Coupling Circuit

Prepared by

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Coupled Circuit

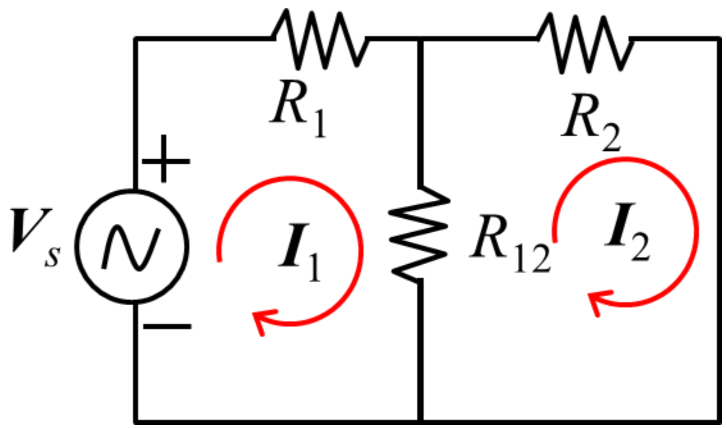
Two circuits are said to be "**Coupled**" when energy transfer takes place from one circuit to the other when one of the circuits is energized.

When interchanging energy takes places from one circuit to other, the circuits are said to be **mutually Coupled**.

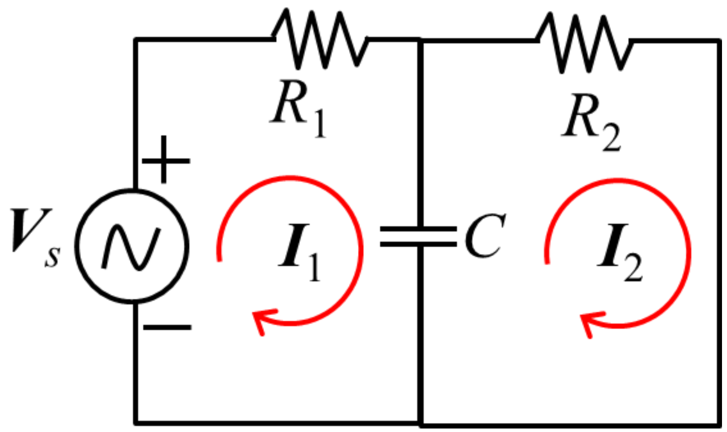
Energy transfer from one circuit to the other can be done by:

Conductive [Conductively Coupled Circuit]
Electrostatic [Electrostatically Coupled or Capacitive Coupled Circuit]
Electromagnetic [Magnetic or Inductive Coupled Circuit]

Two circuits are said to be "**Conductively Coupled**" when energy transfer takes place from one circuit to the other by electrical current conduction that means conductively or electrically.
The energy is transferred by the mutual resistance R_{12}.

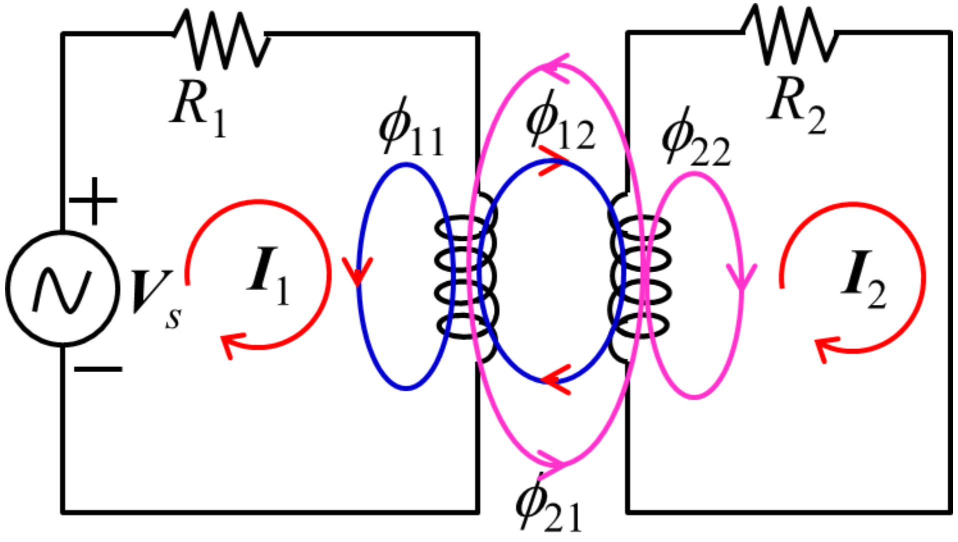


Two circuits are said to be "**Electrostatic or Capacitive Coupled**" when energy transfer takes place from one circuit to the other by electric fields or charges that means electrostatically.
The energy is transferred by the mutual capacitance C.



Two circuits are said to be "**Magnetic or Inductive Coupled**" when energy transfer takes place from one circuit to the other by magnetic field or flux that means magnetically or inductively.

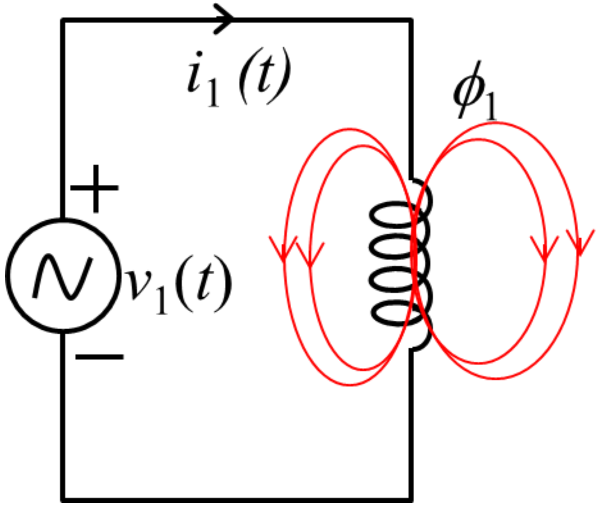
The energy is transferred by the mutual flux (ϕ_{12} or ϕ_{21}) or inductance.



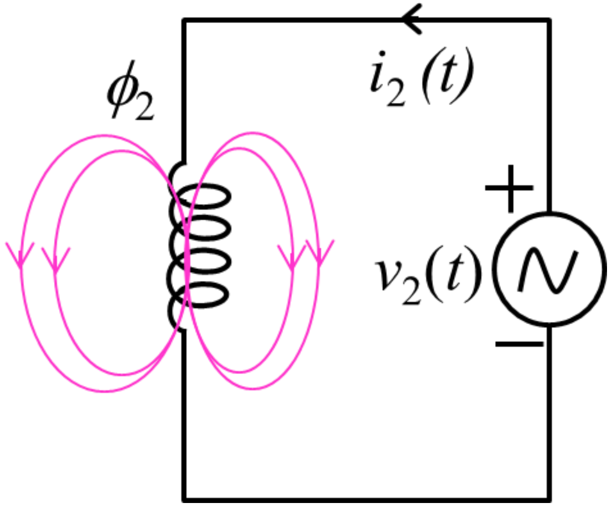
Capacitive coupling favors transfer of the higher frequency components of a signal, whereas inductive coupling favors lower frequency components, and conductive coupling favors neither higher nor lower frequency components.

Self Flux

Self-Flux: The total flux which is generated by supplying a current through a coil is called self flux.



ϕ_1 **is self-flux** since this is the total flux which is produced by the current i_1.

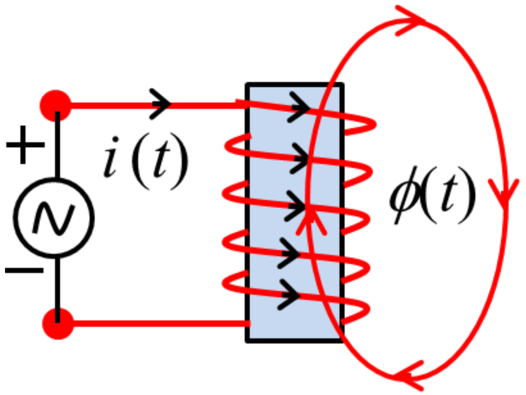


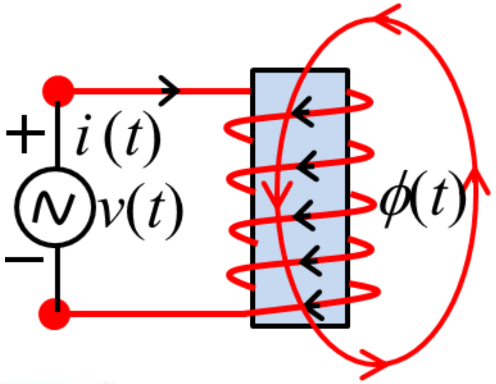
ϕ_2 **is self-flux** since this is the total flux which is produced by the current i_2.

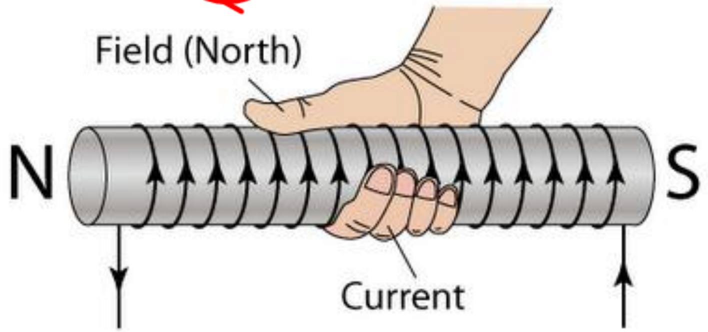
Direction of Flux

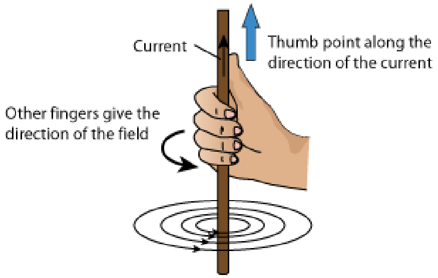
The direction of flux depends on the arrangement of conductor and the direction of flow of current.

An electric current passes through a coil (solenoid), resulting in a magnetic field. When wrapping the right hand around the coil (solenoid) with the fingers in the direction of the conventional current, the thumb points in the direction of the magnetic north pole.

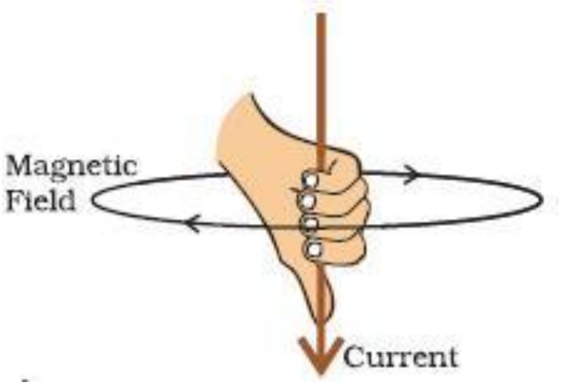








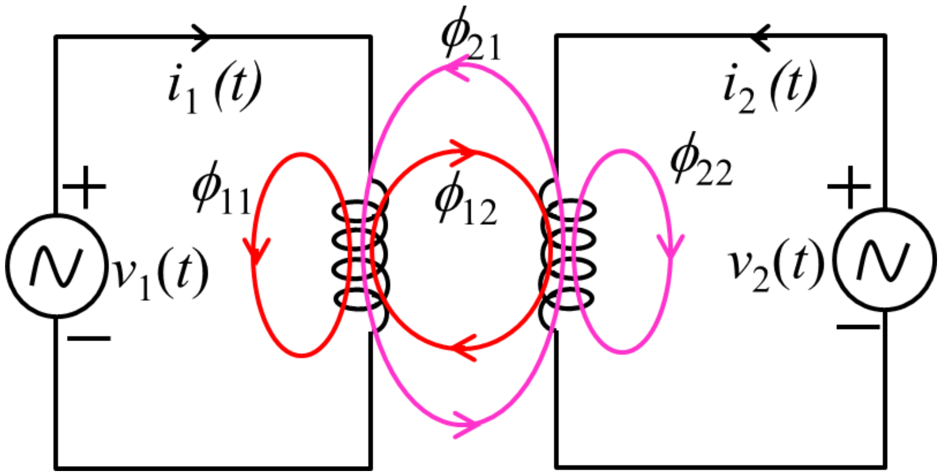
Right Hand Grip Rule



An electric current passes through a straight wire. Grabbing the wire points the thumb in the direction of the conventional current (from positive to negative), while the fingers point in the direction of the magnetic flux lines.

Leakage Flux and Mutual Flux

When two or more coils are brought near some portion of self flux does not link to other coils and some portion of self-flux is linked with other coils as shown in the following figure.



Leakage Flux: The portion of self-flux does not link with other coils is called leakage flux.

ϕ_{11} **is leakage flux** which is the portion of self flux ϕ_1.

ϕ_{22} **is leakage flux** which is the portion of self flux ϕ_2.

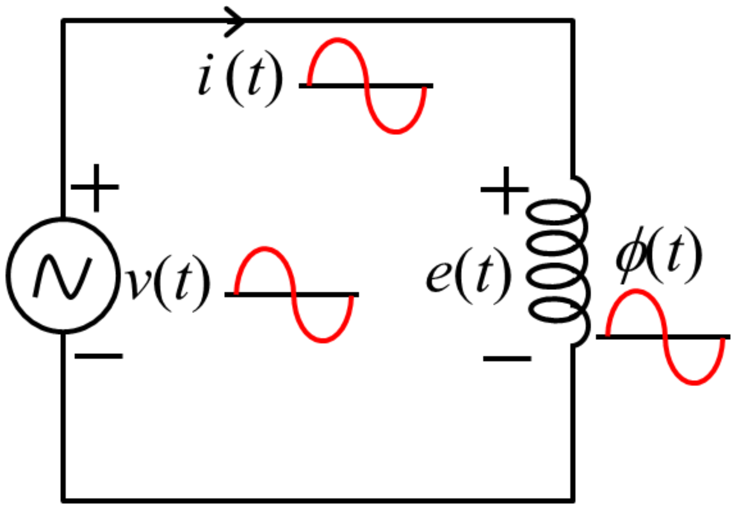
Mutual Flux: The portion of self-flux is linked with other coils is called mutual flux flux.

ϕ_{12} **is mutual flux** which is the portion of self flux ϕ_1 linked to coil 2.

ϕ_{21} **is mutual flux** which is the portion of self flux ϕ_2 linked to coil 1.

