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Fundamental and speculative components of the cryptocurrency pricing dynamics

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Abstract

The driving forces behind cryptoassets' price dynamics are often perceived as being dominated by speculative factors and inherent bubble-bust episodes. Fundamental components are believed to have a weak, if any, role in the price-formation process. This study examines five cryptoassets with different backgrounds, namely Bitcoin, Ethereum, Litecoin, XRP, and Dogecoin between 2016 and 2022. It utilizes the cusp catastrophe model to connect the fundamental and speculative drivers with possible price bifurcation characteristics of market collapse events. The findings show that the price and return dynamics of all the studied assets, except for Dogecoin, emerge from complex interactions between fundamental and speculative components, including episodes of price bifurcations. Bitcoin shows the strongest fundamentals, with on-chain activity and economic factors driving the fundamental part of the dynamics. Investor attention and off-chain activity drive the speculative component for all studied assets. Among the fundamental drivers, the analyzed cryptoassets present their coin-specific factors, which can be tracked to their protocol specifics and are economically sound.

Keywords: Cryptocurrency, Bitcoin, Cusp catastrophe model, Crash

JEL Classification: C52, G12

Introduction

Cryptoasset markets[1] have come a long way from being a fringe curiosity of financial markets to being viewed as players of interest by regulators and central banks. They have attracted, albeit in waves, the attention of both institutional and retail investors. Overall market capitalization increased from $1B in 2013 to almost $3T in 2021 and hovered between $1T–$2T in 2022. As cryptoassets, particularly Bitcoin, have been developed and advertised to challenge standard financial assets and instruments, currencies, and the monetary system, much of the financial research on these assets has focused on their properties as diversifiers, safe havens, and inflation hedges (Bouri et al. 2017; Selmi et al. 2018; Urquhart and Zhang 2019; Dutta et al. 2020; Shahzad et al. 2019). In addition to the above, several studies have also presented rather standard forecasting exercises

[1] Cryptoasset markets, crypto-markets, cryptoassets, and cryptocurrencies are used interchangeably in the text.

utilizing various methodologies (Atsalakis et al. 2019; Wu et al. 2018; Adcock and Gradojevic 2019; Mudassir et al. 2020; Sutiksno et al. 2018; Bedi and Nashier 2020; Alexander and Dakos 2020; Sebastião and Godinho 2021; Gradojevic and Tsiakas 2021; Fang et al. 2021). Xu et al. (2019) and Fang et al. (2022) provide comprehensive surveys of blockchain, its potential, and implications for trading.

Moreover, because cryptoassets are very different from their conventional counterparts, standard pricing and valuation methods are difficult or impossible to implement. Thus, studies on pricing and valuation are relatively scarce. Kristoufek (2015), following the earlier works on speculative attention-driven price dynamics in Kristoufek (2013); Garcia et al. (2014), and Garcia and Schweitzer (2015), is among the first to examine potential fundamental factors in Bitcoin price dynamics via wavelet coherence analysis. The results show that both fundamental (transactions, price level, supply) and speculative attention-based factors drive these dynamics. Hayes (2019) argues that marginal cost of production is essential for explaining Bitcoin prices, thus, challenging the standard economic viewpoint that Bitcoin is worthless. Kristoufek (2019) added the quantity theory of money to the equation, showing that the price dynamics (not necessarily the price itself, as the price level for the US economy is not available in USD terms) closely follow those implied by fundamental economic laws. White et al. (2020) concluded that Bitcoin can be best classified as a technology-based product, an emerging asset class, rather than a currency or security, which has important legal implications. In their heterogeneous agents model, Lee et al. (2020) showed that Bitcoin price dynamics can be explained by the interactions between speculators and tech-savvy investors, each following a different trading strategy. In addition, recurrent pricing bubbles and busts appear to be inherent to the dynamics of cryptoassets (Cheung et al. 2015; Corbet et al. 2018; Kyriazis et al. 2020; Fry 2018; Wheatley et al. 2019).

We build our analytical approach based on three aspects of crypto-markets, as reported in the literature reviewed above: a speculative component, fundamentals, and emergent bubble-burst episodes. The main contribution of the current study stems from connecting these three aspects into a single complex model, instead of studying them separately or in an additive manner. Moreover, we cover a wide range of fundamental and speculative factors, at work inside the crypto-markets and outside. By doing so, we deliver novel insights into pricing dynamics of cryptoassets, connecting the dots between different approaches presented in the current topical literature.

