定义路径跟踪误差

$$\begin{cases}\epsilon_1 = f_1(x(t), y(t), z(t)) \\ \epsilon_2 = f_2(x(t), y(t), z(t))\end{cases} \tag{1.1}$$

则由位置运动学方程，得

$$\begin{cases}\dot{\epsilon}_1 = \dfrac{\partial f_1}{\partial P}\dot{P} = \dfrac{\partial f_1}{\partial P}V \\ \dot{\epsilon}_2 = \dfrac{\partial f_2}{\partial P}\dot{P} = \dfrac{\partial f_2}{\partial P}V\end{cases}, \begin{cases}\ddot{\epsilon}_1 = H_1 + G_1\dot{V} \\ \ddot{\epsilon}_2 = H_2 + G_2\dot{V}\end{cases} \tag{1.2}$$

其中，当$i = 1, 2,$

$$H_i = \frac{\partial^2 f_i}{\partial x^2}\dot{x}^2 + \frac{\partial^2 f_i}{\partial y^2}\dot{y}^2 + \frac{\partial^2 f_i}{\partial z^2}\dot{z}^2 + 2\frac{\partial^2 f_i}{\partial x\partial y}\dot{x}\dot{y} + 2\frac{\partial^2 f_i}{\partial y\partial z}\dot{y}\dot{z} + 2\frac{\partial^2 f_i}{\partial z\partial x}\dot{z}\dot{x}$$

$$G_i = \frac{\partial f_i}{\partial P}$$

通常，需要定义速率误差

$$\epsilon_3 = \tau^T v - v_d \tag{1.3}$$

其中$\tau = ((\partial f_1 / \partial p)\times\partial f_2 / \partial p) / \|(\partial f_1 / \partial p)\times\partial f_2 / \partial p\|$，且

$$\left[\ddot{\epsilon}_1, \ddot{\epsilon}_2, \dot{\epsilon}_3\right]^T = H + G\dot{V} = H + G(-ge_z + \frac{T}{M}Re_z) \tag{1.4}$$

其中，$\dot{V} = (-ge_z + \dfrac{T}{M}Re_z)$是由位置动力学方程得到；向量$H = \left[H_1, H_2, \dot{\tau}^T v\right]^T$，解耦矩阵$G = \left[G_1^T, G_2^T, \tau\right]^T$。

程序中定义的路径跟踪误差为：

$$\begin{cases}\epsilon_1 = x + y - 1 \\ \epsilon_2 = z - z_d \\ \epsilon_3 = \tau^T v - v_d\end{cases} \tag{1.5}$$

对(1.5)求导可得

$$\left[\ddot{\epsilon}_1, \ddot{\epsilon}_2, \dot{\epsilon}_3\right]^T = H + Gu \tag{1.6}$$

其中$H = [h_1, h_2, h_3]^T, G = [g_1^T, g_2^T, g_3^T]^T$，根据(1.2)可知$H = [0,0,0]^T$

$$G=\begin{bmatrix}1 & 1 & 0\\ 0 & 0 & 1\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0\end{bmatrix}$$

令

$$u=G^{-1}(-H+\mu) \tag{1.7}$$

其中 $\mu=[-k_{11}\dot{\epsilon}_1-k_{12}\epsilon_1,-k_{21}\dot{\epsilon}_2-k_{22}\epsilon_2,-k_{31}\epsilon_3]^T$， k_1,k_2,k_3为正整数。

将(1.7)代入(1.6)中，可得

$$\left[\ddot{\epsilon}_1,\ddot{\epsilon}_2,\dot{\epsilon}_3\right]^T=\mu \tag{1.8}$$

则闭环系统(1.8)是 Hurwitz。

四旋翼的动力学方程如下

$$\dot{v}=-ge_z+\frac{T}{M}Re_z \tag{1.9}$$

令 $u=\dot{v}$ 可得

$$\frac{T}{M}Re_z=G^{-1}(-H+\mu)+ge_z \tag{1.10}$$

从而由(1.10)可得

$$\begin{cases}\phi_c=\sin^{-1}\left(\dfrac{U_1\sin\psi_c-U_2\cos\psi_c}{\sqrt{U_1^2+U_2^2+U_3^2}}\right)\\ \theta_c=\tan^{-1}\left(\dfrac{U_1\cos\psi_c+U_2\sin\psi_c}{U_3}\right)\end{cases} \tag{1.11}$$

其中 $[U_1,U_2,U_3]^T=\dfrac{T}{M}Re_z$。