Series Circuits
Part I

Prepared by

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Impedance

According to ohm's Law, Impedance is the ratio of voltage to current. The unit of impedance is **ohm** [Ω].

$$\mathbf{Z} = \frac{v}{i} = \frac{V_m \sin(\omega t + \theta_v)}{I_m \sin(\omega t + \theta_i)} \quad \text{ohm} \ [\Omega]$$

$$\mathbf{Z} = \frac{V_{rms}\angle\theta_v}{I_{rms}\angle\theta_i} = \frac{V_{rms}}{I_{rms}}\angle(\theta_v - \theta_i) = \frac{V}{I}\angle(\theta_v - \theta_i) = |Z|\angle\theta_z \quad \Omega$$

$$\mathbf{Z} = \frac{V_m}{I_m}\angle(\theta_v - \theta_i) = Z\angle\theta_z = R + jX = (\textbf{Resistance}) + j(\textbf{Reactance})$$

Magnitude of impedance, $Z = \dfrac{V_m}{I_m} = \dfrac{V_{rms}}{I_{rms}} = \dfrac{V}{I} \quad \Omega$

Angle of impedance, $\theta_z = \theta_v - \theta_i$

Impedance is always constant that means impedance does not depend on the time that why **impedance is not a phasor quantity**.

$$\textbf{Resistance}: R = Z\cos\theta_Z$$

$$\textbf{Reactance}: X = Z\sin\theta_Z$$

We found that **the angle of impedance is the phase difference between voltage and current.** So the impedance angle can also be called the power factor angle and this angle can be used to calculate the power factor and reactive factor by as follows.

$$\textbf{Angle of power factor}, \theta = \theta_Z = \theta_v - \theta_i$$

$$\textbf{Power factor}, pf = \cos\theta = \cos\theta_Z = \cos(\theta_v - \theta_i)$$

$$\textbf{Reactive factor}, rf = \sin\theta = \sin\theta_Z = \sin(\theta_v - \theta_i)$$

Example 4.1.1: The voltage and current of a circuit are given as follows: $v(t)=100\sin(314t+60^{o})$ V and $i(t)=10\sin(314t+30^{o})$ A. Calculate **(i)** the magnitude of impedance, **(ii)** the angle of impedance, **(iii)** the value of resistance, **(iv)** the value of reactance.

Solution: Here, ω=314 rad/s, V_m = 100 V, I_m = 10 A, θ_v = 60^{o}, and θ_i = 30^{o}.

Magnitude of impedance, $Z=\dfrac{V_m}{I_m}=\dfrac{100}{10}=10\ \Omega$

Angle of impedance, $\theta_Z=\theta_v-\theta_i=60^\circ-30^\circ=30^\circ$

Impedance, $\boldsymbol{Z}=10\angle 30^\circ=8.66+j5\ \Omega$

Resistance, $R=8.66\ \Omega$

Reactance, $X=5\ \Omega$

Example 4.1.2: The voltage and current of a circuit are given as follows: $\boldsymbol{V}=15\angle 30^{o}$ V and $\boldsymbol{I}=1.2\angle 60^{o}$ A. Calculate **(i)** the magnitude of impedance, **(ii)** the angle of impedance, **(iii)** the value of resistance, **(iv)** the value of reactance.

Solution: **Impedance**, $\boldsymbol{Z}=\dfrac{\boldsymbol{V}}{\boldsymbol{I}}=\dfrac{15\angle 30^\circ}{1.2\angle 60^\circ}=12.5\angle -30^\circ=10.83-j6.25\ \Omega$

Magnitude of impedance, $Z=12.5\ \Omega$

Angle of impedance, $\theta_Z=\theta_v-\theta_i=-30^\circ$

Resistance, $R=10.83\ \Omega$

Reactance, $X=6.25\ \Omega$

Example 4.1.3: The voltage and current of a circuit are given as follows: ***V***=10.61+*j*10.61 V and ***I***=2.6-*j*1.5 A. Calculate (**i**) the magnitude of impedance, (**ii**) the angle of impedance, (**iii**) the value of resistance, (**iv**) the value of reactance.

Solution: $\boldsymbol{V} = 10.61 + j10.61 = 15\angle 45°\ \text{V}$ $\qquad \boldsymbol{I} = 2.6 - j1.5 = 3\angle -30°\ \text{A}$

Impedance, $\boldsymbol{Z} = \dfrac{\boldsymbol{V}}{\boldsymbol{I}} = \dfrac{15\angle 45°}{3\angle -30°} = 5\angle 75° = 1.3 - j4.83\ \Omega$

Magnitude of impedance, $Z = 5\ \Omega$ $\qquad$ **Angle of impedance**, $\theta_Z = \theta_V - \theta_i = 75°$

Resistance, $R = 1.3\ \Omega$ $\qquad$ **Reactance**, $X = 4.83\ \Omega$

Example 4.1.4: The magnitude of an impedance is 100 ohm and the angle of impedance is 60°. Calculate (**i**) the value of resistance, (**ii**) the value of reactance, (**iii**) Write the impedance in polar form and Cartesian form.

Solution: Given, $Z = 100\ \Omega$ $\qquad \theta_Z = \theta_V - \theta_i = 60°$

Resistance: $R = Z\cos\theta_Z = 100 \times \cos 60° = 50\ \Omega$

Reactance: $X = Z\sin\theta_Z = 100 \times \sin 60° = 86.6\ \Omega$

Impedance, $\boldsymbol{Z} = 100\angle 60° = 50 + j86.6\ \Omega$

Example 4.1.5: The magnitude of an impedance is 10 ohm and the angle of impedance is 30°. If the applied voltage across the impedance is ***V***=50∠-60° V, write the current in polar form, Cartesian form and instantaneous equation form.

Solution: **Impedance,** $\boldsymbol{Z} = 10\angle 30° \ \Omega$

$$\boldsymbol{I} = \frac{\boldsymbol{V}}{\boldsymbol{Z}} = \frac{50\angle -60°}{10\angle 30°} = 5\angle -90° = -j5 \text{ A}$$

$$i(t) = \sqrt{2}\times 5\sin(\omega t - 90°) = 7.07\sin(\omega t - 90°) \text{ A}$$

Example 4.1.6: The magnitude of an impedance is 5 ohm and the angle of impedance is 60°. If the current flows through the impedance is ***I***=10∠-30° A, write the voltage in polar form, Cartesian form and instantaneous equation form.

Solution: **Impedance,** $\boldsymbol{Z} = 5\angle 60° \ \Omega$

$$\boldsymbol{V} = \boldsymbol{IZ} = (10\angle -30°)(5\angle 60°) = 50\angle 30° = 43.3 + j25 \text{ V}$$

$$v(t) = \sqrt{2}\times 50\sin(\omega t + 30°) = 70.7\sin(\omega t + 90°) \text{ V}$$

Home Work 4.1

Problem 4.1.1: For the following pairs of voltage and current: (**i**) Calculate the magnitude of impedance and angle of impedance. (**ii**) Write the impedance in both polar form and rectangular form.

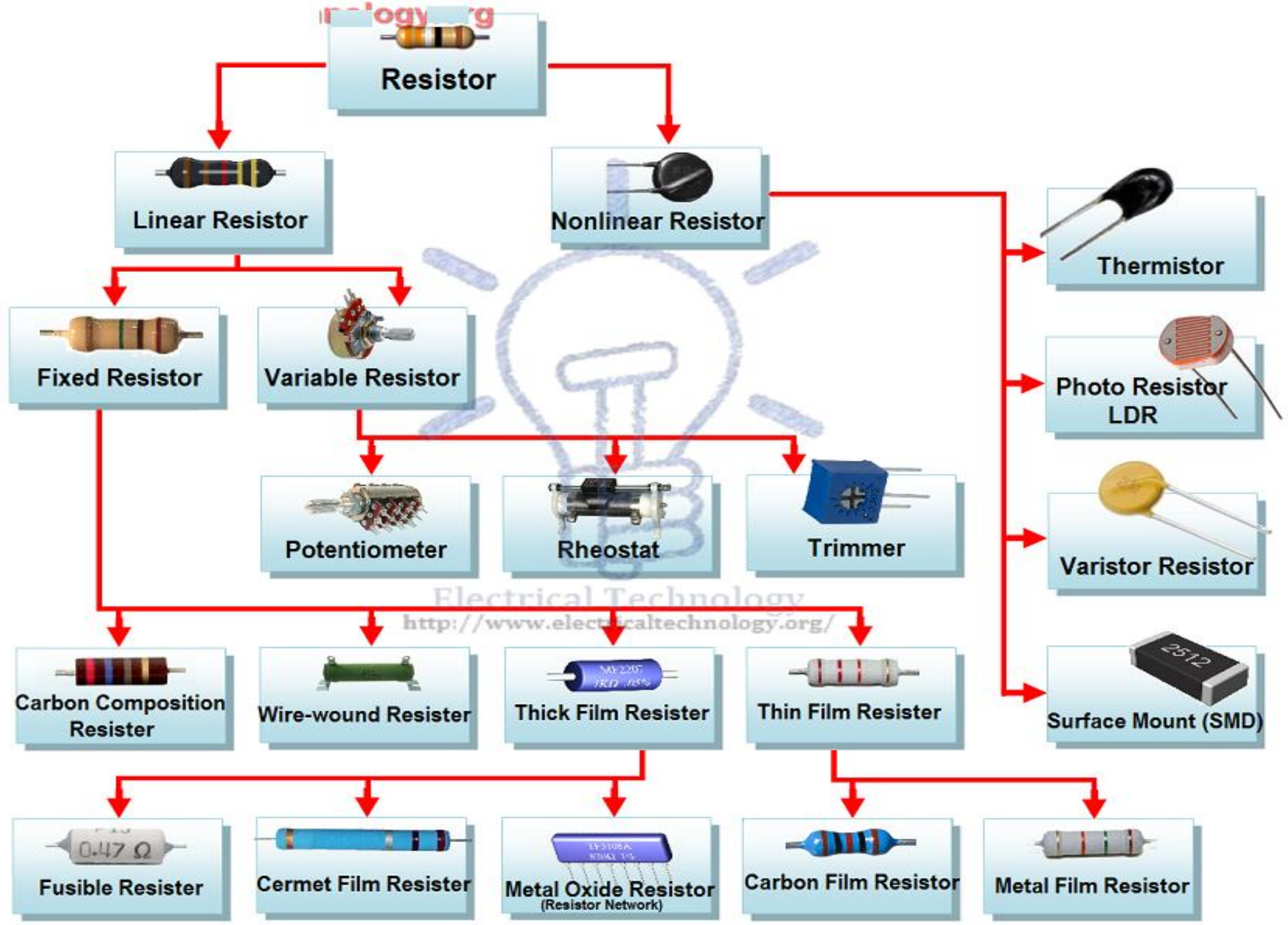
(***i***) $v(t) = 100\sin(\omega t + 40^o)$; $i(t) = 40\sin(\omega t - 20^o)$;

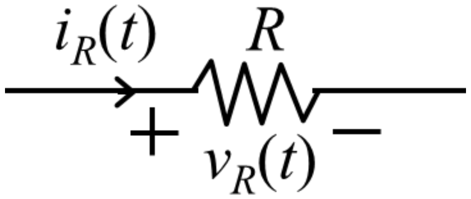
(***ii***) $\boldsymbol{V} = -15\angle 20^o$; $\boldsymbol{I} = -25\angle -45^o$;

(***iii***) $\boldsymbol{V} = 15 - j20$; $\boldsymbol{I} = 5 + j7.5$;

Pure Resistive Circuit

Response of Basic Resistor Element to a Sinusoidal Voltage or Current





Voltage and current relation in a resistor:

$$v_R(t) = Ri_R(t)$$

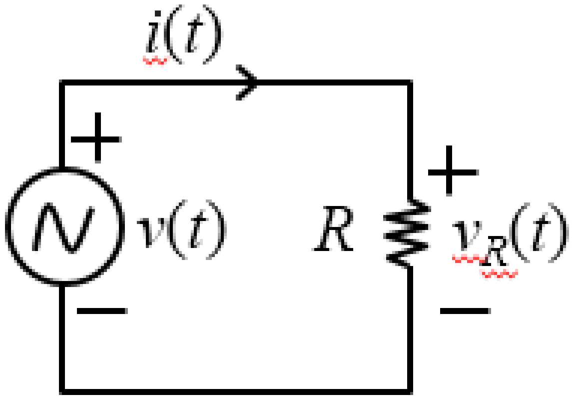
$$i_R(t) = \frac{v_R(t)}{R}$$

Let, the input is $v(t) = V_m \sin\omega t$ V, according to KVL, we have: $v(t) = v_R(t) = V_m \sin\omega t$

For a resistance the relation of voltage and current is: $v_R(t) = Ri(t)$

$$Ri(t) = V_m \sin\omega t$$

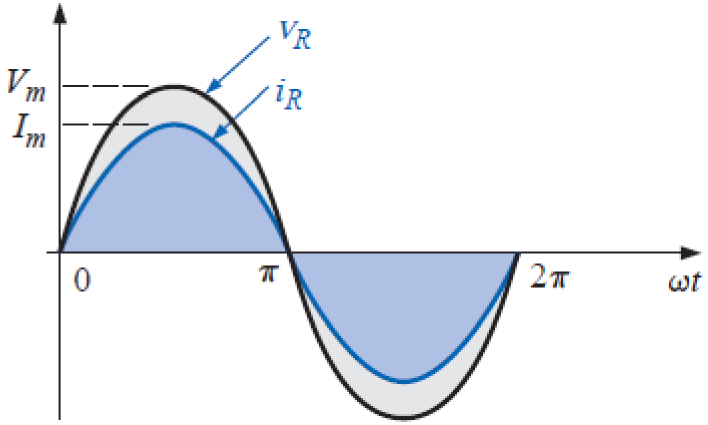
$$i(t) = \frac{V_m}{R}\sin\omega t = I_m \sin\omega t$$



Magnitude of impedance, $Z = \frac{V_m}{I_m} = R \quad \Omega$

Angle of impedance, $\theta_Z = \theta_v - \theta_i = 0°$

Impedance of a Resistor, $\mathbf{Z} = \mathbf{Z}_R = R\angle 0° = R\ \Omega$



The phase difference between voltage across and current through a resistor is zero.