Self Inductance

If a time-changing source is applied to a coil, the produced flux also will be time-changing. According to **Faraday's law of Electromagnetic Induction** an electromotive force (emf) is induced in the coil. The induced emf opposes the supply voltage (according to **Lenz's Law**). The induced emf in coil with *N* number of turns can be given by:



$$e(t) = N\frac{d\phi(t)}{dt} = N\frac{d\phi(t)}{di(t)}\frac{di(t)}{dt} = L\frac{di(t)}{dt}$$

$$L = N\frac{d\phi(t)}{di(t)}$$

If the relation of flux and current is linear then: $L = N\dfrac{\phi}{i}$ $\therefore\ e(t) \propto \dfrac{di(t)}{dt}$

Here, *L* is called self-inductance which is the proportionality constant of an induced emf that is proportional to *di/dt*.

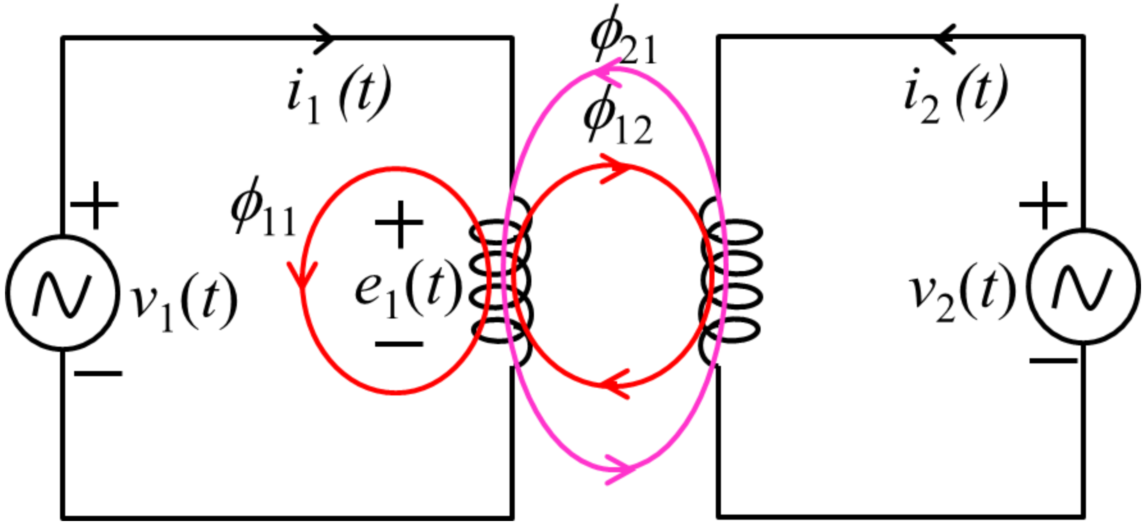
Mutual Inductance

The induced emf of coil 1 is given by:

$$e_1(t) = N_1 \frac{d\phi_1(t)}{dt} + N_1 \frac{d\phi_{21}(t)}{dt} = N_1 \frac{d\phi_1(t)}{di_1(t)} \frac{di_1(t)}{dt} + N_1 \frac{d\phi_{21}(t)}{di_2(t)} \frac{di_2(t)}{dt}$$

$$e_1(t) = L_1 \frac{di_1(t)}{dt} + M_{21} \frac{di_2(t)}{dt}$$

$$L_1 = N_1 \frac{d\phi_1(t)}{di_1(t)} \qquad M_{21} = N_1 \frac{d\phi_{21}(t)}{di_2(t)}$$



If the relation of flux and current is linear then:

$$L_1 = N_1 \frac{\phi_1}{i_1} \qquad M_{21} = N_1 \frac{\phi_{21}}{i_2}$$

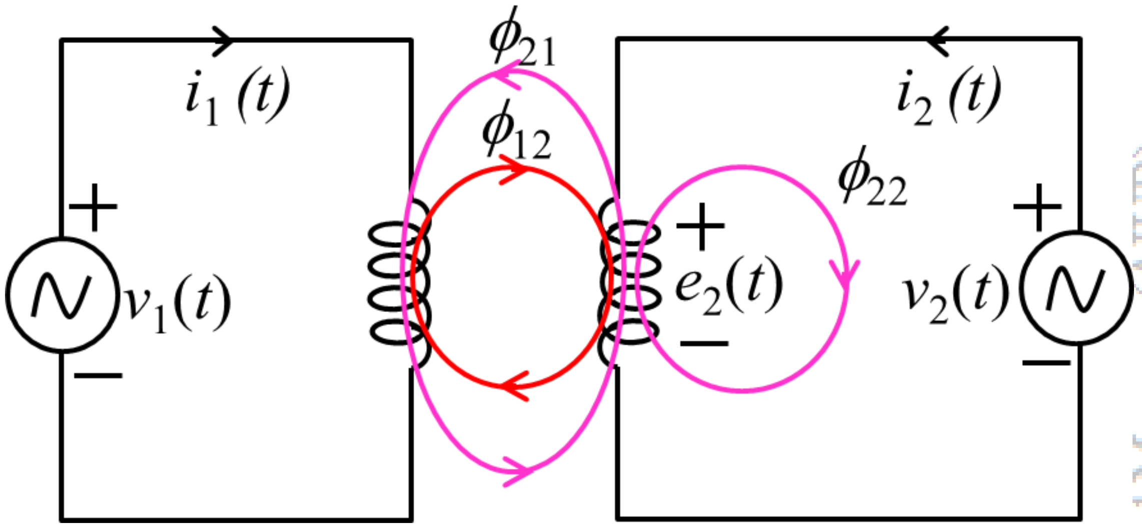
$$\phi_1 = \frac{L_1 i_1}{N_1} \qquad \phi_{21} = \frac{M_{21} i_2}{N_1}$$

M_{12} is called the mutual inductance.

Mutual inductance represents the capability of one inductor to induce an emf in neighbor coil or inductor when a coil is energized.

Similarly, The induced emf of coil 2 is given by:

$$e_2(t) = N_2 \frac{d\phi_2(t)}{dt} + N_2 \frac{d\phi_{12}(t)}{dt}$$



$$e_2(t) = L_2 \frac{di_2(t)}{dt} + M_{12} \frac{di_1(t)}{dt}$$

$$L_2 = N_2 \frac{d\phi_2(t)}{di_2(t)} \qquad M_{12} = N_2 \frac{d\phi_{12}(t)}{di_1(t)}$$

If the relation of flux and current is linear then:

$$L_2 = N_2 \frac{\phi_2}{i_2} \qquad M_{12} = N_2 \frac{\phi_{12}}{i_1} \qquad \phi_2 = \frac{L_2 i_2}{N_2} \qquad \phi_{12} = \frac{M_{12} i_1}{N_2}$$

If the permeability of the mutual flux path is assumed to be constant then

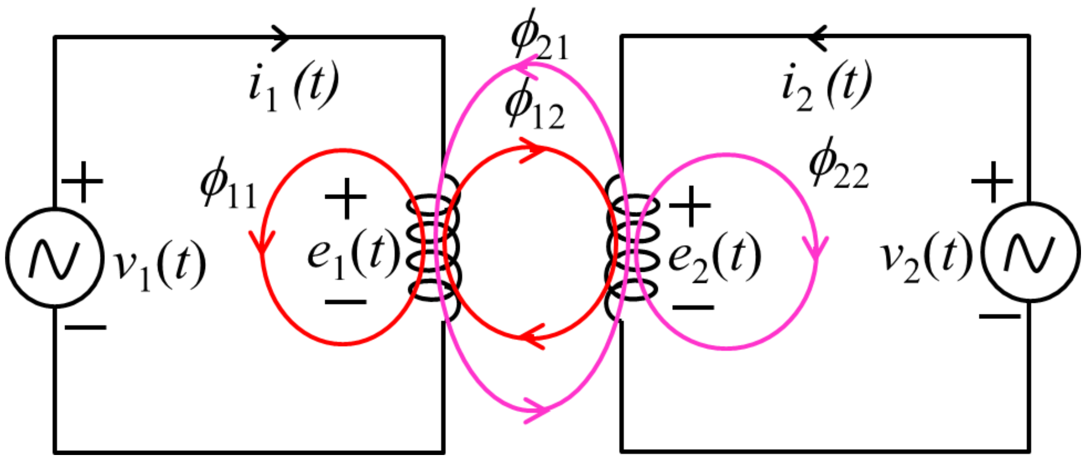
$$M_{12} = M_{21} = M$$

The unit of self-inductance and mutual inductance is henry (H).

The mathematical expression for the two magnetic coupling circuits can be written as follows:

$$v_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt}$$

$$v_2(t) = L_2 \frac{di_2(t)}{dt} + M \frac{di_1(t)}{dt}$$



If $v_1(t)$ and $v_2(t)$ are sinusoidal. Then we have:

$$\boldsymbol{V}_1 = j\omega L_1 \boldsymbol{I}_1 + j\omega M \boldsymbol{I}_2$$

$$\boldsymbol{V}_2 = j\omega L_2 \boldsymbol{I}_2 + j\omega M \boldsymbol{I}_1$$

Let, Self – reactance of coil 1: $X_1 = \omega L_1$ Self – reactance of coil 2: $X_2 = \omega L_2$

Mutual – reactance between coils 1 and 2: $X_m = \omega M$

Then,

$$\boldsymbol{V}_1 = jX_1 \boldsymbol{I}_1 + jX_m \boldsymbol{I}_2$$

$$\boldsymbol{V}_2 = jX_2 \boldsymbol{I}_2 + jX_m \boldsymbol{I}_1$$

Coupling Coefficient or Coefficient of Coupling

The fractional part of ϕ_1 which links with N_2, (ϕ_{12}/ϕ_1), and the fractional part of ϕ_2 which links with N_1, (ϕ_{21}/ϕ_2), are indices of the degree of coupling that exists between two winding.

$$k_m = k = \sqrt{\left(\frac{\phi_{12}}{\phi_1}\right)\left(\frac{\phi_{21}}{\phi_2}\right)} = \sqrt{\left(\frac{M_{12}i_1/N_2}{L_1 i_1/N_1}\right)\left(\frac{M_{21}i_2/N_1}{L_2 i_2/N_2}\right)}$$

$$k_m = k = \sqrt{\left(\frac{M_{12}}{L_1}\right)\left(\frac{M_{21}}{L_2}\right)} = \frac{M}{\sqrt{L_1 L_2}} \qquad \therefore \quad M = k_m\sqrt{L_1 L_2}$$

The **coefficient of coupling** can be defined as the ratio of mutual inductance *M* to the square root of the product of self inductances of coil 1 and coil 2.

The range of **coefficient of coupling** is between 0 and 1. $0 \le k_m \le 1$

If k_m**=1** that means the flux due to one coil is fully linked with the other. In this case, coils are said to be ***perfectly coupled***. This is an **ideal case**.

If k_m**=0** that means the flux due to one coil does not link with the other. In this case, coils are said to be ***magnetically isolated*** from each other.

If $0 < k_m < 0.5$ that means more than 0% and less than 50% of self flux is linked with the other. In this case, coils are said to be ***loosely coupled***.

If $0.5 < k_m < 1$ that means more than 50% and less than 100% of self flux is linked with the other. In this case, coils are said to be ***tightly coupled***.

Example 12.1: Two inductively coupled coils have self-inductances 50 mH and 200 mH. If the coupling coefficient is 0.5 (**i**) find the value of mutual inductance between the coils, and (**ii**) what is the maximum possible mutual inductance?

Solution: (**i**) We know that, $k_m = \dfrac{M}{\sqrt{L_1 L_2}}$

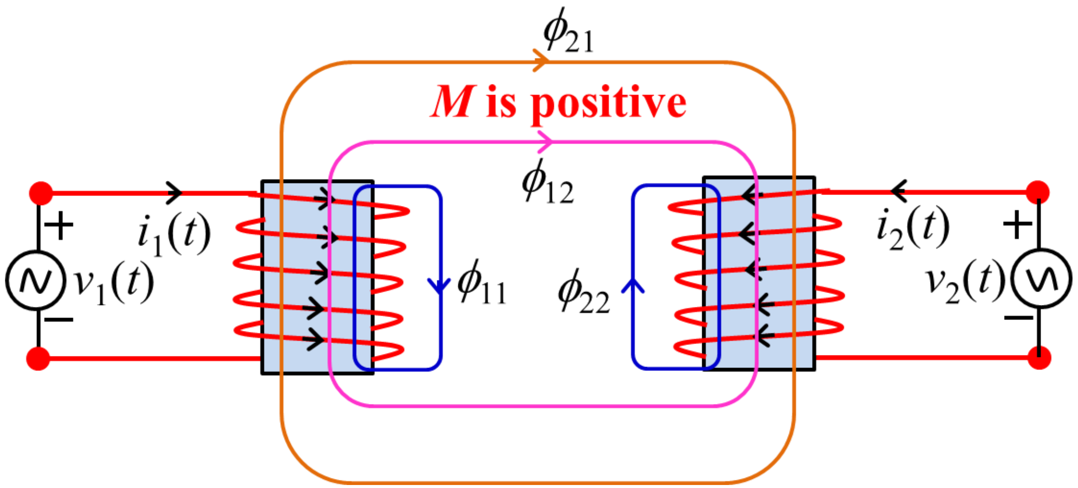
$$M = k_m \sqrt{L_1 L_2} = 0.5 \times \sqrt{\left(50 \times 10^{-3}\right) \times \left(200 \times 10^{-3}\right)} = 50 \times 10^{-3} \text{ H}$$

(**ii**) For maximum value of M, $k_m = 1$ thus

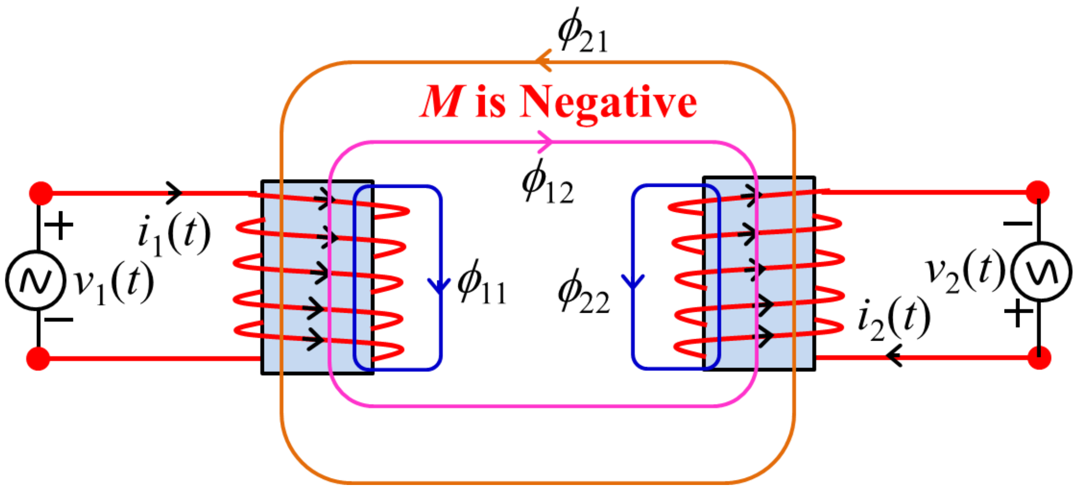
$$M = k_m \sqrt{L_1 L_2} = 1.0 \times \sqrt{\left(50 \times 10^{-3}\right) \times \left(200 \times 10^{-3}\right)} = 100 \times 10^{-3} \text{ H}$$

Circuit Directions and the Sign of M

The sign (either positive or negative) depends on the arrangement of conductor and the direction of flow of flux.



