Parallel Resonance

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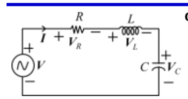
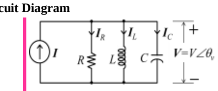
Basic Properties of Resonance Circuit

- The supply voltage and supply current are in phase (that means, the phase difference between supply voltage and current is zero degree; $\theta=\theta_z=\theta_v-\theta_i=0°$) having at least one inductor and one capacitor in circuit.
- Power factor is unity (pf=cosθ=1).
- Reactive factor is zero (rf=sinθ=0).
- Net reactive power is zero ($Q=Q_L-Q_C=0$).
- Power and apparent power are equal ($P=S=V_{rms}I_{rms}$).
- The net reactance (for series circuit *i.e.* $X_L-X_C=0$) or susceptance (for parallel circuit *i.e.* $B_C-B_L=0$) is zero.
- The impedance is purely resistive.

Difference Between Series Resonance Circuit and Ideal Parallel Resonance Circuit

RLC **Series Resonance Circuit**	**Ideal *RLC* Parallel Resonance Circuit**
Circuit Diagram	
	 $G=\frac{1}{R}$ $B_L=\frac{1}{X_L}$ $B_C=\frac{1}{X_C}$
$\boldsymbol{Z}_R=R\angle 0°=R$ $\boldsymbol{Z}_L=X_L\angle 90°=jX_L$	$\boldsymbol{Y}_R=G\angle 0°=G$ $\boldsymbol{Y}_L=B_L\angle -90°=-jX_L$
$\boldsymbol{Z}_C=X_C\angle -90°=-jX_C$	$\boldsymbol{Y}_C=B_C\angle 90°=jB_C$ $\boldsymbol{Y}=G+j(B_C-B_L)$
$\boldsymbol{Z}=R+j(X_L-X_C)$ $\boldsymbol{I}=\frac{\boldsymbol{V}}{\boldsymbol{Z}}$	$\boldsymbol{I}_R=\frac{\boldsymbol{Y}_R}{\boldsymbol{Y}}\boldsymbol{I}$ $\boldsymbol{I}_L=\frac{\boldsymbol{Y}_L}{\boldsymbol{Y}}\boldsymbol{I}$ $\boldsymbol{I}_C=\frac{\boldsymbol{Y}_C}{\boldsymbol{Y}}\boldsymbol{I}$
$\boldsymbol{V}_R=\boldsymbol{IZ}_R$ $\boldsymbol{V}_L=\boldsymbol{IZ}_L$ $\boldsymbol{V}_C=\boldsymbol{IZ}_C$	$\boldsymbol{V}=\frac{\boldsymbol{I}}{\boldsymbol{Y}}$

RLC **Series Resonance Circuit**	**Ideal *RLC* Parallel Resonance Circuit**
Condition for Resonance	
$X_L-X_C=0$ $X_L=X_C$	$B_C-B_L=0$ $B_L=B_C$
Resonance Frequency	
$\omega_{sr}=\frac{1}{\sqrt{LC}}$ $f_{sr}=\frac{1}{2\pi\sqrt{LC}}$	$\omega_{pr}=\frac{1}{\sqrt{LC}}$ $f_{pr}=\frac{1}{2\pi\sqrt{LC}}$
Impedance	**Admittance**
$\boldsymbol{Z}=R$ [minimum]	$\boldsymbol{Y}=G=\frac{1}{R}$ [minimum] $\boldsymbol{Z}=\frac{1}{\boldsymbol{Y}}=R$ [maximum]
Current	**Voltage**
$\boldsymbol{I}=\frac{\boldsymbol{V}}{\boldsymbol{Z}}=\frac{\boldsymbol{V}}{R}$ [maximum] $I_{max}=\frac{V}{R}$	$\boldsymbol{V}=\frac{\boldsymbol{I}}{\boldsymbol{Y}}$ [maximum] $V_{max}=\frac{I}{G}$
$P_{max}=I_{max}^2R$	$P_{max}=V_{max}^2G$

RLC **Series Resonance Circuit**	**Ideal *RLC* Parallel Resonance Circuit**
At Resonance Condition ($f=f_{sr}$)	
$X_L=X_C$ Circuit behaves resistive	$B_L=B_C$ Circuit behaves resistive
$V_L=V_C$	$I_L=I_C$
$Q_L=Q_C$	$Q_L=Q_C$
Below the Resonance Condition ($f<f_{sr}$)	
$X_C>X_L$ Circuit behaves Capacitive	$B_C<B_L$ $X_L>X_C$ Circuit behaves Inductive
Above the Resonance Condition ($f>f_{sr}$)	
$X_L>X_C$ Circuit behaves Inductive	$B_L>B_C$ $X_C>X_L$ Circuit behaves Capacitive

RLC **Series Resonance Circuit**	**Ideal *RLC* Parallel Resonance Circuit**
Quality Factor or Selectivity	
$Q_{sr}=\frac{Q_C}{P}=\frac{Q_L}{P}=\frac{X_C}{R}=\frac{X_L}{R}=\frac{V_L}{V}=\frac{V_C}{V}$	$Q_{pr}=\frac{Q_L}{P}=\frac{R}{X_C}=\frac{R}{X_L}=\frac{I_L}{I}=\frac{I_C}{I}$
$V_L=V_C=Q_{sr}V$	$I_L=I_C=Q_{pr}I$
Since the voltage drop across the inductor and capacitor is quality factor times of supply voltage, the series resonance circuit is also called voltage magnification or voltage amplification or voltage resonance circuit.	Since the current flows through the inductor and capacitor is quality factor times of supply current, the parallel resonance circuit is also called current magnification or current amplification or current resonance circuit.
$Q_{sr}=\frac{1}{R}\sqrt{\frac{L}{C}}$	$Q_{pr}=R\sqrt{\frac{C}{L}}$

RLC **Series Resonance Circuit**	**Ideal *RLC* Parallel Resonance Circuit**
Cut-off Frequency	
$\omega_{sl}=-\frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2+\frac{1}{LC}}$	$\omega_{pl}=-\frac{1}{2RC}+\sqrt{\left(\frac{1}{2RC}\right)^2+\frac{1}{LC}}$
$\omega_{sh}=\frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2+\frac{1}{LC}}$	$\omega_{ph}=\frac{1}{2RC}+\sqrt{\left(\frac{1}{2RC}\right)^2+\frac{1}{LC}}$
$f_{sl}=\frac{\omega_{sl}}{2\pi}=\frac{1}{2\pi}\left[-\frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2+\frac{1}{LC}}\right]$	$f_{pl}=\frac{\omega_{pl}}{2\pi}=\frac{1}{2\pi}\left[-\frac{1}{2RC}+\sqrt{\left(\frac{1}{2RC}\right)^2+\frac{1}{LC}}\right]$
$f_{sh}=\frac{\omega_{sh}}{2\pi}=\frac{1}{2\pi}\left[\frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2+\frac{1}{LC}}\right]$	$f_{ph}=\frac{\omega_{ph}}{2\pi}=\frac{1}{2\pi}\left[\frac{1}{2RC}+\sqrt{\left(\frac{1}{2RC}\right)^2+\frac{1}{LC}}\right]$
At Cut-off Frequency	
$Z=\sqrt{2}R$ $I=\frac{1}{\sqrt{2}}I_{\max}$ $P=\frac{1}{2}P_{\max}$	$Y=\sqrt{2}G$ $V=\frac{1}{\sqrt{2}}V_{\max}$ $P=\frac{1}{2}P_{\max}$

RLC **Series Resonance Circuit**	**Ideal *RLC* Parallel Resonance Circuit**
Bandwidth	
$BW=f_{sh}-f_{sh}=\frac{\omega_{sh}-\omega_{sl}}{2\pi}=\frac{R}{2\pi L}=\frac{f_{sr}}{Q_{sr}}$	$BW=f_{ph}-f_{ph}=\frac{\omega_{ph}-\omega_{pl}}{2\pi}=\frac{1}{2\pi RC}=\frac{f_{pr}}{Q_{pr}}$
$f_{sr}=\sqrt{f_{sh}f_{sl}}$ $\omega_{sr}=\sqrt{\omega_{sh}\omega_{sl}}$	$f_{pr}=\sqrt{f_{ph}f_{pl}}$ $\omega_{pr}=\sqrt{\omega_{ph}\omega_{pl}}$
The series resonant circuit is often described as an *acceptor circuit* since it has its minimum impedance, and thus maximum current/power at the resonant frequency.	The parallel resonant circuit is often described as an *rejector circuit* since it has its maximum impedance, and thus minimum current at the resonant frequency.
Allow to pass a specific frequency	Block or stop to pass a specific frequency

Parallel resonance circuit is generally called a *tank circuit* because of the fact that the circuit stores energy in the magnetic field of the inductor (coil or reactor) in the electric field of the capacitor (condenser). The stored energy is transferred back and forth between the capacitor and inductor and vice-versa.

