1}{Z_5 + Z_1} = \frac{(2.6 + j1.8)(1.5 - j4.5)}{2.6 + j1.8 + 1.5 - j4.5} = 3.05 - j0.18 = 3.06\angle -3.5^\circ \ \Omega$$



$$Z = Z_2 + Z_6 = 6 + j8 + 3.05 - j0.19 = 9.05 + j7.8 = 11.95\angle 40.8^\circ \ \Omega$$

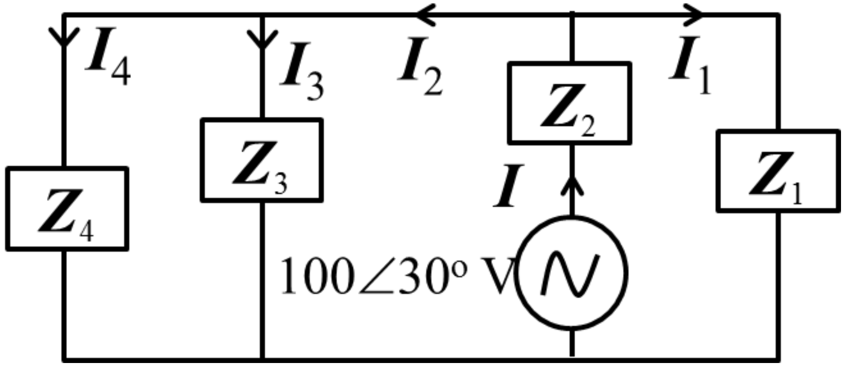
$$I = \frac{V}{Z} = \frac{100\angle 30^\circ}{11.95\angle 40.8^\circ} = 8.4\angle -10.8^\circ \ \text{A}$$



$$I_1 = \frac{Z_5}{Z_1 + Z_5} I = 5.4\angle 57.3^\circ \ \text{A}$$

$$I_2 = \frac{Z_1}{Z_1 + Z_5} I = 8.1\angle -49^\circ \ \text{A}$$

$$I_3 = \frac{Z_4}{Z_3 + Z_4} I_2 = 5.1\angle 22^\circ \ \text{A}$$



$$I_4 = \frac{Z_3}{Z_3 + Z_4} I_2 = 8.1\angle -85.87^\circ \ \text{A}$$

$$V_1 = Z_1 I_1 = 25.5\angle -14.3^\circ \ \text{A}$$

$$P_{1\Omega} = I_4{}^2 \times 1 = 8.1^2 \times 1 = 65.312 \text{ W}$$

$$P_{4\Omega} = I_3{}^2 \times 4 = 5.1^2 \times 4 = 104.5 \text{ W}$$

$$P_{6\Omega} = I\ ^2 \times 6 = 5.4^2 \times 6 = 419.74 \text{ W}$$

$$P_{1.5\Omega} = I_1{}^2 \times 1.5 = 5.4^2 \times 1.5 = 43.54 \text{ W}$$

$$P = P_{1\Omega} + P_{4\Omega} + P_{8\Omega} + P_{1.5\Omega} = 633.1 \text{ W}$$