The catastrophe theory framework brought to empirical finance by Barunik and Vosvrda (2009) and Barunik and Kukacka (2015) provides a coherent playground for the study of cryptoassets as it is specifically transferred from the natural sciences to finance for inspecting the interactions between fundamental and speculative components of the market with endogenous bubbles and busts. In addition, the current study focuses on a larger set of cryptoassets to examine the strength of the speculative and fundamental components, their tendency to fall into price bifurcations, and if and how these aspects are connected to specific properties of such cryptoassets. Bitcoin (BTC), Ethereum (ETH), Litecoin (LTC), XRP, and Dogecoin (DOGE) are studied as they represent different types of cryptoassets. The cusp catastrophe model is the ideal instrument for inspecting the rich dynamics of the above-mentioned interactions.

We find that the price dynamics of all the assets under study (except Dogecoin) emerge from complex interactions between the fundamental and speculative components, passing through price bifurcation episodes and characteristic price surges and collapses. Bitcoin possesses the most solid fundamentals. Generally, the fundamental components driving these dynamics are dominated by on-chain activity, while investor attention mainly represents speculative drivers. Traditional financial markets also drive crypto-markets through both channels via stock market price dynamics and market uncertainty. Thus, both sides of the pricing system—fundamental and speculative—play a unique role in dynamics and price formation, and they are both influenced by the stock market. Thus, the crypto-market is not detached from traditional financial markets.

The remainder of this paper is organized as follows. The next section provides an overview of the literature on catastrophe theory and the following section introduces "Cusp catastrophe model" and details of its application. In the section on "Estimation methodology and evaluation" we discuss the econometric procedures employed in this study. In the next section, "Data description and final sample," we describe the datasets and discuss sample characteristics. The research hypotheses are developed in the next section. The following section "Empirical results" presents and interprets the quantitative and qualitative results of the empirical analysis. Finally, the "Conclusion" summarizes the study, discusses the implications and usefulness of the findings, and suggests potential avenues for future research. Additional details are provided in the Appendix. Supplementary materials associated with this article containing the datasets collected and analyzed during the current study and a sample R code for illustrative replication of the results are available in the GitHub repository: github.com/jirikukacka/Kukacka_Kristoufek_2023 [created 2022-09-15].

Catastrophe theory literature review

While catastrophe theory was developed by Thom (1975) in mathematical biology, it was promptly applied to financial markets by Zeeman (1974) to explain stock market crashes. Unfortunately, the theory was strongly criticized shortly after its inception by Zahler and Sussmann (1977) for the inaccurate use of statistical methods and serious failures to meet its restrictive mathematical assumptions in applications. This led to a significant decline in interest in and development of catastrophe theory until Rosser (2007) re-evaluated the original criticisms in detail and suggested that many of them were misguided. He argued that while its proper use is indeed limited, there are many potential applications in modeling dynamic discontinuities in economic and financial models.

Since then, the application of catastrophe theory has flourished in natural and social sciences (Poston and Stewart 2014), such as ecology (Roopnarine 2008; Wang et al. 2011; Piyaratne et al. 2013), environmental research (Mostafa 2020), hydrology (Ghorbani et al. 2010), physics (Kostomarov et al. 2012), mechanics (Fasoulakis et al. 2015), engineering (She et al. 2020), the building and construction industry (Xiaoping et al. 2010), transportation (Papacharalampous and Vlahogianni 2014), psychology (Stamovlasis and Vaiopoulou 2017; Lv et al. 2017), political sciences (Weidlich and Huebner 2008), medical research (Chen et al. 2014), education (Stamovlasis and Tsaparlis 2012), management research (Alessandri et al. 2018; Guastello et al. 2019), conflict resolution (Chow et al.

2012), and safety research and prevention (Park and Abdel-Aty 2011; Wang et al. 2017; Chen et al. 2018), among other applications.