Example

A current source of 10 mA is connected with an ideal parallel circuit having R = 10 kΩ, L = 1 mH, and C = 1 μF. Calculate (i) the resonance frequency, (ii) the admittance and impedance at resonance condition, (iii) the voltage and power at resonance condition, (iv) the currents pass through the resistance, inductance and capacitance at resonance condition, (v) the lower and higher cut-off frequency, (vi) the admittance and impedance at cut-off frequency, (vii) the voltage and power at cut-off frequency, (viii) the band-width, (ix) the quality factor.

Solution: Given:

$I=10.0\times10^{-3}$ A

$R=10.0\times10^{3}$ Ω

$L=1.0\times10^{-3}$ H

$C=1.0\times10^{-6}$ F

$$f_{pr}=\frac{1}{2\pi\sqrt{LC}}=\frac{1}{2\pi\sqrt{1.0\times10^{-3}\times1.0\times10^{-6}}}=5.03\times10^{3}\ \text{Hz}=5.03\ \text{kHz}$$

The admittance and impedance at resonance are:

$$Y_{pr}=\frac{1}{R}=\frac{1}{10.0\times10^{3}}=1.0\times10^{-4}\ \text{S} \qquad Z_{pr}=R=10.0\times10^{3}\ \Omega=10\ \text{k}\Omega$$

The voltage and power at resonance condition are:

$$V_{pr}=V_{\max}=IZ_{pr}=10.0\times10^{-3}\times10.0\times10^{3}\ \Omega=100\ \text{V} \qquad P_{pr}=P_{\max}=\frac{V_{pr}^{2}}{R}=\frac{100^{2}}{10.0\times10^{3}}=1\ \text{W}$$

The currents pass through the resistance, inductance and capacitance at resonance condition are:

$$G=\frac{1}{R}=\frac{1}{10.0\times10^{3}}=1.0\times10^{-4}\ \text{S}$$

$$B_L=\frac{1}{\omega_{pr}L}=\frac{1}{2\pi f_{pr}L}=\frac{1}{2\pi\times5.03\times10^{3}\times1.0\times10^{-3}}=0.0316\ \text{S}$$

$$B_C=\omega_{pr}C=2\pi f_{pr}C=2\pi\times5.03\times10^{3}\times1.0\times10^{-6}=0.0316\ \text{S}$$

$$I_R=V_{pr}G=100\times1.0\times10^{-4}=0.01\ \text{A} \qquad I_L=V_{pr}B_L=100\times0.0316=3.16\ \text{A}$$

$$I_C=V_{pr}B_C=100\times0.0316=3.16\ \text{A}$$

The lower and higher cut-off frequency are:

$$\alpha_{pr}=\frac{1}{2RC}=\frac{1}{2\times10\times10^{3}\times1.0\times10^{-6}}=50$$

$$\omega_{pr}=2\pi f_{pr}=2\pi\times5.03\times10^{3}=31622.7766\ \text{rad/s}$$

$$\omega_{pl}=-\alpha_{pr}+\sqrt{\alpha_{pr}^{2}+\omega_{pr}^{2}}=-50+\sqrt{50^{2}+(31622.7766)}=31572.8161\ \text{rad/s}$$

$$\omega_{ph}=\alpha_{pr}+\sqrt{\alpha_{pr}^{2}+\omega_{pr}^{2}}=50+\sqrt{50^{2}+(31622.7766)}=31672.8161\ \text{rad/s}$$

$$f_{pl}=\frac{\omega_{pl}}{2\pi}=\frac{31572.8161}{2\pi}=5024.9698\ \text{Hz}$$

$$f_{ph}=\frac{\omega_{ph}}{2\pi}=\frac{31672.8161}{2\pi}=5040.8852\ \text{Hz}$$

The admittance and impedance at cut-off frequency are:

$$Y_c=\sqrt{2}G=\sqrt{2}\times1.0\times10^{-4}=1.4142\times10^{-4}\ \text{S}$$

$$Z_c=\frac{R}{\sqrt{2}}=\frac{10.0\times10^{3}}{\sqrt{2}}\ \Omega=7.071\ \text{k}\Omega$$

The voltage and power at cut-off frequency are:

$$V_c=IZ_c=10.0\times10^{-3}\times7.07\times10^{3}\ \Omega=70.71\ \text{V}$$

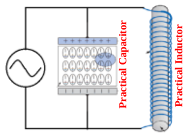
$$P_c=\frac{V_c^{2}}{R}=\frac{70.71^{2}}{10.0\times10^{3}}=0.5\ \text{W}$$

The bandwidth is: $BW=\frac{1}{2\pi RC}=f_{ph}-f_{pl}=5040.8852-5024.9698=15.9155\ \text{Hz}$

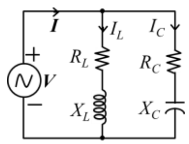
The quality factor is: $Q_{pr}=\frac{R}{X_{L(\text{at resonance})}}=R\sqrt{\frac{C}{L}}=10\times10^{3}\times\sqrt{\frac{1.0\times10^{-6}}{1.0\times10^{-3}}}=316.2278$

Equivalent Parallel Circuit of Parallel Combination of *RL* Series Branch and *RC* Series Branch

Practical *LC* Parallel Circuit



Equivalent Circuit of Practical *LC* Parallel Circuit



Equivalent Parallel Circuit of Parallel Combination of *RL* Series Branch and *RC* Series Branch

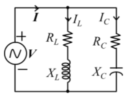
$$\boldsymbol{Z}_L=R_L+jX_L \qquad \boldsymbol{Z}_C=R_C-jX_C \qquad Z_L=\sqrt{R_L^2+X_L^2} \qquad Z_C=\sqrt{R_C^2+X_C^2}$$

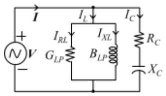
$$\boldsymbol{Y}_L=\frac{1}{\boldsymbol{Z}_L}=\frac{1}{R_L+jX_L}=\frac{R_L}{R_L^2+X_L^2}-j\frac{X_L}{R_L^2+X_L^2}=G_{LP}-jB_{LP}$$

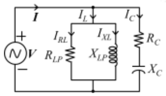
$$G_{LP}=\frac{R_L}{R_L^2+X_L^2} \qquad R_{LP}=\frac{1}{G_{LP}}=\frac{R_L^2+X_L^2}{R_L} \qquad \boldsymbol{I}_{RL}=\frac{\boldsymbol{V}}{R_{LP}}=\boldsymbol{V}G_{LP}=\frac{G_{LP}}{\boldsymbol{Y}_L}\boldsymbol{I}_L$$

$$B_{LP}=\frac{X_L}{R_L^2+X_L^2} \qquad X_{LP}=\frac{1}{B_{LP}}=\frac{R_L^2+X_L^2}{X_L} \qquad \boldsymbol{I}_{XL}=\frac{\boldsymbol{V}}{jX_{LP}}=-j\boldsymbol{V}B_{LP}=\frac{-jB_{LP}}{\boldsymbol{Y}_L}\boldsymbol{I}_L$$

$$\boldsymbol{I}_L=\boldsymbol{I}_{RL}+\boldsymbol{I}_{XL} \qquad \boldsymbol{I}_L=I_{RL}-jI_{XL}$$