$$Q_{3L} = I_4{}^2 \times 3 = 8.1^2 \times 3 = 195.94 \text{ Var}$$

$$Q_{3C} = -I_3{}^2 \times 3 = 5.1^2 \times 3 = -78.37 \text{ Var}$$

$$Q_{8L} = I\ ^2 \times 8 = 8.36^2 \times 3 = 559.65 \text{ Var}$$

$$Q_{4.5C} = -I_1{}^2 \times 4.5 = 5.4^2 \times 4.5 = -130.62 \text{ Var}$$

$$Q = Q_{3L} + Q_{3C} + Q_{8L} + Q_{4.5C} = 546.58 \text{ Var}$$

$$\theta_v = 30°$$

$$\theta_i = -10.8°$$

$$\theta = \theta_v - \theta_i = 40.8°$$

$$pf = \cos\theta = \cos(40.8°) = 0.76$$

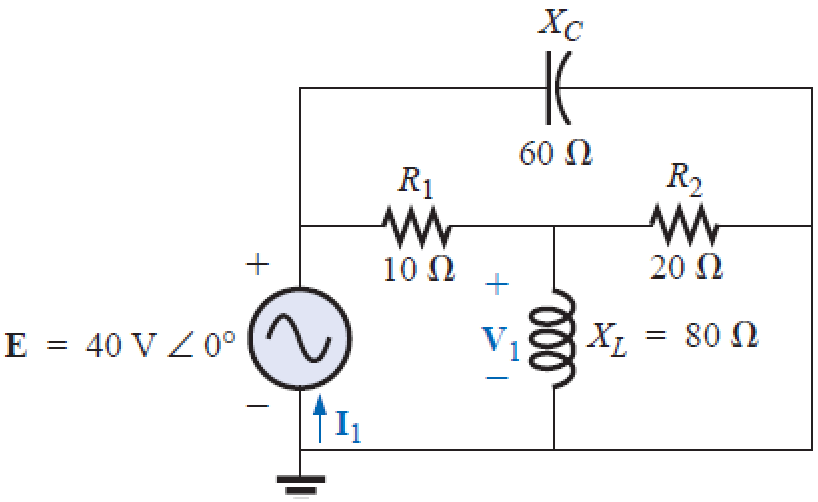
$$rf = \sin\theta = \sin(40.8°) = 0.65$$

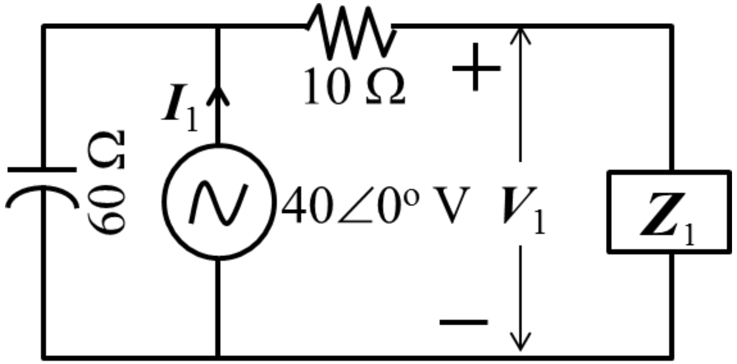
$$P = 100 \times 8.36 \times \cos(40.8°) = 633.1 \text{ W}$$

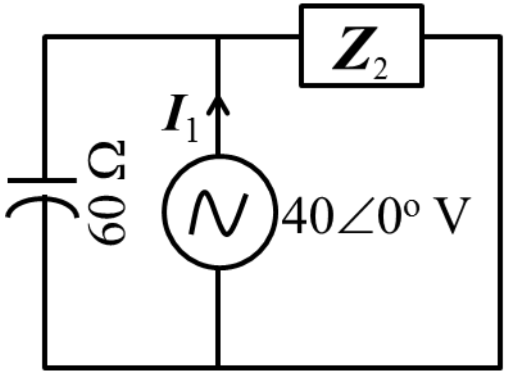
$$Q = 100 \times 8.36 \times \sin(40.8°) = 546.58 \text{ Var}$$

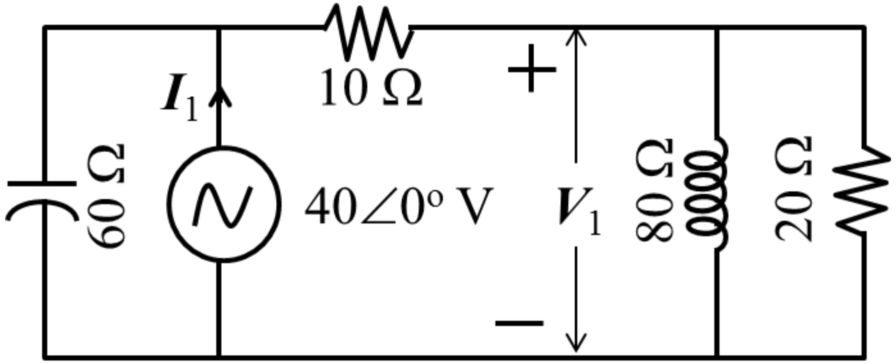
Example

For the following circuits: Calculate the total impedance, $\boldsymbol{I}_1$, $\boldsymbol{V}_1$.