Catastrophe theory has also been successfully applied in various fields of finance. Clark (2006) modeled net flows of US stock mutual funds to understand the determinants of their dynamics and stressed on the importance of sentiment variables in asset pricing. The first empirical application of catastrophe theory to model stock market crashes was provided by Barunik and Vosvrda (2009), where the authors compared two large historic stock market drops, namely, Black Monday on October 19, 1987, and September 11, 2001, following the terrorist attack on the World Trade Center, NY, USA. The authors concluded that internal forces led to the 1987 crash, which can be well explained by catastrophe theory, while the 2001 crash was confirmed to be caused by an external shock. Barunik and Kukacka (2015) extended the original maximum likelihood estimation of the cusp model by Cobb and Watson (1980), as augmented by Wagenmakers et al. (2005) and Grasman et al. (2009), by a two-step approach. It allows for a methodologically rigorous application of stochastic catastrophe theory to more extended periods of stock market data with time-varying volatility. In the first step, daily realized volatility is modeled, which is subsequently used to standardize the 27-year long time series of US stock market returns that enter the estimation routine.

Using two different estimation approaches, Diks and Wang (2016) empirically examined the housing market and interest rate data for six Organization for Economic Cooperation and Development countries using the cusp model. They showed that the behavior of housing prices, which also exhibit turbulent booms and bust periods similar to stocks, can be modeled and predicted by cusp catastrophe theory. Another econometric application, in this case, aimed at explaining the dynamics of the financial crises in the US, is provided by Wesselbaum (2017). Using a catastrophe-augmented bank failure model, the authors showed that the primary triggers of economic crises are concentration of risks via the interaction of banks and their exposure to higher-risk classes and Federal Funds rate hikes, combined with low reserves.

Most recently, Kukacka and Kristoufek (2020) studied the complexity of nine financial agent-based models, including the cusp, and Kukacka and Kristoufek (2021) extended this research topic with a sensitivity analysis concerning model parameter settings. Chen et al. (2021) developed an innovative Bayesian approach based on two different likelihood approximations and estimated the cusp catastrophe model using USD/EUR exchange rate data. Finally, Lux (2021) estimated cusp as a benchmark model using monthly S &P 500 data until 2015 to explain index mispricing compared to the ex-post rational price. As seen from the brief summary of the related literature, this study is, to the best of our knowledge, the first empirical application of the stochastic cusp catastrophe model to explain the dynamics of cryptocurrency markets.

Cusp catastrophe model

Catastrophe theory represents a general theoretical framework describing how gradual, continuous changes in the control variables of a financial system form a bull market, which might lead to an abrupt, discontinuous change known as a market crash. It also suggests the dynamics of the following bear phase of the market and indicates how interactions between the two main types of investors—fundamentalists and speculators—finally result in the market recovering back to its equilibrium state (Zeeman 1974). The following description focuses on the cusp catastrophe model, a topologically simple family of stochastic catastrophe systems with wide applications in behavioral sciences (Cobb 1978; Cobb and Watson 1980; Cobb 1981; Cobb and Zacks 1985; Wagenmakers et al. 2005). Moreover, as the aim is to apply this model to highly volatile cryptocurrency data, the methodology by Barunik and Kukacka (2015) is followed, and in the first step, the returns are standardized by their estimated volatility.

Model description

The model assumes that the market log returns $r_t = \sigma_t y_t, t = 1, \ldots T$ follow a stochastic process y_t:

$$dy_t = -\frac{dV(y_t; \alpha_{x,t}, \beta_{x,t})}{dy_t} dt + \sigma_{y_t} dW_t, \tag{1}$$

where σ_t represents the instantaneous volatility, $dV(y_t; \alpha_{x,t}, \beta_{x,t})/dy_t$ is the deterministic potential function of the state variable y_t describing the equilibrium of the cusp model, and $\alpha_{x,t}$ and $\beta_{x,t}$ are the control functions, called the asymmetry and bifurcation factors, representing the fundamental and speculative sides of the market, respectively. Furthermore, σ_{y_t} denotes the constant diffusion function and W_t denotes the Wiener process, which together represent the stochastic behavior of the system due to market noise. More specifically, the cusp potential function is defined as

$$-V(y_t; \alpha_{x,t}, \beta_{x,t}) = -1/4 y_t^4 + 1/2 \beta_{x,t} y_t^2 + \alpha_{x,t} y_t, \tag{2}$$

where the two dimensions of the control space, $\alpha_{x,t}$ and $\beta_{x,t}$, determine the predicted values of y_t given the realization of n explanatory variables $x_{i,t}, i = 1, \ldots n$ as follows:

$$\alpha_{x,t} = \alpha_0 + \alpha_1 x_{1,t} + \ldots + \alpha_n x_{n,t}, \tag{3}$$

$$\beta_{x,t} = \beta_0 + \beta_1 x_{1,t} + \ldots + \beta_n x_{n,t}. \tag{4}$$

The dynamics of the model are based on the principle that the system returns to the equilibrium state, in which the potential function $V(y_t; \alpha_{x,t}, \beta_{x,t})$ attains the minimum value with respect to y_t. The equilibrium is defined as

$$-\frac{dV(y_t; \alpha_{x,t}, \beta_{x,t})}{dy_t} = -y_t^3 + \beta_{x,t} y_t + \alpha_{x,t} = 0. \tag{5}$$

As a third-order polynomial, it has upto three roots representing three potential predictions regarding equilibrium response surface of the model. The subset of the control space described by $\alpha_{x,t}$ and $\beta_{x,t}$, where (5) has one root, represents the stable unimodal phase of the market around its equilibrium, or the cyclical dynamics of the bull and bear market phases, as first described by Zeeman (1974). In the case of the three solutions of (5), the system enters the bifurcation phase characteristic of a market crash following the bursting of a financial bubble; the state variable y_t becomes bimodal as the model predicts two probable values of y_t accompanied by a so-called anti-prediction,

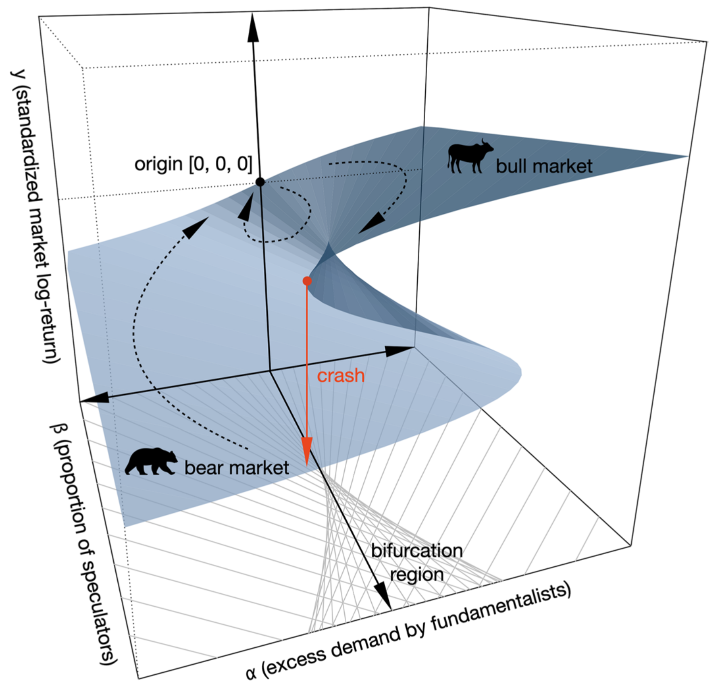

Fig. 1 An illustrative equilibrium response surface of the cusp catastrophe model and the cyclical dynamics of the market. *Source:* produced in R using the `cusp` package version 2.3.3. and completed by the authors

that is, the least probable state of the system. Thus, the deterministic part of the equilibrium response surface creates a typical smooth S-shape within the unstable bifurcation region, supplemented by a simple sheet within the stable domain of the control space, as depicted in Fig. 1.

Application to financial markets

Finally, we discuss the inconspicuous but crucial assumption regarding the constant value of the diffusion function $\sigma_{y_t} = \sigma$ in (1). It originated in the development of catastrophe theory during the 1970s within the natural sciences. Such an assumption is often entirely legitimate in many applications. However, it is completely restrictive when the stochastic cusp model is applied to financial data, where $\sigma_{y_t} = \sigma_t$ represents the volatility of market returns, characterized by strong time-varying dynamics and regular clustering properties. Thus, Barunik and Kukacka (2015) suggested a two-step estimation methodology to overcome this problem rigorously. First, the actual market returns are standardized by their consistently estimated volatility. This effectively leads to simplification $\sigma_{y_t} = 1$, that is, the diffusion term essentially disappears from (1).