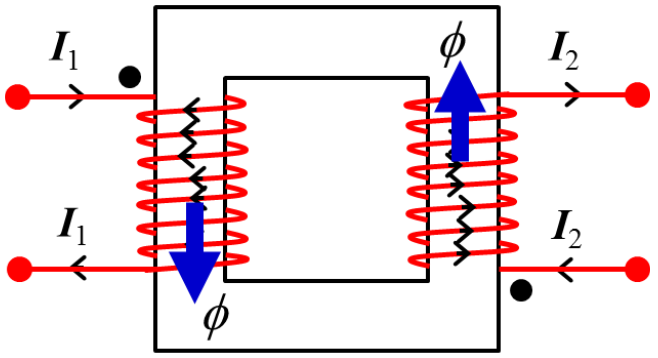
The sign of *M* is positive if the total flux is increased that means the self-flux and the mutual flux are additive.

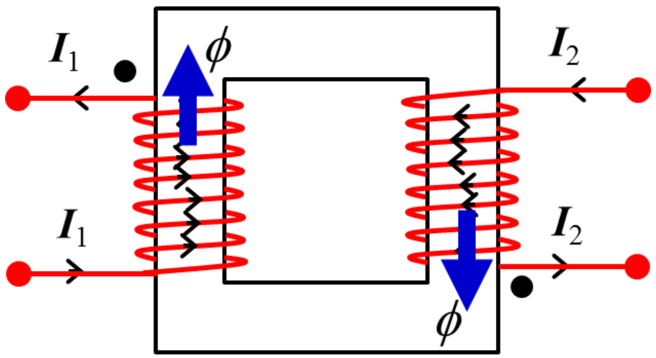


The sign of *M* is negative if the total flux is decreased that means the self-flux and the mutual flux are opposite.

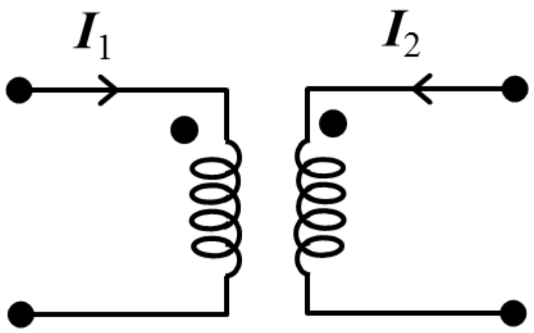
Dot Convention

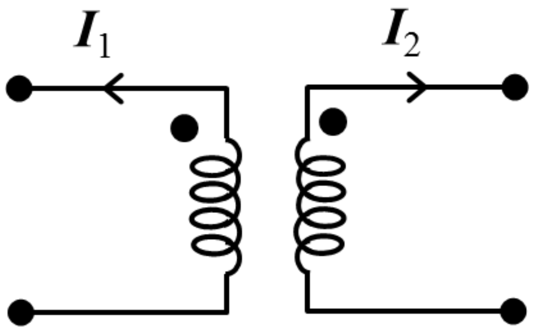
Instead of showing the actual modes of winding, a conventional method employing a ***dot-marked*** terminal, as shown in the following figures. The dots are also known as ***positive polarity marks of mutual voltage***.



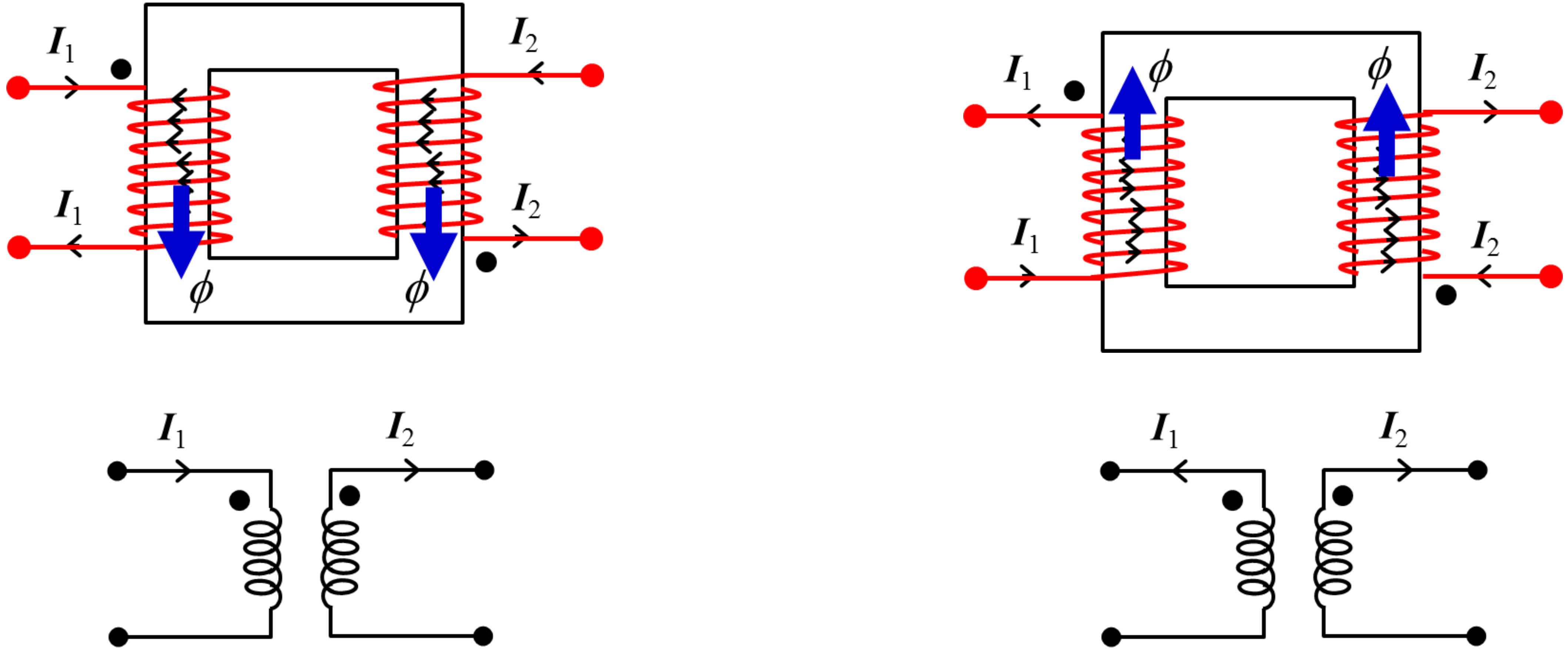


In coupling coils, if both currents are enter or leave to the dotted terminal, the sign of *M* is positive.





If a current enters a dot in one coil, then mutually induced voltage in other coil is positive at the dotted end.

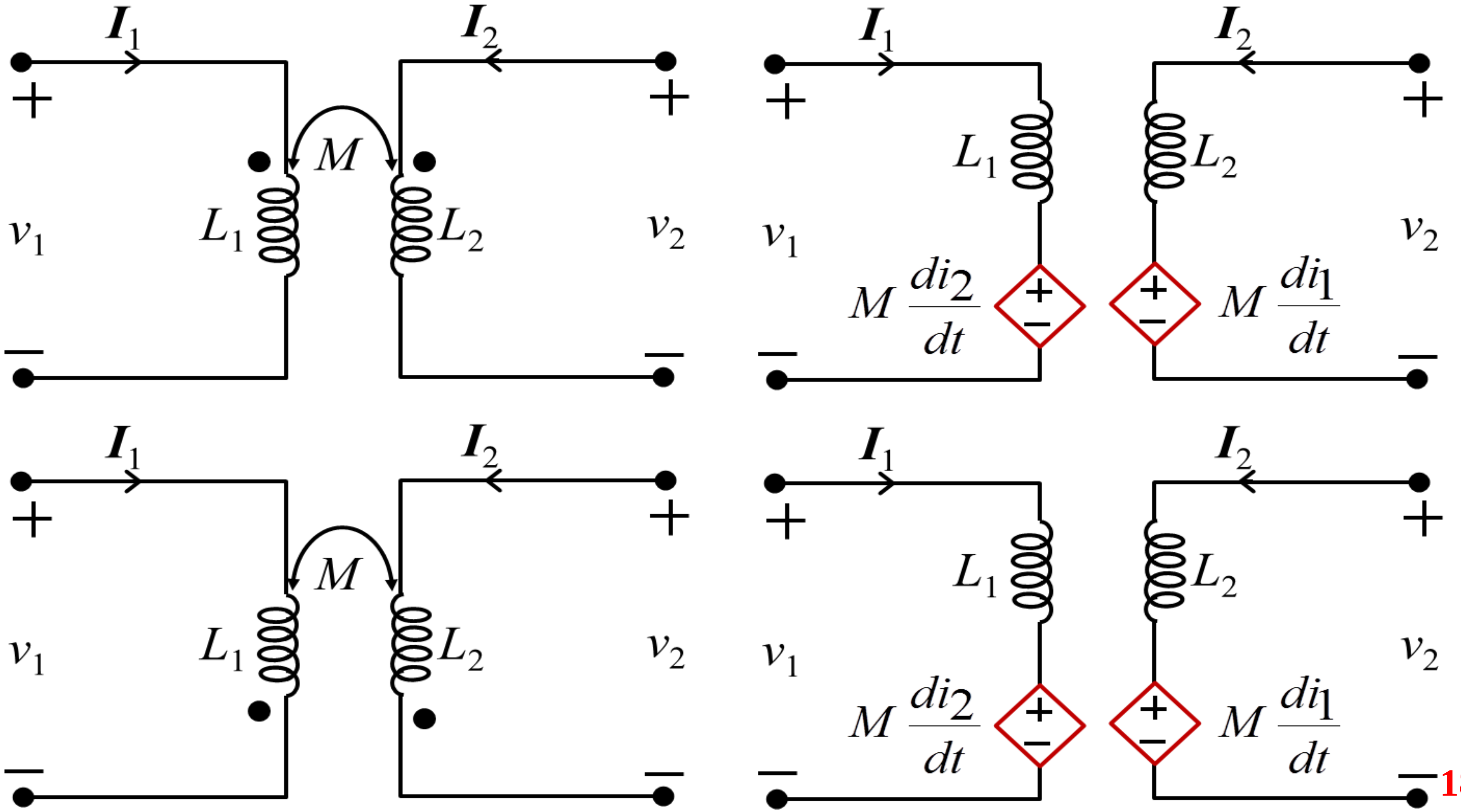


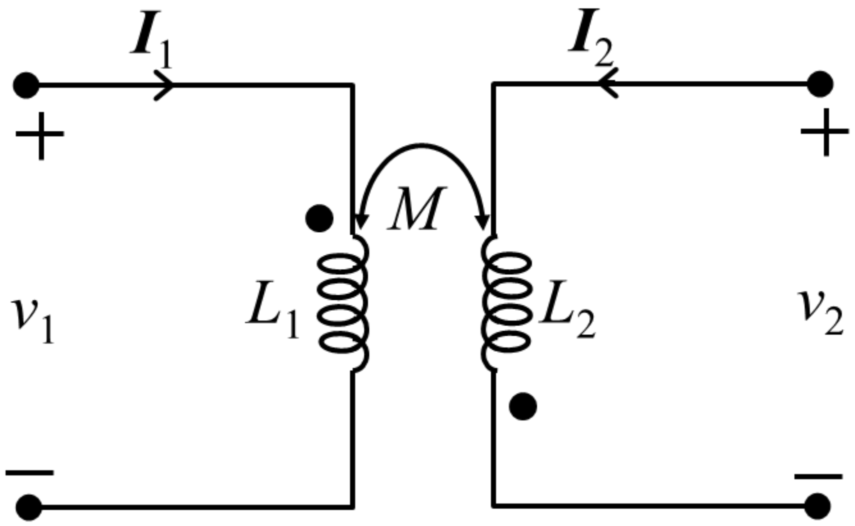
If one current enters to one dotted terminal and other current leaves to another dotted terminal, the sign of *M* is negative.

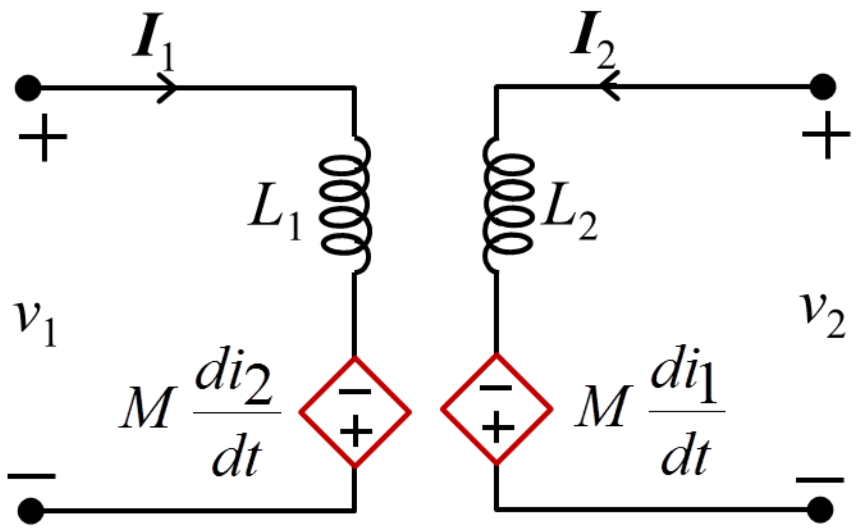
Polarity of Induced Voltage From Dot Convention

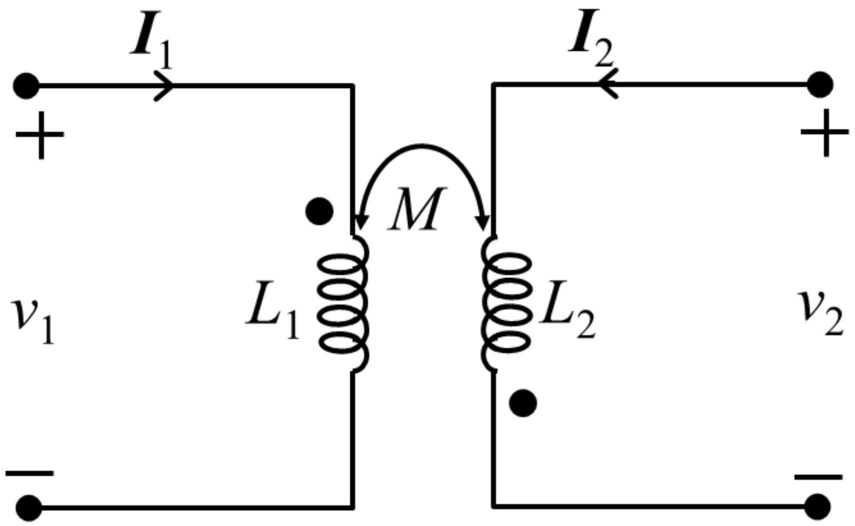
If a current enters a dot in one coil, then mutually induced voltage in other coil is positive at the dotted end.