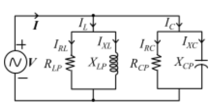


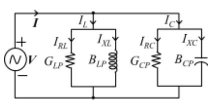


$$\boldsymbol{Y}_C=\frac{1}{\boldsymbol{Z}_C}=\frac{1}{R_C-jX_C}=\frac{R_C}{R_C^2+X_C^2}+j\frac{X_C}{R_C^2+X_C^2}=G_{CP}+jX_{CP}$$

$$G_{CP}=\frac{R_C}{R_C^2+X_C^2} \qquad R_{CP}=\frac{1}{G_{CP}}=\frac{R_C^2+X_C^2}{R_C}$$

$$B_{CP}=\frac{X_C}{R_C^2+X_C^2} \qquad X_{CP}=\frac{1}{B_{CP}}=\frac{R_C^2+X_C^2}{X_C}$$

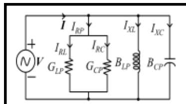




$$\boldsymbol{I}_{RC}=\frac{\boldsymbol{V}}{R_{CP}}=\boldsymbol{V}G_{CP}=\frac{G_{CP}}{\boldsymbol{Y}_C}\boldsymbol{I}_C$$

$$\boldsymbol{I}_{XC}=\frac{\boldsymbol{V}}{-jX_{CP}}=j\boldsymbol{V}B_{CP}=\frac{jB_{CP}}{\boldsymbol{Y}_C}\times\boldsymbol{I}_C$$

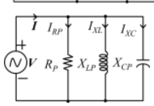
$$\boldsymbol{I}_C=\boldsymbol{I}_{RC}+\boldsymbol{I}_{XC} \qquad \boldsymbol{I}_C=I_{RC}+jI_{XC}$$



Total admittance:

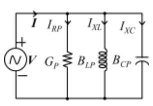
$$\boldsymbol{Y}=\boldsymbol{Y}_L+\boldsymbol{Y}_C=(G_{LP}+G_{CP})-jB_{LP}+jB_{CP}$$

Let, $G_P=G_{LP}+G_{CP}$ $\qquad \boldsymbol{Y}=G_P-jB_{LP}+jB_{CP}$



$$\frac{1}{R_P}=\frac{1}{R_{LP}}+\frac{1}{R_{LP}}$$

$$\boldsymbol{Y}=\frac{1}{R_P}-j\frac{1}{X_{LP}}+j\frac{1}{X_{CP}}$$



$$\boldsymbol{I}_{RP}=\boldsymbol{I}_{RL}+\boldsymbol{I}_{RC}=\frac{\boldsymbol{V}}{R_P}=\boldsymbol{V}G_P$$

$$\boldsymbol{I}=\boldsymbol{I}_{RP}+\boldsymbol{I}_{XL}+\boldsymbol{I}_{XC}$$

Vector Diagram

$$Z_L=\sqrt{R_L^2+X_L^2} \qquad I_{RL}=I_L\cos\theta_L$$

$$\theta_L=\tan^{-1}\left[\frac{X_L}{R_L}\right] \qquad I_{XL}=I_L\sin\theta_L$$

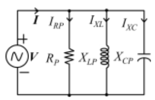
$$I_L=\frac{V}{Z_L} \qquad I_L=\sqrt{I_{RL}^2+I_{XL}^2}$$

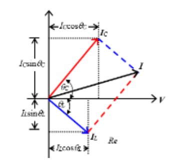
$$Z_C=\sqrt{R_C^2+X_C^2} \qquad I_{RC}=I_C\cos\theta_C$$

$$\theta_C=-\tan^{-1}\left[\frac{X_C}{R_C}\right] \qquad I_{XC}=I_C\sin\theta_C$$

$$I_C=\frac{V}{Z_C}$$

$$I_{RP}=I_{RL}+I_{RC}=I_L\cos\theta_L+I_C\cos\theta_C$$





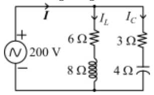
Example

The impedances 6+*j*8 ohm and 3-*j*4 are connected in parallel as shown in the following figure. **(i)** Find the conductance and susceptance of each branch, **(ii)** find the total conductance and susceptance, **(iii)** Draw the equivalent parallel circuit by showing the conductance and capacitive susceptance, and **(iv)** Draw the equivalent parallel circuit by showing the equivalent resistance and capacitive reactance.

Solution: Given: R_L=6 ohm; X_L=8 ohm; R_C=3 ohm; X_C=4 ohm

$$G_{LP}=\frac{R_L}{R_L^2+X_L^2}=\frac{6}{6^2+8^2}=0.06\ \text{S} \qquad X_{LP}=\frac{X_L}{R_L^2+X_L^2}=\frac{8}{6^2+8^2}=0.08\ \text{S}$$

$$G_{CP}=\frac{R_C}{R_C^2+X_C^2}=\frac{3}{3^2+4^2}=0.12\ \text{S} \qquad X_{CP}=\frac{X_C}{R_C^2+X_C^2}=\frac{4}{3^2+4^2}=0.16\ \text{S}$$



Alternative Way:

$$\boldsymbol{Y}_L=\frac{1}{\boldsymbol{Z}_L}=G_{LP}-jB_{LP}=\frac{1}{6+j8}=0.06-j0.08\ \text{S}$$

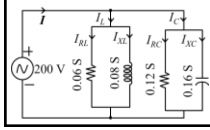
$$\boldsymbol{Y}_C=\frac{1}{\boldsymbol{Z}_C}=G_{CP}+jB_{CP}=\frac{1}{3-j4}=0.12+j0.16\ \text{S}$$

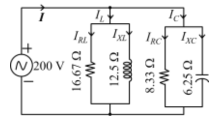
$$R_{LP}=\frac{1}{G_{LP}}=\frac{1}{0.06}=16.67\ \Omega \qquad \boldsymbol{I}_L=I_{RL}-jI_{XL}=\boldsymbol{VY}_L=200(0.06-j0.08)=12-j16\ \text{A}$$

$$X_{LP}=\frac{1}{B_{LP}}=\frac{1}{0.08}=12.5\ \Omega \qquad \boldsymbol{I}_C=I_{RC}+jI_{XC}=\boldsymbol{VY}_C=200(0.12+j0.16)=24+j32\ \text{A}$$

$$R_{CP}=\frac{1}{G_{CP}}=\frac{1}{0.12}=8.33\ \Omega \qquad I_{RL}=12\ \text{A} \qquad I_{RC}=24\ \text{A}$$

$$X_{CP}=\frac{1}{B_{CP}}=\frac{1}{0.16}=6.25\ \Omega \qquad I_{XL}=16\ \text{A} \qquad I_{CX}=32\ \text{A}$$





Parallel Resonance of Practical Parallel inductor and Capacitor Circuit

The total admittance of a practical parallel inductor and capacitor circuit is given by:

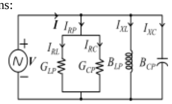
$$\boldsymbol{Y}=\boldsymbol{Y}_L+\boldsymbol{Y}_C=(G_{LP}+G_{CP})+j(B_{CP}-B_{LP})$$

The practical parallel inductor and capacitor circuit will be resonance if the imaginary part of admittance is zero that means:

$$B_{CP}-B_{LP}=0$$

$$B_{LP}=B_{CP}$$

$$\frac{X_L}{R_L^2+X_L^2}=\frac{X_C}{R_C^2+X_C^2}$$



$$\frac{1}{X_{CP}}-\frac{1}{X_{LP}}=0$$

$$\boldsymbol{X}_{LP}=\boldsymbol{X}_{CP}$$

$$\frac{R_L^2+X_L^2}{X_L}=\frac{R_C^2+X_C^2}{X_C}$$

$$\boldsymbol{Y}_{pr}=G_{LP}+G_{CP}=\left(\frac{1}{R_{LP}}+\frac{1}{R_{CP}}\right)$$

$$\boldsymbol{Z}_{pr}=\frac{1}{\boldsymbol{Y}_{pr}}=\frac{1}{G_{LP}+G_{CP}}=\left(\frac{1}{R_{LP}}+\frac{1}{R_{CP}}\right)^{-1}$$

$$I_{XL}=I_{XC} \quad \therefore I_L\sin\theta_L=I_C\sin\theta_C$$

$$\boldsymbol{I}=\boldsymbol{I}_{RP}=\frac{\boldsymbol{V}}{R_P}=I_L\cos\theta_L+I_C\cos\theta_C$$

