Balanced Poly-Phase System

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Poly-phase or Multi-phase Systems

Definition of Poly-phase System:
Circuits or systems in which AC sources operate at the same frequency but different phases are known as ***polyphase***.

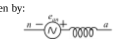
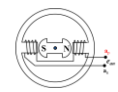
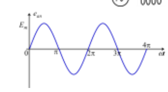
Poly means **many** and **phases** mean **windings** so that a polyphase generator has many phases or windings.

Definition of Poly-Phase Alternator:
A **polyphase alternator** (synchronous generator) has two or more separate but identical windings (called **phases**) displaced from each other by equal electrical angle and acted upon by the common uniform magnetic field.
Each winding or phase produces a single alternating voltage of the same magnitude and frequency.

Single-Phase System

The following figure shows a ***single phase system***. It has one winding or coil $\boldsymbol{a_p a_n}$ rotating in anticlockwise or counter-clock wise direction with and angular velocity ω in the 2-pole field. Consider the induced emf in the coil is e_{an}.
The equation of the emf induced in the coil is given by:

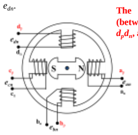
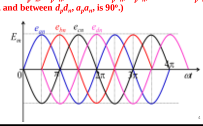
$e_{an} = E_m \sin \omega t$ $\quad$ $\mathbf{E}_{an} = E_m \angle 0°$

Four-Phase System

The following figure shows a ***four phase system***. It has four windings or coils $\boldsymbol{a_p a_n}$, $\boldsymbol{b_p b_n}$, $\boldsymbol{c_p c_n}$, and $\boldsymbol{d_p d_n}$, rotating in anticlockwise or counter-clock wise direction with and angular velocity ω in the 2-pole field. Consider the induced emfs in these coils are e_{an}, e_{bn}, e_{cn}, and e_{dn}.

The phase difference between two adjacent phases (between $a_p a_n$, $b_p b_n$, between $b_p b_n$, $c_p c_n$, between $c_p c_n$, $d_p d_n$, and between $d_p d_n$, $a_p a_n$, is 90°.)

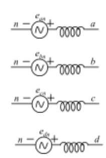
Four-Phase System Continue…….

The equations of the emfs induced in these coils are given by:

$$e_{an} = E_m \sin \omega t$$
$$e_{bn} = E_m \sin(\omega t - 90°)$$
$$e_{cn} = E_m \sin(\omega t - 180°) = -E_m \sin \omega t = e_{an}$$
$$e_{dn} = E_m \sin(\omega t - 270°) = -E_m(\omega t - 90°) = e_{bn}$$

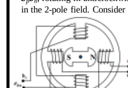
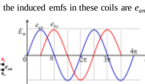
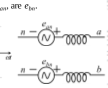
The equations of the emfs in phasor form are given by:

$$\mathbf{E}_{an} = E_m \angle 0°$$
$$\mathbf{E}_{bn} = E_m \angle -90°$$
$$\mathbf{E}_{cn} = E_m \angle -180° = -E_m \angle 0° = \mathbf{E}_{an}$$
$$\mathbf{E}_{dn} = E_m \angle -270° = -E_m \angle -90° = \mathbf{E}_{bn}$$



Two-Phase System

The following figure shows a ***two phase system***. It has two windings or coils $\boldsymbol{a_p a_n}$, and $\boldsymbol{b_p b_n}$, rotating in anticlockwise or counter-clock wise direction with and angular velocity ω in the 2-pole field. Consider the induced emfs in these coils are e_{an}, are e_{bn}.

The phase difference between two adjacent phases (between $a_p a_n$, $b_p b_n$, and between $b_p b_n$, $a_p a_n$, is 90°.)

The equations of the emfs induced in these coils are given by:

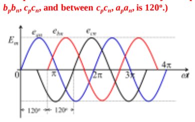
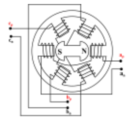
$e_{an} = E_m \sin \omega t$ $\quad$ $\mathbf{E}_{an} = E_m \angle 0°$

$e_{bn} = E_m \sin(\omega t - 90°)$ $\quad$ $\mathbf{E}_{bn} = E_m \angle -90°$

Three-Phase System

The following figure shows a ***three phase system***. It has three windings or coils $\boldsymbol{a_p a_n}$, $\boldsymbol{b_p b_n}$, and $\boldsymbol{c_p c_n}$, rotating in anticlockwise or counter-clock wise direction with and angular velocity ω in the 2-pole field. Consider the induced emfs in these coils are e_{an}, e_{bn}, and e_{cn}.

The phase difference between two adjacent phases (between $a_p a_n$, $b_p b_n$, between $b_p b_n$, $c_p c_n$, and between $c_p c_n$, $a_p a_n$, is 120°.)

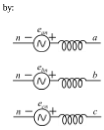
Three-Phase System Continue…….

The equations of the emfs induced in these coils are given by:

$$e_{an} = E_m \sin \omega t$$
$$e_{bn} = E_m \sin(\omega t - 120°)$$
$$e_{cn} = E_m \sin(\omega t - 240°) = E_m \sin(\omega t + 120°)$$

The equations of the emfs in phasor form are given by:

$$\mathbf{E}_{an} = E_m \angle 0°$$
$$\mathbf{E}_{bn} = E_m \angle -120°$$
$$\mathbf{E}_{cn} = E_m \angle -240° = E_m \angle 120°$$



Phase Displacement Between Two Adjacent Phases or Windings

The electrical displacement between the windings is determined by the number of phases or windings.
For **other polyphase** systems (e.g. three-phase, four-phase, six-phase) the electrical displacement between different phases or windings can be calculated by:

$\boxed{\theta_p = \frac{360°}{n}}$ $\quad$ Where, n is the number of phases.

For two phase n=2 but $\quad \theta_p = 90°$ $\quad$ This is an exceptional case.

For three phase n=3 thus

$$\theta_p = \frac{360°}{n} = \frac{360°}{3} = 120°$$

For four phase n=4 thus

$$\theta_p = \frac{360°}{n} = \frac{360°}{4} = 90°$$

For five phase n=5 thus

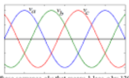
$$\theta_p = \frac{360°}{n} = \frac{360°}{5} = 72°$$

For six phase n=6 thus

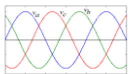
$$\theta_p = \frac{360°}{n} = \frac{360°}{6} = 60°$$

Phase Order or Phase Sequence

The order or sequence in which the voltages in three phases reach their maximum positive values is called **phase order** or **phase sequence**.
The phase sequence is important in determining direction of rotation of ac motors, parallel operation of alternators etc.



Phase sequence *abc* that means *b* lags *a* by 120º and *c* lags *b* by 120º.

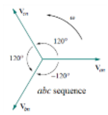


Phase sequence *acb* that means *c* lags *a* by 120º and *b* lags *c* by 120º.

Three phase system has mainly two basic phase order or phase sequence:

abc sequence same as: ***bca*** sequence or ***cab*** sequence
acb sequence same as: ***cba*** sequence or ***bac*** sequence

Phase Order or Phase Sequence Continue…….



$$e_{an} = E_m \sin \omega t$$
$$e_{bn} = E_m \sin(\omega t - 120°)$$
$$e_{cn} = E_m \sin(\omega t - 240°) = E_m \sin(\omega t + 120°)$$

$$\boldsymbol{E}_{an} = E_m \angle 0° = E_m (1 + j0)$$
$$\boldsymbol{E}_{bn} = E_m \angle -120° = E_m (-0.5 - j0.866)$$
$$\boldsymbol{E}_{cn} = E_m \angle -240° = E_m \angle 120° = E_m (-0.5 + j0.866)$$

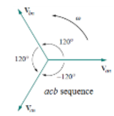
***abc* sequence**: the emf of phase *b* lags that of *a* by 120º. Also, the emf of phase *c* lags that of phase *b* by 120º that means the emf of phase *c* lags that of phase *a* by 240º.

$\boldsymbol{E}_{an} = E_m \angle 0°$ $\quad \boldsymbol{E}_{bn} = E_m \angle -120°$ $\quad \boldsymbol{E}_{cn} = E_m \angle 120°$

Phase Order or Phase Sequence Continue…….

$$e_{an} = E_m \sin \omega t$$
$$e_{cn} = E_m \sin(\omega t - 120°)$$
$$e_{bn} = E_m \sin(\omega t - 240°) = E_m \sin(\omega t + 120°)$$

$$\boldsymbol{E}_{an} = E_m \angle 0° = E_m (1 - j0)$$
$$\boldsymbol{E}_{cn} = E_m \angle -120° = E_m (-0.5 - j0.866)$$
$$\boldsymbol{E}_{bn} = E_m \angle -240° = E_m \angle 120° = E_m (-0.5 + j0.866)$$



***acb* sequence**: the emf of phase *c* lags that of *a* by 120º. Also, the emf of phase *b* lags that of phase *c* by 120º that means the emf of phase *b* lags that of phase *a* by 240º.