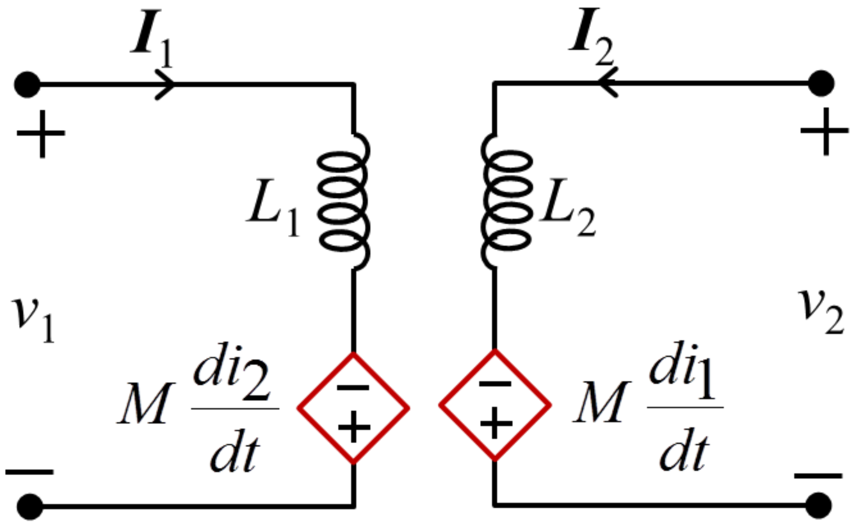
If a current leaves a dot in one coil, then mutually induced voltage in other coil is negative at the dotted end.



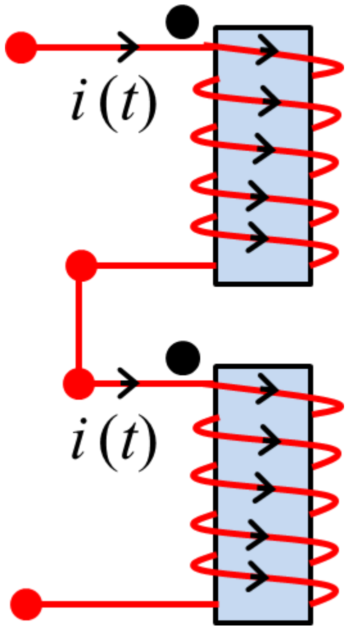

I_1
I_2
+
+
M
L_1
L_2
v_1
v_2
−
−

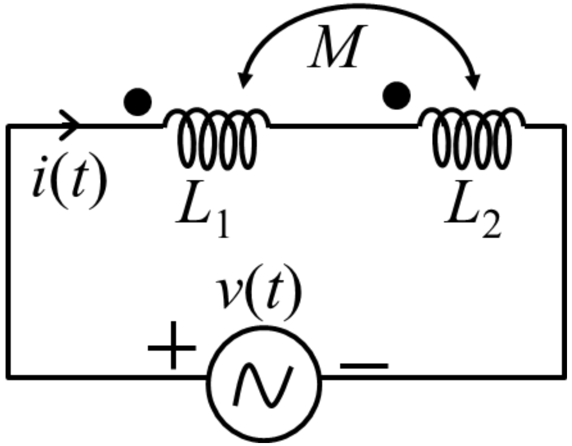

I_1
I_2
+
+
L_1
L_2
v_1
v_2
$M\frac{di_2}{dt}$
$M\frac{di_1}{dt}$
−
−


I_1
I_2
+
+
M
L_1
L_2
v_1
v_2
−
−


I_1
I_2
+
+
L_1
L_2
v_1
v_2
$M\frac{di_2}{dt}$
$M\frac{di_1}{dt}$
−
−

Equivalent Inductance When Two Coils are Connected in Series





M is Positive

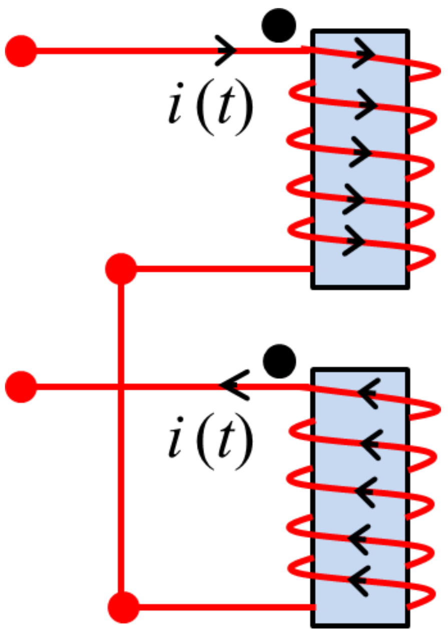
$$v(t) = \left[L_1 \frac{di(t)}{dt} + M\frac{di(t)}{dt}\right] + \left[L_2 \frac{di(t)}{dt} + M\frac{di(t)}{dt}\right]$$

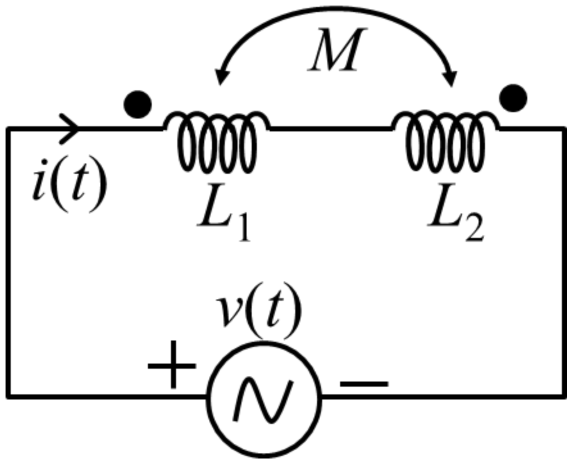
$$v(t) = [L_1 + L_2 + 2M]\frac{di(t)}{dt}$$

$$v(t) = L_{eq}\frac{di(t)}{dt} = L_{eff}\frac{di(t)}{dt} = L_{sp}\frac{di(t)}{dt}$$

$$L_{eq} = L_{eff} = L_{sp} = L_1 + L_2 + 2M$$

Equivalent Inductance When Two Coils are Connected in Series





M is Negative

$$v(t) = \left[L_1 \frac{di(t)}{dt} - M\frac{di(t)}{dt}\right] + \left[L_2 \frac{di(t)}{dt} - M\frac{di(t)}{dt}\right]$$

$$v(t) = [L_1 + L_2 - 2M]\frac{di(t)}{dt}$$

$$v(t) = L_{eq}\frac{di(t)}{dt} = L_{eff}\frac{di(t)}{dt} = L_{sn}\frac{di(t)}{dt}$$

$$L_{eq} = L_{eff} = L_{sn} = L_1 + L_2 - 2M$$

Equivalent Inductance When Two Coils are Connected in Parallel

Applying KVL:

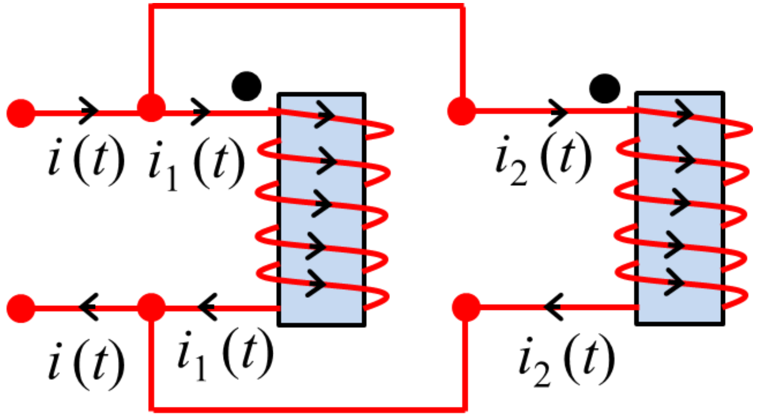
$$v(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt} \qquad (1)$$

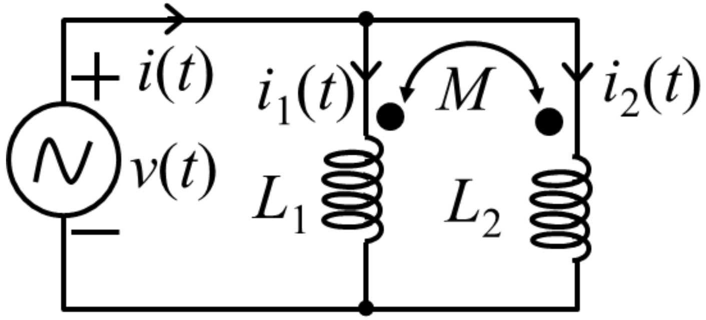
$$v(t) = L_2 \frac{di_2(t)}{dt} + M \frac{di_1(t)}{dt} \qquad (2)$$

Applying KCL:

$$i(t) = i_1(t) + i_2(t) \qquad (3)$$

M is Positive





Solving Eqs. (1) to (3) we have:

$$v(t) = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} \frac{di(t)}{dt}$$

$$v(t) = L_{eq} \frac{di(t)}{dt} = L_{eff} \frac{di(t)}{dt} = L_{pp} \frac{di(t)}{dt}$$

$$\boxed{L_{eq} = L_{eff} = L_{pp} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}}$$

Equivalent Inductance When Two Coils are Connected in Parallel

M is Negative

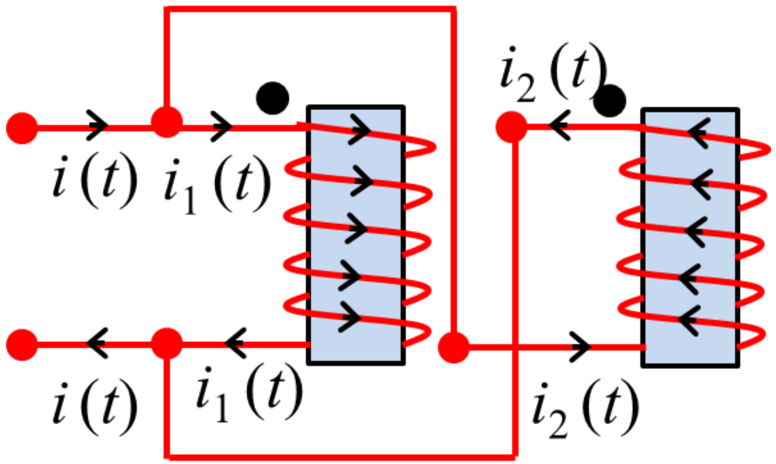
Applying KVL:

$$v(t) = L_1 \frac{di_1(t)}{dt} - M \frac{di_2(t)}{dt} \qquad (1)$$

$$v(t) = L_2 \frac{di_2(t)}{dt} - M \frac{di_1(t)}{dt} \qquad (2)$$

Applying KCL:

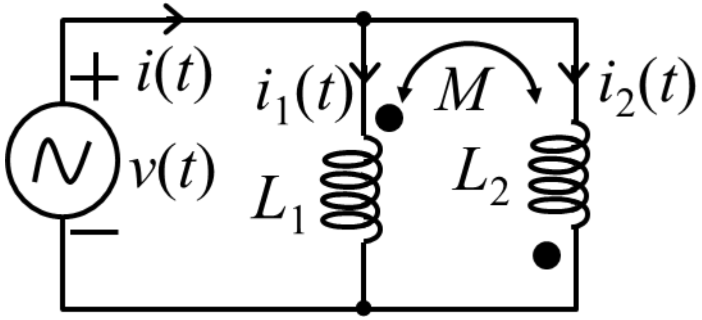
$$i(t) = i_1(t) + i_2(t) \qquad (3)$$



Solving Eqs. (1) to (3) we have:

$$v(t) = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} \frac{di(t)}{dt}$$

$$v(t) = L_{eq} \frac{di(t)}{dt} = L_{eff} \frac{di(t)}{dt} = L_{pn} \frac{di(t)}{dt}$$



$$\boxed{Leq = L_{eff} = Lpn = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}}$$

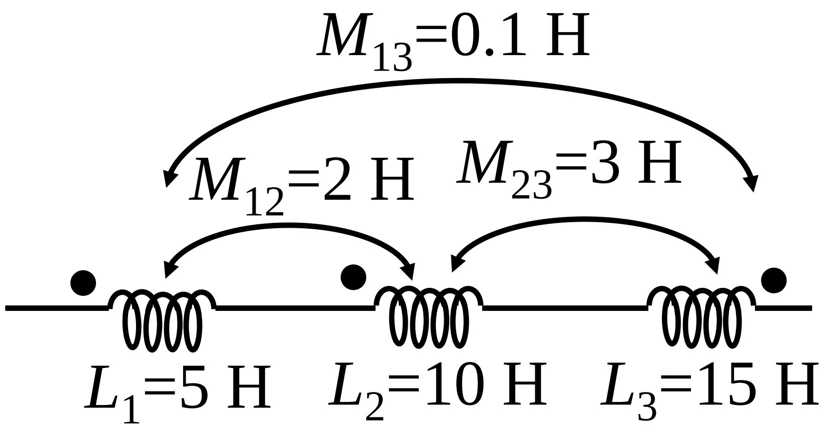
Example 12.2: Two coils are connected in series having an equivalent inductance of 1.8 H when connecting in aiding, and an equivalent inductance 0.6 H when the connection is opposing. Calculate the mutual inductance of the coils.

Solution: Given: $L_{sp} = 1.8\ \text{H}$ $L_{sn} = 0.6\ \text{H}$

We know that: $L_{sp} = L_1 + L_2 + 2M = 1.8\ \text{H}$ (i) $L_{sn} = L_1 + L_2 - 2M = 0.6\ \text{H}$ (ii)

Subtracting (ii) from (i) $4M = 1.2\ \text{H}$ $\therefore M = 0.3\ \text{H}$

Example 12.3: Find the total inductance of the series coils of following Figure.



For Coil 1: $L_{eq1} = L_1 + M_{12} - M_{13}$

For Coil 2: $L_{eq2} = L_2 + M_{12} - M_{23}$

For Coil 3: $L_{eq3} = L_3 - M_{13} - M_{23}$

Total: $L_{eq} = L_{eq1} + L_{eq2} + L_{eq3}$

$$L_{eq} = L_1 + L_2 + L_3 + 2M_{12} - 2M_{23} - 2M_{13}$$

$$L_{eq} = 5 + 10 + 15 + 2\times 2 - 2\times 3 - 2\times 1 = 26\ \text{H}$$

Example 12.4: The coefficient of coupling between two coils is 0.6. When the two coils are connected in series such that their fluxes are in the same direction, the net inductance is 1.8 H. However, when connected in series such that fluxes are in opposite directions, the net inductance is 0.8 H. Determine the mutual inductance and the self-inductances of the two coils.