Dynamic Impedance:

The impedance of a parallel resonance circuit is called **Dynamic Impedance**. The dynamic impedance for the practical parallel *L* and *C* circuit is given by:

$$Z_d=Z_{pr}=R_P=\frac{1}{G_{LP}+G_{CP}}=\left(\frac{1}{R_{LP}}+\frac{1}{R_{CP}}\right)^{-1}$$

Dynamic Admittance: $Y_d=\frac{1}{Z_d}=\frac{1}{R_P}=G_{LP}+G_{CP}$

At resonance condition the magnitude of currents are given by:

$$I_{XL}=VB_{LP}=\frac{V}{X_{LP}} \qquad I_{XC}=VB_{CP}=\frac{V}{X_{CP}} \qquad I=VY_d=\frac{V}{Z_d}$$

Quality Factor

$$Q_{pr}=\frac{I_{XL}}{I}=\frac{V/X_{LP}}{V/Z_d}=\frac{Z_d}{X_{LP}} \qquad Q_{pr}=\frac{I_{XC}}{I}=\frac{V/X_{CP}}{V/Z_d}=\frac{Z_d}{X_{CP}}$$

Resonance Frequency of Practical Parallel inductor and Capacitor Circuit

Let f_{rp} and ω_{pr} are the parallel resonance frequency and angular frequency at the condition of resonance. The condition of parallel resonance can be written as follows:

$$\frac{X_C}{R_C^2+X_C^2}=\frac{X_L}{R_L^2+X_L^2} \qquad R_L^2X_C+X_L^2X_C=R_C^2X_L+X_LX_C^2 \qquad R_L^2+X_L^2=\frac{R_C^2X_L}{X_C}+X_LX_C$$

$$R_L^2\frac{1}{\omega_{pr}C}+\omega_{pr}^2L^2\frac{1}{\omega_{pr}C}=R_C^2\omega_{pr}L+\omega_{pr}L\frac{1}{\omega_{pr}^2C^2}$$

$$\omega_{pr}=\omega_{sr}\sqrt{\frac{R_L^2-(L/C)}{R_C^2-(L/C)}}$$

$$f_{pr}=f_{sr}\sqrt{\frac{R_L^2-(L/C)}{R_C^2-(L/C)}}$$

The frequency will be real if any one of the following condition is satisfied:

(*i*) $R_L^2>(L/C)$ *and* $R_C^2>(L/C)$

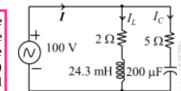
(*ii*) $R_L^2<(L/C)$ *and* $R_C^2<(L/C)$

If R_L=R_C we have

$$\omega_{pr}=\frac{1}{\sqrt{LC}}=\omega_{sr}$$

$$f_{pr}=\frac{1}{2\pi\sqrt{LC}}=f_{sr}$$

Example: For the following circuit, calculate **(i)** the resonance frequency, **(ii)** the dynamic impedance and the dynamic admittance, **(iii)** the *RL* branch current, **(iv)** the *RC* branch current, **(v)** the inductive branch current (I_{XL}) and the capacitive branch current (I_{XC}), **(vi)** the total current, **(vii)** the quality factor, and **(viii)** the bandwidth.



Solution: Given: R_L=2 ohm; L=24.3 mH; R_C=5 ohm; C=200 μF

(i) $$f_{sr}=\frac{1}{2\pi\sqrt{LC}}=\frac{1}{2\pi\sqrt{24.3\times10^{-3}\times200\times10^{-6}}}=74.2\ \text{Hz}$$

$$f_{pr}=f_{sr}\sqrt{\frac{R_L^2-(L/C)}{R_C^2-(L/C)}}=f_{sr}\sqrt{\frac{2^2-\left[24.3\times10^{-3}/\left(200\times10^{-6}\right)\right]}{5^2-\left[24.3\times10^{-3}/\left(200\times10^{-6}\right)\right]}}=82.41\ \text{Hz}$$

(ii) $$X_L=2\pi f_{pr}L=11.91\ \Omega \qquad X_C=\frac{1}{2\pi f_{pr}C}=9.66\ \Omega$$

$$\boldsymbol{Z}_L=2+j11.91\ \Omega \qquad \boldsymbol{Z}_C=5-j9.66\ \Omega$$

$$\boldsymbol{Z}_d=\frac{\boldsymbol{Z}_L\boldsymbol{Z}_C}{\boldsymbol{Z}_L+\boldsymbol{Z}_C}=17.87+j0\ \Omega$$

(iii) $$\boldsymbol{I}_L=\frac{\boldsymbol{V}}{\boldsymbol{Z}_L}=\frac{100}{2+j11.91}=1.37-j8.16\ \text{A}$$

(iv) $$\boldsymbol{I}_C=\frac{\boldsymbol{V}}{\boldsymbol{Z}_C}=\frac{100}{5-j9.66}=4.23+j8.16\ \Omega$$

(v) $$I_{LX}=\text{Im}[\boldsymbol{I}_L]=8.16\ \text{A}$$

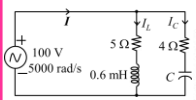
$$I_{CX}=\text{Im}[\boldsymbol{I}_C]=8.16\ \text{A}$$

(vi) $$\boldsymbol{I}=\frac{\boldsymbol{V}}{\boldsymbol{Z}_d}=\boldsymbol{I}_L+\boldsymbol{I}_C=\frac{100}{17.87}=5.6\ \text{A}$$

(vii) $$Q_{pr}=\frac{I_{LX}}{I}=\frac{8.16}{5.6}=1.46$$

(viii) $$BW=\frac{f_{pr}}{Q_{pr}}=\frac{82.41}{1.46}=56.44\ \text{Hz}$$

Example: A *RL* series circuit having the resistive value is 5 ohm and the inductance value is 0.6 mH is connected in parallel with a *RC* series circuit having the resistance value is 4 ohm and the capacitance is variable. If the applied source is 100 V with 5000 rad/s, Calculate **(i)** the value of capacitance for the resonance, **(ii)** the dynamic impedance, **(iii)** the total current at resonance condition.



Solution: Given, R_L= 5 Ω; and L= 0.6 mH, R_C= 4 Ω; V= 100 V and ω=5000 rad/s.

$$X_L=\omega L=5000\times0.6\times10^{-3}=3\ \Omega \qquad \boldsymbol{Z}_L=5+j3\ \Omega$$

At resonance condition: $$\frac{X_L}{R_L^2+X_L^2}=\frac{X_C}{R_C^2+X_C^2} \qquad \frac{X_C}{4^2+X_C^2}=\frac{3}{5^2+3^2} \qquad \frac{X_C}{16+X_C^2}=\frac{3}{34}$$

$$3X_C^2-34X_C+48=0 \qquad X_C=\frac{-(-34)\pm\sqrt{(-34)^2-4\times48\times3}}{2\times3}=9.68\ \Omega\text{ or }1.65\ \Omega$$

For X_C=9.68 Ω:

$$C=\frac{1}{\omega X_C}=\frac{1}{5000\times9.68}=20.66\,\mu F \qquad \boldsymbol{Z}_C=4-j9.68\ \Omega$$

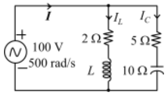
$$\boldsymbol{Z}_d=\frac{\boldsymbol{Z}_L\boldsymbol{Z}_C}{\boldsymbol{Z}_L+\boldsymbol{Z}_C}=5.4466\ \Omega \qquad I=\frac{V}{Z_d}=\frac{100}{5.4466}=18.36\ \text{A}$$

For X_C=1.65 Ω:

$$C=\frac{1}{\omega X_C}=\frac{1}{5000\times1.65}=121\,\mu F \qquad \boldsymbol{Z}_C=4-j1.65\ \Omega$$

$$\boldsymbol{Z}_d=\frac{\boldsymbol{Z}_L\boldsymbol{Z}_C}{\boldsymbol{Z}_L+\boldsymbol{Z}_C}=2.7732\ \Omega \qquad I=\frac{V}{Z_d}=\frac{100}{2.7732}=36.06\ \text{A}$$

Example: A *RL* series circuit having the resistive value is 2 ohm and the inductance is variable is connected in parallel with a *RC* series circuit having the resistive value is 5 ohm and the reactance of capacitor is 10 ohm. If the applied source angular frequency is 500 rad/s, calculate the value of inductance for the resonance.