$\boldsymbol{E}_{an} = E_m \angle 0°$ $\quad \boldsymbol{E}_{bn} = E_m \angle 120°$ $\quad \boldsymbol{E}_{cn} = E_m \angle -120°$

Example 9.1

Determine the phase sequence of the set of voltages: v_{an}=√2×200sin(ωt+10º); v_{bn}=√2×200sin(ωt-230º); and v_{cn}=√2×200sin(ωt-110º)

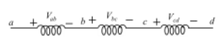
Solution: The voltages can be expressed in phasor form as:

$\boldsymbol{V}_{an} = 200\angle 10°$ V $\quad \boldsymbol{V}_{bn} = 200\angle -230°$ V $\quad \boldsymbol{V}_{cn} = 200\angle -110°$ V

We note that $\boldsymbol{V}_{cn}$ lags $\boldsymbol{V}_{an}$ by 120º and $\boldsymbol{V}_{bn}$ lags $\boldsymbol{V}_{cn}$ by 120º, hence we have *acb* sequence.

Double-Subscript Notation of Voltage

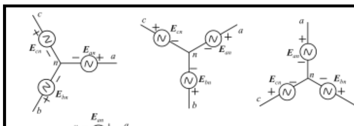
When a voltage is represented by a double subscripts (such as V_{ab}) its first letter (such as *a*) represents the **positive terminal** and second letter (such as *b*) represents the **negative terminal**.

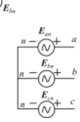


$$V_{ac} = V_{ab} + V_{bc} = V_{ab} - V_{cb}$$

$$V_{bd} = V_{bc} + V_{cd} = V_{bc} - V_{dc}$$

$$V_{ad} = V_{ab} + V_{bc} + V_{cd} = V_{ab} - V_{cb} - V_{dc}$$





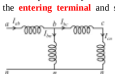
$$\boldsymbol{E}_{ab} = \boldsymbol{E}_{an} + \boldsymbol{E}_{nb} = \boldsymbol{E}_{an} - \boldsymbol{E}_{bn}$$

$$\boldsymbol{E}_{bc} = \boldsymbol{E}_{bn} + \boldsymbol{E}_{nc} = \boldsymbol{E}_{bn} - \boldsymbol{E}_{cn}$$

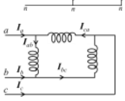
$$\boldsymbol{E}_{ca} = \boldsymbol{E}_{cn} + \boldsymbol{E}_{na} = \boldsymbol{E}_{cn} - \boldsymbol{E}_{an}$$

Double-Subscript Notation of Current

When a current is represented by a double subscripts (such as I_{ab}) its first letter (such as *a*) represents the **entering terminal** and second letter (such as *b*) represents the **leaving terminal**.



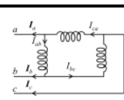
$$I_{ab} = I_{bn} + I_{bc} = I_{bn} - I_{cb}$$
$$I_{bc} = I_{cn} = -I_{nc}$$

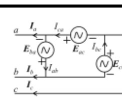


$\boldsymbol{I}_a + \boldsymbol{I}_{ca} = \boldsymbol{I}_{ab}$ $\quad \boldsymbol{I}_a = \boldsymbol{I}_{ab} - \boldsymbol{I}_{ca}$

$\boldsymbol{I}_b + \boldsymbol{I}_{ab} = \boldsymbol{I}_{bc}$ $\quad \boldsymbol{I}_b = \boldsymbol{I}_{bc} - \boldsymbol{I}_{ab}$

$\boldsymbol{I}_c + \boldsymbol{I}_{bc} = \boldsymbol{I}_{ca}$ $\quad \boldsymbol{I}_c = \boldsymbol{I}_{ca} - \boldsymbol{I}_{bc}$





$\boldsymbol{I}_a + \boldsymbol{I}_{ab} = \boldsymbol{I}_{ca}$ $\quad \boldsymbol{I}_a = \boldsymbol{I}_{ca} - \boldsymbol{I}_{ab}$

$\boldsymbol{I}_b + \boldsymbol{I}_{bc} = \boldsymbol{I}_{ab}$ $\quad \boldsymbol{I}_b = \boldsymbol{I}_{ab} - \boldsymbol{I}_{bc}$

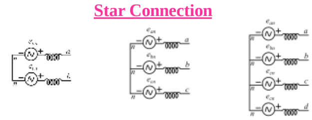
$\boldsymbol{I}_c + \boldsymbol{I}_{ca} = \boldsymbol{I}_{bc}$ $\quad \boldsymbol{I}_c = \boldsymbol{I}_{bc} - \boldsymbol{I}_{ca}$

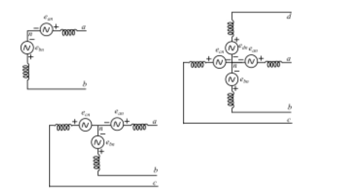
Connections of Poly-Phase

Connection of poly-phase systems are two types:

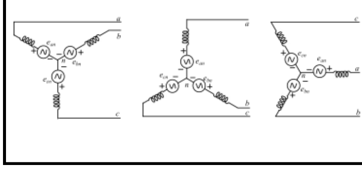
1. **Star Connection**
2. **Mesh Connection**

Star Connection

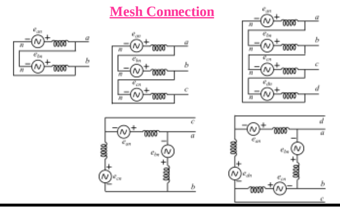




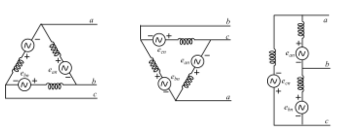
The three-phase star connection looks like a Y that's why three-phase star connection is also called wye-connection or Y-connection.



Mesh Connection



The three-phase mesh connection looks like a delta (Δ) that's why three-phase star connection is also called delta-connection or Δ-connection.



Line Voltage of Star Connection

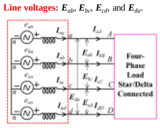
Let, a four-phase star connected source is connected with a load.

Phase voltages: $\boldsymbol{E}_{an}$, $\boldsymbol{E}_{bn}$, $\boldsymbol{E}_{cn}$, and $\boldsymbol{E}_{dn}$.

Phase currents: $\boldsymbol{I}_{na}$, $\boldsymbol{I}_{nb}$, $\boldsymbol{I}_{nc}$, and $\boldsymbol{I}_{nd}$.

Line voltages: $\boldsymbol{E}_{ab}$, $\boldsymbol{E}_{bc}$, $\boldsymbol{E}_{cd}$, and $\boldsymbol{E}_{da}$.

Line currents: $\boldsymbol{I}_{aA}$, $\boldsymbol{I}_{bB}$, $\boldsymbol{I}_{cC}$, and $\boldsymbol{I}_{dD}$.



$$\boldsymbol{I}_{aA} = \boldsymbol{I}_{na} \qquad \boldsymbol{I}_{bB} = \boldsymbol{I}_{nb}$$
$$\boldsymbol{I}_{cC} = \boldsymbol{I}_{nc} \qquad \boldsymbol{I}_{dD} = \boldsymbol{I}_{nd}$$

The phase current and line current magnitudes of star connection are equal.

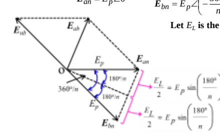
$$\boldsymbol{E}_{ab} = \boldsymbol{E}_{an} + \boldsymbol{E}_{nb} = \boldsymbol{E}_{an} - \boldsymbol{E}_{bn}$$
$$\boldsymbol{E}_{bc} = \boldsymbol{E}_{bn} + \boldsymbol{E}_{nc} = \boldsymbol{E}_{bn} - \boldsymbol{E}_{cn}$$
$$\boldsymbol{E}_{cd} = \boldsymbol{E}_{cn} + \boldsymbol{E}_{nd} = \boldsymbol{E}_{cn} - \boldsymbol{E}_{dn}$$
$$\boldsymbol{E}_{da} = \boldsymbol{E}_{dn} + \boldsymbol{E}_{na} = \boldsymbol{E}_{dn} - \boldsymbol{E}_{an}$$

The phase voltage and line voltage magnitudes of star connection are not equal.

$\boxed{I_L = I_p}$

Consider n-phase system where $\boldsymbol{E}_{an}$ and $\boldsymbol{E}_{bn}$ are phase voltage. Thus,

$$\boldsymbol{E}_{an} = E_p\angle 0^\circ \qquad \boldsymbol{E}_{bn} = E_p\angle\left(-\frac{360^\circ}{n}\right) \qquad \boldsymbol{E}_{ab} = \boldsymbol{E}_{an} + \boldsymbol{E}_{nb} = \boldsymbol{E}_{an} - \boldsymbol{E}_{bn}$$



Let E_L is the magnitude of line voltage thus:

$$\boxed{E_L = 2E_p \sin\left(\frac{180^\circ}{n}\right)}$$

$$\boxed{\text{For four-phase: } E_L = \sqrt{2}E_p}$$

The picture can't be displayed.

Exceptional for two-phase:

The picture can't be displayed.

Line Current of Mesh Connection

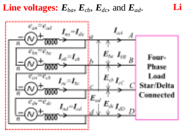
Let, a four-phase mesh connected source is connected with a load.