Solution: Given: $L_{sp} = 1.8\ \text{H}$ $\quad L_{sn} = 0.8\ \text{H}$ $\quad k_m = 0.8\ \text{H}$

We know that: $L_{sp} = L_1 + L_2 + 2M = 1.8\ \text{H} \quad \text{(i)}$ $\qquad L_{sn} = L_1 + L_2 - 2M = 0.8\ \text{H} \quad \text{(ii)}$

Subtracting (ii) from (i) $\quad 4M = 1.0\ \text{H} \quad \therefore\ M = 0.25\ \text{H}$

Adding (i) from (ii) $\quad 2(L_1 + L_2) = 2.6 \qquad L_1 + L_2 = 1.3 \quad \text{(iii)}$

$$k_m = \frac{M}{\sqrt{L_1 L_2}} = 0.8\ \text{H} \qquad \sqrt{L_1 L_2} = \frac{M}{0.8} = \frac{0.25}{0.8} \qquad L_1 L_2 = \left(\frac{0.25}{0.8}\right)^2 = 0.1736 \quad (iv)$$

By solving (iii) and (iv) we have: $(L_1 - L_2)^2 = (L_1 + L_2)^2 - 4L_1 L_2$

$$L_1 - L_2 = \sqrt{(L_1 + L_2)^2 - 4L_1 L_2} = 0.9978$$

$$L_1 = 1.149\ \text{H} \qquad L_2 = 0.151\ \text{H}$$

Homework 12.1

Problem 12.1.1: Two coupled coils with coefficient of coupling 0.433, have self inductances of 8 H and 6 H. Determine the equivalent inductance of the combination when they are connected in parallel such that **(i)** the mutual inductance assists the self-inductance, and **(ii)** the mutual inductance opposes the self-inductance.

Problem 12.1.2: Two identical coupled coils are connected inn series. When the mutual inductance assists the self-inductance the effective inductance is 4 H and when the mutual inductance opposes the self-inductance the effective inductance is 0.8 H. Calculate the self-inductance, the mutual inductance and the coefficient of coupling.

Problem 12.1.3: A coil of inductance 200 mH is magnetically coupled with another coil of inductance 800 mH. The coefficient of coupling between two coils is 0.5. Calculate the equivalent inductance of (i) series aiding, (ii) series opposing, (iii) parallel aiding, and (iv) parallel opposing combinations.

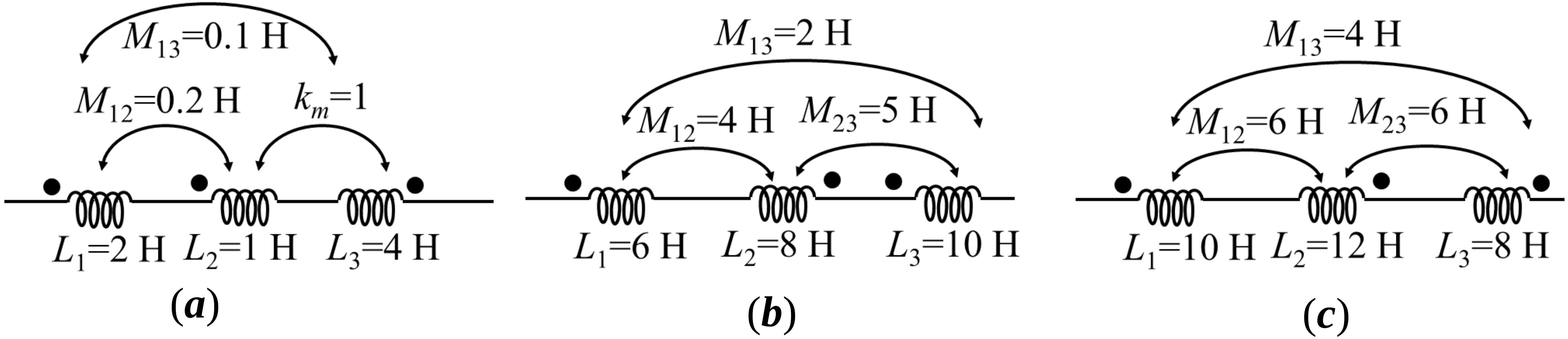
Problem 12.1.4: The combined inductance of two coils connected inn series is either 0.75 H or 0.25 H, depending on the relative direction of the current in the two coils. If one of the coils, when isolated, has a self inductance of 0.5 H. Calculate (i) the mutual inductance, (ii) the coefficient of coupling between the coils.

Continue of Homework ………………...

Problem 12.1.5: Two similar coils have a coupling coefficient of 0.25. When they are connected cumulatively in series, the total inductance is 80 mH. Calculate the self-inductance inductance of each coil. Also, calculate the total inductance when the coils are differentially connected in series.

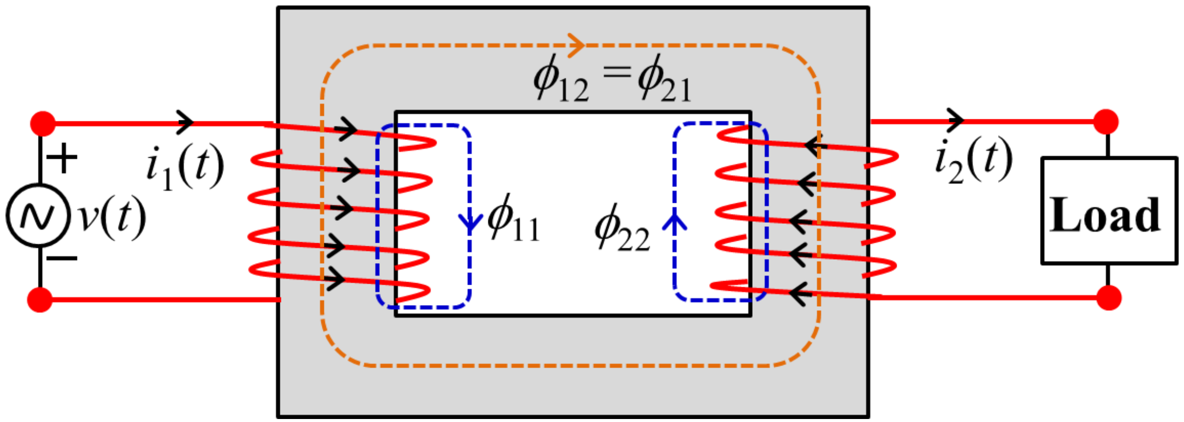
Problem 12.1.6: Two inductively coupled coils have self-inductances 7.5 H and 25 H. If the coupling coefficient is 0.8 (i) find the value of mutual inductance between the coils, and (ii) what is the maximum possible mutual inductance?

Problem 12.1.7: Find the total inductance of the series coils of following Figures.

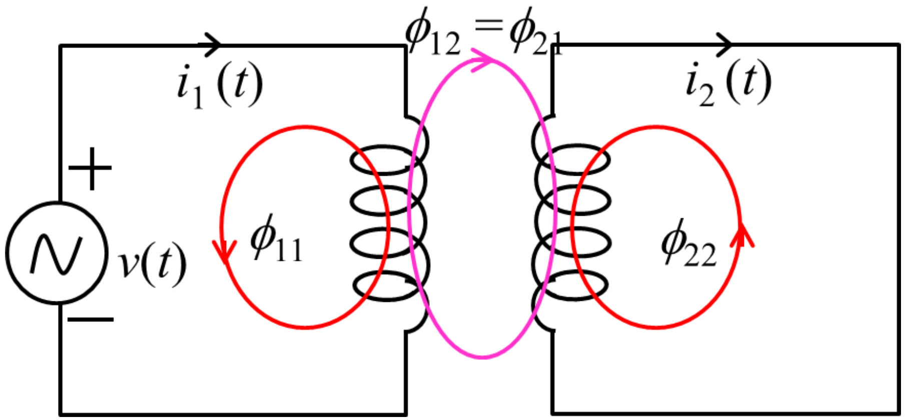


Transformer

Transformer is a static device which transfers the electrical energy from one circuit to another circuit without changing frequency.

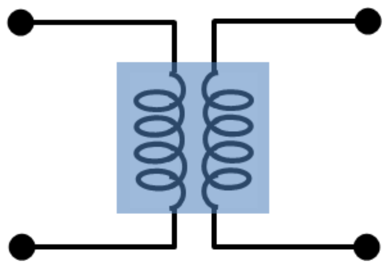


Iron-core Transformer

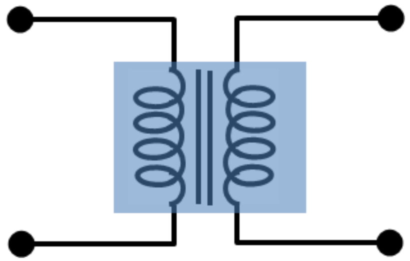


Air-core Transformer

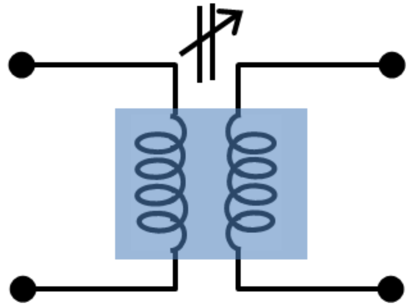
Symbol of Transformer



Air-core

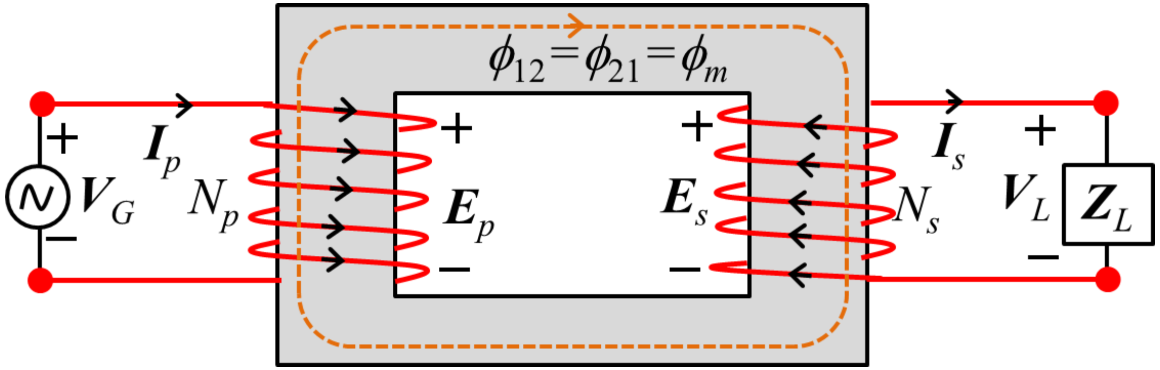


Iron-core



Variable Iron-core

The winding of a transformer at which the source is connected is called ***primary side*** and the winding of a transformer at which the load is connected is called ***secondary side***.



$\boldsymbol{V_G}$: Source voltage; $\boldsymbol{V_L}$: Load voltage; $\boldsymbol{V_L}$: Load impedance;
$\boldsymbol{E_p}$: Primary-side induced emf; $\boldsymbol{I_p}$: Primary-side current;
N_p: Number of turns in primary coil; $\boldsymbol{E_s}$: Secondary-side induced emf;
$\boldsymbol{I_s}$: Secondary-side current; N_s: Number of turns in secondary coil

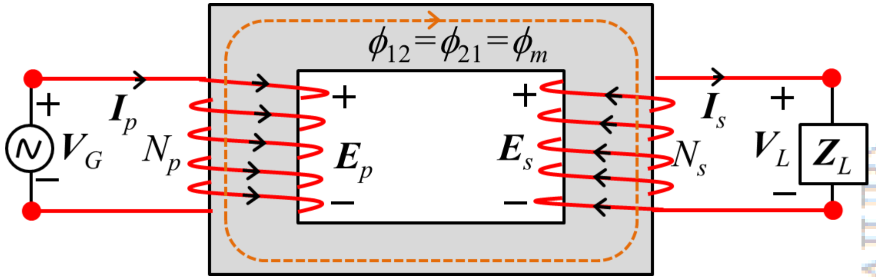
The expression of induced voltages:

$$E_p = 4.44\, fN_p\phi_m \qquad E_s = 4.44\, fN_s\phi_m$$

Relation between primary and secondary voltage: $$\frac{E_s}{E_p} = \frac{N_s}{N_p}$$

The ratio of the magnitudes of the induced voltages is the same as the ratio of the corresponding turns.

In ideal case (neglecting losses): $\boldsymbol{E}_p = \boldsymbol{V}_G$; $\boldsymbol{E}_s = \boldsymbol{V}_L$



Thus: $\dfrac{\boldsymbol{V}_L}{\boldsymbol{V}_G} = \dfrac{\boldsymbol{E}_s}{\boldsymbol{E}_p} = \dfrac{N_s}{N_p} = a$ Where, $a = \dfrac{N_s}{N_p}$

a is called **Transformation** (or turns) **ratio**.

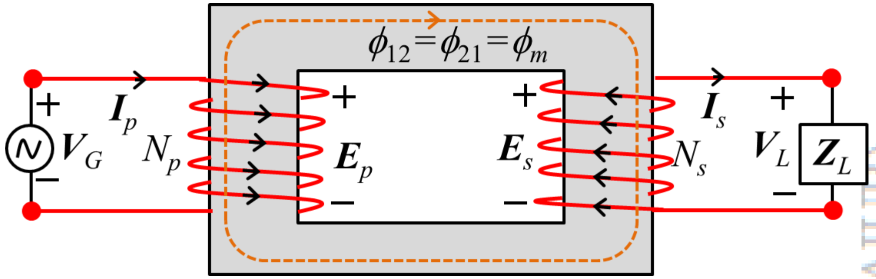
If $a>1$ that means secondary side voltage is greater than primary side voltage, the transformer is called step-up transformer.
If $a<1$ that means secondary side voltage is smaller than primary side voltage, the transformer is called step-down transformer.

In ideal case (neglecting losses) the input power equals to output power that means:

$$P_{in} = P_{out} \qquad P_p = P_s \qquad \boldsymbol{E}_p\boldsymbol{I}_p = \boldsymbol{E}_s\boldsymbol{I}_s \qquad \frac{\boldsymbol{I}_s}{\boldsymbol{I}_p} = \frac{\boldsymbol{E}_p}{\boldsymbol{E}_s} = \frac{N_p}{N_s} = \frac{1}{a}$$

The primary and secondary currents of a transformer are therefore related by the inverse ratios of the turns.