***For a purely resistive element, the voltage across and the current through the element are in phase*.**

Power of Resistive Load

The power of a resistive load:

$$p(t) = (V_m \sin \omega t)(I_m \sin \omega t) = V_m I_m \sin^2 \omega t = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$$

$$p(t) = V_{rms} I_{rms} - V_{rms} I_{rms} \cos 2\omega t$$

Power Factor :

$$\text{pf} = \cos(\theta_Z) = \cos(0) = 1$$

For resistive load, the power factor is 1 which called **unity power factor**.

Reactive Factor :

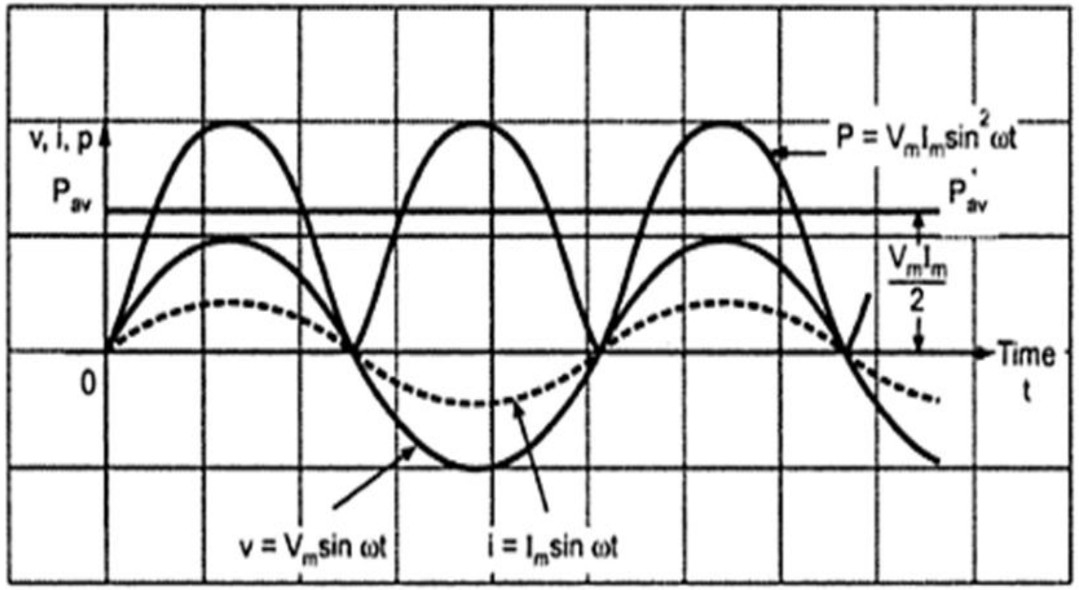
$$\text{rf} = \sin(\theta_Z) = \sin(0) = 0$$

Average or Real Power :

$$P = P_{ave} = V_{rms} I_{rms} = \frac{V_{rms}^2}{R} = I_{rms}^2 R \quad \text{W}$$

Reactive Power :

$$Q = P_X = V_{rms} I_{rms} \sin \theta_Z = 0 \quad \text{VAR}$$



v, i and p for purely resistive circuit

Apparent Power :

$$S=\sqrt{P_r^2+P_x^2}=\sqrt{P^2+Q^2}=P=\frac{V_m I_m}{2}=V_{rms}I_{rms}=VI \quad \text{VA}$$

The **instantaneous real power** for resistive load:

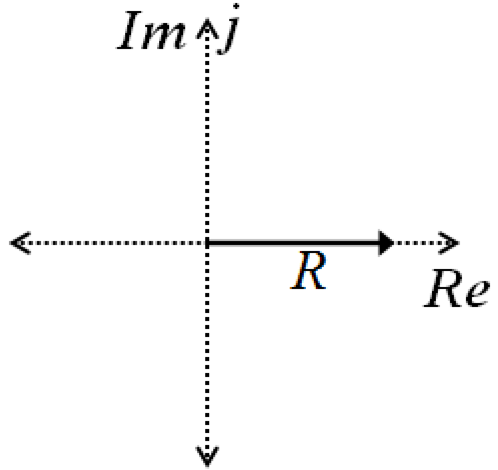
$$p_r(t)=P-P\cos 2\omega t=\frac{V_m I_m}{2}-\frac{V_m I_m}{2}\cos 2\omega t \qquad \text{W}$$

The **instantaneous reactive power** for resistive load:

$$p_i(t)=Q\sin 2\omega t=0 \qquad VAR$$

Complex Algebra for Only Resistive Circuit

Impedance for a resistance is: $\boldsymbol{Z}_R = R\angle 0° = R\ \Omega$



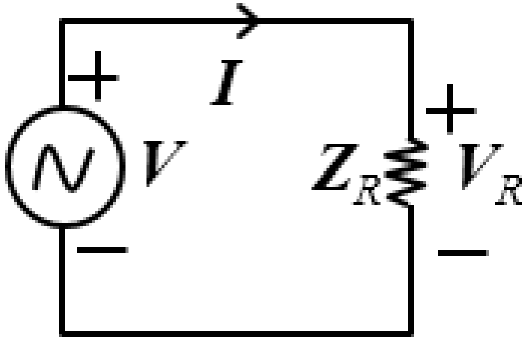
Impedance diagram for resistive load

Applied Voltage in Complex form: $\boldsymbol{V} = \dfrac{V_m}{\sqrt{2}}\angle\theta_v = V_{rms}\angle\theta_v = V\angle\theta_v$ V

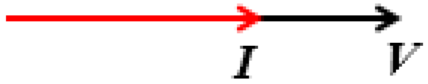
Current can be calculated as:

$$\boldsymbol{I} = \frac{\boldsymbol{V}}{\boldsymbol{Z}_R} = \frac{V_{rms}\angle\theta_v}{R\angle 0°} = \frac{V_{rms}}{R}\angle\theta_v = I_{rms}\angle\theta_i \quad \text{A}$$

Where, $I_{rms} = \dfrac{V_{rms}}{R} = \dfrac{I_m}{\sqrt{2}}; \qquad \theta_i = \theta_v$



Phasor diagram:



Summary For a Resistive Load

Magnitude of impedance, $Z = \frac{V_m}{I_m} = R \quad \Omega$

$\theta_i = \theta_v = 0$

Angle of impedance, $\theta_z = \theta_v - \theta_i = 0°$

Impedance of a Resistor, $\mathbf{Z} = \mathbf{Z}_R = R\angle 0° = R + j0\ \Omega$

The phase difference between voltage across and current through a resistor is zero.

That means, the voltage across and the current through the element are in phase.

The power factor is 1 that means **unity**.

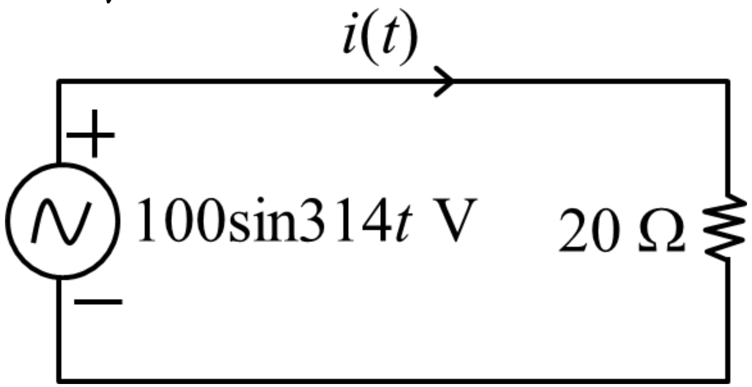
The reactive factor is 0 that means **zero**.

The reactive power is 0 that means **zero**.

The apparent power equals to real power.

Example 4.2.1: The voltage $v(t)$=100sin314t is applied to a 20 ohm resistance. (**i**) Find the expression of current. (**ii**) Calculate the impedance. (**iii**) Calculate the power factor, reactive factor, the real power, reactive power, and apparent power. (**iv**) Sketch the waveform of $v(t)$ and $i(t)$. (**v**) Draw the phasor or vector diagram.

$$v(t) = 100\sin 314t \ \ \text{V} \qquad v(t) = v_R(t) = 100\sin 314t \ \ \text{V} \qquad \theta_v = 0$$



We know that, the voltage drop across the resistance is:

$$v_R(t) = Ri(t) = v(t)$$

$$i(t) = \frac{v(t)}{R} = \frac{100\sin 314t}{20} = 5\sin 314t \ \ \text{A}$$

$$I_m = \frac{V_m}{R} = \frac{100}{20} = 5 \ \text{A} \qquad \theta_i = \theta_v = 0 \qquad Z = \frac{V_m}{I_m} = R = 20 \quad \Omega \qquad \theta_z = \theta_v - \theta_i = 0°$$

Impedance, $Z_R = 20\angle 0° = 20 + j0 \ \Omega$

$$\theta = \theta_z = \theta_v - \theta_i = 0$$

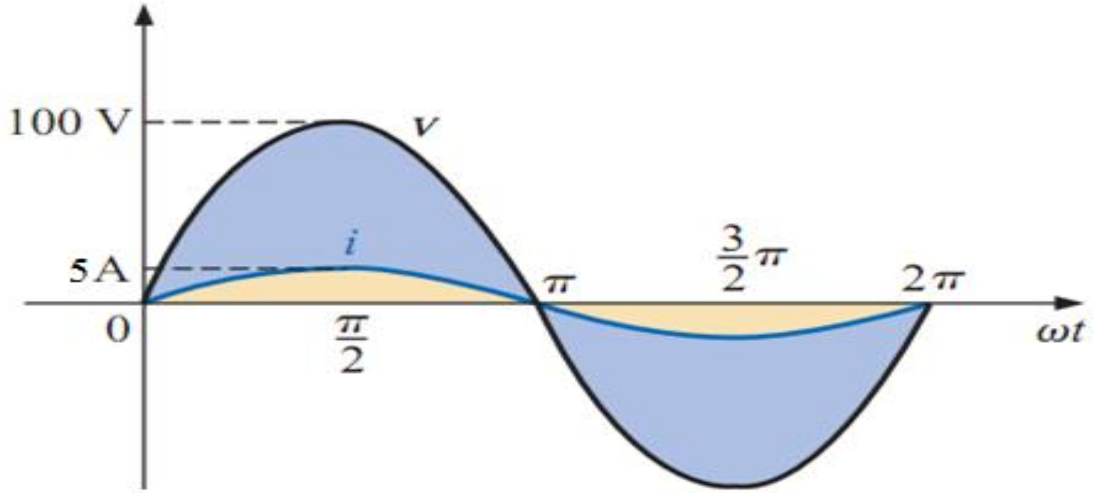
$$\textbf{Power Factor}: \mathrm{pf} = \cos(\theta_Z) = \cos(0) = 1$$

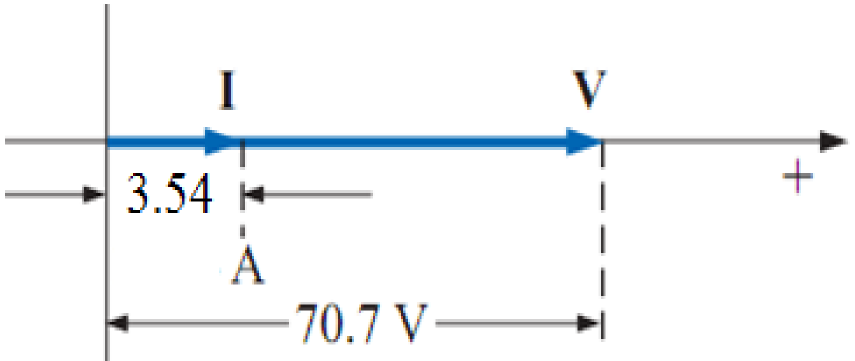
$$\textbf{Reactive Factor}: \mathrm{rf} = \sin(\theta_Z) = \sin(0) = 0$$

$$\textbf{Average or Real Power}: \ P = \frac{V_m I_m}{2} = \frac{100 \times 5}{2} = 250 \ \ \mathrm{W}$$

$$\textbf{Reactive Power}: \ Q = P_X = V_{rms} I_{rms} \sin\theta_Z = 0 \ \ \mathrm{VAR}$$

$$\textbf{Apparent Power}: \ S = \sqrt{P^2 + Q^2} = P = 250 \ \ \mathrm{VA}$$





Phasor or Vector diagram

Solution using Complex Algebra

Impedance, $\boldsymbol{Z}_R = 20\angle 0° = 20\ \Omega$

$\theta_v = 0$

$$\boldsymbol{V} = \frac{V_m}{\sqrt{2}}\angle\theta_v = \frac{100}{\sqrt{2}}\angle 0° = 70.7\angle 0°\ \text{V}$$

$$\boldsymbol{I} = \frac{\boldsymbol{V}}{\boldsymbol{Z}_R} = \frac{70.7\angle 0°}{20\angle 0°} = 3.54\angle 0°\ \text{A}$$

$$I_m = \sqrt{2}I_{rms} = \sqrt{2}\times 3.54 = 5\ \text{A}$$

$i(t) = 5\sin 314t\ \text{A}$

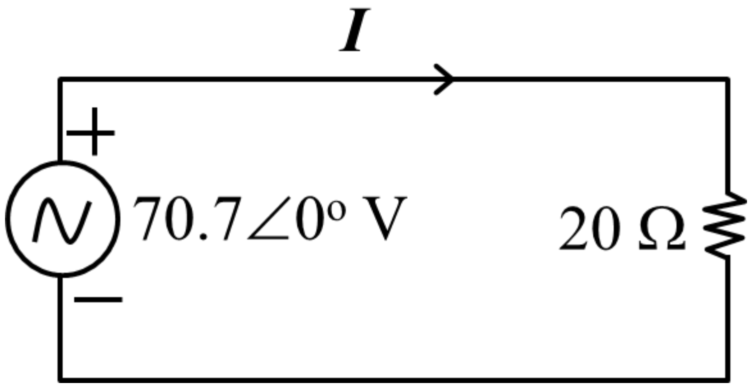
$\theta_i = 0$

$\theta = \theta_z = \theta_v - \theta_i = 0$

Power Factor : $\text{pf} = \cos(\theta_z) = \cos(0) = 1$

Reactive Factor : $\text{rf} = \sin(\theta_z) = \sin(0) = 0$

Average or Real Power : $P = \frac{V_m I_m}{2} = \frac{100\times 5}{2} = 250\ \text{W}$