$$\boldsymbol{Z}_1 = \frac{1}{\dfrac{1}{20} + \dfrac{1}{j80}} = 18.82 + j4.7 = 19.4\angle 14^\circ\ \Omega$$

$$\boldsymbol{Z}_2 = 10 + \boldsymbol{Z}_1 = 28.82 + j4.7 = 29.2\angle 9.27^\circ\ \Omega$$

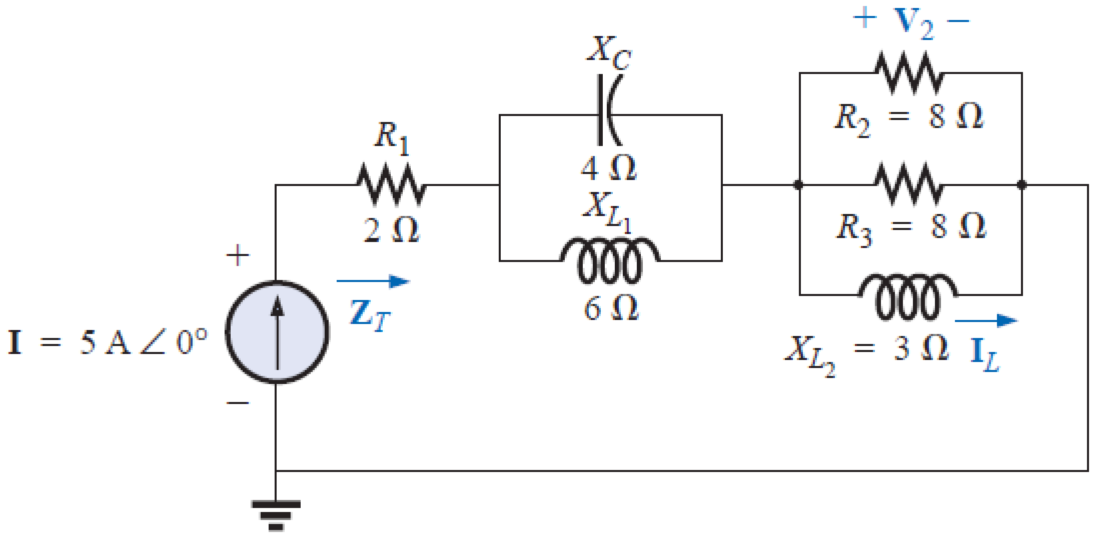
$$\boldsymbol{Z} = \frac{-j60 \times \boldsymbol{Z}_2}{-j60 + \boldsymbol{Z}_2} = 26.7 - j8.8 = 28.1\angle -18.3^\circ\ \Omega$$

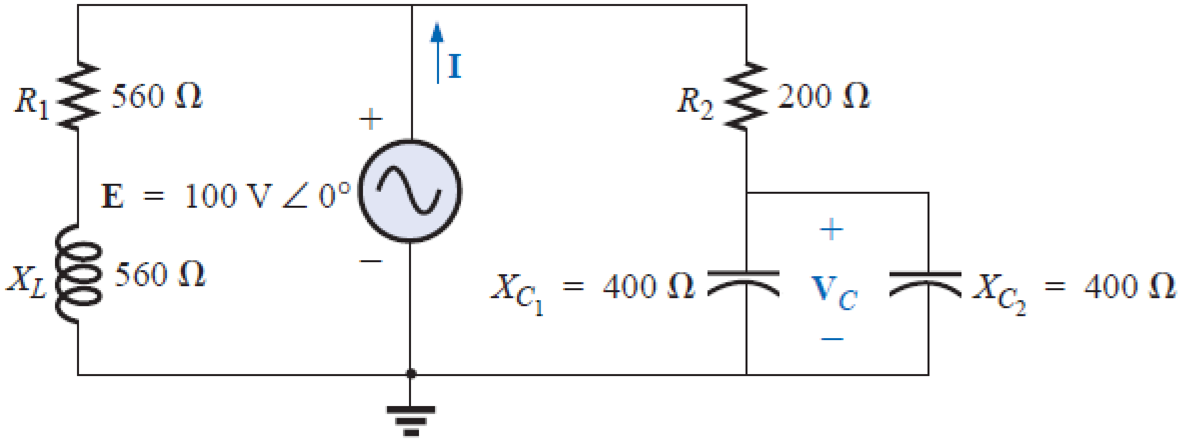
$$\boldsymbol{I} = \frac{\boldsymbol{V}}{\boldsymbol{Z}} = 1.35 + j0.446 = 1.42\angle 18.3^\circ\ \text{A}$$

$$\boldsymbol{V}_1 = \frac{\boldsymbol{Z}_1}{10 + \boldsymbol{Z}_1}\boldsymbol{V} = 26.48 + j2.2 = 26.57\angle 4.8^\circ\ \text{V}$$