Reflected Impedance



$$\frac{\boldsymbol{V}_L}{\boldsymbol{V}_G} = \frac{N_s}{N_p} = a \qquad \frac{\boldsymbol{I}_s}{\boldsymbol{I}_p} = \frac{N_p}{N_s} = \frac{1}{a}$$

$$\frac{\boldsymbol{V}_L/\boldsymbol{V}_G}{\boldsymbol{I}_s/\boldsymbol{I}_p} = \frac{a}{1/a} = a^2 \qquad \frac{\boldsymbol{V}_L}{\boldsymbol{V}_G}\frac{\boldsymbol{I}_p}{\boldsymbol{I}_s} = a^2 \qquad \frac{\boldsymbol{V}_L/\boldsymbol{I}_s}{\boldsymbol{V}_G/\boldsymbol{I}_p} = a^2$$

Since, $\boldsymbol{Z}_L = \dfrac{\boldsymbol{V}_L}{\boldsymbol{I}_s}$ $\quad \boldsymbol{Z}_p = \dfrac{\boldsymbol{V}_G}{\boldsymbol{I}_p}$ $\quad$ then $\quad \boldsymbol{Z}_L = a^2\boldsymbol{Z}_p$ $\quad \boldsymbol{Z}_p = \dfrac{1}{a^2}\boldsymbol{Z}_L$

The impedance of the primary circuit of an ideal transformer is the inverse of transformation ratio squared times the impedance of the load.

If a transformer is used, therefore, an impedance can be made to appear larger or smaller at the primary by placing it in the secondary of a step-down (a < 1) or step-up (a > 1) transformer, respectively.

If the load is capacitive or inductive, the reflected impedance will also be capacitive or inductive.

Example 12.5: For a transformer the following information are given: primary-side induced voltage 200V, secondary side induced voltage 2400 V, primary side turns number 50 and frequency 50 Hz. Calculate (i) the maximum flux, ϕ_m, (ii) the secondary turns, N_s.

Solution: We know that: $E_p = 4.44\, f N_p \phi_m$

$$\therefore \quad \phi_m = \frac{E_p}{4.44\, f N_p} = \frac{200}{4.44 \times 50 \times 50} = 15.02 \text{ mWb}$$

$$\frac{E_s}{E_p} = \frac{N_s}{N_p}$$

$$N_s = \frac{E_s}{E_p} N_p = \frac{2400}{200} \times 50 = 600 \text{ turns}$$

Example 12.6: For the transformer as shown in the following figure, find (i) the magnitude of the current in the primary and the impressed voltage across the primary, and (ii) the input resistance of the transformer.

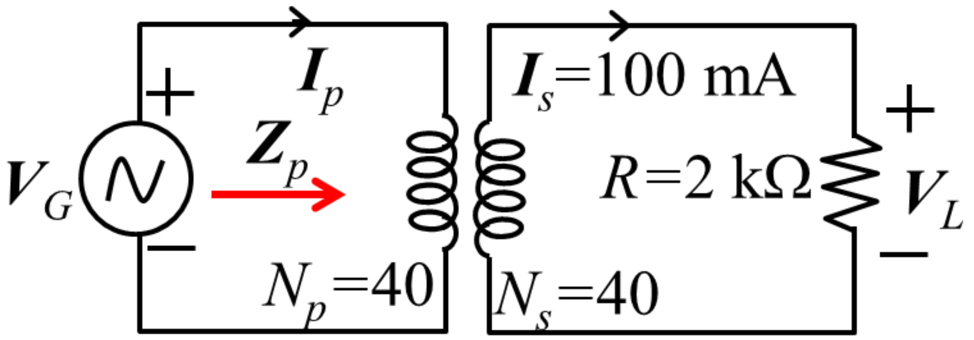
Solution: We know that: $\frac{I_p}{I_s} = \frac{N_s}{N_p}$ $\quad \therefore I_p = \frac{N_s}{N_p} I_s = \frac{5}{40} \times 100 \text{ mA} = 12.5 \text{ mA}$

$$V_L = I_s R = 2000 \times 0.1 = 200 \text{ V}$$

$$\frac{V_g}{V_L} = \frac{N_p}{N_s} \qquad \therefore V_g = \frac{N_p}{N_s} V_L = \frac{40}{5} \times 200 = 1600 \text{ V}$$

$$a = \frac{N_s}{N_p} = \frac{5}{40} = \frac{1}{8} \qquad \frac{1}{a} = 8$$

$$\mathbf{Z}_p = \frac{1}{a^2} \mathbf{Z}_L = (8)^2 \times 2 \text{ k}\Omega = 128 \text{ k}\Omega$$



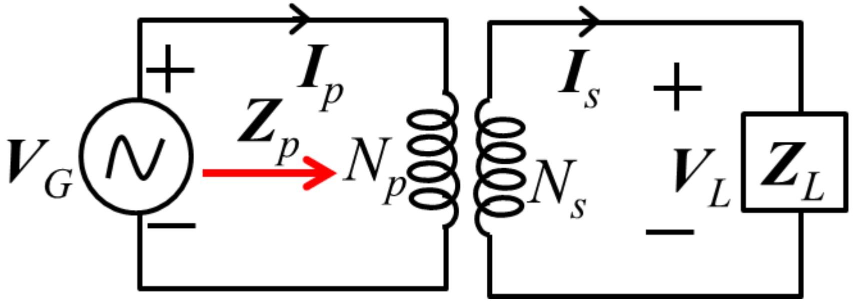
Homework 12. 2

Problem 12.2.1: In a air-core transformer the self inductance of primary coil is 50 mH, the mutual inductance is 80 mH. Calculate the self inductance of secondary coil for (***i***) k_m =0.8, (***ii***) k_m = 1, and (**iii**) k_m= 0.2.

Problem 12.2.2: For a transformer the following information are given: primary-side induced voltage 25V, primary and secondary sides turns number 8 and 64 and frequency 60 Hz. Calculate (**i**) the maximum flux, ϕ_m, (**ii**) the secondary-side induced voltage E_s, (***iii***) if the maximum flux is 12.5 mWb, calculate the frequency.

Problem 12.2.3: If V_L = 240 V, Z_L= 20 ohm, I_p= 0.05 A, and N_s=50 turns, find the number of turns in the primary side.

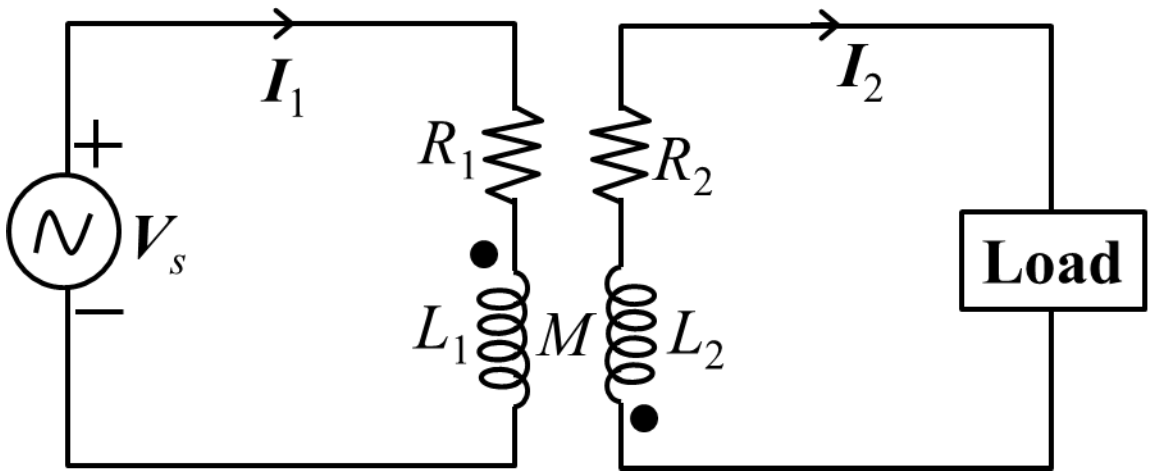
Problem 12.2.4: If N_p = 400 turns, N_s = 1200 turns, V_G= 100 V, Z_L= 9+j12 ohm, find (***i***) the magnitude of load current I_L, (ii) the magnitude of load voltage V_L, and (**iii**) the magnitude of primary side current I_p.



Equivalent Circuit of a Transformer

The following figure shows the equivalent circuit of a transformer. The winding of a transformer at which the source is connected is called ***primary side*** and the winding of a transformer at which the load is connected is called ***secondary side***.

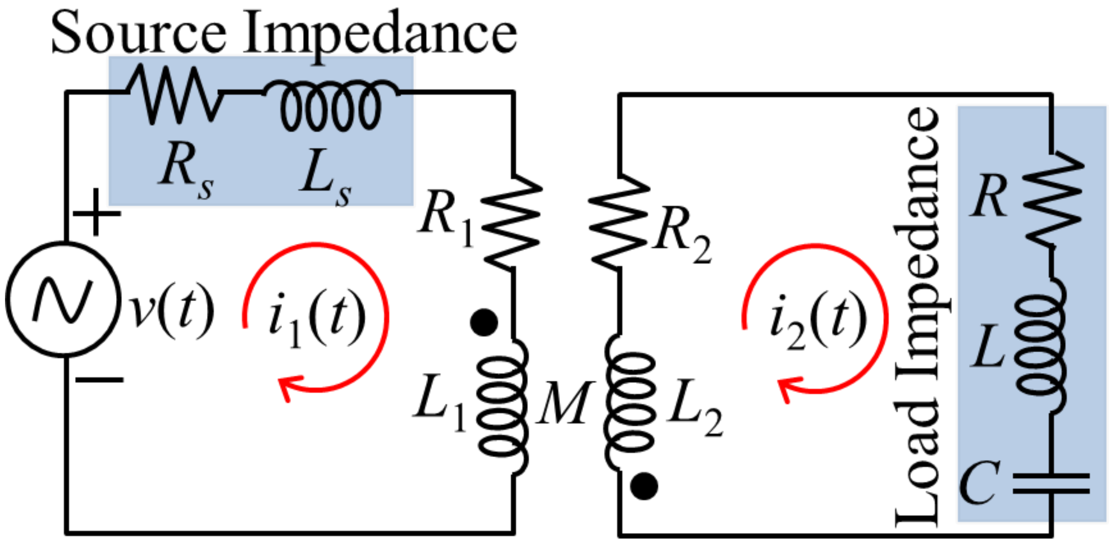
R_1: Resistance of primary-side
R_2: Resistance of secondary-side
L_1: Self-inductance of primary-side
L_2: Self-inductance of secondary-side
M: Mutual inductance between two coils



Mathematical Expression of a Transformer

$$v = (R_s + R_1)i_1 + (L_s + L_1)\frac{di_1}{dt} + M\frac{di_2}{dt}$$

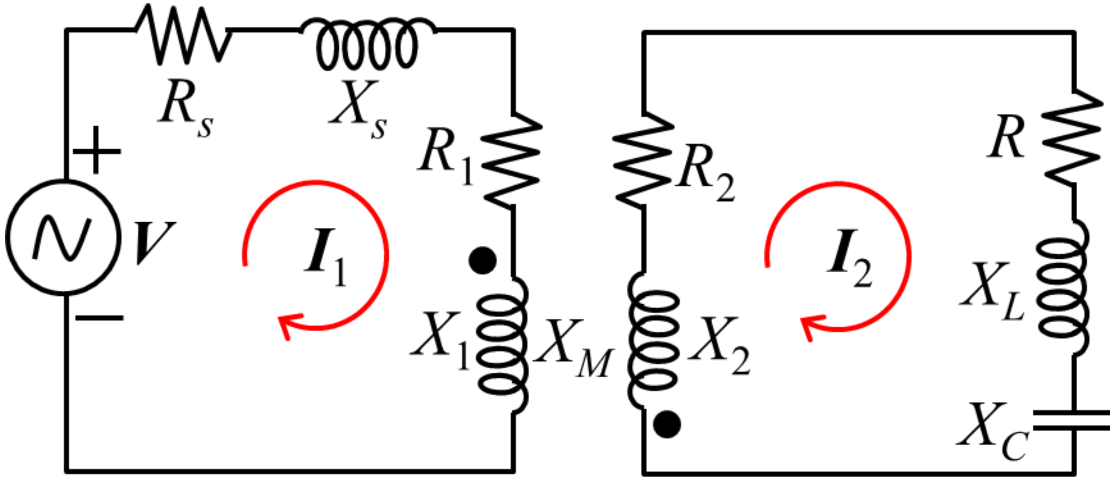
$$0 = (R_2 + R)i_2 + (L_2 + L_s)\frac{di_2}{dt} + \frac{1}{C}\int i_2 dt + M\frac{di_1}{dt}$$



If $v = V_m \sin(\omega t + \theta_v)$ then

$$X_1 = \omega L_1 \quad X_2 = \omega L_2 \quad X_s = \omega L_s \quad X_M = \omega M$$

$$X_L = \omega L \qquad X_C = \frac{1}{\omega C}$$



$$\boldsymbol{V} = (R_s + R_1)\boldsymbol{I}_1 + j(X_s + X_1)\boldsymbol{I}_1 + jX_M \boldsymbol{I}_2$$

$$0 = (R + R_2)\boldsymbol{I}_2 + j(X_2 + X_L - X_C)\boldsymbol{I}_2 + jX_M \boldsymbol{I}_1$$

Let, $\boldsymbol{Z}_s = R_s + jX_s \qquad \boldsymbol{Z}_1 = R_1 + jX_1 \qquad \boldsymbol{Z}_2 = R_2 + jX_2 \qquad \boldsymbol{Z}_L = R + jX_L - jX_C$

$\boldsymbol{Z}_M = jX_M$ then

$$(\boldsymbol{Z}_s + \boldsymbol{Z}_1)\boldsymbol{I}_1 + \boldsymbol{Z}_M \boldsymbol{I}_2 = \boldsymbol{V} \qquad \boldsymbol{Z}_M \boldsymbol{I}_1 + (\boldsymbol{Z}_2 + \boldsymbol{Z}_L)\boldsymbol{I}_2 = 0$$

$$\boldsymbol{I}_2 = -\frac{\boldsymbol{Z}_M}{(\boldsymbol{Z}_2 + \boldsymbol{Z}_L)}\boldsymbol{I}_1 \qquad (\boldsymbol{Z}_s + \boldsymbol{Z}_1)\boldsymbol{I}_1 - \frac{\boldsymbol{Z}_M^2}{(\boldsymbol{Z}_2 + \boldsymbol{Z}_L)}\boldsymbol{I}_1 = \boldsymbol{V}$$

$$\boldsymbol{I}_1 = \frac{(\boldsymbol{Z}_2 + \boldsymbol{Z}_L)\boldsymbol{V}}{(\boldsymbol{Z}_s + \boldsymbol{Z}_1)(\boldsymbol{Z}_2 + \boldsymbol{Z}_L) - \boldsymbol{Z}_M^2} \qquad \boldsymbol{I}_2 = -\frac{\boldsymbol{Z}_M \boldsymbol{V}}{(\boldsymbol{Z}_s + \boldsymbol{Z}_1)(\boldsymbol{Z}_2 + \boldsymbol{Z}_L) - \boldsymbol{Z}_M^2}$$