Phase voltages: $\boldsymbol{E}_{an}$, $\boldsymbol{E}_{bn}$, $\boldsymbol{E}_{cn}$, and $\boldsymbol{E}_{dn}$.

Phase currents: $\boldsymbol{I}_{na}$, $\boldsymbol{I}_{nb}$, $\boldsymbol{I}_{nc}$, and $\boldsymbol{I}_{nd}$.

Line voltages: $\boldsymbol{E}_{ba}$, $\boldsymbol{E}_{cb}$, $\boldsymbol{E}_{dc}$, and $\boldsymbol{E}_{ad}$.

Line currents: $\boldsymbol{I}_{aA}$, $\boldsymbol{I}_{bB}$, $\boldsymbol{I}_{cC}$, and $\boldsymbol{I}_{dD}$.



$$\boldsymbol{E}_{an} = \boldsymbol{E}_{ad} \qquad \boldsymbol{E}_{bn} = \boldsymbol{E}_{ba}$$
$$\boldsymbol{E}_{cn} = \boldsymbol{E}_{cb} \qquad \boldsymbol{E}_{dn} = \boldsymbol{E}_{dc}$$

The phase voltage and line voltage magnitudes of mesh connection are equal.

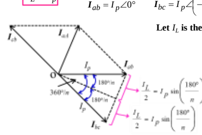
$$\boldsymbol{I}_{aA} + \boldsymbol{I}_{ab} = \boldsymbol{I}_{da}$$
$$\boldsymbol{I}_{bB} + \boldsymbol{I}_{bc} = \boldsymbol{I}_{ab}$$
$$\boldsymbol{I}_{cC} + \boldsymbol{I}_{cd} = \boldsymbol{I}_{bc}$$
$$\boldsymbol{I}_{dD} + \boldsymbol{I}_{da} = \boldsymbol{I}_{cd}$$

The phase voltage and line voltage magnitudes of star connection are not equal.

$\boxed{E_L = E_p}$

Consider n-phase system where $\boldsymbol{I}_{ab}$ and $\boldsymbol{I}_{bc}$ are phase current. Thus,

$$\boldsymbol{I}_{ab} = I_p\angle 0^\circ \qquad \boldsymbol{I}_{bc} = I_p\angle\left(-\frac{360^\circ}{n}\right) \qquad \boldsymbol{I}_{bB} = \boldsymbol{I}_{ab} - \boldsymbol{I}_{bc} = \boldsymbol{I}_{ab} + \boldsymbol{I}_{cb}$$



Let I_L is the magnitude of line current thus:

$$\boxed{I_L = 2I_p \sin\left(\frac{180^\circ}{n}\right)}$$

$$\boxed{\text{For four-phase: } I_L = \sqrt{2}I_p}$$

$$\boxed{\text{For three-phase: } I_L = \sqrt{3}I_p}$$

Exceptional for two-phase:

$$\boxed{\text{For two-phase: } I_L = 2I_p \sin\left(\frac{90^\circ}{2}\right) = \sqrt{2}I_p}$$

Example

The phase voltage and phase current of a five-phase system are 400 V and 20 A. Calculate the line voltage and line current for (***i***) star-connection, and (***ii***) mesh-connection.

Solution: Given: V_P =400 V, I_P = 20 A, n=5

(***i***) For star connection:

$$I_L = I_P = 20 \text{ A} \qquad V_L = 2V_P \sin\left(\frac{360^\circ}{5}\right) = 2\times 440\times\sin(72^\circ) = 837 \text{ V}$$

(***ii***) For mesh connection:

$$V_L = V_P = 440 \text{ V} \qquad I_L = 2I_P \sin\left(\frac{360^\circ}{5}\right) = 2\times 20\times\sin(72^\circ) = 38 \text{ A}$$

Home Work 9.1

Problem 9.1.1: Determine the phase sequence for the following sets of three-phase voltages:

(*a*) $e_{an} = 25\sin(\omega t - 50^\circ)$, $e_{bn} = 25\sin(\omega t + 70^\circ)$, $e_{cn} = 25\sin(\omega t - 170^\circ)$

(*b*) $e_{an} = 20\sin(\omega t + 40^\circ)$, $e_{bn} = 20\sin(\omega t - 80^\circ)$, $e_{cn} = 20\sin(\omega t + 160^\circ)$

Problem 9.1.2: The line voltage and line current of a five-phase system are 1500 V and 200 A. Calculate the phase voltage and phase current for (*i*) star-connection, and (*ii*) mesh-connection.

Problem 9.1.3: If the phase current is 100 A and phase voltage is 330 V for a balanced four-phase system, find the magnitude of the line voltage and the line current for (**i**) the star-connection, and (ii) the mesh-connection.

Problem 9.1.4: If the line current is 120 A and line voltage is 440 V for a balanced five-phase system, find the magnitude of the phase voltage and the phase current for (**i**) the star-connection, and (**ii**) the mesh-connection.

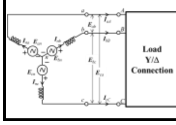
Three-Phase Star or Y Connection

Phase Voltages: E_{an}, E_{bn}, E_{cn}.

Line Voltages: E_{ab}, E_{bc}, E_{ca}.

Phase Currents: I_{na}, I_{nb}, I_{nc}.

Line Currents: I_{aA}, I_{bB}, I_{cC}.



$I_{aA} = I_{na}$; $I_{bB} = I_{nb}$; $I_{cC} = I_{nc}$

Line Currents = Phase Currents

$E_{an} = E_p\angle 0^\circ$

$E_{bn} = E_p\angle -120^\circ$

$E_{cn} = E_p\angle -240^\circ = E_p\angle 120^\circ$

$E_{ab} = E_{an} + E_{nb} = E_{an} - E_{bn} = \sqrt{3}E_p\angle 30^\circ$

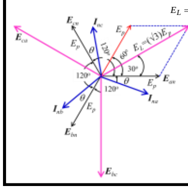
$E_{bc} = E_{bn} + E_{nc} = E_{bn} - E_{cn} = \sqrt{3}E_p\angle -90^\circ$

$E_{ca} = E_{cn} + E_{na} = E_{cn} - E_{an} = \sqrt{3}E_p\angle -210^\circ$

$E_{ca} = \sqrt{3}E_p\angle 150^\circ$

Vector Diagram for Three-Phase Star or Y Connected Source

$$E_L = E_{ab} = \sqrt{E_p^2 + E_p^2 + 2E_pE_p\cos 60^\circ} = \sqrt{3}E_p$$



For three-phase Y-connection or Star-connection the magnitude of line voltage is √3 time of the magnitude of phase voltage.

For three-phase Y-connection or Star-connection the line voltage leads phase voltage by 30°.

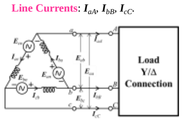
Three-Phase Mesh or Δ Connection

Phase Voltages: E_{an}, E_{bn}, E_{cn}.

Line Voltages: E_{ab}, E_{bc}, E_{ca}.

Phase Currents: I_{ba}, I_{cb}, I_{ac}.

Line Currents: I_{aA}, I_{bB}, I_{cC}.



$E_{ab} = E_{an}$; $E_{bc} = E_{bn}$; $E_{ca} = E_{cn}$

Line voltages = Phase Voltages

$I_{ba} = I_p\angle 0^\circ$

$I_{cb} = I_p\angle -120^\circ$

$I_{ac} = I_p\angle -240^\circ = I_p\angle 120^\circ$

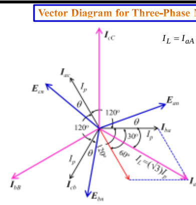
$I_{aA} = I_{ba} - I_{ac} = \sqrt{3}I_p\angle -30^\circ$

$I_{bB} = I_{cb} - I_{ba} = \sqrt{3}I_p\angle -150^\circ$

$I_{cC} = I_{ac} - I_{cb} = \sqrt{3}I_p\angle -270^\circ = \sqrt{3}I_p\angle 90^\circ$

Vector Diagram for Three-Phase Mesh or Δ Connected Source

$$I_L = I_{aA} = \sqrt{I_p^2 + I_p^2 + 2I_pI_p\cos 60^\circ} = \sqrt{3}I_p$$



For three-phase Δ-connection or Mesh-connection the magnitude of line current is √3 time of the magnitude of phase current.

For three-phase Δ-connection or Mesh-connection the line current lags phase current by 30°.

Balanced Source System

A set of three phase quantities (phase voltages or phase currents or line voltages or line currents), all the same frequency with equal peak (and hence equal rms) values and shifted (or displaced) successively by 120° is defined as a ***Balanced Three-Phase Quantity***.