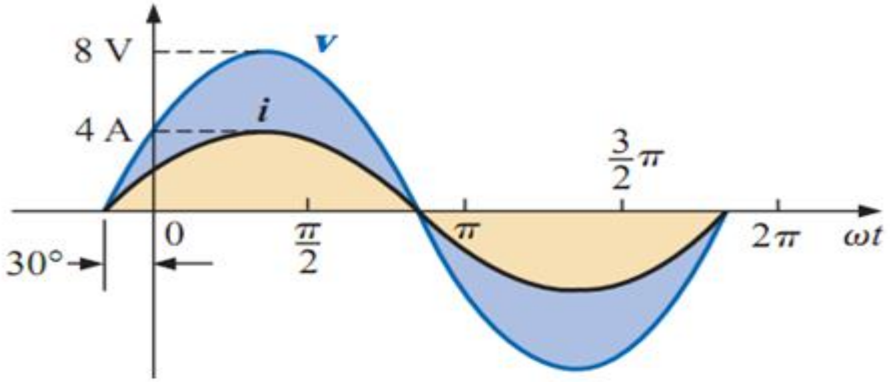


Reactive Power : $Q = P_X = V_{rms}I_{rms}\sin\theta_z = 0\ \text{VAR}$

Apparent Power : $S = \sqrt{P^2 + Q^2} = P = 250\ \text{VA}$

Example 4.2.2: The current $i(t)$=4sin(ωt+30°) of is flowing through a 2 Ω resistance. (**i**) Find the applied voltage v. (**ii**) Calculate the impedance. (**iii**) Calculate the power factor, reactive factor, the real power, reactive power, and apparent power. (**iv**) Sketch the waveform of $v(t)$ and $i(t)$. (**v**) Draw the phasor or vector diagram.

$$i(t) = 4\sin(\omega t + 30°)\ \text{A}$$



We know that, the voltage drop across the resistance is:

$$v(t) = v_R(t) = Ri(t) = 2 \times 4\sin(\omega t + 30°) = 8\sin(\omega t + 30°)\ \text{V}$$

$$V_m = RI_m = 4 \times 2 = 8\ \text{V}$$

$$\theta_i = 30° \qquad \theta_v = \theta_i = 30° \qquad Z = \frac{V_m}{I_m} = R = 2\ \ \Omega \qquad \theta_z = \theta_v - \theta_i = 0°$$

Impedance, $\boldsymbol{Z}_R = 2\angle 0° = 2\ \Omega$ $\qquad \theta = \theta_z = \theta_v - \theta_i = 0$

Power Factor : $\text{pf} = \cos(\theta_Z) = \cos(0) = 1$

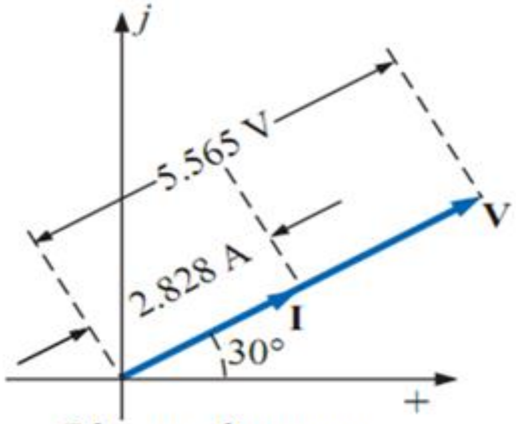
Reactive Factor : $\text{rf} = \sin(\theta_Z) = \sin(0) = 0$

Average or Real Power : $P = \dfrac{V_m I_m}{2} = \dfrac{8 \times 4}{2} = 16 \ \text{W}$

Reactive Power : $Q = P_X = V_{rms} I_{rms} \sin\theta_Z = 0 \ \text{VAR}$

Apparent Power : $S = \sqrt{P^2 + Q^2} = P = 16 \ \text{VA}$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{8}{\sqrt{2}} = 5.565 \ \text{V} \qquad I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2.828 \ \text{A}$$



Phasor diagram

Solution using Complex Algebra

Impedance, $\boldsymbol{Z}_R = 2\angle 0° = 2\ \Omega$ $\qquad \theta_v = 0$

$$\boldsymbol{I} = \frac{I_m}{\sqrt{2}}\angle\theta_i = \frac{4}{\sqrt{2}}\angle 30° = 2.828\angle 30°\ \text{A} \qquad \boldsymbol{V} = \boldsymbol{Z}_R\boldsymbol{I} = (2\angle 0°)(2.828\angle 30°) = 5.565\angle 30°\ \text{V}$$

$$V_m = \sqrt{2}V_{rms} = \sqrt{2}\times 5.565 = 8\ \text{V}$$

$$v(t) = 8\sin(\omega t + 30°)\ \text{V} \qquad \theta_i = 30° \qquad \theta = \theta_z = \theta_v - \theta_i = 0$$

Power Factor : $\text{pf} = \cos(\theta_z) = \cos(0) = 1$

Reactive Factor : $\text{rf} = \sin(\theta_z) = \sin(0) = 0$

Average or Real Power : $P = \dfrac{V_m I_m}{2} = \dfrac{8\times 4}{2} = 16\ \text{W}$

Reactive Power : $Q = P_x = V_{rms} I_{rms} \sin\theta_z = 0\ \text{VAR}$

Apparent Power : $S = \sqrt{P^2 + Q^2} = P = 16\ \text{VA}$

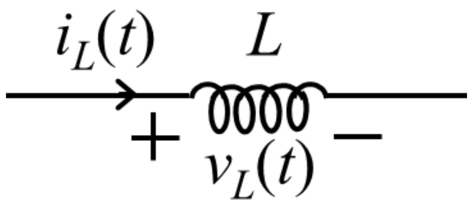
Home Work 4.2

Problem 4.2.1: The current $i(t)= 4\sin(\omega t-20^o)$ A flows through a 8 Ω resistor. (***i***) What is the sinusoidal expression for the voltage? (***ii***) Calculate the real power, reactive power, power factor, reactive factor. (***iii***) write the expression of instantaneous power. (***iv***) Sketch the $v(t)$ and $i(t)$ sinusoidal waveforms on the same axis. (***v***) Draw the phasor diagram.

Problem 4.2.2: The voltage $v(t)= 20\sin(\omega t+30^o)$ V is applied to a 4 Ω resistor. (***i***) What is the sinusoidal expression for the current? (**ii**) Calculate the real power, reactive power, power factor, reactive factor. (**iii**) write the expression of instantaneous power. (**iv**) Sketch the $v(t)$ and $i(t)$ sinusoidal waveforms on the same axis. (**v**) Draw the phasor diagram.

Pure Inductive Circuit

Response of Basic Inductor or Choke or Reactor Element to a Sinusoidal Voltage or Current



Voltage and current relation in a inductor:

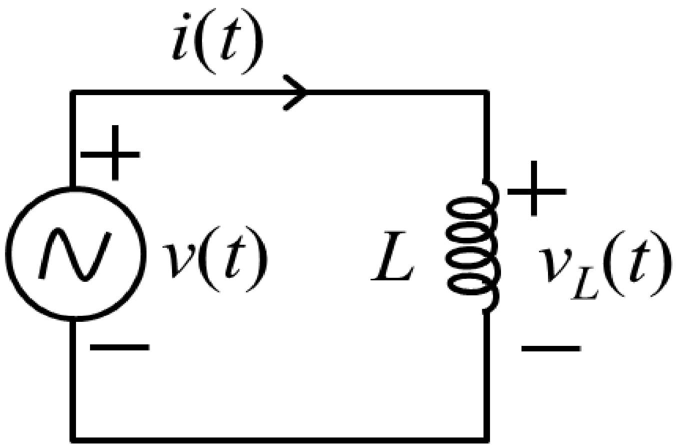
$$v_L(t) = L\frac{di_L(t)}{dt}$$

$$i_L(t) = \frac{1}{L}\int v_L(t)dt$$

Inductance opposes the rate of change current, and for this reason it is sometimes called *electrical inertia*.

Let, the input is $v(t) = V_m \sin\omega t$ V, according to KVL, we have: $v(t) = v_L(t) = V_m \sin\omega t$

For an inductance the relation of voltage and current is:



$$i_L(t) = \frac{1}{L}\int v_L(t)dt = \frac{V_m}{L}\int \sin\omega t dt = -\frac{V_m}{\omega L}\cos\omega t$$

$$i_L(t) = \frac{V_m}{\omega L}\sin(\omega t - 90°) = I_m \sin(\omega t + \theta_i)$$

Magnitude of impedance, $Z = \dfrac{V_m}{I_m} = \omega L = X_L \quad \Omega$

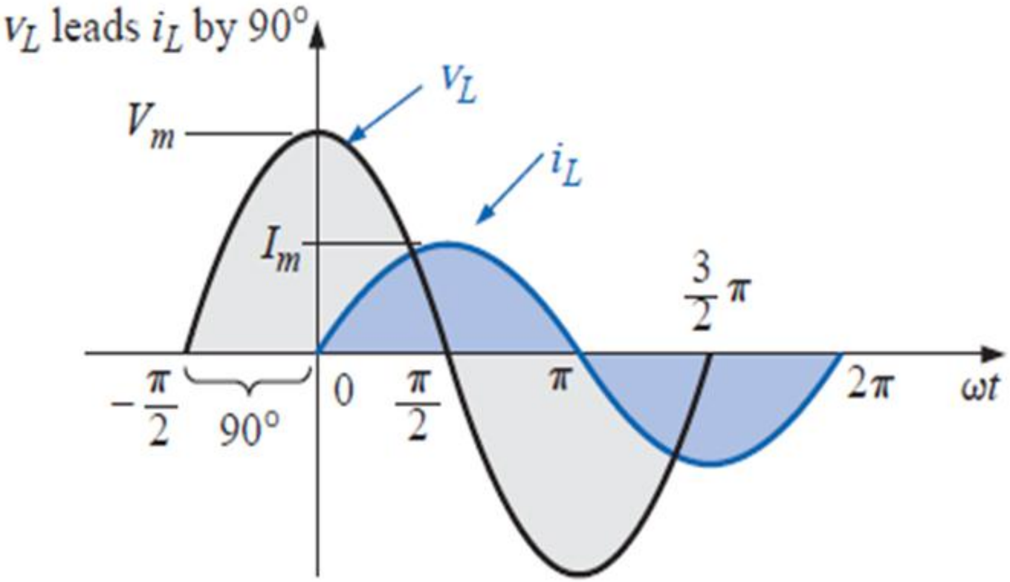
Inductive reactance, $X_L = \omega L = 2\pi f L \quad \Omega$

Angle of current, $\theta_i = -90°$

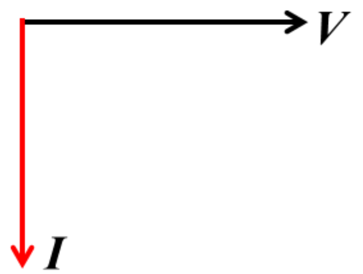
Angle of impedance, $\theta_Z = \theta_v - \theta_i = 90°$

Impedance of a Inuctor, $\mathbf{Z} = \mathbf{Z}_L = X_L \angle 90°$

$= jX_L \ \Omega$



The phase difference between voltage across and current through an inductor is 90º.
For a purely inductive element, the voltage leads the current through the inductive element by 90º. **Or,** ***the current lags the voltage in an inductive element by 90º***.

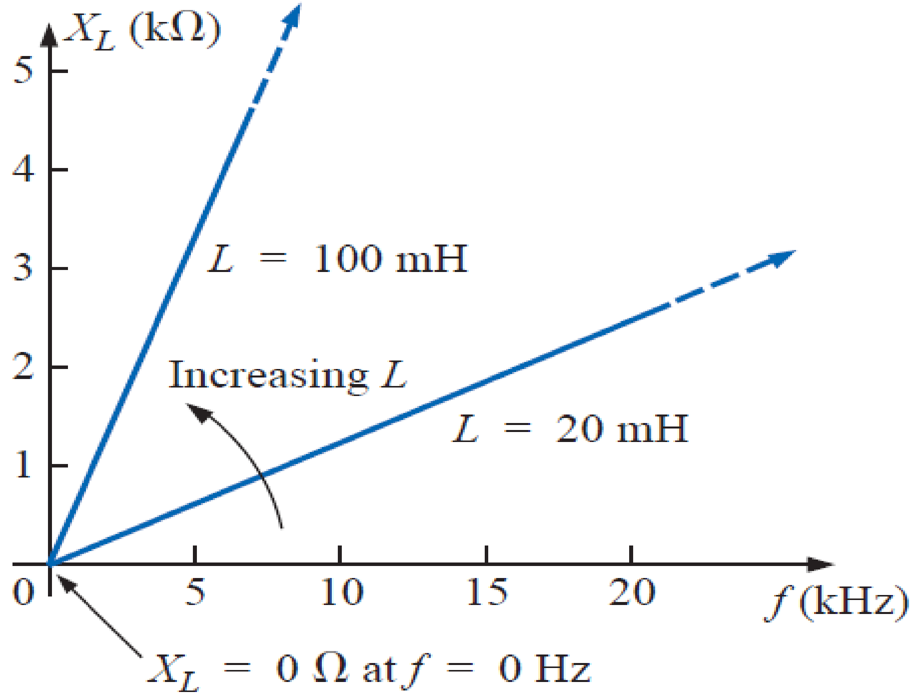


Phasor Diagram

Inductive Reactance vs Frequency

Inductive Reactance is defined as the opposition offered by the inductance of a circuit to the flow of an alternating sinusoidal current.

The inductive reactance is directly proportional to the frequency, i.e. $X_L \propto f$.
So, graph of X_L vs f is a **straight line** passing through the origin.
If frequency is zero, which is do far dc voltage, the inductive reactance is zero. Therefore, it is said that the inductance offers zero reactance (short-circuit) for the dc or steady current.