Solution: Given, R_L= 2 Ω; and R_C= 5 Ω; X_C= 10 Ω; V= 100 V and ω=500 rad/s.

$$Z_C = 5 - j10\ \Omega$$

At resonance condition: $\dfrac{X_L}{R_L^2+X_L^2} = \dfrac{X_C}{R_C^2+X_C^2}$ $\dfrac{X_L}{2^2+X_L^2} = \dfrac{10}{5^2+10^2}$ $\dfrac{X_L}{4+X_L^2} = \dfrac{10}{125}$

$$10X_C^2 - 125X_C + 40 = 0 \qquad X_L = \frac{-(-125)\pm\sqrt{(-125)^2-4\times40\times10}}{2\times10} = 12.1714\ \Omega \text{ or } 0.3286\ \Omega$$

For X_L=12.1714 Ω:

$$L = \frac{X_L}{\omega} = \frac{12.1714}{500} = 24.3\text{mH}$$

For X_L=0.3286 Ω:

$$L = \frac{X_L}{\omega} = \frac{0.3286}{500} = 0.65728\text{mH}$$

Example: A *RL* series branch is connected with a *RC* series branch. The current of *RL* branch and *RC* branch are $\boldsymbol{I}_L$=10-j20 A and $\boldsymbol{I}_C$=5+j20 A. (a) Is this circuit is resonance? Justify your answer. (b) Calculate the parameters (R_L, R_C, L, and C) of the circuit if the supplied voltage is 220∠0° V with 60 Hz.

Solution: Given, $\boldsymbol{I}_L$=10-j20 A; $\boldsymbol{I}_C$=5+j20 A; V= 220∠0° V and f=60 Hz.

(a) Yes, this circuit is resonance circuit. From the given data we have

$$I_{LX} = 20\ \text{A and } I_{CX} = 20\ \text{A}$$

Since the imaginary parts of current are equal ($I_{LX}=I_{CX}$) the circuit is resonance.

(b) The impedance and admittance of *RL* and *RC* series circuit can be calculated as follows:

$$\boldsymbol{Z}_L = \frac{\boldsymbol{V}}{\boldsymbol{I}_L} = \frac{220}{10-j20} = 4.4 + j8.8\ \Omega \qquad \boldsymbol{Z}_C = \frac{\boldsymbol{V}}{\boldsymbol{I}_C} = \frac{220}{5+j20} = 2.6 - j10.35\ \Omega$$

$$\boldsymbol{Y}_L = \frac{1}{\boldsymbol{Z}_L} = \frac{1}{4.4+j8.8} = 0.0455 - j0.0909\ \Omega \qquad \boldsymbol{Y}_C = \frac{1}{\boldsymbol{Z}_C} = \frac{1}{2.6-j10.35} = 0.0227 + j0.0909\ \Omega$$

From impedance and admittance we have:

$$R_L = 4.4\ \Omega \qquad X_L = 8.8\ \Omega$$
$$R_C = 2.6\ \Omega \qquad X_C = 10.35\ \Omega$$
$$G_L = 0.0455\ \text{S} \qquad B_L = 0.0909\ \text{S}$$
$$G_C = 0.0277\ \text{S} \qquad B_C = 0.0909\ \text{S}$$

$$\omega = 2\pi\times60 = 377\ \text{rad/s}$$

$$L = \frac{X_L}{\omega} = \frac{8.8}{377} = 23.3\text{mH}$$

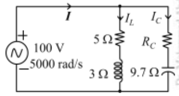
$$C = \frac{1}{\omega X_C} = \frac{1}{377\times10.35} = 256.28\mu\text{F}$$

Example: A *RL* series circuit having the resistive value is 5 ohm and the inductive reactance is 3 ohm is connected in parallel with a *RC* series circuit having the variable resistance (R_C) and the capacitive reactance is 9.7 ohm. If the applied source is 100 V with 5000 rad/s, Calculate **(i)** the value of capacitive branch resistance (R_C) for the resonance, **(ii)** the dynamic impedance.

Solution: Given, R_L= 5 Ω; and X_L= 3 ohm, X_C= 9.7 Ω; V= 100 V and ω=5000 rad/s.

$$\boldsymbol{Z}_L = 5 + j3\ \Omega \quad \boldsymbol{Y}_L = \frac{1}{\boldsymbol{Z}_L} = \frac{1}{5+j3} = 0.1471 - j0.0882\ \text{S}$$

$$G_{LP} = 0.1471\ \text{S} \qquad B_{LP} = 0.0882\ \text{S}$$



At resonance condition: $X_{CP} = X_{LP}$

$$\frac{R_C^2+X_C^2}{X_C} = \frac{R_L^2+X_L^2}{X_L} \qquad \frac{R_C^2+9.7^2}{9.7} = \frac{5^2+3^2}{3} = 11.33 \qquad R_C^2 + 94.09 = 109.9$$

$$R_C^2 = 109.9 - 94.09 = 15.81 \qquad R_C = 109.9 - 94.09 = \sqrt{15.81} = 3.98\ \Omega$$

$$\boldsymbol{Z}_C = 3.98 + j9.7\ \Omega \qquad \boldsymbol{Y}_C = \frac{1}{\boldsymbol{Z}_C} = \frac{1}{3.98-j9.7} = 0.0362 + j0.0882\ \text{S}$$

At resonance condition: $\boldsymbol{Y} = \boldsymbol{Y}_L + \boldsymbol{Y}_C = 0.1471 + 0.0362 = 0.1833\ \text{S}$

Dynamic impedance: $Z_d = \dfrac{1}{Y} = \dfrac{1}{0.1833} = 5.455\ \Omega$

Homework

Problem 1: Two impedances $\boldsymbol{Z}_1$ = 25 - j1 ohm and $\boldsymbol{Z}_2$ = 100 + jX_L ohm are connected in parallel across a voltage source. Find the value of X_L which will produce resonance.

Ans: X_L = 609.59 ohm or 16.4043 ohm

Problem 2: A *RL* series circuit having the resistance (R) is variable and the inductance value is 1 mH is connected with a 20 µF capacitor. If the resonance is occurred at 300 Hz, calculate the value of R.

Ans: R = 6.82 ohm

Problem 3: Two impedances $\boldsymbol{Z}_1$ = R_L + j10 ohm and $\boldsymbol{Z}_2$ = 10 – j5 ohm are connected in parallel across a voltage source. Find the value of R_L which will produce resonance.

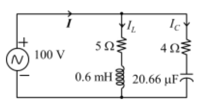
Ans: R_L = 12.25 ohm

Problem 4: Two impedances $\boldsymbol{Z}_1$ = 8 + j6 ohm and $\boldsymbol{Z}_2$ = 8 – jX_C ohm are connected in parallel across a voltage source which angular frequency is 5000 rad/s. Find the value of X_C and the value of capacitance which will produce resonance.

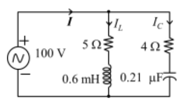
Ans: X_C = 10.67 or 6 ohm, and C= 18.75 or 33.33 µF.

Homework

Problem 1: For the following circuits, calculate **(i)** the resonance frequency, **(ii)** the total current, the inductive branch current, the capacitive branch current and quality factor at resonance condition.



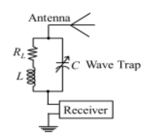
(a)



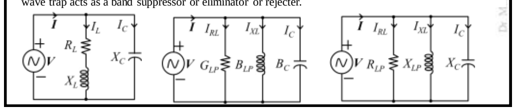
(b)

Wave Trap Circuit

Resonance phenomenon forms the basis of many circuits **used in wire and wireless communication systems**. They are specially adapted to selective circuits used for **filters and oscillators (repetitive wave generator)**. A parallel combination of inductor and capacitor can work as **band eliminator or rejctor, suppressor or wave trap**. A wave trap is connected in series with antenna and receiver as shown in the following figure. By proper design the dynamic impedance at resonance frequency can be made about 10 times the impedance at frequencies ±20 kHz away from resonant frequency within the standard broadcast band. Thus wave trap acts as a band suppressor or eliminator or rejecter.