$E_{an} = E_{bn} = E_{cn} = E_p$ $\quad$ $I_{an} = I_{bn} = I_{cn} = I_p$

$E_{ab} = E_{bc} = E_{ca} = E_L$ $\quad$ $I_{aA} = I_{bB} = I_{cC} = I_L$

$\boldsymbol{E}_{an} + \boldsymbol{E}_{bn} + \boldsymbol{E}_{cn} = 0$ $\quad$ $\boldsymbol{I}_{an} + \boldsymbol{I}_{bn} + \boldsymbol{I}_{cn} = \boldsymbol{I}_n = 0$

$\boldsymbol{E}_{ab} + \boldsymbol{E}_{bc} + \boldsymbol{E}_{ca} = 0$ $\quad$ $\boldsymbol{I}_{aA} + \boldsymbol{I}_{bB} + \boldsymbol{I}_{cC} = 0$

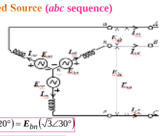
Y-Connected Balanced Source (*abc* sequence)

$\boldsymbol{E}_{an} = E_p\angle\theta_v$

$\boldsymbol{E}_{bn} = E_p\angle(\theta_v - 120^\circ) = \boldsymbol{E}_{an}(1\angle -120^\circ)$

$\boldsymbol{E}_{cn} = E_p\angle(\theta_v - 240^\circ) = E_p\angle(\theta_v + 120^\circ)$

$\boldsymbol{E}_{cn} = \boldsymbol{E}_{an}(1\angle -240^\circ) = \boldsymbol{E}_{an}(1\angle 120^\circ)$



$\boldsymbol{E}_{ab} = \sqrt{3}E_p\angle(\theta_v + 30^\circ) = \boldsymbol{E}_{an}(\sqrt{3}\angle 30^\circ)$

$\boldsymbol{E}_{bc} = \boldsymbol{E}_{ab}(1\angle -120^\circ) = \sqrt{3}E_p\angle(\theta_v + 30^\circ - 120^\circ) = \boldsymbol{E}_{bn}(\sqrt{3}\angle 30^\circ)$

$\boldsymbol{E}_{ca} = \boldsymbol{E}_{ab}(1\angle 120^\circ) = \sqrt{3}E_p\angle(\theta_v + 30^\circ + 120^\circ) = \boldsymbol{E}_{cn}(\sqrt{3}\angle 30^\circ)$

$\boldsymbol{I}_{na} = \boldsymbol{I}_{aA} = I_p\angle\theta_i$ $\quad$ $\boldsymbol{I}_{nb} = \boldsymbol{I}_{bB} = I_p\angle(\theta_i - 120^\circ)$

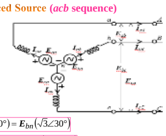
$\boldsymbol{I}_{nc} = \boldsymbol{I}_{cC} = I_p\angle(\theta_i - 240^\circ) = I_p\angle(\theta_i + 120^\circ)$

Y-Connected Balanced Source (*acb* sequence)

$\boldsymbol{E}_{an} = E_p\angle\theta_v$

$\boldsymbol{E}_{bn} = E_p\angle(\theta_v + 120^\circ) = \boldsymbol{E}_{an}(1\angle 120^\circ)$

$\boldsymbol{E}_{cn} = \boldsymbol{E}_{an}(1\angle -120^\circ)$



$\boldsymbol{E}_{ab} = \sqrt{3}E_p\angle(\theta_v + 30^\circ) = \boldsymbol{E}_{an}(\sqrt{3}\angle 30^\circ)$

$\boldsymbol{E}_{bc} = \boldsymbol{E}_{ab}(1\angle 120^\circ) = \sqrt{3}E_p\angle(\theta_v + 30^\circ + 120^\circ) = \boldsymbol{E}_{bn}(\sqrt{3}\angle 30^\circ)$

$\boldsymbol{E}_{ca} = \boldsymbol{E}_{ab}(1\angle -120^\circ) = \sqrt{3}E_p\angle(\theta_v + 30^\circ - 120^\circ) = \boldsymbol{E}_{cn}(\sqrt{3}\angle 30^\circ)$

$\boldsymbol{I}_{na} = \boldsymbol{I}_{aA} = I_p\angle\theta_i$ $\quad$ $\boldsymbol{I}_{nb} = \boldsymbol{I}_{bB} = I_p\angle(\theta_i - 120^\circ)$

$\boldsymbol{I}_{nc} = \boldsymbol{I}_{cC} = I_p\angle(\theta_i - 240^\circ) = I_p\angle(\theta_i + 120^\circ)$

Example

For Y-Connected Source (*abc* sequence)

Let, $\boldsymbol{E}_{an} = 100\angle 10^\circ$

$\boldsymbol{E}_{bn} = (100\angle 10^\circ)(1\angle -120^\circ) = 100\angle -110^\circ$

$\boldsymbol{E}_{cn} = (100\angle 10^\circ)(1\angle 120^\circ) = 100\angle 130^\circ$

Line Voltage

$\boldsymbol{E}_{ab} = (100\angle 10^\circ)(\sqrt{3}\angle 30^\circ) = 173.2\angle 40^\circ$

$\boldsymbol{E}_{bc} = (173.2\angle 40^\circ)(1\angle -120^\circ) = 173.2\angle -80^\circ$

$\boldsymbol{E}_{ca} = (173.2\angle 40^\circ)(1\angle 120^\circ) = 173.2\angle 160^\circ$

For Y-Connected Source (*acb* sequence)

Let, $\boldsymbol{E}_{an} = 100\angle 10^\circ$

$\boldsymbol{E}_{bn} = (100\angle 10^\circ)(1\angle 120^\circ) = 100\angle 130^\circ$

$\boldsymbol{E}_{cn} = (100\angle 10^\circ)(1\angle -120^\circ) = 100\angle -110^\circ$

Line Voltage

$\boldsymbol{E}_{ab} = (100\angle 10^\circ)(\sqrt{3}\angle 30^\circ) = 173.2\angle 40^\circ$

$\boldsymbol{E}_{bc} = (173.2\angle 40^\circ)(1\angle 120^\circ) = 173.2\angle 160^\circ$

$\boldsymbol{E}_{ca} = (173.2\angle 40^\circ)(1\angle -120^\circ) = 173.2\angle -80^\circ$

Example

For Y-Connected Source (*abc* sequence)

Let, $E_{ab} = 260\angle 30^\circ$

$E_{bc} = (260\angle 30^\circ)(1\angle -120^\circ) = 260\angle -90^\circ$

$E_{ca} = (260\angle 30^\circ)(1\angle 120^\circ) = 260\angle 150^\circ$

Phase Voltage

$E_{an} = (260\angle 30^\circ)\left(\frac{1}{\sqrt{3}}\angle -30^\circ\right) = 150\angle 0^\circ$

$E_{bn} = (150\angle 0^\circ)(1\angle -120^\circ) = 150\angle -120^\circ$

$E_{cn} = (150\angle 0^\circ)(1\angle 120^\circ) = 150\angle 120^\circ$

For Y-Connected Source (*acb* sequence)

Let, $E_{ab} = 260\angle 30^\circ$

$E_{bc} = (260\angle 30^\circ)(1\angle 120^\circ) = 260\angle 150^\circ$

$E_{ca} = (260\angle 30^\circ)(1\angle -120^\circ) = 260\angle -90^\circ$

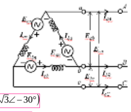
Phase Voltage

$E_{an} = (260\angle 30^\circ)\left(\frac{1}{\sqrt{3}}\angle -30^\circ\right) = 150\angle 0^\circ$

$E_{bn} = (150\angle 0^\circ)(1\angle 120^\circ) = 150\angle 120^\circ$

$E_{cn} = (150\angle 0^\circ)(1\angle -120^\circ) = 150\angle -120^\circ$

Δ-Connected Balanced Source (*abc* sequence)



$I_{ba} = I_p\angle\theta_i$

$I_{cb} = I_p\angle(\theta_i - 120^\circ) = I_{ba}(1\angle -120^\circ)$

$I_{ac} = I_p\angle -240^\circ = I_{ba}(1\angle -240^\circ) = I_{ba}(1\angle 120^\circ)$

$I_{aA} = \sqrt{3}I_p\angle(\theta_i - 30^\circ) = I_{ba}(\sqrt{3}\angle -30^\circ)$

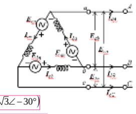
$I_{bB} = I_{aA}(1\angle -120^\circ) = \sqrt{3}I_p\angle(\theta_i + 30^\circ - 120^\circ) = I_{cb}(\sqrt{3}\angle -30^\circ)$

$I_{cC} = I_{aA}(1\angle 120^\circ) = \sqrt{3}I_p\angle(\theta_i - 30^\circ + 120^\circ) = I_{ac}(\sqrt{3}\angle -30^\circ)$

$E_{an} = E_{ab} = E_p\angle\theta_v$ $\quad E_{bn} = E_{bc} = E_{an}(1\angle -120^\circ) = E_p\angle(\theta_v - 120^\circ)$

$E_{cn} = E_{ca} = E_{an}(1\angle 120^\circ) = E_p\angle(\theta_v + 120^\circ)$

Δ-Connected Balanced Source (*acb* sequence)



$I_{ba} = I_p\angle\theta_i$

$I_{cb} = I_p\angle(\theta_i + 120^\circ) = I_{ba}(1\angle 120^\circ)$

$I_{ac} = I_p\angle -120^\circ = I_{ba}(1\angle -120^\circ) = I_{ba}(1\angle -120^\circ)$