Home Work 5.5

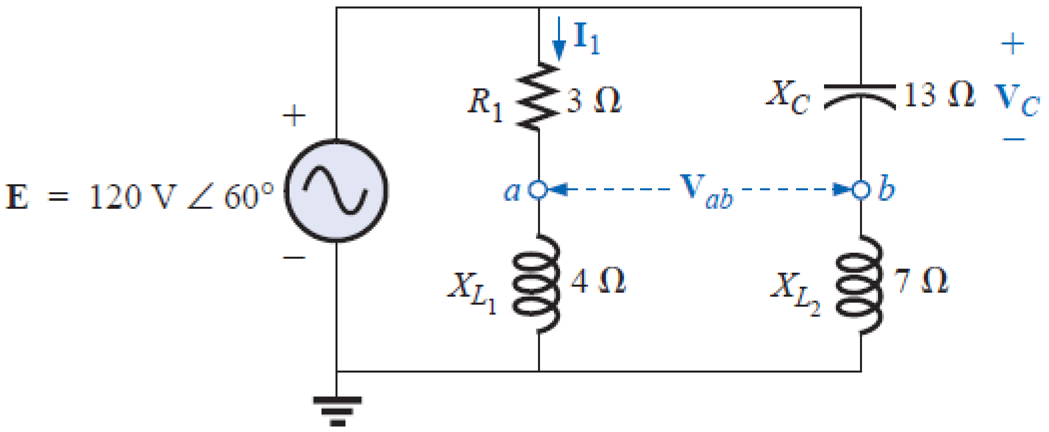
Problem 1: Calculate $\boldsymbol{Z_T}$, $\boldsymbol{V_2}$, $\boldsymbol{I_L}$ for the following network. Also calculate the power factor, reactive factor, power, reactive power and apparent power.

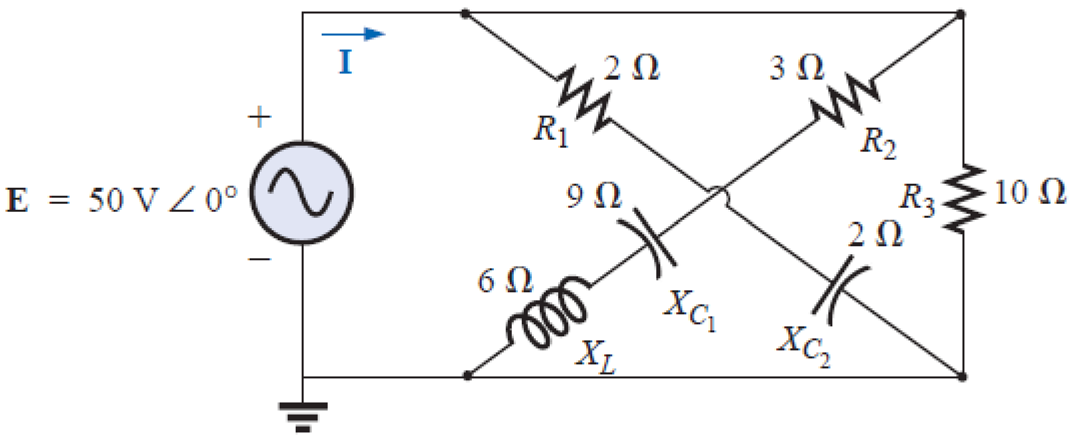




Problem 2: Calculate $\boldsymbol{I}$, and $\boldsymbol{V_C}$ for the following network. Also calculate the power factor, reactive factor, power, reactive power and apparent power.

Problem 3: Calculate $\boldsymbol{I_1}$, $\boldsymbol{V_C}$, $\boldsymbol{V_{ab}}$ for the following network.





Problem 4: Calculate $\boldsymbol{I}$ for the following network. Also calculate the power factor, reactive factor, power, reactive power and apparent power.