Wave Trap Circuit



Resonance for a Parallel of *RL* series with Lossless *C* [Wave Trap]

Total Admittance of wave trap circuit: $\boldsymbol{Y} = \boldsymbol{Y}_L + \boldsymbol{Y}_C = G_{LP} + j(B_C - B_{LP}) = \dfrac{1}{R_{LP}} + j\left(\dfrac{1}{X_C} - \dfrac{1}{X_{LP}}\right)$

The wave trap circuit will be resonance if the imaginary part of admittance is zero that means:

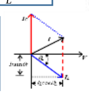
$$B_{LP} = B_C \qquad \frac{X_L}{R_L^2+X_L^2} = \frac{1}{X_C}$$

$$X_{LP} = X_C \qquad \frac{R_L^2+X_L^2}{X_L} = \frac{Z_L^2}{X_L} = X_C$$

At resonance condition of a Wave Trap Circuit

$$Z_L = \sqrt{X_L X_C} = \sqrt{\frac{L}{C}} \qquad \boldsymbol{Y}_{pr} = G_{LP} = \frac{1}{R_{LP}} \qquad \boldsymbol{Z}_{pr} = \frac{1}{\boldsymbol{Y}_{pr}} = \frac{1}{G_{LP}} = R_{LP}$$

$$I_{XL} = I_C \qquad \therefore I_L \sin\theta_L = I_C \qquad \boldsymbol{I} = \boldsymbol{I}_{RL} = \frac{\boldsymbol{V}}{R_{LP}} = I_L\cos\theta_L$$



Resonance Frequency for a Wave Trap Circuit

$$\omega_{pr} = \omega_{sr}\sqrt{1-\frac{R_L^2 C}{L}} \qquad f_{pr} = f_{sr}\sqrt{1-\frac{R_L^2 C}{L}}$$

Dynamic Impedance and Currents of a Wave Trap Circuit

$$Z_d = Z_{pr} = R_{LP} = \frac{1}{G_{LP}} = \frac{R_L^2 + X_L^2}{R_L} = \frac{Z_L^2}{R_L} = \frac{L}{R_L C} \qquad I = \frac{V}{Z_d} \qquad I_{XL} = \frac{V}{X_{LP}} \qquad I_C = \frac{V}{X_C}$$

Quality Factor of a Wave Trap Circuit

$$Q_{pr} = \frac{I_{XL}}{I} = \frac{I_C}{I} = \frac{Z_d}{X_{LP}} = \frac{Z_d}{X_C} = \frac{Z_L^2}{R_L X_C} = \frac{X_L X_C}{R_L X_C} = \frac{X_L}{R_L} \qquad Z_d = Q_{pr} X_{LP} = Q_{pr} X_C$$

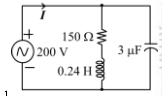
Special Case of a Wave Trap Circuit

If $R_L \approx 0$ then $Q_{pr} >> 10$ thus:

$$\omega_{pr} = \frac{1}{\sqrt{LC}} = \omega_{sr} \qquad f_{pr} = \frac{1}{2\pi\sqrt{LC}} = f_{sr} \qquad Q_{pr} = \frac{X_L}{R_L} = \frac{\omega_{pr} L}{R_L} = \frac{1}{R_L}\sqrt{\frac{L}{C}}$$

Example

A practical resonant circuit consists of a coil, having a resistance of 150 Ω and 0.24 H, in parallel with a lossless capacitor of capacitance 3 μF. (**a**) Find the resonance frequency. (**b**) Find the impedance and current at resonance condition. (**c**) Find the impedance and current if the frequency is half of resonance frequency. (**d**) Find the impedance and current if the frequency is double of resonance frequency.



(**a**) $f_{pr} = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{R_L^2 C}{L}}$

$$= \frac{1}{2\pi\sqrt{0.24\times 3\times 10^{-6}}}\sqrt{1 - \frac{(150)^2 \times 3\times 10^{-6}}{0.24}} = 159.02 \text{ Hz}$$

(**b**) $X_L = 2\pi\times 159.02\times 0.24 = 239.8\ \Omega \qquad X_C = \frac{1}{2\pi\times 159.02\times 3\times 10^{-6}} = 333.6\ \Omega$

$\mathbf{Z}_L = 150 + j239.8\ \Omega \qquad \mathbf{Z}_C = -j333.6\ \Omega \qquad \mathbf{Y}_L = 1/\mathbf{Z}_L = 0.0019 - j0.003\ S$

$\mathbf{Y}_C = 1/\mathbf{Z}_C = j0.003\ S \qquad \mathbf{Y}_T = \mathbf{Y}_L + \mathbf{Y}_C = 0.0019\ S \qquad \mathbf{Z}_T = 1/\mathbf{Y}_T = 1/0.0019 = 533.35\ \Omega$

$\mathbf{I}_T = \mathbf{V}/\mathbf{Z}_T = 200/533.35 = 0.375$ A

(**c**) $f = \frac{159.02}{2} = 79.545$ Hz $\qquad X_L = 2\pi\times 79.545\times 0.24 = 119.951\ \Omega$

$X_C = \frac{1}{2\pi\times 79.545\times 3\times 10^{-6}} = 666.94\ \Omega \qquad \mathbf{Z}_L = 150 + j119.951\ \Omega$

$\mathbf{Z}_C = -j666.94\ \Omega \qquad \mathbf{Y}_L = 1/\mathbf{Z}_L = 0.0041 - j0.0033\ S \qquad \mathbf{Y}_C = 1/\mathbf{Z}_C = j0.0015\ S$

$\mathbf{Y}_T = \mathbf{Y}_L + \mathbf{Y}_C = 0.0041 - j0.0018\ S \qquad \mathbf{Z}_T = 1/\mathbf{Y}_T = 207.4 + j89.38 = 226\angle 23.3°\ \Omega$

$\mathbf{I}_T = \mathbf{V}/\mathbf{Z}_T = 200/226\angle 23.3° = 0.885\angle -23.3°$ A

(**d**) $f = 159.02\times 2 = 318.18$ Hz $\qquad X_L = 2\pi\times 318.18\times 0.24 = 480\ \Omega$

$X_C = \frac{1}{2\pi\times 318.18\times 3\times 10^{-6}} = 166.73\ \Omega \qquad \mathbf{Z}_L = 150 + j480\ \Omega$

$\mathbf{Z}_C = -j166.73\ \Omega \qquad \mathbf{Y}_L = 1/\mathbf{Z}_L = 0.0006 - j0.0019\ S \qquad \mathbf{Y}_C = 1/\mathbf{Z}_C = j0.006\ S$

$\mathbf{Y}_T = \mathbf{Y}_L + \mathbf{Y}_C = 0.0006 + j0.0041\ S \qquad \mathbf{Z}_T = 1/\mathbf{Y}_T = 34.6 - j238.9 = 241.4\angle -82°\ \Omega$

$\mathbf{I}_T = \mathbf{V}/\mathbf{Z}_T = 200/241.4\angle 23.3° = 0.83\angle 82°$ A

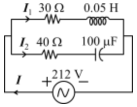
Example: A practical resonant circuit consists of a coil, having a resistance of 15 Ω and 0.05 H, in parallel with a RC series circuit of resistance 20 Ω and capacitance 100 μF is connected across 212 V. (**a**) Find the resonance frequency. (**b**) Find the impedance, currents, quality factor and bandwidth at resonance condition.