$I_{aA} = \sqrt{3}I_p\angle(\theta_i - 30^\circ) = I_{ba}(\sqrt{3}\angle -30^\circ)$

$I_{bB} = I_{aA}(1\angle 120^\circ) = \sqrt{3}I_p\angle(\theta_i + 30^\circ + 120^\circ) = I_{cb}(\sqrt{3}\angle -30^\circ)$

$I_{cC} = I_{aA}(1\angle -120^\circ) = \sqrt{3}I_p\angle(\theta_i - 30^\circ - 120^\circ) = I_{ac}(\sqrt{3}\angle -30^\circ)$

$E_{an} = E_{ab} = E_p\angle\theta_v$ $\quad E_{bn} = E_{bc} = E_{an}(1\angle 120^\circ) = E_p\angle(\theta_v + 120^\circ)$

$E_{cn} = E_{ca} = E_{an}(1\angle -120^\circ) = E_p\angle(\theta_v - 120^\circ)$

Example

For Δ-Connected Source (*abc* sequence)

Let, $I_{ba} = 10\angle -20^\circ$

$I_{cb} = (10\angle -20^\circ)(1\angle -120^\circ) = 10\angle -140^\circ$

$I_{ac} = (10\angle -20^\circ)(1\angle 120^\circ) = 10\angle 100^\circ$

Line Current

$I_{aA} = (10\angle -20^\circ)(\sqrt{3}\angle 30^\circ) = 17.3\angle 10^\circ$

$I_{bB} = (17.3\angle 10^\circ)(1\angle -120^\circ) = 17.3\angle -110^\circ$

$I_{cC} = (17.3\angle 10^\circ)(1\angle 120^\circ) = 17.3\angle 130^\circ$

For Δ-Connected Source (*acb* sequence)

Let, $I_{ba} = 10\angle -20^\circ$

$I_{cb} = (10\angle -20^\circ)(1\angle 120^\circ) = 10\angle 100^\circ$

$I_{ac} = (10\angle -20^\circ)(1\angle -120^\circ) = 10\angle -140^\circ$

Line Current

$I_{aA} = (10\angle -20^\circ)(\sqrt{3}\angle 30^\circ) = 17.3\angle 10^\circ$

$I_{bB} = (17.3\angle 10^\circ)(1\angle 120^\circ) = 17.3\angle 130^\circ$

$I_{cC} = (17.3\angle 10^\circ)(1\angle -120^\circ) = 17.3\angle -110^\circ$

Example

For Δ-Connected Source (*abc* sequence)

Let, $I_{aA} = 260\angle -30^\circ$

$I_{bB} = (260\angle -30^\circ)(1\angle -120^\circ) = 260\angle -150^\circ$

$I_{cC} = (260\angle -30^\circ)(1\angle 120^\circ) = 260\angle 90^\circ$

Phase Current

$I_{ba} = (260\angle -30^\circ)\left(\frac{1}{\sqrt{3}}\angle 30^\circ\right) = 150\angle 0^\circ$

$I_{cb} = (150\angle 0^\circ)(1\angle -120^\circ) = 150\angle -120^\circ$

$I_{ac} = (150\angle 0^\circ)(1\angle 120^\circ) = 150\angle 120^\circ$

For Δ-Connected Source (*acb* sequence)

Let, $I_{aA} = 260\angle -30^\circ$

$I_{bB} = (260\angle -30^\circ)(1\angle 120^\circ) = 260\angle 90^\circ$

$I_{cC} = (260\angle -30^\circ)(1\angle -120^\circ) = 260\angle -150^\circ$

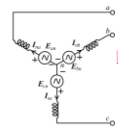
Phase Current

$I_{ba} = (260\angle -30^\circ)\left(\frac{1}{\sqrt{3}}\angle 30^\circ\right) = 150\angle 0^\circ$

$I_{cb} = (150\angle 0^\circ)(1\angle 120^\circ) = 150\angle 120^\circ$

$I_{ac} = (150\angle 0^\circ)(1\angle -120^\circ) = 150\angle -120^\circ$

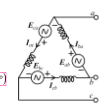
Conversion from Y-Connected Source to Δ-Connected Balanced Source



$E_{ab} = E_{an}(\sqrt{3}\angle 30^\circ)$

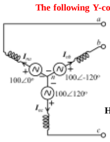
$E_{bc} = E_{ab}(1\angle 120^\circ) = E_{bn}(\sqrt{3}\angle 30^\circ)$

$E_{ca} = E_{ab}(1\angle -120^\circ) = E_{cn}(\sqrt{3}\angle 30^\circ)$



Example

The following Y-connected source convert to Δ-Connected Source

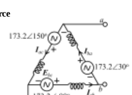


Here, for Y-connected source

$E_{an} = 100\angle 0^\circ$

$E_{bn} = 100\angle -120^\circ$

$E_{cn} = 100\angle 120^\circ$



Here, for Δ-connected source

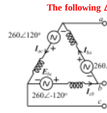
$E_{ab} = (100\angle 0^\circ)(\sqrt{3}\angle 30^\circ) = 173.2\angle 30^\circ$

$E_{bc} = (173.2\angle 30^\circ)(1\angle -120^\circ) = 173.2\angle -90^\circ$

$E_{ca} = (173.2\angle 30^\circ)(1\angle 120^\circ) = 173.2\angle 150^\circ$

Example

The following Δ-connected source convert to Y-Connected Source

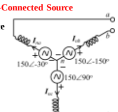


Here, for Δ-connected source

$E_{ab} = 260\angle 0^\circ$

$E_{bc} = 260\angle -120^\circ$

$E_{ca} = 260\angle 120^\circ$



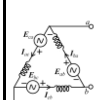
Here, for Y-connected source

$E_{an} = (260\angle 0^\circ)\left(\frac{1}{\sqrt{3}}\angle -30^\circ\right) = 150\angle -30^\circ$

$E_{bc} = (150\angle -30^\circ)(1\angle -120^\circ) = 150\angle -150^\circ$

$E_{ca} = (150\angle -30^\circ)(1\angle 120^\circ) = 150\angle 90^\circ$

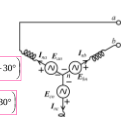
Conversion from Δ -Connected Source to Y-Connected Balanced Source



$E_{an} = E_{ab}\left(\frac{1}{\sqrt{3}}\angle -30^\circ\right)$

$E_{bn} = E_{an}(1\angle -120^\circ) = E_{bc}\left(\frac{1}{\sqrt{3}}\angle -30^\circ\right)$

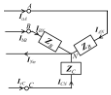
$E_{cn} = E_{an}(1\angle 120^\circ) = E_{ca}\left(\frac{1}{\sqrt{3}}\angle -30^\circ\right)$



Balanced Load

A set of three phase load impedances, all the same value is defined as a *Balanced Three-Phase load*.

Y-Connected load



For Balanced load

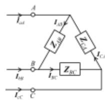
$\mathbf{Z}_A = \mathbf{Z}_B = \mathbf{Z}_C = \mathbf{Z}_Y$

$\mathbf{Z}_{AB} = \mathbf{Z}_{BC} = \mathbf{Z}_{CA} = \mathbf{Z}_\Delta$

$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3}$

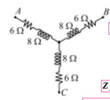
$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y$

Δ-Connected load



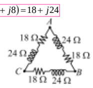
Example

Conversion from Y-connected load convert to Δ-Connected load and vice versa



$\mathbf{Z}_A = \mathbf{Z}_B = \mathbf{Z}_C = \mathbf{Z}_Y = 6 + j8$

$\mathbf{Z}_{AB} = \mathbf{Z}_{BC} = \mathbf{Z}_{CA} = \mathbf{Z}_\Delta = 3\mathbf{Z}_{CA} = 3(6 + j8) = 18 + j24$



$\mathbf{Z}_{AB} = \mathbf{Z}_{BC} = \mathbf{Z}_{CA} = \mathbf{Z}_\Delta = 18 + j24$

$\mathbf{Z}_A = \mathbf{Z}_B = \mathbf{Z}_C = \mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = \frac{18 + j24}{3} = 6 + j8$

Three-Phase Power

Let, the current and voltage of a three-phase system are:

$i_{an} = I_m \sin \omega t;\ \ i_{bn} = I_m \sin(\omega t - 120°);\ \ i_{cn} = I_m \sin(\omega t + 120°)$

$v_{an} = V_m \sin(\omega t + \theta);\ \ v_{bn} = V_m \sin(\omega t + \theta - 120°);\ \ v_{cn} = V_m \sin(\omega t + \theta + 120°)$

The power of each individual phase are given by:

$p_a(t) = v_{an} i_{an};\ \ p_b(t) = v_{bn} i_{bn};\ \ p_c(t) = v_{cn} i_{cn}$

The total three-phase power is given by:

$p_{3\phi}(t) = p_a(t) + p_b(t) + p_c(t) = v_{an} i_{an} + v_{bn} i_{bn} + v_{cn} i_{cn}$

The total three-phase power for a balanced system is given by:

$p_{3\phi}(t) = 3p_a(t) = 3p_b(t) = 3p_c(t) = 3v_{an} i_{an} = 3v_{bn} i_{bn} = 3v_{cn} i_{cn}$

$p_{3\phi}(t) = \frac{3}{2} V_m I_m \cos\theta$

Let, V_p is the rms value of phase voltage and I_p is the rms value of phase current that means:

$V_p = \frac{V_m}{\sqrt{2}};\ \ I_p = \frac{I_m}{\sqrt{2}}$

Thus, the total three-phase power for a balanced system is given by:

$p_{3\phi}(t) = 3V_p I_p \cos\theta$

The power of a single phase system is given by:

$p_{1\phi}(t) = P[1 - \cos 2\omega t] + Q \sin 2\omega t$

The single-phase power changes with respect to the time, so the single phase machine vibrates and makes noise. On the other hand, the three-phase power does not depend on time so it is constant and the three-phase machine is not vibrated and does not make noise.