Equivalent Impedance: $$\boldsymbol{Z}_{eq} = \boldsymbol{Z}_i = \frac{\boldsymbol{V}}{\boldsymbol{I}_1} = \frac{(\boldsymbol{Z}_s + \boldsymbol{Z}_1)(\boldsymbol{Z}_2 + \boldsymbol{Z}_L) - \boldsymbol{Z}_M^2}{(\boldsymbol{Z}_2 + \boldsymbol{Z}_L)}$$

$$\boldsymbol{Z}_{eq} = \boldsymbol{Z}_i = (\boldsymbol{Z}_s + \boldsymbol{Z}_1) - \frac{\boldsymbol{Z}_M^2}{(\boldsymbol{Z}_2 + \boldsymbol{Z}_L)}$$

Alternative Way:

$$(\mathbf{Z}_s + \mathbf{Z}_1)\mathbf{I}_1 + \mathbf{Z}_M \mathbf{I}_2 = \mathbf{V} \qquad \mathbf{Z}_M \mathbf{I}_1 + (\mathbf{Z}_2 + \mathbf{Z}_L)\mathbf{I}_2 = 0$$

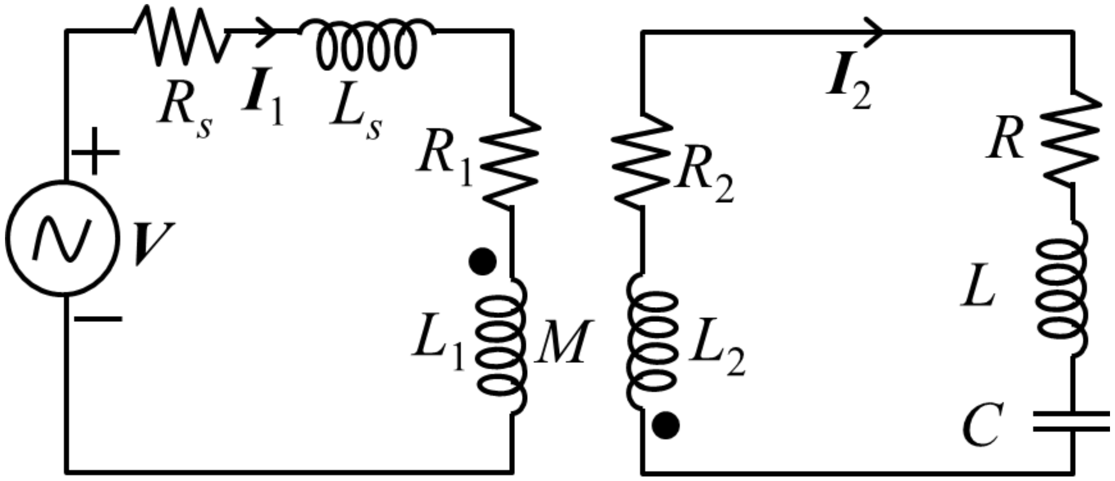
$$\begin{bmatrix} \mathbf{Z}_s + \mathbf{Z}_1 & \mathbf{Z}_M \\ \mathbf{Z}_M & \mathbf{Z}_2 + \mathbf{Z}_L \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{V} \\ 0 \end{bmatrix}$$

$$D = \begin{vmatrix} \mathbf{Z}_s + \mathbf{Z}_1 & \mathbf{Z}_M \\ \mathbf{Z}_M & \mathbf{Z}_2 + \mathbf{Z}_L \end{vmatrix} = (\mathbf{Z}_s + \mathbf{Z}_1)(\mathbf{Z}_2 + \mathbf{Z}_L) - \mathbf{Z}_M^2$$

$$D_1 = \begin{vmatrix} \mathbf{V} & \mathbf{Z}_M \\ 0 & \mathbf{Z}_2 + \mathbf{Z}_L \end{vmatrix} = (\mathbf{Z}_2 + \mathbf{Z}_L)\mathbf{V} \qquad D_2 = \begin{vmatrix} \mathbf{Z}_s + \mathbf{Z}_1 & \mathbf{V} \\ \mathbf{Z}_M & 0 \end{vmatrix} = -\mathbf{Z}_M \mathbf{V}$$

$$\mathbf{I}_1 = \frac{D_1}{D} = \frac{(\mathbf{Z}_2 + \mathbf{Z}_L)\mathbf{V}}{(\mathbf{Z}_s + \mathbf{Z}_1)(\mathbf{Z}_2 + \mathbf{Z}_L) - \mathbf{Z}_M^2} \qquad \mathbf{I}_2 = \frac{D_2}{D} = \frac{-\mathbf{Z}_M \mathbf{V}}{(\mathbf{Z}_s + \mathbf{Z}_1)(\mathbf{Z}_2 + \mathbf{Z}_L) - \mathbf{Z}_M^2}$$

Example 12.7: A 100∠0° V, 314 rad/s supply voltage is applied to the primary circuit of an air-core transformer as shown in following figure with the parameters of: R_s= R_s= 2 ohms, R_2 = R = 3 ohms, L_s= 0.0032 H, L_1= 0.0064 H, L_2 = 0.0127 H, M = 0.0048 H, L= 0.0255 H and C=0.08 μF. Find (i) the primary current, $\boldsymbol{I}_1$, and the secondary current $\boldsymbol{I}_2$, and (ii) the equivalent impedance, and (iii) the consumed power.



$$X_S = 314 \times 0.0032 = 1.0\Omega$$
$$X_1 = 314 \times 0.0064 = 2.0\Omega$$
$$X_2 = 314 \times 0.0127 = 4.0\Omega$$
$$X_M = 314 \times 0.0048 = 1.5\Omega$$
$$X_L = 314 \times 0.0255 = 8.0\Omega$$
$$X_C = \frac{1}{314 \times 0.08 \times 10^{-6}} = 4.0\Omega$$

$$\boldsymbol{Z}_S = 2 + j1$$
$$\boldsymbol{Z}_1 = 2 + j2$$
$$\boldsymbol{Z}_2 = 3 + j4$$
$$\boldsymbol{Z}_M = j1.5$$
$$\boldsymbol{Z}_L = 3 + j8 - j4 = 3 + j4$$

$$(\boldsymbol{Z}_S + \boldsymbol{Z}_1)\boldsymbol{I}_1 + \boldsymbol{Z}_M \boldsymbol{I}_2 = \boldsymbol{V} \qquad \boldsymbol{Z}_M \boldsymbol{I}_1 + (\boldsymbol{Z}_2 + \boldsymbol{Z}_L)\boldsymbol{I}_2 = 0$$

$$\boldsymbol{Z}_S + \boldsymbol{Z}_1 = 4 + j3$$
$$\boldsymbol{Z}_2 + \boldsymbol{Z}_L = 6 + j8$$

$$\begin{bmatrix} \boldsymbol{Z}_S + \boldsymbol{Z}_1 & \boldsymbol{Z}_M \\ \boldsymbol{Z}_M & \boldsymbol{Z}_2 + \boldsymbol{Z}_L \end{bmatrix} \begin{bmatrix} \boldsymbol{I}_1 \\ \boldsymbol{I}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{V} \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 4 + j3 & j1.5 \\ j1.5 & 6 + j8 \end{bmatrix} \begin{bmatrix} \boldsymbol{I}_1 \\ \boldsymbol{I}_2 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$$

$$D=\begin{vmatrix}4+j3 & j1.5\\ j1.5 & 6+j8\end{vmatrix}=2.25+j50 \qquad D_1=\begin{vmatrix}100 & j1.5\\ 0 & 6+j8\end{vmatrix}=600+j800$$

$$D_2=\begin{vmatrix}4+j3 & 100\\ j1.5 & 0\end{vmatrix}=-j150$$

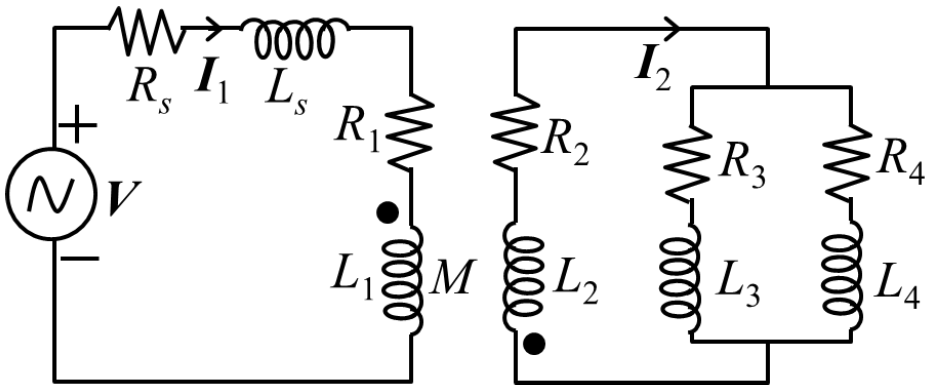
$$\boldsymbol{I}_1=\frac{D_1}{D}=\frac{600+j800}{2.25-j50}=16.51-j11.26=19.98\angle-34.3^\circ \ \text{ A}$$

$$\boldsymbol{I}_2=\frac{D_2}{D}=\frac{-j150}{2.25-j50}=-2.99-j0.135=2.997\angle-177.42^\circ \quad \text{A}$$

$$\boldsymbol{Z}_{eq}=\boldsymbol{Z}_i=\frac{\boldsymbol{V}}{\boldsymbol{I}_1}=\frac{100}{16.51-j11.26}=4.14+j2.82 \ \Omega$$

$$P=VI_1\cos(34.3^\circ)=100\times19.98\cos(34.3^\circ)=1650.7 \ \text{ W}$$

Example 12.8: A 100∠0° V, 314 rad/s supply voltage is applied to the primary circuit of an air-core transformer as shown in following figure with the parameters of: $R_s = R_s = 2$ ohms, $R_2 = 3$ ohms, $R_3 = R_4 = 6$ ohms, $L_s = 0.0032$ H, $L_1 = 0.0064$ H, $L_2 = 0.0127$ H, $M = 0.0048$ H, and $L_3 = L_4 = 0.0255$ H. Find (i) the primary current, $\boldsymbol{I}_1$, and the secondary current $\boldsymbol{I}_2$, and (ii) the equivalent impedance, and (iii) the consumed power.



$$X_S = 314 \times 0.0032 = 1.0\,\Omega$$
$$X_1 = 314 \times 0.0064 = 2.0\,\Omega$$
$$X_2 = 314 \times 0.0127 = 4.0\,\Omega$$
$$X_M = 314 \times 0.0048 = 1.5\,\Omega$$
$$X_{L3} = X_{L4} = 314 \times 0.0255 = 8.0\,\Omega$$
$$\boldsymbol{Z}_S = 2 + j1$$

$$\boldsymbol{Z}_1 = 2 + j2$$
$$\boldsymbol{Z}_2 = 3 + j4$$
$$\boldsymbol{Z}_M = j1.5$$
$$\boldsymbol{Z}_3 = 6 + j8$$
$$\boldsymbol{Z}_4 = 6 + j8$$

$$\boldsymbol{Z}_L = \frac{\boldsymbol{Z}_3\boldsymbol{Z}_4}{\boldsymbol{Z}_3 + \boldsymbol{Z}_4} = \frac{6 + j8}{2} = 3 + j4$$

$$(\boldsymbol{Z}_S + \boldsymbol{Z}_1)\boldsymbol{I}_1 + \boldsymbol{Z}_M\boldsymbol{I}_2 = \boldsymbol{V} \qquad \boldsymbol{Z}_M\boldsymbol{I}_1 + (\boldsymbol{Z}_2 + \boldsymbol{Z}_L)\boldsymbol{I}_2 = 0$$

$$\boldsymbol{Z}_S + \boldsymbol{Z}_1 = 4 + j3$$
$$\boldsymbol{Z}_2 + \boldsymbol{Z}_L = 6 + j8$$

$$\begin{bmatrix} \boldsymbol{Z}_S + \boldsymbol{Z}_1 & \boldsymbol{Z}_M \\ \boldsymbol{Z}_M & \boldsymbol{Z}_2 + \boldsymbol{Z}_L \end{bmatrix} \begin{bmatrix} \boldsymbol{I}_1 \\ \boldsymbol{I}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{V} \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 4 + j3 & j1.5 \\ j1.5 & 6 + j8 \end{bmatrix} \begin{bmatrix} \boldsymbol{I}_1 \\ \boldsymbol{I}_2 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$$

$$D = \begin{vmatrix} 4+j3 & j1.5 \\ j1.5 & 6+j8 \end{vmatrix} = 2.25 + j50 \qquad D_1 = \begin{vmatrix} 100 & j1.5 \\ 0 & 6+j8 \end{vmatrix} = 600 + j800$$

$$D_2 = \begin{vmatrix} 4+j3 & 100 \\ j1.5 & 0 \end{vmatrix} = -j150$$

$$\boldsymbol{I}_1 = \frac{D_1}{D} = \frac{600 + j800}{2.25 - j50} = 16.51 - j11.26 = 19.98\angle -34.3^\circ \ \ \text{A}$$

$$\boldsymbol{I}_2 = \frac{D_2}{D} = \frac{-j150}{2.25 - j50} = -2.99 - j0.135 = 2.997\angle -177.42^\circ \quad \text{A}$$

$$\boldsymbol{Z}_{eq} = \boldsymbol{Z}_i = \frac{\boldsymbol{V}}{\boldsymbol{I}_1} = \frac{100}{16.51 - j11.26} = 4.14 + j2.82 \ \ \Omega$$

$$P = VI_1 \cos(34.3^\circ) = 100 \times 19.98 \cos(34.3^\circ) = 1650.7 \ \ \text{W}$$

Example 12.9: For the following circuit write the loop equations and calculate (i) the primary current, $\boldsymbol{I}_1$, and the secondary current $\boldsymbol{I}_2$, and (ii) the equivalent impedance. and (iii) the consumed power.

Here, M is negative.