Power of Inductive Load

The power of an inductive load:

$$p(t) = V_m I_m \sin\omega t \sin(\omega t - 90°) = -V_m I_m \sin\omega t \cos\omega t = -\frac{V_m I_m}{2}\sin 2\omega t$$

Power Factor : $\text{pf} = \cos(\theta_z) = \cos(90°) = 0$

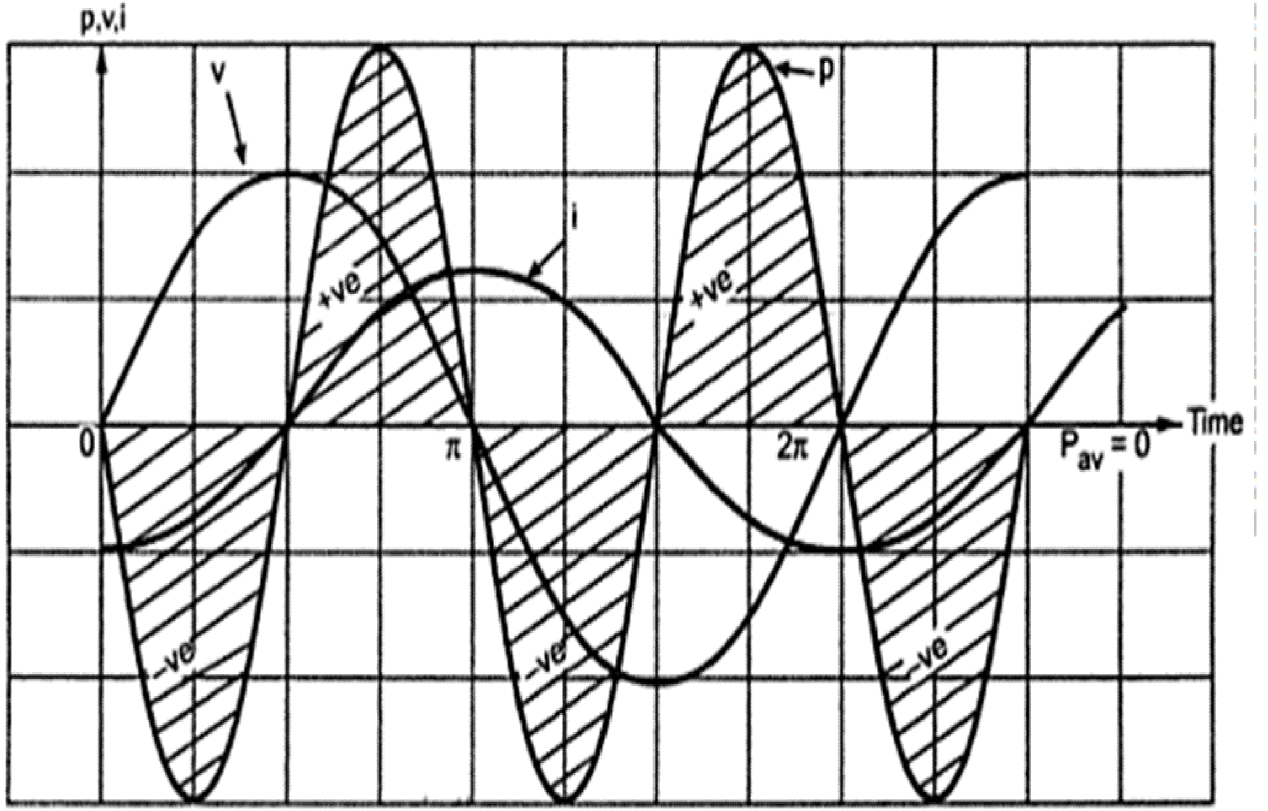
Reactive Factor : $\text{rf} = \sin(\theta_z) = \sin(90°) = 1$

For inductive load, the power factor is zero which called **zero lagging power factor**.

Average or Real Power :

$$P = P_{ave} = V_{rms} I_{rms} \cos\theta_z = 0$$

Inductor never consumes power.



Waveforms of voltage, current and power

Reactive Power : $Q_L = P_{Lx} = V_{rms} I_{rms} \sin\theta_Z = V_{rms} I_{rms}$ VAR

$$Q_L = P_{Lx} = I_{rms}^2 \frac{V_{rms}}{I_{rms}} = I_{rms}^2 X_L = \frac{V_{rms}^2}{X_L} \text{ VAR}$$

The **positive sign of reactive power** indicates that the inductor is **absorbed or consumed the reactive power**.

Apparent Power :

$$S = \sqrt{P_r^2 + P_x^2} = \sqrt{P^2 + Q^2} = Q = \frac{V_m I_m}{2} = V_{rms} I_{rms} = VI \quad \text{VA}$$

The **instantaneous real power** for inductive load: $p_{r(t)} = P - P\cos 2\omega t = 0 \quad$ W

The **instantaneous reactive power** for inductive load:

$$p_{i(t)} = Q\cos 2\omega t = \frac{V_m I_m}{2} \sin 2\omega t \qquad \text{VAR}$$

Energy of Inductive Load

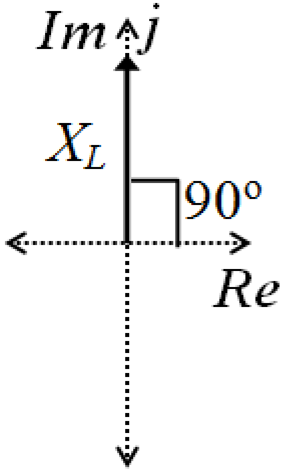
The inductive element receives energy from the source during one-quarter of a cycle of the applied voltage and returns exactly the same amount of energy to the driving source during the next one-quarter of a cycle. The exact amount of energy deliver to the circuit during a quarter of a cycle is obtained by:

$$W_L = \int_{T/4}^{T/2} -\frac{V_m I_m}{2}\sin 2\omega t dt = \frac{V_m I_m}{2\left(\frac{4\pi}{T}\right)}\left[\cos\frac{4\pi}{T}t\right]_{T/4}^{T/2} = \frac{V_m I_m}{2\omega} = \frac{\omega L I_m^2}{2\omega} = \frac{1}{2}L I_m^2 \ \text{J}$$

W_L is the maximum amount of energy stored in the magnetic field of the inductor at any one time.

Complex Algebra for Only Inductive Circuit

Impedance for a inductance is: $\boldsymbol{Z}_L = X_L\angle 90° = jX_L\ \Omega$



Impedance diagram for inductive load

Applied Voltage in Complex form:

$$\boldsymbol{V} = \frac{V_m}{\sqrt{2}}\angle\theta_v = V_{rms}\angle\theta_v = V\angle\theta_v \quad \text{V}$$

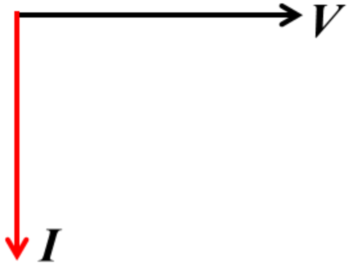
Current can be calculated as:

$$\boldsymbol{I} = \frac{\boldsymbol{V}}{\boldsymbol{Z}_L} = \frac{V_{rms}\angle\theta_v}{X_L\angle 90°} = \frac{V_{rms}}{X_L}\angle(\theta_v - 90°) = I_{rms}\angle\theta_i \quad \text{A}$$

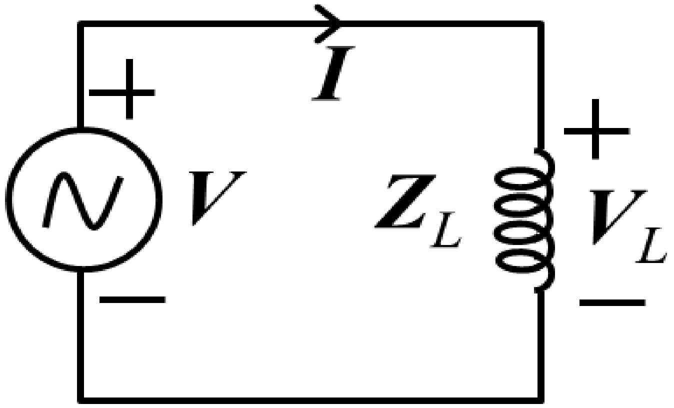
Where,

$$I_{rms} = \frac{V_{rms}}{X_L} = \frac{I_m}{\sqrt{2}};$$

$$\theta_i = \theta_v - 90°; \quad \theta_v = \theta_i + 90°$$



Phasor Diagram



Summary For a pure Inductive Load

Magnitude of impedance, $Z = \frac{V_m}{I_m} = X_L \quad \Omega$ $\qquad \theta_i = \theta_v - 90° \qquad \theta_v = \theta_i + 90°$

Angle of impedance, $\theta_z = \theta_v - \theta_i = 90°$

Impedance of a Resistor, $\mathbf{Z} = \mathbf{Z}_L = X_L \angle 90° = jX_L \ \Omega$

The phase difference between voltage across and current through an inductor is 90°.

The voltage leads the current in an inductor by 90°.

The current lags the voltage in an inductor by 90°.

The power factor is 0 which is called **zero lagging power factor**.

The reactive factor is 1.

The active power is 0 that means **zero**.

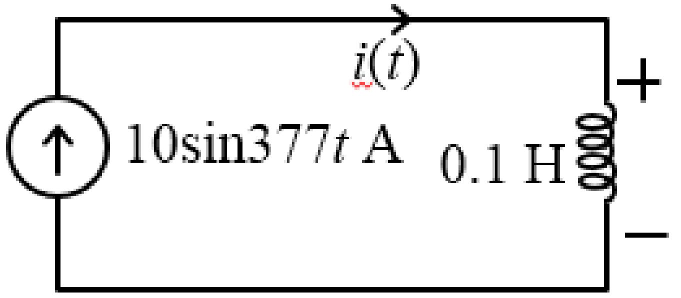
The apparent power equals to reactive power.

Inductor consumed the reactive power.

Example 4.3.1: The current through a 0.1 H coil is $i(t)=10\sin 377t$ A. (**i**) Find the applied voltage v. (**ii**) Calculate the impedance. (**iii**) Calculate the power factor, reactive factor, the real power, reactive power, and apparent power. (**iv**) Sketch the waveform of $v(t)$ and $i(t)$. (**v**) Draw the phasor or vector diagram.

$i(t)=10\sin 377t\ \text{ A}$ $\quad\theta_i=0°\quad$ $\omega=377\text{ rad/s}\quad$ $I_m=10\text{ A}$

We know that: $\quad v(t)=v_L(t)=L\dfrac{di_L(t)}{dt}=L\dfrac{di(t)}{dt}$



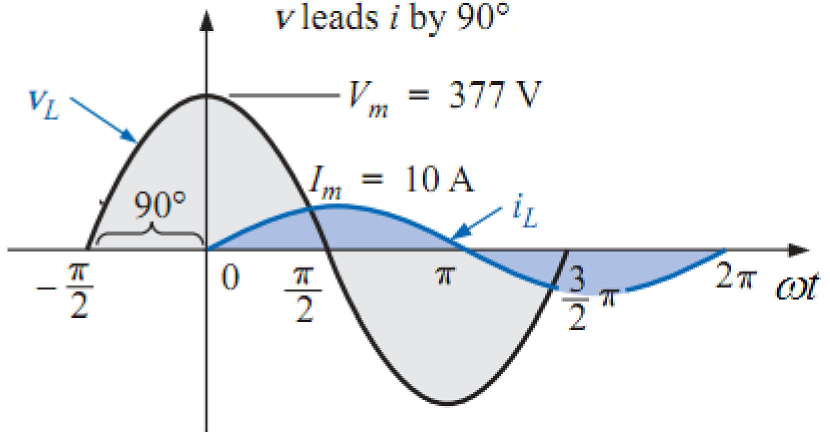
$$v(t)=0.1\frac{d[10\sin 377t]}{dt}=0.1\times10\times377\cos 377t$$

$$v(t)=377\sin(377t+90°)\qquad \theta_v=90°$$

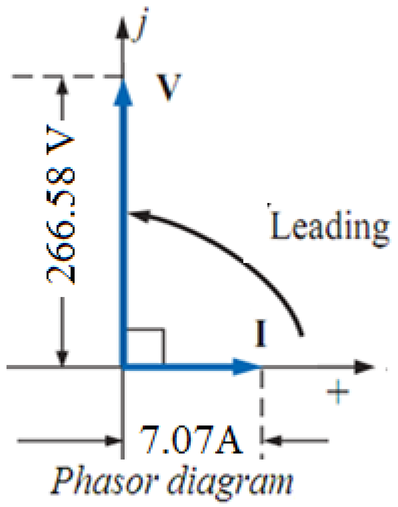
$$X_L=\omega L=377\times0.1=37.7\ \Omega$$

$$Z=\frac{V_m}{I_m}=X_L=37.7\quad\Omega\qquad \theta_z=\theta_v-\theta_i=90°$$

$$\mathbf{Z}_L=37.7\angle 90°=j37.7\ \Omega$$



$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{377}{\sqrt{2}} = 266.58 \text{ V} \qquad I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.07 \text{ A}$$



Power Factor : $\text{pf} = \cos(\theta_Z) = \cos(90°) = 0$

Reactive Factor : $\text{rf} = \sin(\theta_Z) = \sin(90°) = 1$

Average or Real Power : $P = \frac{V_m I_m}{2} \cos\theta = 0 \text{ W}$

Reactive Power : $Q = P_X = 266.58 \times 7.07 \times 1 = 1884.7 \text{VAR}$

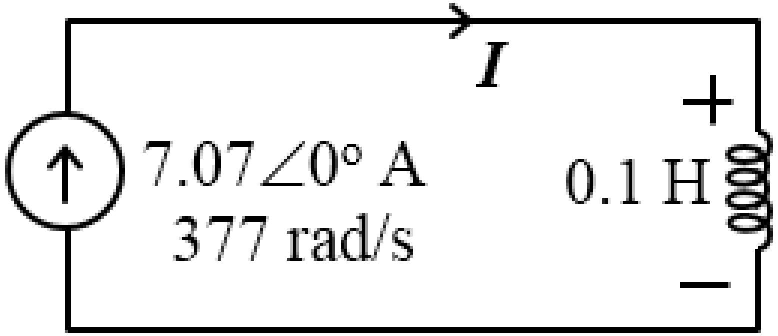
Apparent Power : $S = \sqrt{P^2 + Q^2} = Q = 1884.7 \text{ VA}$