Let, V_L is the rms value of line voltage and I_L is the rms value of line current.

For Y-connection

$V_L = \sqrt{3} V_p;\ \ I_L = I_p$

For Δ-connection

$I_L = I_p;\ \ V_L = \sqrt{3} V_p$

$P = 3V_p I_p \cos\theta = \sqrt{3} V_L I_L \cos\theta$

$Q = 3V_p I_p \sin\theta = \sqrt{3} V_L I_L \sin\theta$

$S = 3V_p I_p = \sqrt{3} V_L I_L$

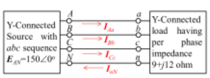
$pf = \cos\theta = \frac{P}{S}$

$rf = \sin\theta = \frac{Q}{S}$

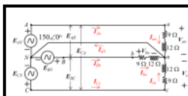
Example (Y→Y)

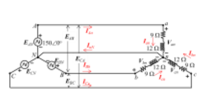
Example (Y→Y)

A three-phase Y-connected balanced source with *abc* sequence is connected with a Y-connected balanced load having the per phase impedance 9+*j*12 Ω. The neutrals of both source and load are connected. The phase voltage of source is given by $\mathbf{E}_{an}$=150∠0° V. Calculate **(i)** the phase voltages ($\mathbf{V}_{an}$, $\mathbf{V}_{bn}$, $\mathbf{V}_{cn}$), **(ii)** the line voltages ($\mathbf{E}_{AB}$, $\mathbf{E}_{BC}$, $\mathbf{E}_{CA}$), **(iii)** the phase currents ($\mathbf{I}_{an}$, $\mathbf{I}_{bn}$, $\mathbf{I}_{cn}$), **(iv)** the line currents ($\mathbf{I}_{Aa}$, $\mathbf{I}_{Bb}$, $\mathbf{I}_{Cc}$), **(v)** the power factor and reactive factor, and **(vi)** the power, reactive power and apparent power.



$\mathbf{Z}_a = \mathbf{Z}_b = \mathbf{Z}_c = \mathbf{Z}_Y = 9 + j12 = 15\angle 53.13°\ \Omega$





Phase Voltage:

$\mathbf{V}_{an} = \mathbf{E}_{AN} = 150\angle 0°$

$\mathbf{V}_{bn} = \mathbf{E}_{BN} = \mathbf{E}_{AN}(1\angle -120°)$
$= (150\angle 0°)(1\angle -120°)$
$= 150\angle -120°\ \text{V}$

$\mathbf{V}_{cn} = \mathbf{E}_{CN} = \mathbf{E}_{AN}(1\angle 120°)$
$= (150\angle 0°)(1\angle 120°)$
$= 150\angle 120°\ \text{V}$

Line Voltage:

$\mathbf{E}_{AB} = \mathbf{E}_{AN} - \mathbf{E}_{BN} = \mathbf{E}_{AN}(\sqrt{3}\angle 30°) = (150\angle 0°)(\sqrt{3}\angle 30°) = 260\angle 30°\ \text{V}$

$\mathbf{E}_{BC} = \mathbf{E}_{BN} - \mathbf{E}_{CN} = \mathbf{E}_{BN}(\sqrt{3}\angle 30°) = \mathbf{E}_{AB}(1\angle -120°) = 260\angle -90°\ \text{V}$

$\mathbf{E}_{CA} = \mathbf{E}_{CN} - \mathbf{E}_{AN} = \mathbf{E}_{CN}(\sqrt{3}\angle 30°) = \mathbf{E}_{AB}(1\angle 120°) = 260\angle 150°\ \text{V}$

For Y connection: Phase Current = Line Current

$\mathbf{I}_{an} = \mathbf{I}_{Aa} = \frac{\mathbf{V}_{an}}{\mathbf{Z}_a} = \frac{150\angle 0°}{15\angle 53.13°} = 10\angle -53.13°\ \text{A}$

$\mathbf{I}_{bn} = \mathbf{I}_{Bb} = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_b} = \mathbf{I}_{an}(1\angle -120°) = (10\angle -53.13°)(1\angle -120°) = 10\angle -173.13°\ \text{A}$

$\mathbf{I}_{cn} = \mathbf{I}_{Cc} = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_c} = \mathbf{I}_{an}(1\angle 120°) = (10\angle -53.13°)(1\angle 120°) = 10\angle 66.87°\ \text{A}$

Power factor and Reactive factor:

$\theta = \theta_v - \theta_i = 0° - (-53.13°) = 53.13°$

$pf = \cos\theta = \cos(53.13°) = 0.6$ $\quad rf = \sin\theta = \sin(53.13°) = 0.8$

Apparent Power, Real Power and Reactive Power

$V_L = 260$ V; $\quad I_L = 10$ A

$S = \sqrt{3} V_L I_L = \sqrt{3} \times 260 \times 10 = 4503.3$ VA

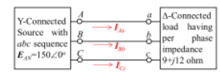
$P = \sqrt{3} V_L I_L \cos\theta = S\cos\theta = 4503.3 \times 0.6 = 2702$ W

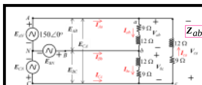
$Q = \sqrt{3} V_L I_L \sin\theta = S\sin\theta = 4503.3 \times 0.8 = 3602.7$ VAR

Example (Y→Δ)

Example (Y→Δ)

A three-phase Y-connected balanced source with *abc* sequence is connected with a Δ-connected balanced load having the per phase impedance 9+*j*12 Ω. The phase voltage of source is given by E_{an}=150∠0° V. Calculate **(i)** the phase voltage, **(ii)** the line voltage, **(iii)** the phase current, **(iv)** the line current, **(v)** the power factor and reactive factor, and **(vi)** the power, reactive power and apparent power.





$\boxed{Z_{ab} = Z_{bc} = Z_{ca} = Z_\Delta = 9 + j12 = 15\angle 53.13°\ \Omega}$

Phase Voltage:

$V_{an} = E_{AN} = 150\angle 0°$

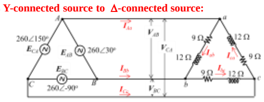
$$V_{bn} = E_{BN} = E_{AN}(1\angle -120°) = (150\angle 0°)(1\angle -120°) = 150\angle -120°\ \text{V}$$

$$V_{cn} = E_{CN} = E_{AN}(1\angle 120°) = (150\angle 0°)(1\angle 120°) = 150\angle 120°\ \text{V}$$

Line Voltage:

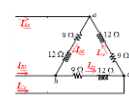
$$E_{AB} = E_{AN} - E_{BN} = E_{AN}(\sqrt{3}\angle 30°) = (150\angle 0°)(\sqrt{3}\angle 30°) = 260\angle 30°\ \text{V}$$
$$E_{BC} = E_{BN} - E_{CN} = E_{BN}(\sqrt{3}\angle 30°) = E_{AB}(1\angle -120°) = 260\angle -90°\ \text{V}$$
$$E_{CA} = E_{CN} - E_{AN} = E_{CN}(\sqrt{3}\angle 30°) = E_{AB}(1\angle 120°) = 260\angle 150°\ \text{V}$$

Converts Y-connected source to Δ-connected source:



Phase Current:

$$I_{ab} = \frac{E_{AB}}{Z_{ab}} = \frac{260\angle 30°}{15\angle 53.13°} = 17.33\angle -23.13°\ \text{A}$$
$$I_{bc} = \frac{E_{BC}}{Z_{bc}} = I_{ab}(1\angle -120°) = 17.33\angle -143.13°\ \text{A}$$
$$I_{ca} = \frac{E_{CA}}{Z_{ca}} = I_{ab}(1\angle 120°) = 17.33\angle 96.87°\ \text{A}$$



Line Current:

$$I_{Aa} = I_{ab} - I_{ca} = I_{ab}(\sqrt{3}\angle -30°) = (17.33\angle -23.13°)(\sqrt{3}\angle -30°) = 30\angle -53.13°\ \text{A}$$
$$I_{Bb} = I_{bc} - I_{ab} = I_{bc}(\sqrt{3}\angle -30°) = I_{Aa}(1\angle -120°) = 30\angle -173.13°\ \text{A}$$
$$I_{Cc} = I_{ca} - I_{bc} = I_{ca}(\sqrt{3}\angle -30°) = I_{Aa}(1\angle 120°) = 30\angle 66.87°\ \text{A}$$