Loop equations: $(8+j6)\boldsymbol{I}_1 - j10\boldsymbol{I}_2 = 100 \quad (i)$ $\qquad -j10\boldsymbol{I}_1 + (8+j8)\boldsymbol{I}_2 = 0 \quad (ii)$

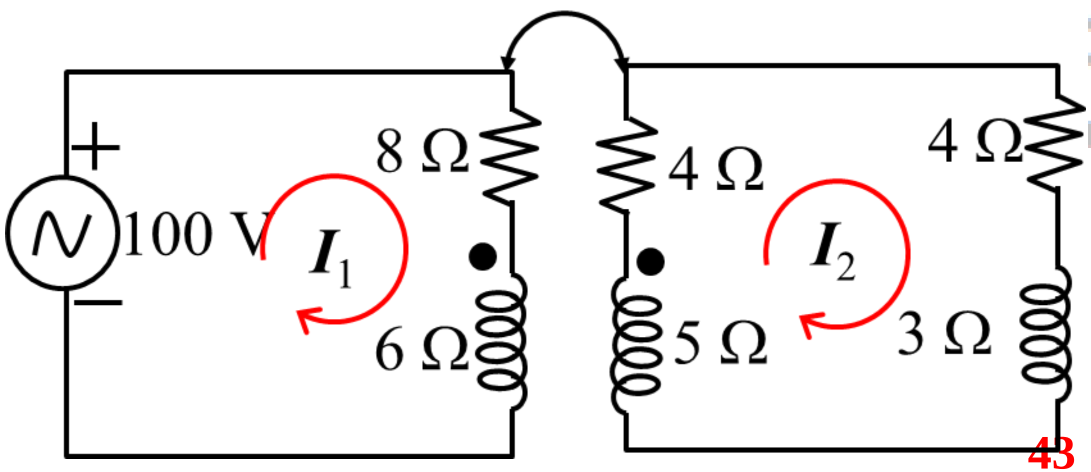
From Eq. (ii): $\boldsymbol{I}_2 = \dfrac{j10}{8+j8}\boldsymbol{I}_1 = \dfrac{j10}{8+j8}\boldsymbol{I}_1 = (0.625+j0.625)\boldsymbol{I}_1 \quad (iii)$

$(8+j6)\boldsymbol{I}_1 - j10(0.625+j0.625)\boldsymbol{I}_1 = 100$ $\qquad (14.25-j0.25)\boldsymbol{I}_1 = 100$

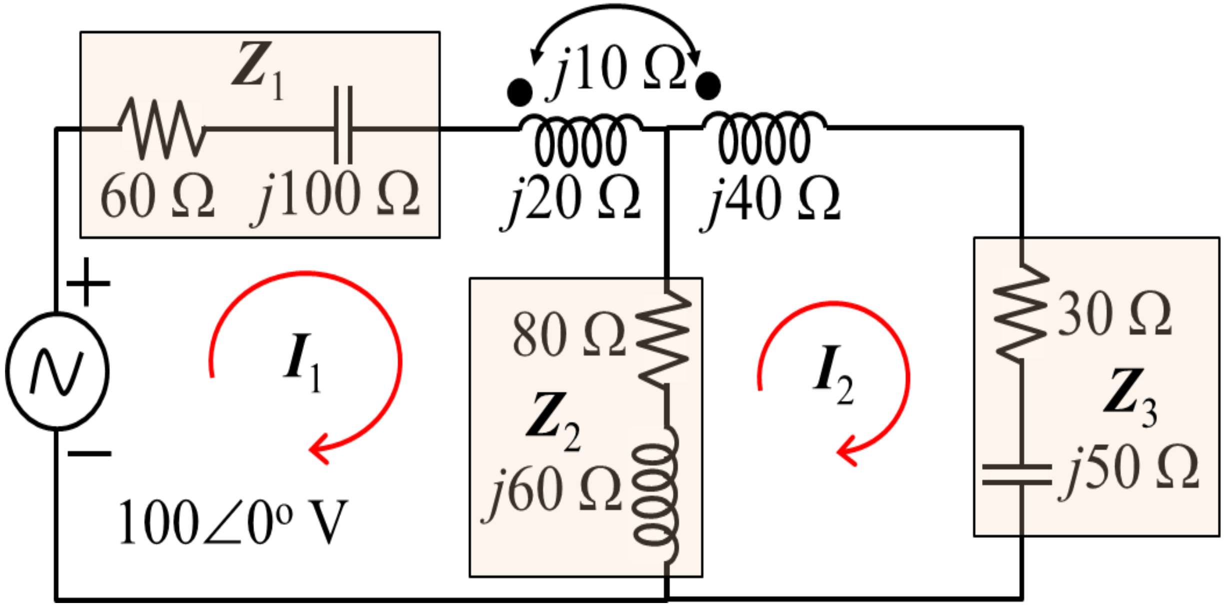
$$\boldsymbol{I}_1 = \frac{100}{14.25-j0.25} = 7.0+j0.123 = 7.0\angle 1.0°$$

$$\boldsymbol{I}_2 = \frac{j10}{8+j8}(7.0+j0.123) = 4.3+j4.46 = 6.2\angle 46°$$

$$P = 100\times 7\cos(1.0°) = 701.53 \text{ W}$$



Example 12.10: For the following circuit write the loop equations and calculate (i) the currents, $\boldsymbol{I}_1$, and $\boldsymbol{I}_2$, and (ii) the equivalent impedance. and (iii) the consumed power.



Let, $\boldsymbol{Z}_{L1} = j20\ \Omega$

$\boldsymbol{Z}_{L2} = j40\ \Omega \qquad \boldsymbol{Z}_M = j10\ \Omega$

$\boldsymbol{Z}_1 = 60 - j100\ \Omega \qquad \boldsymbol{Z}_2 = 80 + j60\ \Omega$

$\boldsymbol{Z}_3 = 30 - j50\ \Omega \qquad \boldsymbol{V}_S = 100\angle 0° = 100\ \text{V}$

$$(\boldsymbol{Z}_1 + \boldsymbol{Z}_{L1} + \boldsymbol{Z}_2)\boldsymbol{I}_1 + (\boldsymbol{Z}_M - \boldsymbol{Z}_2)\boldsymbol{I}_2 = \boldsymbol{V}_s$$

$$(\boldsymbol{Z}_M - \boldsymbol{Z}_2)\boldsymbol{I}_1 + (\boldsymbol{Z}_2 + \boldsymbol{Z}_{L2} + \boldsymbol{Z}_3)\boldsymbol{I}_2 = 0$$

$$\boldsymbol{Z}_1 + \boldsymbol{Z}_{L1} + \boldsymbol{Z}_2 = 60 - j100 + j20 + 80 + j60 = 140 - j20\ \Omega$$

$$\boldsymbol{Z}_2 + \boldsymbol{Z}_{L2} + \boldsymbol{Z}_3 = 80 + j60 + j40 + 30 - j50 = 110 + j50\ \Omega$$

$$\boldsymbol{Z}_M - \boldsymbol{Z}_2 = j10 - 80 - j60 = -80 - j50\ \Omega$$

$$(140 - j20)\boldsymbol{I}_1 + (-80 - j50)\boldsymbol{I}_2 = 100$$

$$(-80 - j50)\boldsymbol{I}_1 + (110 + j50)\boldsymbol{I}_2 = 0$$

$$D = \begin{vmatrix} 140 - j20 & -80 - j50 \\ -80 - j50 & 110 + j50 \end{vmatrix} = 12500 - j3200 \qquad D_1 = \begin{vmatrix} 100 & -80 - j50 \\ 0 & 110 + j50 \end{vmatrix} = 11000 + j5000$$

$$D_2 = \begin{vmatrix} 140 - j20 & 100 \\ -80 - j50 & 0 \end{vmatrix} = 8000 + j5000$$

$$\boldsymbol{I}_1 = \frac{D_1}{D} = \frac{11000 + j5000}{12500 - j3200} = 0.9364\angle -38.8^\circ \ \text{A}$$

$$\boldsymbol{I}_2 = \frac{D_2}{D} = \frac{8000 + j5000}{12500 - j3200} = 0.73\angle 46.36^\circ \quad \text{A}$$

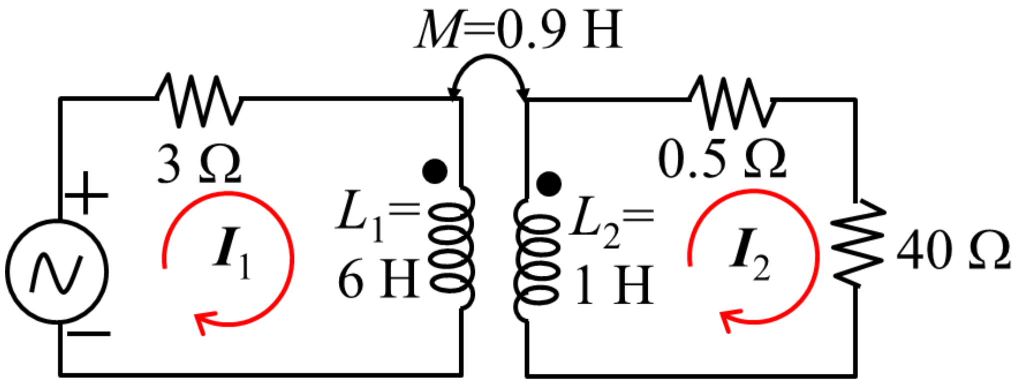
$$\boldsymbol{Z}_{eq} = \boldsymbol{Z}_i = \frac{\boldsymbol{V}}{\boldsymbol{I}_1} = \frac{100}{0.9364\angle -38.8^\circ} = 83.22 - j67 \ \Omega$$

$$P = 100 \times 0.9364 \cos(38.8^\circ) = 73 \ \text{W}$$

Example 12. 11: For the following circuit write the loop equations.

$$X_{L1} = 400 \times 6 = 2400\ \Omega \qquad X_{L2} = 400 \times 1 = 400\ \Omega$$

$$X_M = 400 \times 0.8 = 360\ \Omega$$



$$(3 + j2400)\boldsymbol{I}_1 - j360\boldsymbol{I}_2 = 100$$

$$-j360\boldsymbol{I}_1 + (40.5 + j400)\boldsymbol{I}_2 = 0$$

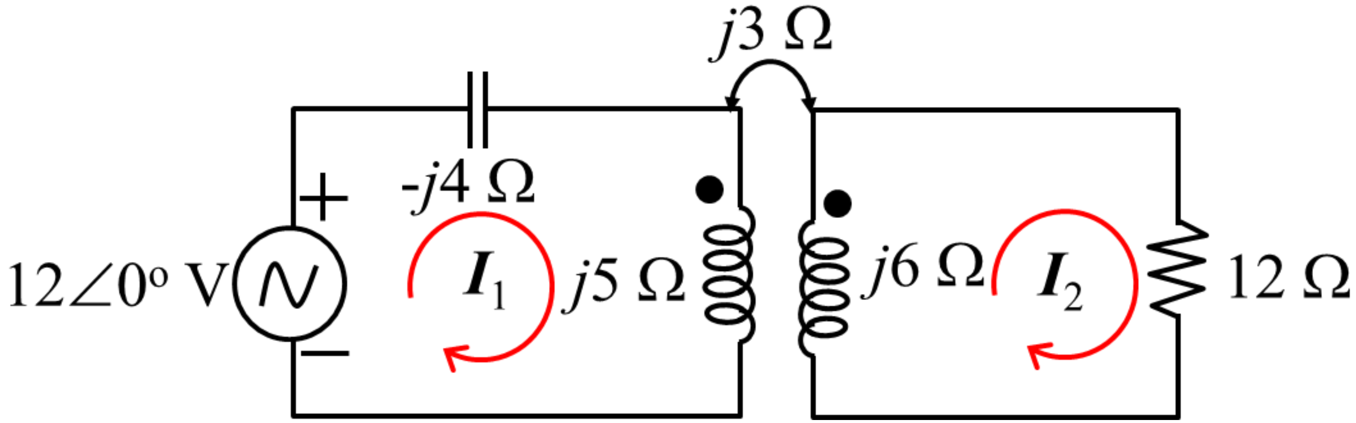
$$\boldsymbol{I}_2 = \frac{j360}{(40.5 + j400)}\boldsymbol{I}_1 \qquad \left[(3 + j2400) - \frac{(j360)(j360)}{(40.5 + j400)}\right]\boldsymbol{I}_1 = 100$$

$$[35.47 + j2079.3]\boldsymbol{I}_1 = 100 \qquad \boldsymbol{I}_1 = \frac{100}{35.47 + j2079.3} = 0.0008 - j0.0481$$

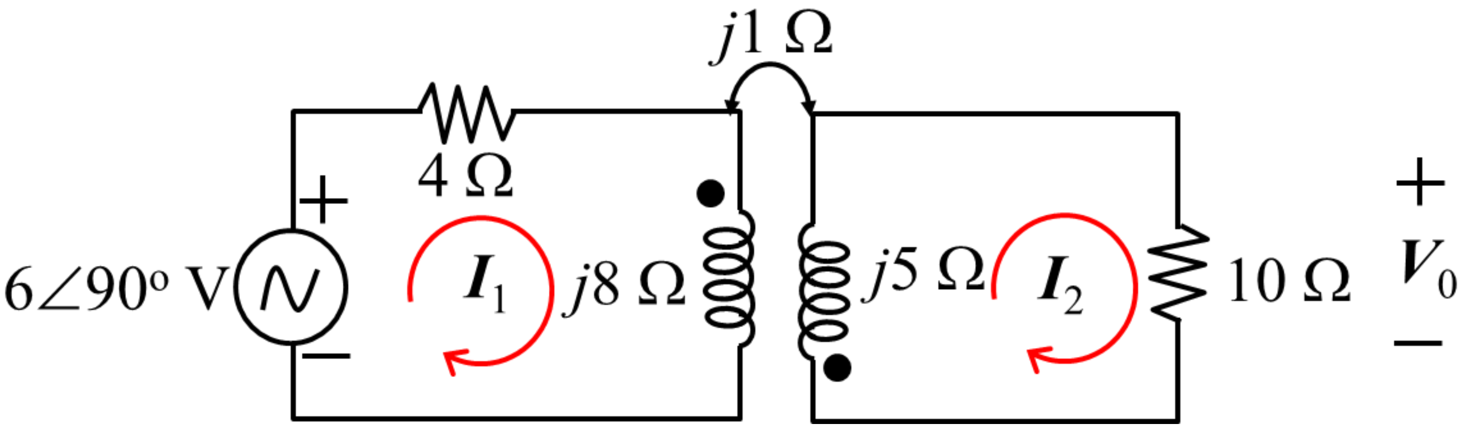
$$\boldsymbol{Z}_{eq} = \frac{V}{\boldsymbol{I}_1} = 35.47 + j2079.3 \qquad \boldsymbol{I}_2 = \frac{j360}{(40.5 + j400)}(0.0008 - j0.0481) = 0.0051 - j0.0428$$

Homework 12.3

Problem 12.3.1: For the following circuits write the loop equations and calculate (i) the currents, $\boldsymbol{I}_1$, and $\boldsymbol{I}_2$, and (ii) the equivalent impedance. and (iii) the consumed power.



Problem 12.3.2: For the following circuits write the loop equations and calculate (i) the currents, $\boldsymbol{I}_1$, $\boldsymbol{I}_2$, and $\boldsymbol{V}_0$, and (ii) the equivalent impedance. and (iii) the consumed power.



Problem 12.3.3: For the following circuits write the loop equations and calculate (i) the currents, $\boldsymbol{I}_1$, and $\boldsymbol{I}_2$, and (ii) the equivalent impedance. and (iii) the consumed power.