$f_{sr} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.05\times 100\times 10^{-6}}} = 71.18$ Hz $\qquad f_{pr} = f_{sr}\sqrt{\frac{R_L^2 - (L/C)}{R_C^2 - (L/C)}} = 42.92$ Hz

$X_L = 2\pi f_{pr} L = 13.48\ \Omega \qquad X_C = \frac{1}{2\pi f_{pr} C} = 37.08\ \Omega$

$\mathbf{Z}_1 = 30 + j13.48\ \Omega \qquad \mathbf{Y}_1 = \frac{1}{30 + j13.48} = 0.028 - j0.0125\ S$

$\mathbf{Z}_2 = 40 - j37.08\ \Omega \qquad \mathbf{Y}_2 = \frac{1}{40 - j37.08} = 0.0134 + j0.0125\ S$



$\mathbf{Y}_T = \mathbf{Y}_1 + \mathbf{Y}_2 = \frac{1}{30 + j13.48} = 0.0412\ S$

$\mathbf{Y}_T = \mathbf{Y}_1 + \mathbf{Y}_2 = \frac{1}{30 + j13.48} = 0.0412\ S$

$\mathbf{Z}_d = \frac{1}{\mathbf{Y}_T} = 17.87\ \Omega$

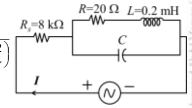
$\mathbf{I}_1 = \mathbf{V}\mathbf{Y}_1 = 5.88 - j2.64$ A $\qquad \mathbf{I}_2 = \mathbf{V}\mathbf{Y}_2 = 2.85 + j2.64$ A $\qquad \mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_d} = 11.86$ A

$Q_{pr} = \frac{I_{1X}}{I} = \frac{2.64}{11.86} = 0.2226$

$BW = \frac{f_{pr}}{Q_{pr}} = \frac{42.92}{0.2226} = 192.699$ Hz

Example

A series-parallel circuit is shown in the following figure. Calculate (i) the capacitor for resonance, (ii) the Q-factor, (iii) the dynamic impedance, (iv) the total equivalent impedance and (v) the total line current.



230 V, 1 MHz

$X_L = 2\pi f L = 2\pi\times 1\times 10^6 \times 0.2\times 10^{-3} = 1256.6\ \Omega$

At resonance: $\frac{R^2 + X_L^2}{X_L} = X_C \qquad \therefore\ C = \frac{X_L}{2\pi f\left(R^2 + X_L^2\right)}$

$C = \frac{1256.6}{2\pi\times 1\times 10^6 \times\left(20^2 + 1256.6^2\right)}$

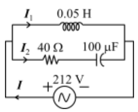
$= 12.66\times 10^{-12}\ F = 12.66\ pF$

$Q_{pr} = \frac{X_L}{R_L} = \frac{1256.6}{20} = 62.83 \qquad Z_d = \frac{L}{RC} = \frac{200\times 10^{-6}}{20\times 126.6\times 10^{-12}} = 78989\ \Omega$

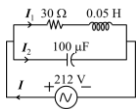
$Z_T = Z_d + R_s = 78989 + 8000 = 86989\ \Omega \qquad I = \frac{V}{Z_T} = \frac{230}{86989} = 2.644$ mA

Homework

Problem 1: For the following circuits: (**a**) Find the resonance frequency. (**b**) Find the impedance, currents, quality factor and bandwidth at resonance condition.



(**a**)

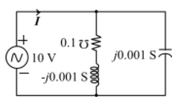


(**b**)

Problem 2: Two impedances: $\mathbf{Z}_L = R_L + j0.24\omega$ and $\mathbf{Z}_C = R_C + j[1/(3\times 10^{-6}\omega)]$ are connected in parallel with a voltage source of 200 V. Determine the resonant frequency, the source current and the input impedance for the following cases:

Case I:	R_L=150 ohm	R_C=100 ohm	**Case II**:	R_L=150 ohm	R_C=0 ohm
Case III:	R_L=0 ohm	R_C=100 ohm	**Case IV**:	R_L=0 ohm	R_C=0 ohm

Problem 3: The following circuit impedance are given at resonance condition. (**a**) Find the impedance and current at resonance condition. (**b**) Find the impedance and current if the frequency is half of resonance frequency. (**c**) Find the impedance and current if the frequency is double of resonance frequency.



$$Q_{pr} = \frac{X_L}{R_L}$$

$$Z_d = Q_{pr} X_{LP} = Q_{pr} X_C$$

Parallel Resonance by Varying the Inductor

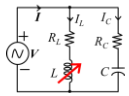
Capacitor and Supply Voltage and Frequency are Fixed

The magnitude of inductive and capacitive branches current are given by:

$$I_L = \frac{V}{\sqrt{R_L^2 + X_L^2}} \qquad I_C = \frac{V}{\sqrt{R_C^2 + X_C^2}}$$

The angle of inductive and capacitive branches impedance are given by:

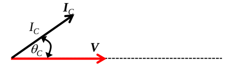
$$\theta_L = \tan^{-1}\left[\frac{X_L}{R_L}\right] \qquad \theta_C = -\tan^{-1}\left[\frac{X_C}{R_C}\right]$$



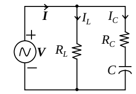
Since the capacitor and supply voltage and frequency are fixed, the fixed quantities are: I_C, θ_C.
The quantities which are varies with the variation of inductor are: I_L, θ_L.

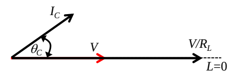
Now, the drawing of locus of inductive branch current and total current is going to discuss.

By considering the supply voltage $\boldsymbol{V}$ as a reference the vector of capacitive branch I_C (which is fixed) can be drawn as following figure.

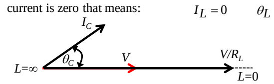


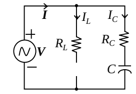
Let, L=0 then X_L=0 (short-circuit), thus inductive branch current is given by: $I_L = \frac{V}{R_L}$ $\quad \theta_L = 0$





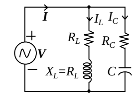
Let, $L=\infty$ then $X_L=\infty$ (open-circuit), thus inductive branch current is zero that means: $I_L = 0$ $\quad \theta_L = 0$

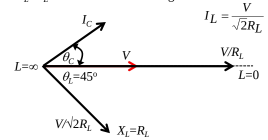




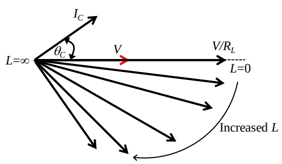
If $X_L=R_L$ then the current and angle of inductor branch current are:

$$I_L = \frac{V}{\sqrt{2}R_L} \qquad \theta_L = 45°$$

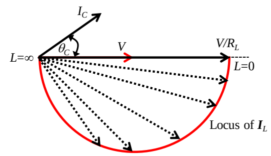




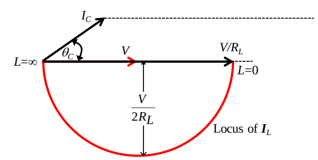
By changing the value of L and draw the vector of $\boldsymbol{I_L}$ it look like as following Figure:



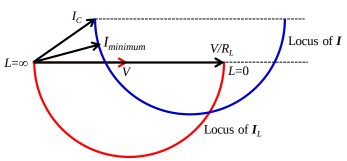
By connecting the all end points of vectors of I_L it will make a semicircle which diameter is V/R_L and radius is $V/2R_L$. This semicircle is called the locus (or path) of $\boldsymbol{I_L}$.



In order to draw the locus of total current $\boldsymbol{I=I_C+I_L}$, first draw a parallel line reference at the end point of $\boldsymbol{I_C}$ vector.

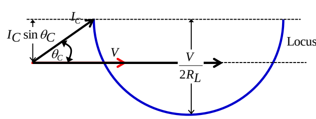


Now just shift the locus of I_L at the end of $\boldsymbol{I_C}$ vector. The new semicircle represents the locus of $\boldsymbol{I}$.

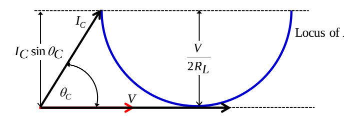


Three cases is possible here.

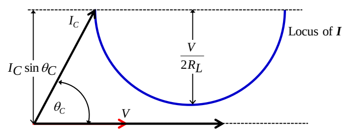
Case I: If $(V/2R_L) > (I_C \sin\theta_C)$, there will be two resonant points.



Case II: If $(V/2R_L) = (I_C \sin\theta_C)$, there will be only one resonant point.



Case III: If $(V/2R_L) < (I_C \sin\theta_C)$, parallel resonant cannot be obtained regardless of the value of L.