Power factor and Reactive factor:

$\theta = \theta_v - \theta_i = 30° - (-23.13°) = 53.13°$

$pf = \cos\theta = \cos(53.13°) = 0.6$ $\quad rf = \sin\theta = \sin(53.13°) = 0.8$

Apparent Power, Real Power and Reactive Power

$V_L = 260$ V; $\quad I_L = 10$ A

$S = \sqrt{3} V_L I_L = \sqrt{3} \times 260 \times 30 = 1351$ VA

$P = \sqrt{3} V_L I_L \cos\theta = S\cos\theta = 1351 \times 0.6 = 8106$ W

$Q = \sqrt{3} V_L I_L \sin\theta = S\sin\theta = 1351 \times 0.8 = 1080.8$ VAR

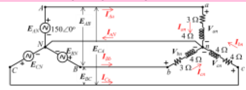
For same supply voltage and same per-phase impedance the power consumed by delta-connected load is 3 times higher than for wye-connected load. Thus we have

$\boxed{P_{\Delta L} = 3P_{YL}}$

Converts Δ-connected load to Y-connected load:

$\boxed{Z_{ab} = Z_{bc} = Z_{ca} = Z_\Delta = 9 + j12 = 15\angle 53.13°\ \Omega}$

$\boxed{Z_a = Z_b = Z_c = Z_Y = \frac{Z_\Delta}{3} = \frac{9 + j12}{3} = \frac{15\angle 53.13°}{3} = 5\angle 53.13° = 3 + j4\ \Omega}$

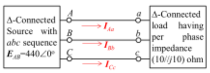


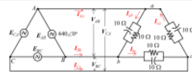
Solve same as the Example (Y→Y)

Example (Δ→Δ)

Example (Δ→Δ)

A three-phase Δ-connected balanced source with abc sequence is connected with a Δ-connected balanced load. In each phase, a 10 ohm resistance is connected in parallel with a capacitor having 10 ohm reactance. The phase voltage of source is given by E_{AB}=440∠0º V. Calculate **(i)** the phase voltage, **(ii)** the line voltage, **(iii)** the phase current, **(iv)** the line current, **(v)** the power factor and reactive factor, and **(vi)** the power, reactive power and apparent power.





$$\boxed{Z_{ab} = Z_{bc} = Z_{ca} = Z_\Delta = \frac{10\times(-j10)}{10-j10} = 5-j5 = 7.07\angle-45°\ \Omega}$$

For Δ connection: **Phase Voltage = Line Voltage**

$E_{AB} = 440\angle 0°$ V

$E_{BC} = E_{AB}(1\angle-120°) = 440\angle-120°$ V

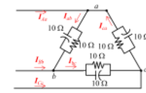
$E_{CA} = E_{AB}(1\angle 120°) = 440\angle 120°$ V

Phase Current:

$$I_{ab} = \frac{E_{AB}}{Z_{ab}} = \frac{440\angle 0°}{7.07\angle-45°} = 62.23\angle 45°\ \text{A}$$

$$I_{bc} = \frac{E_{BC}}{Z_{bc}} = I_{ab}(1\angle-120°) = 62.23\angle-75°\ \text{A}$$

$$I_{ca} = \frac{E_{CA}}{Z_{ca}} = I_{ab}(1\angle 120°) = 62.23\angle 165°\ \text{A}$$



Line Current:

$I_{Aa} = I_{ab} - I_{ca} = I_{ab}(\sqrt{3}\angle-30°) = 108\angle 15°$ A

$I_{Bb} = I_{bc} - I_{ab} = I_{bc}(\sqrt{3}\angle-30°) = I_{Aa}(1\angle-120°) = 108\angle-105°$ A

$I_{Cc} = I_{ca} - I_{bc} = I_{ca}(\sqrt{3}\angle-30°) = I_{Aa}(1\angle 120°) = 108\angle 125°$ A

Power factor and Reactive factor:

$\theta = \theta_v - \theta_i = 0° - (45°) = -45°$

$pf = \cos\theta = \cos(-45°) = 0.707$ $\qquad rf = \sin\theta = \sin(-45°) = -0.707$

Apparent Power, Real Power and Reactive Power

$V_L = 440$ V; $\quad I_L = 108$ A

$S = \sqrt{3}V_L I_L = \sqrt{3}\times 440\times 108 = 82.3$ kVA

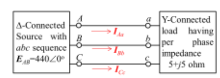
$P = \sqrt{3}V_L I_L\cos\theta = S\cos\theta = 82.3\text{kVA}\times 0.707 = 58.2$ kW

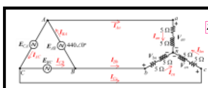
$Q = \sqrt{3}V_L I_L\cos\theta = S\cos\theta = 82.3\text{kVA}\times -0.707 = -58.2$ kVar

Example (Δ→Y)

Example (Δ→Y)

A three-phase Δ-connected balanced source with abc sequence is connected with a Y-connected balanced load having the per-phase impedance 5+j5 ohm. The phase voltage of source is given by E_{AB}=440∠0º V. Calculate **(i)** the phase voltage, **(ii)** the line voltage, **(iii)** the phase current, **(iv)** the line current, **(v)** the power factor and reactive factor, and **(vi)** the power, reactive power and apparent power.

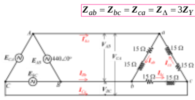




$$\boxed{Z_a = Z_b = Z_c = Z_Y = 5+j5 = 7.07\angle 45°\ \Omega}$$

Converts Y-connected load to Δ-connected load:

$$\boxed{Z_{ab} = Z_{bc} = Z_{ca} = Z_\Delta = 3Z_Y = 15+j15 = 21.21\angle 45°\ \Omega}$$



For Δ connection: **Phase Voltage = Line Voltage**

$E_{AB} = 440\angle 0°$ V

$E_{BC} = E_{AB}(1\angle-120°) = 440\angle-120°$ V

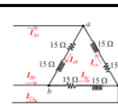
$E_{CA} = E_{AB}(1\angle 120°) = 440\angle 120°$ V

Phase Current:

$$I_{ab} = \frac{E_{AB}}{Z_{ab}} = \frac{440\angle 0°}{21.21\angle 45°} = 20.74\angle-45°\ \text{A}$$

$$I_{bc} = \frac{E_{BC}}{Z_{bc}} = I_{ab}(1\angle-120°) = 20.74\angle-165°\ \text{A}$$

$$I_{ca} = \frac{E_{CA}}{Z_{ca}} = I_{ab}(1\angle 120°) = 20.74\angle 75°\ \text{A}$$



Line Current:

$I_{Aa} = I_{ab} - I_{ca} = I_{ab}(\sqrt{3}\angle-30°) = 36\angle-75°$ A

$I_{Bb} = I_{bc} - I_{ab} = I_{bc}(\sqrt{3}\angle-30°) = I_{Aa}(1\angle-120°) = 36\angle-195° = 36\angle 165°$ A

$I_{Cc} = I_{ca} - I_{bc} = I_{ca}(\sqrt{3}\angle-30°) = I_{Aa}(1\angle 120°) = 36\angle 45°$ A

Power factor and Reactive factor:

$\theta = \theta_v - \theta_i = 0° - (-45°) = 45°$

$pf = \cos\theta = \cos(45°) = 0.707$ $\qquad rf = \sin\theta = \sin(45°) = 0.707$

Apparent Power, Real Power and Reactive Power

$V_L = 440$ V; $\quad I_L = 36$ A

$S = \sqrt{3}V_L I_L = \sqrt{3}\times 440\times 36 = 27.45$ kVA

$P = \sqrt{3}V_L I_L\cos\theta = S\cos\theta = 27.45\text{kVA}\times 0.707 = 19.4$ kW

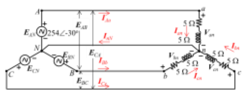
$Q = \sqrt{3}V_L I_L\cos\theta = S\cos\theta = 27.45\text{kVA}\times 0.707 = 19.4$ kVar

Converts Δ-connected source to Y-connected source:

$\boldsymbol{E}_{AB} = 440\angle 0°\ \text{V}$ $\quad \boldsymbol{E}_{BC} = \boldsymbol{E}_{AB}(1\angle -120°) = 440\angle -120°\ \text{V}$

$\boldsymbol{E}_{CA} = \boldsymbol{E}_{AB}(1\angle 120°) = 440\angle 120°\ \text{V}$

$\boxed{\boldsymbol{E}_{AN} = \boldsymbol{E}_{ab}\left(\frac{1}{\sqrt{3}}\angle -30°\right) = 254\angle -30°}$ $\quad \boxed{\boldsymbol{E}_{BN} = 254\angle -150°}$ $\quad \boxed{\boldsymbol{E}_{CN} = 254\angle 90°}$



Solve same as the Example (Y→Y)

Example

A 220 volts three-phase supply is connected with a Y-connected load having R = 6 Ω and X_L = 8 Ω. (**i**) Find the line current, power per phase and total power. (**ii**) Find the phase voltages, phase currents and line voltages of the load in polar form considering *abc* sequence.