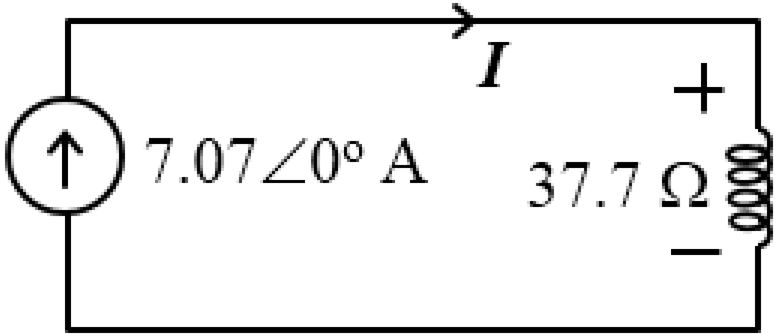
Maximum Stored Energy : $W_L = \frac{1}{2} L I_m^2 = \frac{1}{2} \times 0.1 \times (10)^2 = 5 \text{ J}$

The instantaneous real power for inductive load: $p_{r(t)} = P - P\cos 2\omega t = 0 \quad \text{W}$

The instantaneous reactive power for inductive load: $p_i(t) = 1884.7 \sin 754t \quad \text{VAR}$

Solution using Complex Algebra





Impedance, $\boldsymbol{Z}_L = 37.7\angle 90° = j37.7\ \Omega$ $\qquad \theta_i = 0$

$$\boldsymbol{I} = \frac{I_m}{\sqrt{2}}\angle\theta_i = \frac{10}{\sqrt{2}}\angle 0° = 7.07\angle 0°\ \text{A}$$

$$\boldsymbol{V} = \boldsymbol{Z}_L\boldsymbol{I} = (37.7\angle 90°)(7.07\angle 0°) = 266.58\angle 90°\ \text{V}$$

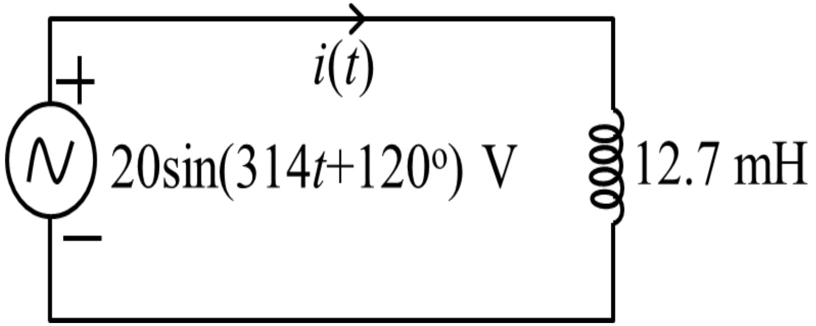
$$V_m = \sqrt{2}V_{rms} = \sqrt{2}\times 266.58 = 377\ \text{V}$$

$$v(t) = 300\sin(\omega t + 90°)\ \text{V} \qquad \theta_i = 0° \qquad \theta = \theta_z = \theta_v - \theta_i = 90°$$

Example 4.3.2: The applied voltage in a 12.7 mH coil is $v(t)$=20sin(314t+120º) V. (**i**) Find the applied current i. (**ii**) Calculate the impedance. (**iii**) Calculate the power factor, reactive factor, the real power, reactive power, and apparent power. (**iv**) Sketch the waveform of $v(t)$ and $i(t)$. (**v**) Draw the phasor or vector diagram.

$v(t) = 20\sin(314t + 120°)$ V $\quad \theta_v = 120° \quad \omega = 314$ rad/s $\quad V_m = 20$ V

$$i_L(t) = i(t) = \frac{1}{L}\int v_L(t)dt = \frac{1}{L}\int v(t)dt$$



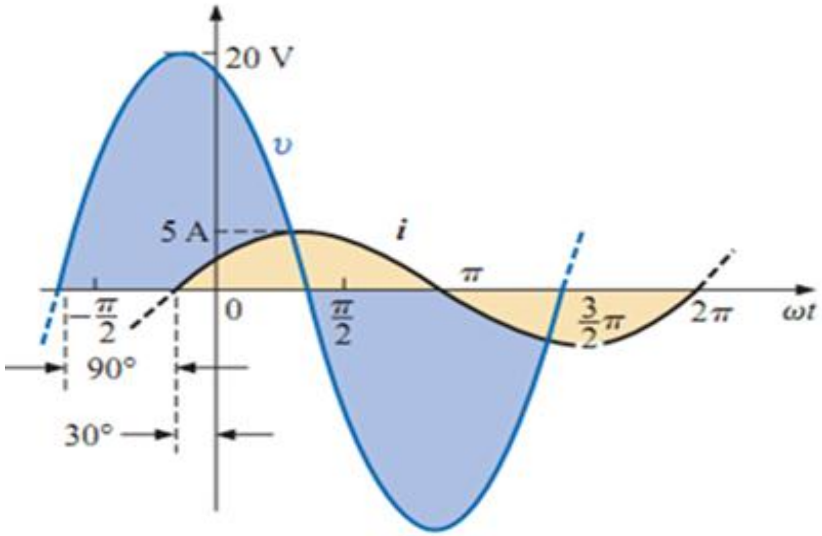
$$i(t) = \frac{20}{12.7\times10^{-3}}\int \sin(314t + 120°)dt$$

$$i(t) = -\frac{20}{12.7\times10^{-3}\times314}\cos(314t + 120°) = 5\sin(314t + 30°)$$

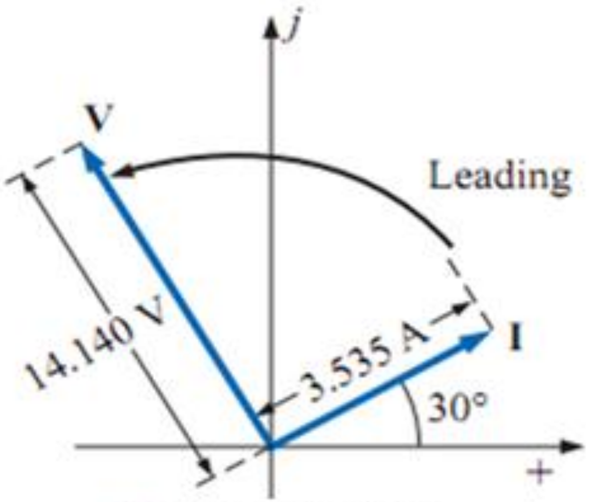
$\therefore\ \theta_i = 30° \qquad X_L = \omega L = 314\times0.0127 = 4\ \Omega$

$$Z = \frac{V_m}{I_m} = X_L = 4 \quad \Omega \qquad \theta_z = \theta_v - \theta_i = 90°$$

$$\mathbf{Z}_L = 4\angle90° = j37.7\ \Omega$$



$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{20}{\sqrt{2}} = 14.14 \text{ V} \qquad I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{5}{\sqrt{2}} = 3.535 \text{ A}$$



Phasor diagram

Power Factor : $\text{pf} = \cos(\theta_Z) = \cos(90°) = 0$

Reactive Factor : $\text{rf} = \sin(\theta_Z) = \sin(90°) = 1$

Average or Real Power : $P = \dfrac{V_m I_m}{2}\cos\theta = 0 \text{ W}$

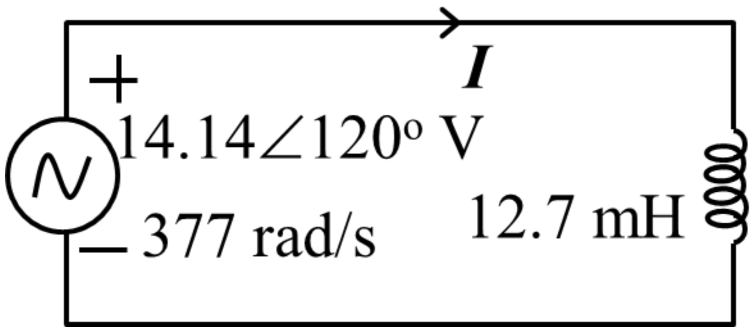
Reactive Power : $Q = P_X = 14.14 \times 3.535 \times 1 = 50 \text{VAR}$

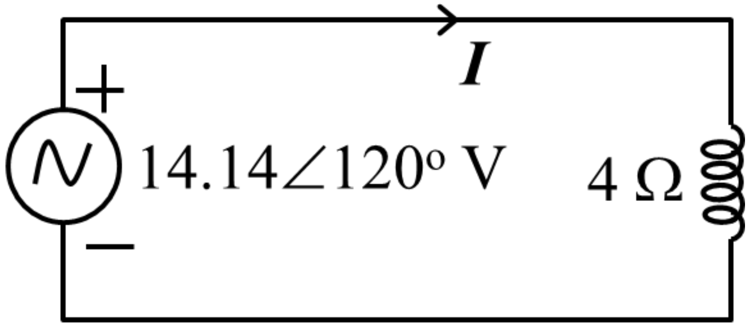
Apparent Power : $S = \sqrt{P^2 + Q^2} = Q = 50 \text{ VA}$

The **instantaneous real power** **for** **inductive** **load:** $p_r(t) = P - P\cos 2\omega t = 0 \quad \text{W}$

The **instantaneous reactive power** **for** **inductive** **load:** $p_i(t) = 50\sin 628t \quad \text{VAR}$

Solution using Complex Algebra





Impedance, $\boldsymbol{Z}_L = 4\angle 90° = j4\ \Omega$

$\theta_v = 120°$

$$\boldsymbol{V} = \frac{V_m}{\sqrt{2}}\angle\theta_v = \frac{20}{\sqrt{2}}\angle 120° = 14.13\angle 120°\ \text{ V}$$

$$\boldsymbol{I} = \frac{\boldsymbol{V}}{\boldsymbol{Z}_L} = \frac{14.14\angle 120°}{4\angle 90°} = 3.535\angle 30°\ \text{ A}$$

$\theta_i = 30°$

$$I_m = \sqrt{2} I_{rms} = \sqrt{2}\times 3.535 = 5\ \text{ A}$$

$$i(t) = 5\sin(\omega t + 30°)\ \text{ A}$$

$$\theta = \theta_z = \theta_v - \theta_i = 90°$$

Home Work 4.3

Problem 1: Determine the inductive reactance (in ohms) of a 2-H coil for the following frequencies: (i) 25 Hz, (ii) 50 Hz, (iii) 60 Hz, (iv) 2000 Hz, and (iv) 100000 Hz.

Problem 2: Determine the inductance of a coil that has a reactance of (i) 20 Ω at f=2 Hz, (ii) 1000 Ω at f=60 Hz, (iii) 5280 Ω at f=1 kHz.

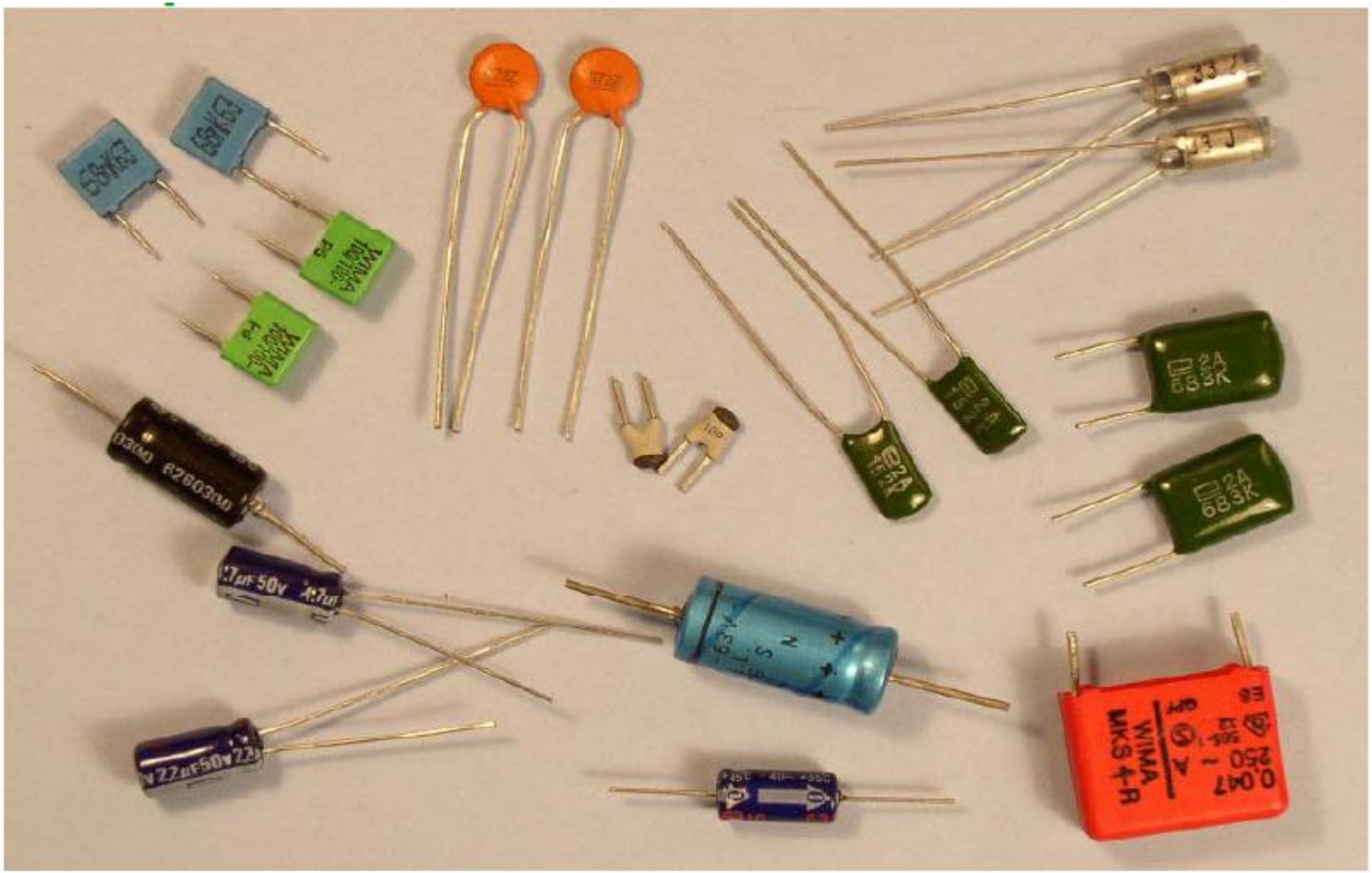
Problem 3: Determine the frequency at which a 10-H inductances has the following inductive reactances: (i) 50 Ω, (ii) 3770 Ω, (iii) 15.7 Ω, and (iii) 243 Ω.