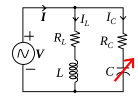


Parallel Resonance by Varying the Capacitor

Inductor and Supply Voltage and Frequency are Fixed

The magnitude of inductive and capacitive branches current are given by:

$$I_L = \frac{V}{\sqrt{R_L^2 + X_L^2}} \qquad I_C = \frac{V}{\sqrt{R_C^2 + X_C^2}}$$



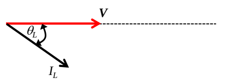
The angle of inductive and capacitive branches impedance are given by:

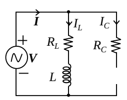
$$\theta_L = \tan^{-1}\left[\frac{X_L}{R_L}\right] \qquad \theta_C = -\tan^{-1}\left[\frac{X_C}{R_C}\right]$$

Since the inductor and supply voltage and frequency are fixed, the fixed quantities are: I_L, θ_L.
The quantities which are varies with the variation of capacitor are: I_C, θ_C.

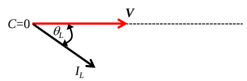
Now, the drawing of locus of inductive branch current and total current is going to discuss.

By considering the supply voltage $\boldsymbol{V}$ as a reference the vector of inductive branch $\boldsymbol{I}_C$ (which is fixed) can be drawn as following figure.

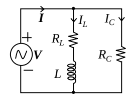


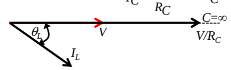


Let, C=0 then $X_C=\infty$ (open-circuit), thus capacitive branch current is given by: $I_C = 0 \qquad \theta_C = 0$

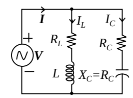


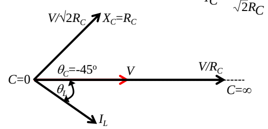
Let, $C=\infty$ then $X_C= 0$ (short-circuit), thus capacitive branch current is as follows: $I_C = \frac{V}{R_C} \qquad \theta_C = 0$



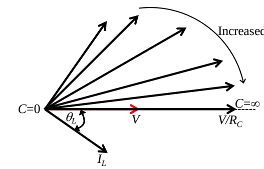


If $X_C=R_C$ then the current and angle of capacitor branch current are: $I_C = \frac{V}{\sqrt{2}R_C} \qquad \theta_C = -45°$

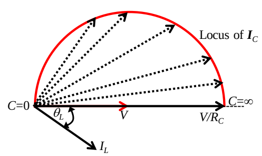




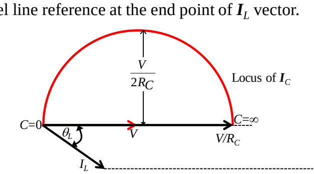
By changing the value of C and draw the vector of $\boldsymbol{I}_C$ it look like as following Figure:

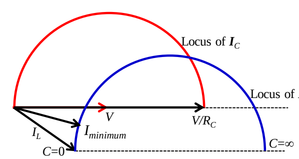


By connecting the all end points of vectors of $\boldsymbol{I}_C$ it will make a semicircle which diameter is V/R_C and radius is $V/2R_C$. This semicircle is called the locus (or path) of $\boldsymbol{I}_C$.



In order to draw the locus of total current $\boldsymbol{I}=\boldsymbol{I}_C+\boldsymbol{I}_L$, first draw a parallel line reference at the end point of $\boldsymbol{I}_L$ vector.





Now just shift the locus of I_C at the end of $\boldsymbol{I}_L$ vector. The new semicircle represents the locus of $\boldsymbol{I}$.

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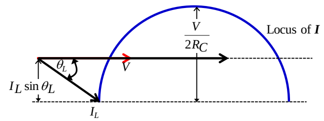
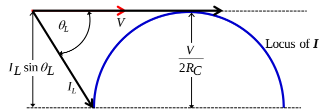
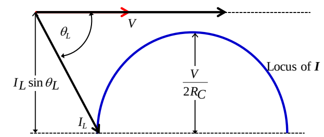




Case II: If ($V/2R_C$) = ($I_L \sin\theta_L$), there will be only one resonant point.



Case III: If ($V/2R_C$)< ($I_L \sin\theta_L$), parallel resonant cannot be obtained regardless of the value of C.

Homework

Problem 1: Draw the locus of total current by varying the inductor for a parallel circuit where two impedances $\mathbf{Z}_1 = R_L + jX_L$ and $\mathbf{Z}_2 = R_C - jX_C$ are connected in parallel and then conclude the comments regarding the resonance points.

Problem 2: Draw the locus of total current by varying the capacitor for a parallel circuit where two impedances $\mathbf{Z}_1 = R_L + jX_L$ and $\mathbf{Z}_2 = R_C - jX_C$ are connected in parallel and then conclude the comments regarding the resonance points.

Series-Parallel Tuning (or Resonance) Circuit

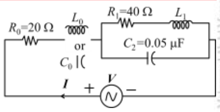
Series-Parallel Tuning (or Resonance) Circuit

- Since the impedance is minimum and the current is maximum, the series resonance circuit is used to pass a specific band of frequency. . **The parameters of series circuit for resonance can be calculated by using the passing frequency.**
- Since the impedance is maximum and the current is minimum, the parallel resonance circuit is used to block a specific band of frequency. **The parameters of parallel circuit for resonance can be calculated by using the blocking frequency.**

Example

Design the following circuit as shown in the following figure to pass any wave of 45 kHz and block 15 kHz wave. Find the value of L_1. What type of reactance (inductive or capacitive) must be placed in series with the source? Calculate the value of L_0 or C_0 which is required to put.

To block 15 kHz wave the parallel resonance circuit is used. Thus at 15 kHz



$$X_{C2} = \frac{1}{2\pi f C_2} = \frac{1}{2\pi \times 15000 \times 0.05 \times 10^{-6}}$$
$$= 212.3142\ \Omega$$

At resonance: $$\frac{R_1^2 + X_{L1}^2}{X_{L1}} = X_{C2}$$

$$X_{L1}^2 - X_{C2}X_{L1} + R_1^2 = 0 \qquad X_{L1} = \frac{1}{2}\left[X_{C2} \pm \sqrt{X_{C2}^2 - 4R_1^2}\right]$$

$$L_1 = \frac{1}{4\pi f}\left[X_{C2} \pm \sqrt{X_{C2}^2 - 4R_1^2}\right] = \frac{1}{4\pi \times 15000}\left[212.3142 \pm \sqrt{(212.3142)^2 - 4 \times (40)^2}\right]$$

$$L_1 = \frac{1}{4\pi \times 15000}[212.3142 \pm 196.6655] = 2.2\ \text{mH} \ \text{ or } \ 0.083061\ \text{mH}$$

For lower conductance consider: $L_1 = 2.2$ mH

To pass 45 kHz wave the series resonance circuit is used. Thus at 45 kHz

$$X_{C2} = \frac{1}{2\pi f C_2} = \frac{1}{2\pi \times 45000 \times 0.05 \times 10^{-6}} = 70.7714\ \Omega$$

$$X_{L1} = 2\pi f L_1 = 2\pi \times 45000 \times 2.2 \times 10^{-3} = 621.72\ \Omega$$

$$\mathbf{Z}_p = \frac{(R_1 + jX_{L1})(-jX_{C2})}{R_1 + jX_{L1} - jX_{C2}} = \frac{(40 + j621.72)(-j70.7714)}{40 + j621.72 - j70.7714} = 0.66 - j79.81\ \Omega$$

The parallel impedance is capacitive thus inductor (L_0) is required.

The inductive reactance should be:

$$X_{L0} = 2\pi f L_0 = 79.81 \qquad L_0 = \frac{79.81}{2\pi \times 45000} = 0.282\ \text{mH}$$

The dynamic impedance at 45 kHz will be: $Z_{d(45\ \text{kHz})} = 20 + 0.66 = 20.66\ \Omega$

Homework

Problem 1: Design the following circuit as shown in the following figure to pass any wave of 15 kHz and block 45 kHz wave. Find the value of C_2. What type of reactance (inductive or capacitive) must be placed in series with the source? Calculate the value of L_0 or C_0 which is required to put.