Solution

$V_L = 220\ \text{V} \quad V_p = \frac{V_L}{\sqrt{3}} = \frac{220}{\sqrt{3}} = 127\ \text{V} \quad \boldsymbol{Z} = \boldsymbol{Z}_{an} = \boldsymbol{Z}_{bn} = \boldsymbol{Z}_{cn} = 6 + j8 = 10\angle 53.13°\ \Omega$

$I_L = I_p = \frac{127}{10} = 12.7\ \text{A} \quad P = 3I_p^2 R = 3\times 12.7^2 \times 6 = 3\times 968 = 2904\ \text{W}$

$\boldsymbol{V}_{an} = 127\angle 0°\ \text{V} \quad \boldsymbol{V}_{bn} = 127\angle -120°\ \text{V} \quad \boldsymbol{V}_{cn} = 127\angle 120°\ \text{V}$

$\boldsymbol{V}_{ab} = 220\angle 30°\ \text{V} \quad \boldsymbol{V}_{bc} = 220\angle -90°\ \text{V} \quad \boldsymbol{V}_{ca} = 220\angle 150°\ \text{V}$

$\boldsymbol{I}_{an} = \boldsymbol{I}_{Aa} = \frac{\boldsymbol{V}_{an}}{\boldsymbol{Z}_a} = \frac{127\angle 0°}{10\angle 53.13°} = 12.7\angle -53.13°\ \text{A}$

$\boldsymbol{I}_{bn} = \boldsymbol{I}_{Bb} = 10\angle -173.13°\ \text{A} \quad \boldsymbol{I}_{cn} = \boldsymbol{I}_{Cc} = 10\angle 66.87°\ \text{A}$

Example

A 220 volts three-phase supply is connected with a Δ-connected load having R = 6 Ω and X_L = 8 Ω. (**i**) Find the line current, power per phase and total power. (**ii**) Find the phase voltages, phase currents and line voltages of the load in polar form considering *abc* sequence.

Solution

$V_L = V_p = 220\ \text{V} \quad \boldsymbol{Z}_{ab} = \boldsymbol{Z}_{bc} = \boldsymbol{Z}_{ca} = 6 + j8 = 10\angle 53.13°\ \Omega \quad I_p = \frac{220}{10} = 22\ \text{A}$

$I_L = \sqrt{3}\times 22 = 38.1\ \text{A} \quad P = 3I_p^2 R = 3\times 22^2 \times 6 = 8712\ \text{W}$

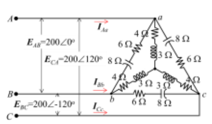
$\boldsymbol{V}_{an} = \boldsymbol{V}_{ab} = 127\angle 0°\ \text{V} \quad \boldsymbol{V}_{bn} = \boldsymbol{V}_{bc} = 127\angle -120°\ \text{V} \quad \boldsymbol{V}_{cn} = \boldsymbol{V}_{ca} = 127\angle 120°\ \text{V}$

$\boldsymbol{I}_{ab} = \frac{\boldsymbol{V}_{ab}}{\boldsymbol{Z}_{ab}} = \frac{220\angle 0°}{10\angle 53.13°} = 22\angle -53.13°\ \text{A} \quad \boldsymbol{I}_{bc} = 22\angle -173.13°\ \text{A} \quad \boldsymbol{I}_{ca} = 22\angle 66.87°\ \text{A}$

$\boldsymbol{I}_{Aa} = \left[\sqrt{3}\angle -30°\right]\boldsymbol{I}_{ab} = 38.1\angle -83.13°\ \text{A} \quad \boldsymbol{I}_{Bb} = 38.1\angle -203.13°\ \text{A} \quad \boldsymbol{I}_{Cc} = 38.1\angle 36.87°\ \text{A}$

Example

A balanced Y-connected load is connected in parallel with a balanced delta-connected load. The impedance of Y-load is $\boldsymbol{Z}_Y$ = 4 + j3 and the impedance of Δ-load is $\boldsymbol{Z}_\Delta$ = 6 + j8. Find the total average, reactive, and apparent power. In addition, find the power factor of the load.



Solution: $\boldsymbol{Z}_\Delta = 6 - j8 = 10\angle -53.13°\ \Omega \quad \boldsymbol{Z}_Y = 4 + j3 = 5\angle 36.87°\ \Omega$

$\boldsymbol{Z}_{Y\to\Delta} = 3\boldsymbol{Z}_Y = 12 + j9 = 15\angle 36.87°\ \Omega$

$$\boldsymbol{Z}_{P\Delta} = \frac{(10\angle -53.13°)(15\angle 36.87°)}{10\angle -53.13° + 15\angle 36.87°} = 8.3\angle -19.44° = 7.85 - j2.77\ \Omega$$

$V_L = V_p = 200\ \text{V} \quad I_p = \frac{200}{8.3} = 24.1\ \text{A} \quad I_L = \sqrt{3}\times 24.1 = 42.74\ \text{A}$

$P = 3I_p^2 R = 3\times 24.1^2 \times 7.85 = 13.68\ \text{kW}$

$Q = 3I_p^2 X = 3\times 24.1^2 \times 2.77 = 4.84\ \text{kVar (Capacitive)}$

$S = \sqrt{P^2 + Q^2} = \sqrt{(13.68)^2 + (4.84)^2} = 14.51\ \text{kVA}$

$pf = \frac{P}{S} = \frac{13.68}{14.51} = 0.94$

Example

A three-phase motor takes 10 kVA to 0.6 power factor lagging from a source of 220 volts. It is in parallel with a balanced delta load having 16 ohms resistance and 12 ohms capacitive reactance in series in each phase. Find the total volt-amperes, power, line current and power factor of the combination.

Solution: **For Motor** $S_m = 10\ \text{kVA} \quad \cos\theta_m = 0.6 \quad \sin\theta_m = \sqrt{1 - 0.6^2} = 0.8$

$P_m = S_m \cos\theta_m = 10\times 0.6 = 6\ \text{kW} \quad Q_m = S_m \sin\theta_m = 10\times 0.8 = 8\ \text{kVar}$

$I_{Lm} = \frac{S_m}{\sqrt{3}V_L} = \frac{10\times 10^3}{\sqrt{3}\times 220} = 26.24\ \text{A}$

For Delta Load $Z_\Delta = \sqrt{16^2 + 12^2} = 20\ \Omega \quad I_{p\Delta} = \frac{220}{20} = 11\ \text{A} \quad I_{p\Delta} = \sqrt{3}\times 11 = 19.1\ \text{A}$

$P_\Delta = 3\times 11^2 \times 16 = 5.81\ \text{kW} \quad Q_\Delta = -3\times 11^2 \times 12 = -4.36\ \text{kVar}$

Total

$P_T = P_m + P_\Delta = 6 + 5.81 = 11.81\ \text{kW}$

$Q_T = Q_m + Q_\Delta = 8 - 4.36 = 3.64\ \text{kW}$

$S_T = \sqrt{P_T^2 + Q_T^2} = \sqrt{(11.81)^2 + (3.64)^2} = 12.37\ \text{kVA}$

$pf = \frac{P_T}{S_T} = \frac{11.81}{12.37} = 0.955\ \text{(lagging)}$

$I_{LT} = \frac{S_T}{\sqrt{3}V_L} = \frac{12.37\times 10^3}{\sqrt{3}\times 220} = 32.5\ \text{A}$

Example

A three-phase motor takes 10 kVA to 0.6 power factor lagging from a source of 220 volts. Calculate the per phase impedance if the motor is connected (**i**) Y-connected, and (**ii**) Δ-connected.

$\cos\theta = 0.6 \quad \sin\theta = \sqrt{1 - 0.6^2} = 0.8$

For Y-connection: $V_p = \frac{V_L}{\sqrt{3}} = \frac{220}{\sqrt{3}} = 127\ \text{V} \quad I_L = I_p = \frac{S}{\sqrt{3}V_L} = \frac{10\times 10^3}{\sqrt{3}\times 220} = 26.24\ \text{A}$

$Z = \frac{V_p}{I_L} = \frac{127}{26.25} = 4.84\ \Omega \quad R = Z\cos\theta = 4.84\times 0.6 = 2.9\ \Omega \quad X = Z\sin\theta = 4.84\times 0.8 = 3.9\ \Omega \quad \boldsymbol{Z} = 2.9 + j3.9\ \Omega$

For Δ-connection: $V_p = V_L = 220\ \text{V} \quad I_L = \frac{S}{\sqrt{3}V_L} = \frac{10\times 10^3}{\sqrt{3}\times 220} = 26.24\ \text{A}$

$I_p = \frac{I_L}{\sqrt{3}} = \frac{26.25}{\sqrt{3}} = 15.14\ \text{A} \quad Z = \frac{V_p}{I_L} = \frac{220}{15.14} = 14.52\ \Omega$

$R = Z\cos\theta = 14.52\times 0.6 = 8.7\ \Omega \quad X = Z\sin\theta = 14.52\times 0.8 = 11.6\ \Omega \quad \boldsymbol{Z} = 8.7 + j11.6\ \Omega$