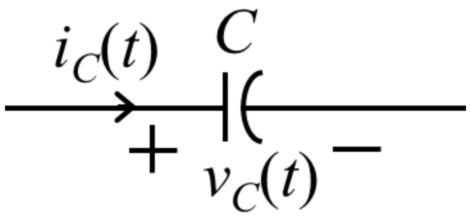
Problem 4: The current $i(t)=5\sin(\omega t+30^o)$ A flows through a 4 Ω inductive reactance. (***i***) What is the sinusoidal expression for the voltage? (***ii***) Calculate the real power, reactive power, power factor, reactive factor. (***iii***) write the expression of instantaneous power. (***iv***) Sketch the $v(t)$ and $i(t)$ sinusoidal waveforms on the same axis. (***v***) Draw the phasor diagram.

Problem 5: The voltage $v(t)=24\sin\omega t$ V is applied to a 3 Ω inductive reactance. (***i***) What is the sinusoidal expression for the current? (**ii**) Calculate the real power, reactive power, power factor, reactive factor. (**iii**) write the expression of instantaneous power. (**iv**) Sketch the $v(t)$ and $i(t)$ sinusoidal waveforms on the same axis. (**v**) Draw the phasor diagram.

Pure Capacitive Circuit

Response of Basic Capacitor or Condenser Element to a Sinusoidal Voltage or Current



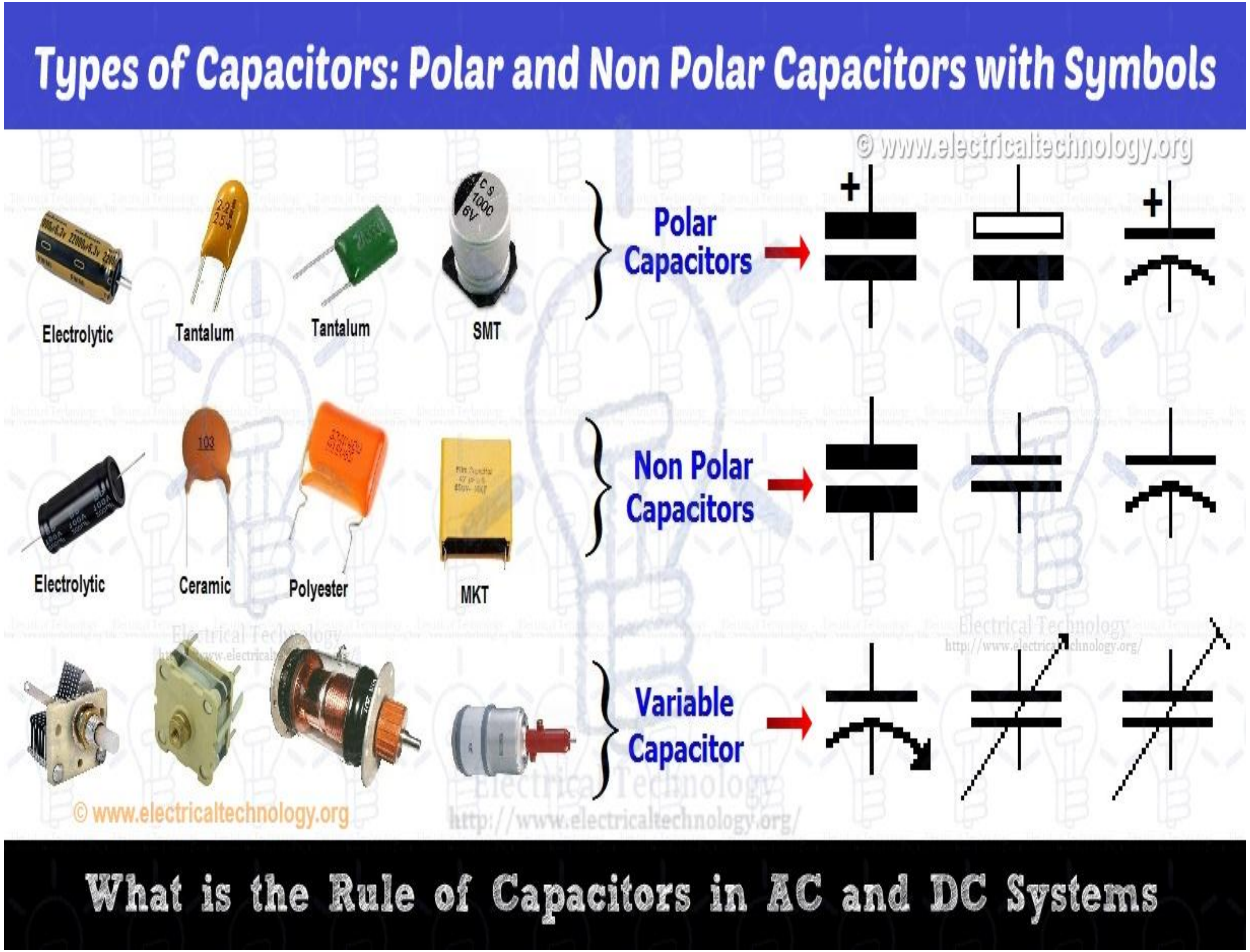


Voltage and current relation in a capacitor:

$$v_C(t) = \frac{q}{C} = \frac{1}{C}\int i_C(t)dt$$

$$q = Cv_C(t)$$

$$i_C(t) = \frac{dq}{dt} = C\frac{dv_C(t)}{dt}$$


Types of Capacitors: Polar and Non Polar Capacitors with Symbols
© www.electricaltechnology.org
Electrolytic
Tantalum
Tantalum
SMT
Polar Capacitors
+
+
Electrolytic
Ceramic
Polyester
MKT
Non Polar Capacitors
Variable Capacitor
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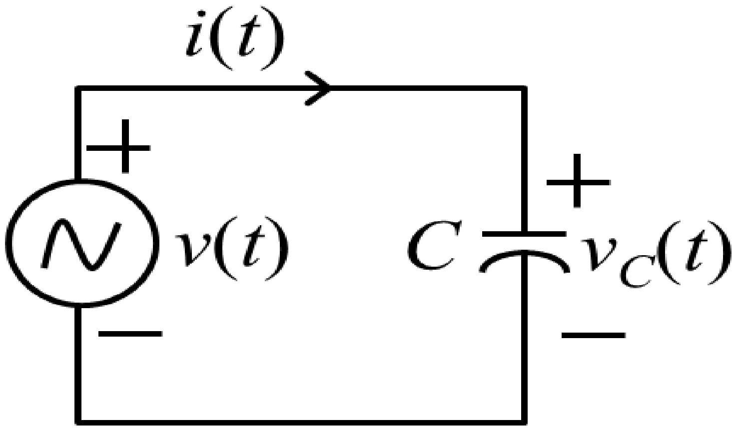
What is the Rule of Capacitors in AC and DC Systems

Let, the input is $v(t) = V_m \sin\omega t$ V, according to KVL, we have: $v(t) = v_C(t) = V_m \sin\omega t$

For a capacitance the relation of voltage and current is:



$$i(t) = i_C(t) = C\frac{dv_C(t)}{dt} = CV_m \frac{d(\sin\omega t dt)}{dt} = \omega L V_m \cos\omega t$$

$$i(t) = \omega C V_m \sin(\omega t + 90^\circ) = I_m \sin(\omega t + \theta_i)$$

Magnitude of impedance, $Z = \frac{V_m}{I_m} = \frac{1}{\omega C} = X_C \quad \Omega$

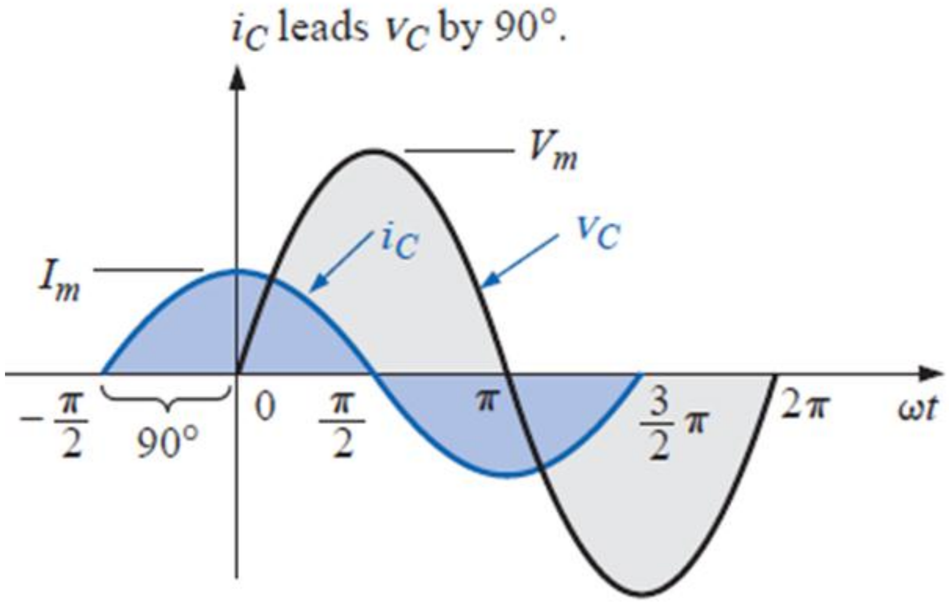
Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \quad \Omega$

Angle of current, $\theta_i = 90^\circ$

Angle of impedance, $\theta_z = \theta_v - \theta_i = -90^\circ$

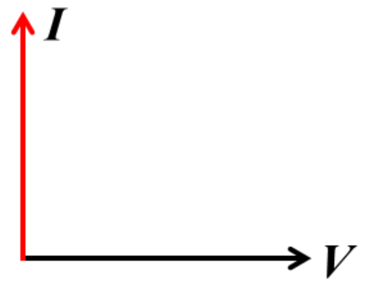
Impedance of a Capacitor, $\mathbf{Z} = \mathbf{Z}_C = X_C\angle -90^\circ$

$$= -jX_C \ \Omega$$



The phase difference between voltage across and current through a capacitor is 90°.

***For a purely capacitive element, the voltage lags the current through the capacitive element by 90°*. Or, *the current leads the voltage in an capacitive element by 90°*.**



Phasor Diagram

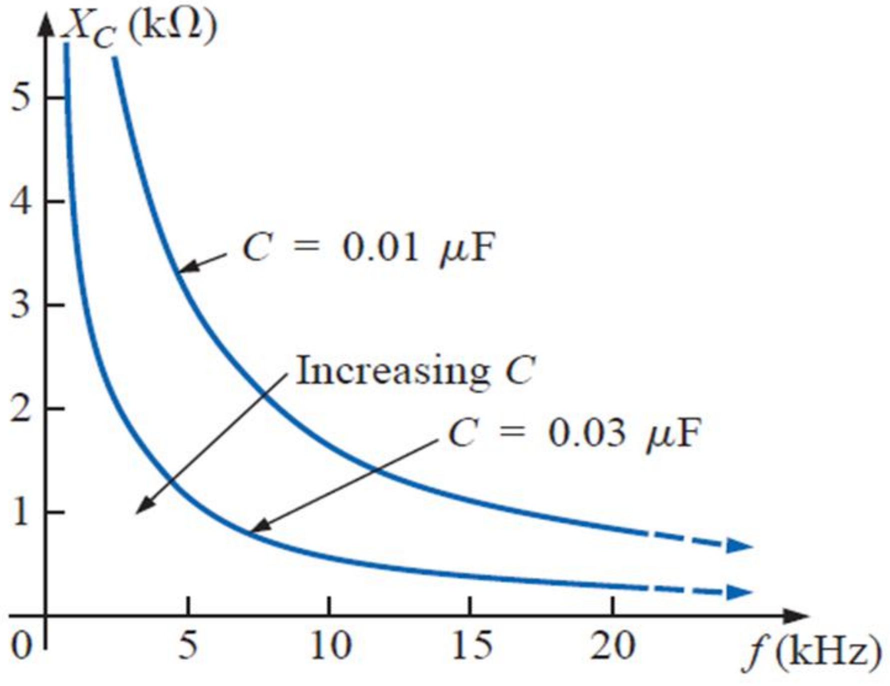
Capacitive Reactance vs Frequency

Capacitive Reactance is defined as the opposition offered by the capacitance of a circuit to the flow of an alternating sinusoidal current.

The capacitive reactance is inversely proportional to the frequency, i.e. $X_C \propto (1/f)$.

So, graph of X_C vs f is a **rectangular hyperbola**.

***If frequency is zero, which is do far dc voltage, the capacitive reactance is infinite. Therefore, it is said that the capacitance offers infinite (open circuit) reactance for the dc or steady current*.**



Power of Capacitive Load

The power of an inductive load:

$$p(t) = V_m I_m \sin\omega t \sin(\omega t + 90°) = V_m I_m \sin\omega t \cos\omega t = \frac{V_m I_m}{2}\sin 2\omega t$$

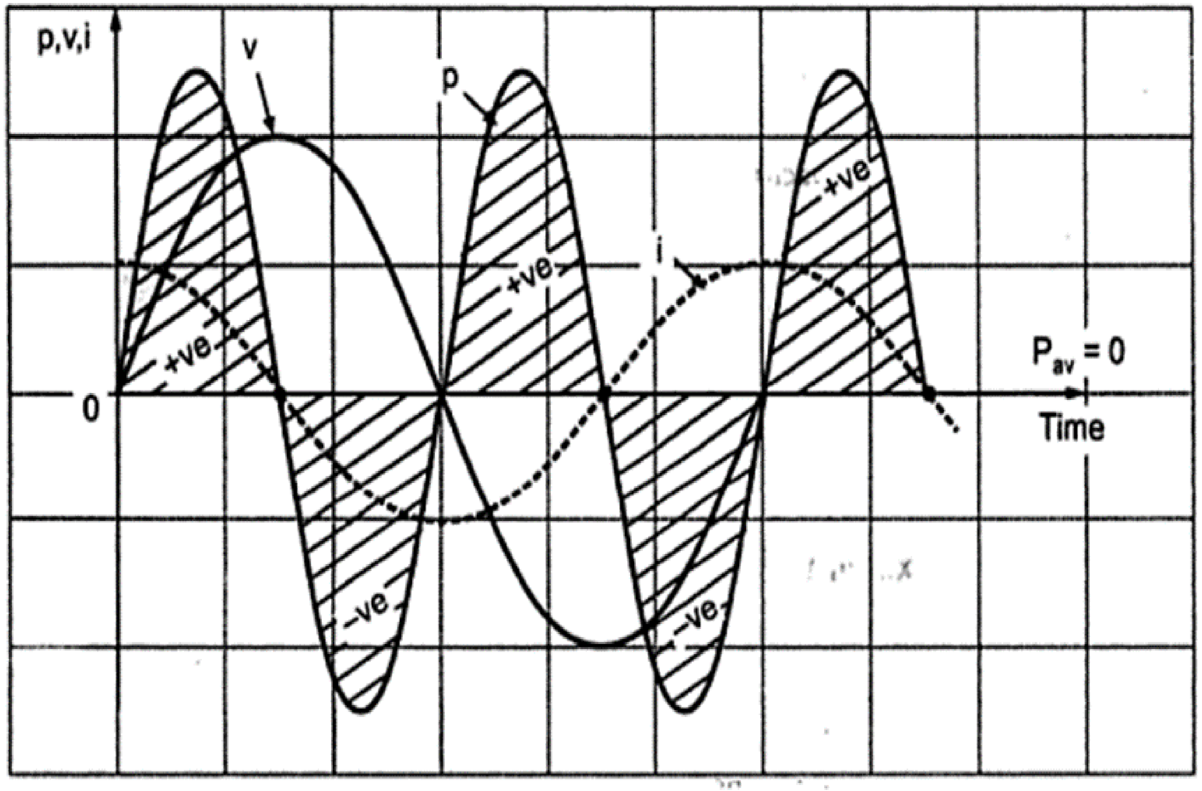
Power Factor : $\mathrm{pf} = \cos(\theta_Z) = \cos(-90°) = 0$ **Reactive Factor** : $\mathrm{rf} = \sin(\theta_Z) = \sin(-90°) = -1$

For capacitive load, the power factor is zero which called **zero leading power factor**.

Average or Real Power :

$$P = P_{ave} = V_{rms} I_{rms} \cos\theta_Z = 0$$

Capacitor never consumes power.



Waveforms of voltage, current and power

Reactive Power : $Q_C = P_{Cx} = V_{rms} I_{rms} \sin\theta_z = -V_{rms} I_{rms}$ VAR

$$Q_C = P_{Cx} = I_{rms}^2 \frac{V_{rms}}{I_{rms}} = I_{rms}^2 X_C = \frac{V_{rms}^2}{X_C} \text{ VAR}$$

The **negative sign of reactive power** indicates that the capacitor is **supplied the reactive power**.

Apparent Power :

$$S = \sqrt{P_r^2 + P_x^2} = \sqrt{P^2 + Q^2} = Q = \frac{V_m I_m}{2} = V_{rms} I_{rms} = VI \quad \text{VA}$$

The **instantaneous real power** for capacitive load: $p_{r(t)} = P - P\cos 2\omega t = 0 \quad$ W

The **instantaneous reactive power** for capacitive load:

$$p_{i(t)} = Q\cos 2\omega t = -\frac{V_m I_m}{2}\sin 2\omega t \qquad \text{VAR}$$

Energy of Capacitive Load

The amount of energy received by the capacitor during one-quarter of a cycle is obtained by:

$$W_C = \int_0^{T/4} \frac{V_m I_m}{2} \sin 2\omega t dt = \frac{V_m I_m}{2\left(\frac{4\pi}{T}\right)} \left[-\cos \frac{4\pi}{T} t \right]_0^{T/4} = \frac{V_m I_m}{2\omega}$$

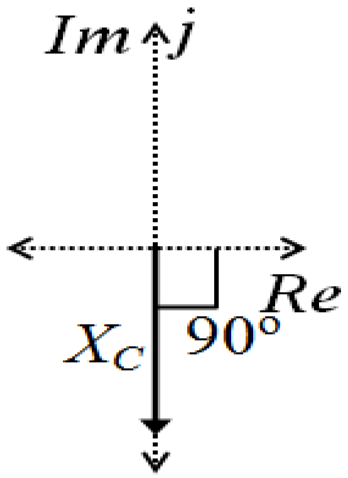
$$\because \quad I_m = \omega C V_m$$

$$W_C = \frac{\omega C V_m V_m}{2\omega} = \frac{1}{2} C V_m^2 \text{ J}$$

W_C is the maximum amount of energy stored in the electrical field of the capacitor at any one time.

Complex Algebra for Only Capacitive Circuit

Impedance for a capacitance is: $\boldsymbol{Z}_C = X_C\angle -90^\circ = -jX_C\ \Omega$



Impedance diagram for capacitive load

Applied Voltage in Complex form:

$$\boldsymbol{V} = \frac{V_m}{\sqrt{2}}\angle\theta_v = V_{rms}\angle\theta_v = V\angle\theta_v \quad \text{V}$$

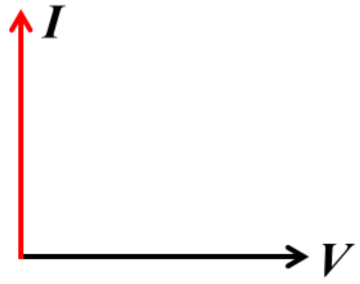
Current can be calculated as:

$$\boldsymbol{I} = \frac{\boldsymbol{V}}{\boldsymbol{Z}_C} = \frac{V_{rms}\angle\theta_v}{X_C\angle -90^\circ} = \frac{V_{rms}}{X_C}\angle(\theta_v + 90^\circ) = I_{rms}\angle\theta_i \quad \text{A}$$

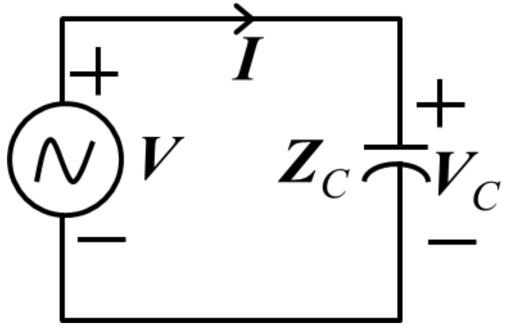
Where,

$$I_{rms} = \frac{V_{rms}}{X_C} = \frac{I_m}{\sqrt{2}};$$

$$\theta_i = \theta_v + 90^\circ; \qquad \theta_v = \theta_i - 90^\circ$$



Phasor Diagram



Summary For a pure Capacitive Load

Magnitude of impedance, $Z = \dfrac{V_m}{I_m} = X_C \quad \Omega$ $\qquad \theta_i = \theta_v + 90^\circ \qquad \theta_v = \theta_i - 90^\circ$

Angle of impedance, $\theta_Z = \theta_v - \theta_i = -90^\circ$

Impedance of a Capacitor, $\mathbf{Z} = \mathbf{Z}_C = X_C\angle -90^\circ = -jX_C \ \Omega$

The phase difference between voltage across and current through a capacitor is 90°.

The voltage lags the current in an inductor by 90°.

The current leads the voltage in an inductor by 90°.

The power factor is 0 which is called **zero leading power factor**.

The reactive factor is -1.

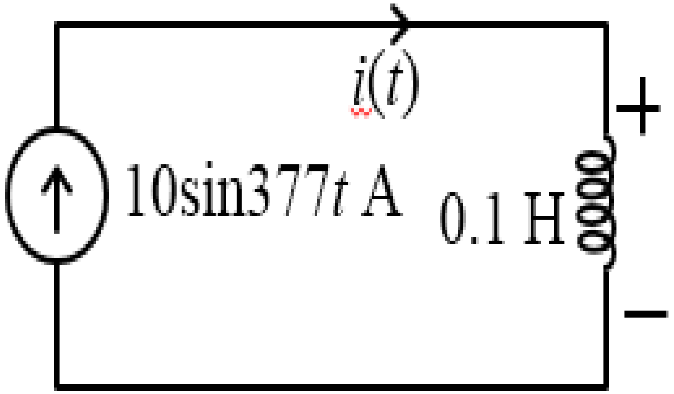
The active power is 0 that means **zero**.

The apparent power equals to reactive power.

Capacitor supply the reactive power.

Example 4.3.1: The current through a 0.0053 F capacitor is $i(t)=6\sin(377t-60^{o})$ A. (**i**) Find the applied voltage $v(t)$. (**ii**) Calculate the impedance. (**iii**) Calculate the power factor, reactive factor, the real power, reactive power, and apparent power. (**iv**) Sketch the waveform of $v(t)$ and $i(t)$. (**v**) Draw the phasor or vector diagram.

$$I_m = 6 \text{ A} \qquad \theta_i = -60^\circ \qquad \omega = 377 \text{ rad/s}$$



$$v(t) = v_C(t) = \frac{q}{C} = \frac{1}{C}\int i_C(t)dt = \frac{1}{C}\int i(t)dt$$

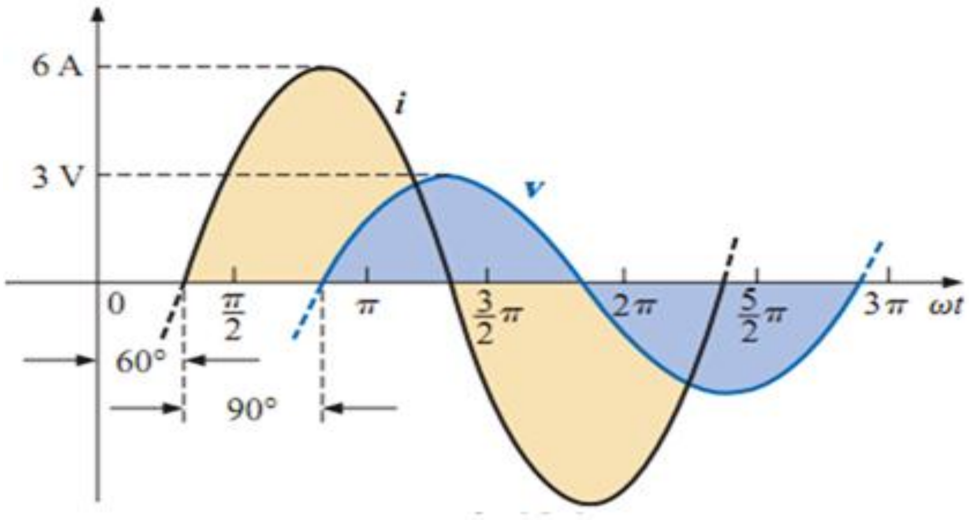
$$v(t) = \frac{6}{0.0053}\int \sin(377t - 60^\circ)dt = -\frac{6}{0.0053\times 377}\cos(377t - 60^\circ)$$

$$v(t) = 3\sin(377t - 150^\circ) \text{ V}$$

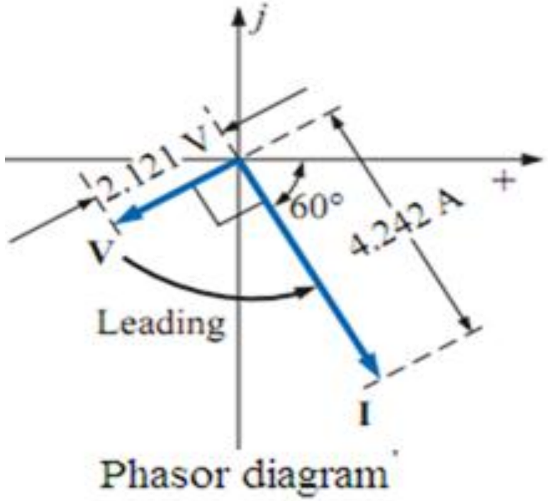
$$\therefore \ \theta_v = -150^\circ \qquad \theta_z = \theta_v - \theta_i = -90^\circ$$

$$Z = \frac{V_m}{I_m} = X_C = \frac{1}{\omega C} = \frac{1}{377\times 0.0053} = 0.5\ \Omega$$

Impedance, $Z_C = 0.5\angle -90^\circ = j0.5\ \Omega$



$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{3}{\sqrt{2}} = 2.121 \text{ V} \qquad I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{6}{\sqrt{2}} = 4.242 \text{ A}$$



Power Factor : $\text{pf} = \cos(\theta_Z) = \cos(90°) = 0$

Reactive Factor : $\text{rf} = \sin(\theta_Z) = \sin(-90°) = -1$

Average or Real Power : $P = \frac{V_m I_m}{2} \cos\theta = 0 \text{ W}$

Reactive Power : $Q = P_X = 2.121 \times 4.242 \times -1 = -9 \text{ VAR}$

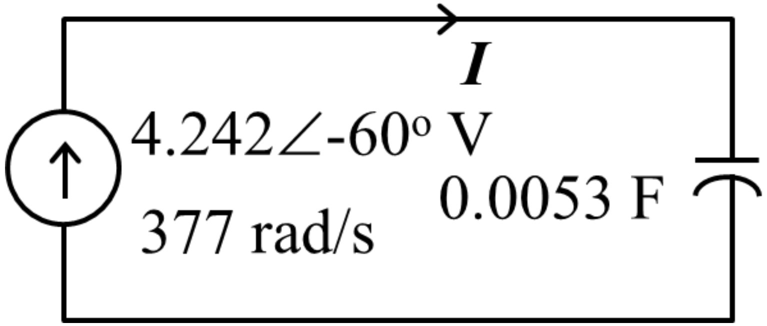
Apparent Power : $S = \sqrt{P^2 + Q^2} = Q = 9 \text{ VA}$

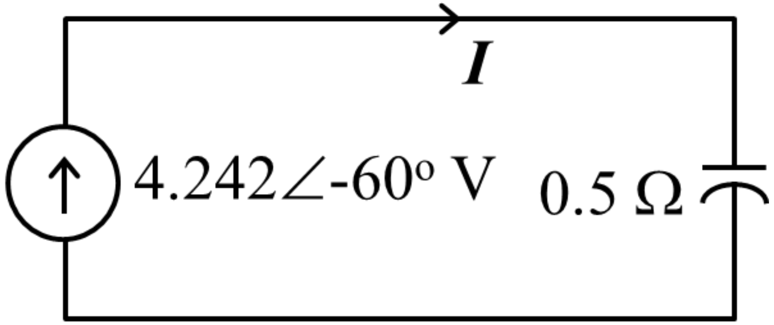
Maximum Stored Energy : $W_C = \frac{1}{2} C V_m^2 = \frac{1}{2} \times 0.0053 \times (3)^2 = 0.0239 \text{ J}$

The instantaneous real power for capacitive load: $p_{r(t)} = P - P\cos 2\omega t = 0 \quad \text{W}$

The instantaneous reactive power for capacitive load: $p_i(t) = -9\sin 754t \quad \text{VAR}$

Solution using Complex Algebra





Impedance, $\boldsymbol{Z}_C = 0.5\angle -90° = j0.5\ \Omega$

$\theta_i = -60°$

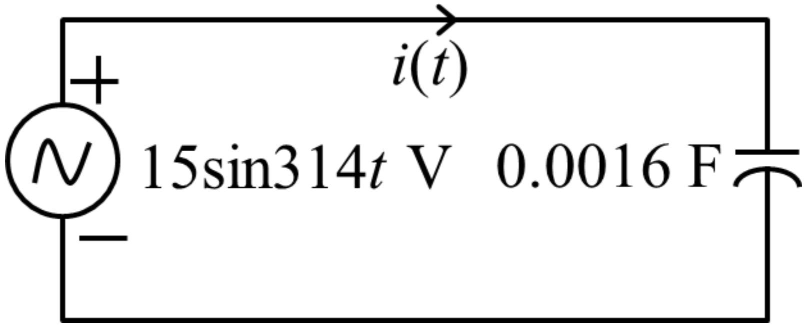
$$\boldsymbol{I} = \frac{I_m}{\sqrt{2}}\angle\theta_i = \frac{6}{\sqrt{2}}\angle -60° = 4.242\angle -60°\ \text{A}$$

$$\boldsymbol{V} = \boldsymbol{Z}_C\boldsymbol{I} = (0.5\angle -90°)(4.242\angle -60°) = 2.121\angle -150°\ \text{V}$$

$$V_m = \sqrt{2}V_{rms} = \sqrt{2}\times 2.121 = 3\ \text{V}$$

$$v(t) = 3\sin(\omega t - 150°)\ \text{V} \qquad \theta_i = 0° \qquad \theta = \theta_z = \theta_v - \theta_i = -90°$$

Example 4.4.2: The applied voltage in a 0.0016 F capacitor is $v(t)$=15sin314t V. (**i**) Find the current $i(t)$. (**ii**) Calculate the impedance. (**iii**) Calculate the power factor, reactive factor, the real power, reactive power, and apparent power. (**iv**) Sketch the waveform of $v(t)$ and $i(t)$. (**v**) Draw the phasor or vector diagram.



$V_m = 15$ V $\qquad \theta_v = 0°$ $\qquad \omega = 314$ rad/s

$$i(t) = i_C(t) = \frac{dq}{dt} = C\frac{dv_C(t)}{dt} = C\frac{dv(t)}{dt}$$

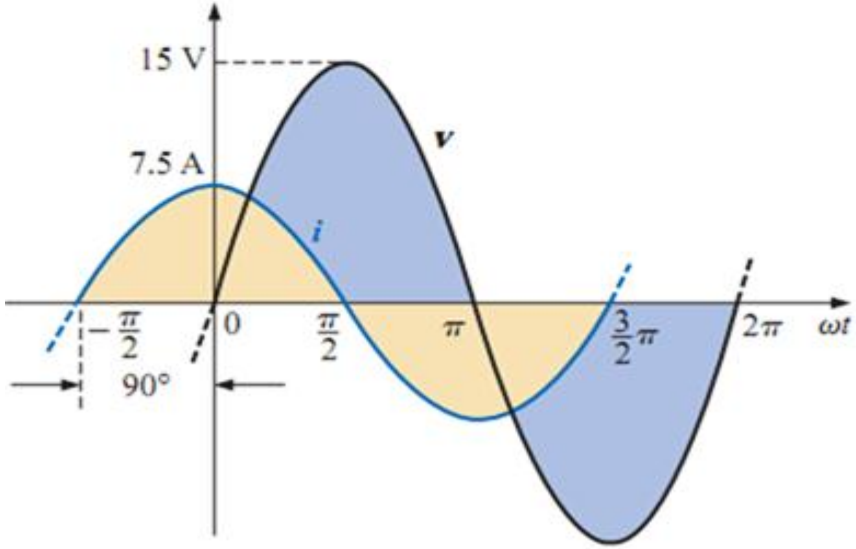
$$i(t) = 0.0016\frac{d(15\sin 314t)}{dt} = 0.0016\times 15\times 314\cos 314t$$

$$i(t) = 7.5\sin(314t + 90°)\ \text{A}$$

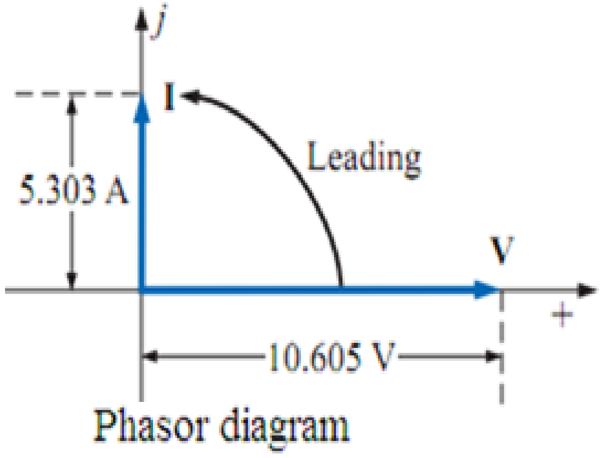
$\theta_i = 90°$ $\qquad \theta_z = \theta_v - \theta_i = -90°$

$$Z = \frac{V_m}{I_m} = X_C = \frac{1}{\omega C} = \frac{1}{314\times 0.0016} = 2\ \Omega$$

Impedance, $Z_C = 2\angle -90° = -j2\ \Omega$



$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{15}{\sqrt{2}} = 10.605 \text{ V} \qquad I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{7.5}{\sqrt{2}} = 5.303 \text{ A}$$



Power Factor : $\text{pf} = \cos(\theta_Z) = \cos(90°) = 0$

Reactive Factor : $\text{rf} = \sin(\theta_Z) = \sin(90°) = 1$

Average or Real Power : $P = \dfrac{V_m I_m}{2} \cos\theta = 0 \text{ W}$

Reactive Power : $Q = P_X = 14.14 \times 3.535 \times 1 = 50 \text{VAR}$

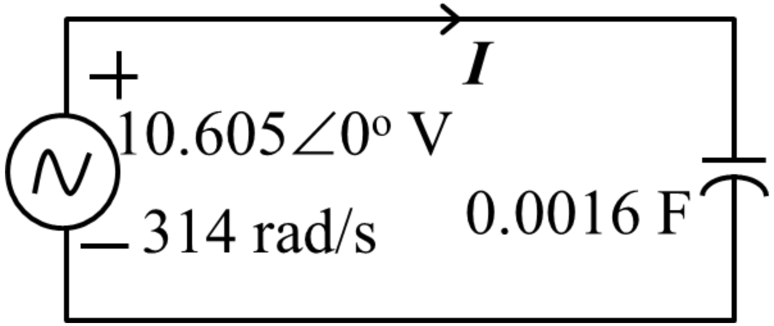
Apparent Power : $S = \sqrt{P^2 + Q^2} = Q = 50 \quad \text{VA}$

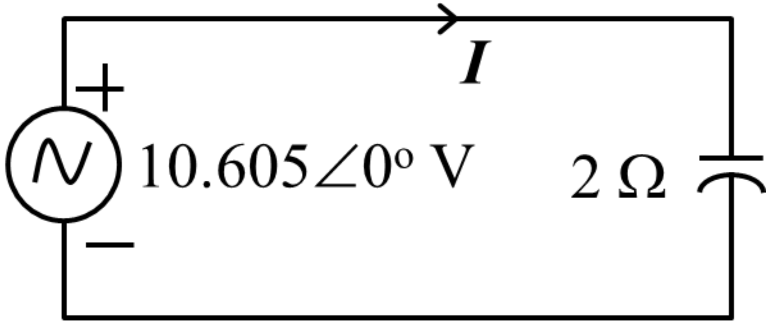
Maximum Stored Energy : $W_L = \dfrac{1}{2} L I_m^2 = \dfrac{1}{2} \times 0.0127 \times (5)^2 = 0.158 \quad \text{J}$

The instantaneous real power for capacitive load: $p_r(t) = P - P\cos 2\omega t = 0 \quad \text{W}$

The instantaneous reactive power for capacitive load: $p_i(t) = 50 \sin 628t \qquad \text{VAR}$

Solution using Complex Algebra





Impedance, $\boldsymbol{Z}_C = 2\angle -90° = -j2\ \Omega$

$$\theta_v = 0°$$

$$\boldsymbol{V} = \frac{V_m}{\sqrt{2}}\angle\theta_v = \frac{15}{\sqrt{2}}\angle 0° = 10.605\angle 0°\ \text{V}$$

$$\boldsymbol{I} = \frac{\boldsymbol{V}}{\boldsymbol{Z}_C} = \frac{10.605\angle 0°}{2\angle -90°} = 5.303\angle 90°\ \text{A}$$

$$\theta_i = 90°$$

$$I_m = \sqrt{2}I_{rms} = \sqrt{2}\times 5.303 = 7.5\ \text{A}$$

$$i(t) = 7.5\sin(\omega t + 90°)\ \text{A}$$

$$\theta = \theta_z = \theta_v - \theta_i = -90°$$

Home Work 4.4

Problem 1: Determine the capacitive reactance (in ohms) of a 5-μF capacitor for the following frequencies: (i) 60 Hz, (ii) 120 Hz, (iii) 1800 Hz, (iv) 2000 Hz, and (iv) 24000 Hz.

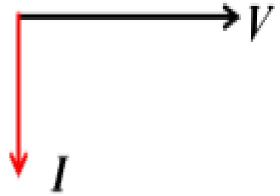
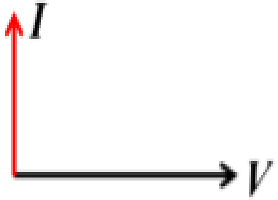
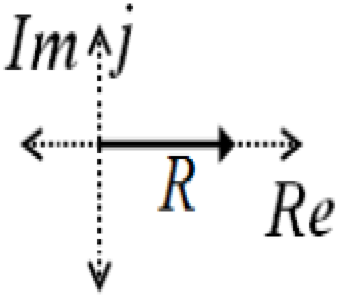
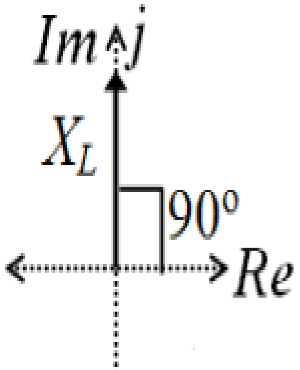
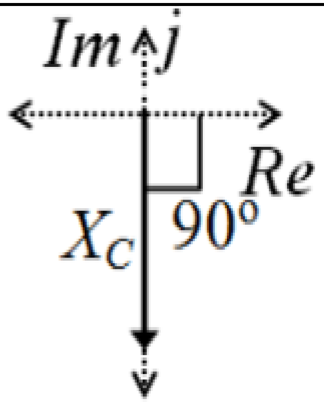
Problem 2: Determine the capacitance in microfarads if a capacitor has a reactance of (i) 250 Ω at f=60 Hz, (ii) 55 Ω at f=312 Hz, (iii) 10 Ω at f=25 Hz.

Problem 3: Determine the frequency at which a 50-μF capacitor has the following inductive reactances: (i) 342 Ω, (ii) 684 Ω, (iii) 171 Ω, and (iii) 2000 Ω.

Problem 4: The current $i(t)=5\sin(\omega t+30^o)$ A flows through a 4 Ω capacitive reactance. (***i***) What is the sinusoidal expression for the voltage? (***ii***) Calculate the real power, reactive power, power factor, reactive factor. (***iii***) write the expression of instantaneous power. (***iv***) Sketch the v and i sinusoidal waveforms on the same axis. (***v***) Draw the phasor diagram.

Problem 5: The voltage $v(t)=24\sin\omega t$ V is applied to a 3 Ω capacitive reactance. (***i***) What is the sinusoidal expression for the current? (**ii**) Calculate the real power, reactive power, power factor, reactive factor. (**iii**) write the expression of instantaneous power. (**iv**) Sketch the v and i sinusoidal waveforms on the same axis. (**v**) Draw the phasor diagram.

	Resistance	**Inductance**	**Capacitance**
Magnitude of impedance(Z) [Ω]	R	X_L	X_C
Angle of impedance($\theta=\theta_z$)	0°	90°	-90°
Impedance ($\mathbf{Z}$) [Ω]	$\mathbf{Z}_R=R\angle 0°=R+j0$	$\mathbf{Z}_L= X_L\angle 90°=0+jX_L$	$\mathbf{Z}_C= X_C\angle -90°=0-jX_L$
Phase difference between voltage and current	0°	90°	-90°
Relation between voltage and current	Voltage and current are in phase	Voltage leads current Current **lags** voltage	Voltage lags current Currents **leads** voltage
Power factor (pf=cosθ)	Unity (1)	Zero **lagging** power factor (pf=0)	Zero **leading** power factor (pf=0)
Reactive factor (rf=sinθ)	0	1	-1
Power (P) [W]	$V_{rms}I_{rms}=I_{rms}^2R=V_{rms}^2/R$	0	0
Reactive power ($Q=P_x$) [Var]	0	$V_{rms}I_{rms}=I_{rms}^2X_L$ $=V_{rms}^2/X_L$	$-V_{rms}I_{rms}=-I_{rms}^2X_C$ $=-V_{rms}^2/X_C$
Apparent power (S) [VA]	$S= V_{rms}I_{rms}$	$S= V_{rms}I_{rms}$	$S= V_{rms}I_{rms}$

	Resistance	Inductance	Capacitance
Phasor diagram			
Impedance Diagram			
Power Triangle