The estimation might be based on the well-known concept of daily realized variance if high-frequency market data are available or can even be based on a simple GARCH-type model for which only daily closing prices are necessary. In this study, the popular Garman–Klass volatility estimator is utilized (Garman and Klass 1980), which provides daily volatility estimates $\widehat{\sigma}_{GK,t}$ using high, low, opening, and closing prices:

$$\widehat{\sigma}_{GK,t} = \sqrt{0.5 \log\left(\frac{h_t}{l_t}\right)^2 - (2\log 2 - 1)\log\left(\frac{c_t}{o_t}\right)^2}, \quad (6)$$

where h_t, l_t, c_t, and o_t denote high, low, opening, and closing prices, respectively. The second step then follows the usual maximum likelihood estimation of the cusp model under assumptions that are fully compatible with stochastic catastrophe theory.

Estimation methodology and evaluation

Under these assumptions, the probability distribution corresponding to the solution of (1) converges to that of a limiting stationary stochastic process because changes in the explanatory variables $x_{i,t}$ are assumed to be much slower than the reactions of the state variable y_t (Cobb 1981; Wagenmakers et al. 2005). Thus, the model parameters can be estimated using the maximum likelihood approach, first proposed by Cobb (1978); Cobb and Watson (1980).

Software packages and estimation methods

Several packages are available for estimating the cusp catastrophe model. The `cuspfit` FORTRAN program was developed by Cobb (1978) and subsequently modified by Hartelman (1997). Oliva et al. (1987) suggested `GEMCAT` (General Multivariate Methodology for Estimating Catastrophe Models), which was later extended to `GEMCAT II`, implemented in Delphi by Lange et al. (2000). Diks and Wang (2016) applied an alternative numerical method to that of Cobb (1978), which is based on Euler discretization to approximate the model dynamics and obtain estimates of the parameters using nonlinear least squares. Recently, there has been an innovative attempt at Bayesian estimation of this model by Chen et al. (2021). However, the "industry standard" used in this study is the `cusp` package version 2.3.3, available in R (Grasman et al. 2009), which implements and extends the method in Cobb et al. (1983). Its main advantages include the stability of the software for fitting the cusp probability density, simplicity of use, and its optimization routine based on the well-known limited-memory BFGS algorithm.

Estimation setup and model selection

Estimated parameters in (3) and (4) represent the reaction coefficients of the individual empirical explanatory variables, that is, $\{\alpha_0, \ldots, \alpha_n, \beta_0, \ldots, \ldots, \beta_n\}$. Moreover, according to Grasman et al. (2009), two additional parameters, ω_0 and ω_1, are estimated from

$$y_t = \omega_0 + \omega_1 r_t / \widehat{\sigma}_t, \quad (7)$$

which first-order approximates the true, smooth transformation of the measured market returns, standardized by their estimated volatility $r_t/\widehat{\sigma}_t$.

The subsequent model selection procedure follows the standard stepwise elimination of individual explanatory variables based on their statistical significance until all remaining variables in the model are significant at least at the 10% level. This technique alleviates potential multi-collinearity problems between the explanatory variables, leading to higher statistical support for the resulting parsimonious models.

Evaluation of the fit

The `cusp` package produces several empirical results summarized in Table 1, accompanied by various diagnostic tools to evaluate the quality of the estimation fit. First, parameter ω_1 from (7) should be statistically significant and of a reasonable magnitude. Otherwise, equation (7) would only trivially represent a constant ω_0 and the cusp model would hardly explain the data. Additionally, at least one of the coefficients (except for the intercepts) in each of equations (3) and (4) should be statistically significant at a standard level. Second, the log-likelihood for the given sample labeled "LL" is standardly based on logarithms of the probability distribution corresponding to the solution of (1) accumulated over $t = 1, \ldots T$. As a result, the higher the log-likelihood, the better the estimated model fits. Moreover, the nonlinear cusp model should exhibit a significantly better fit than multiple linear regression with the same set of n explanatory variables. This means that this log-likelihood should be significantly higher compared to ordinary regression, which can be statistically assessed rigorously using a likelihood ratio test. The null is defined as the log-likelihoods of the two models being at the same level, and the statistics for this test are directly reported under "χ_2^2." This is compared to a critical value based on the chi-squared distribution with two degrees of freedom, as parameters ω_0 and ω_1 are not estimated for the linear model.

Finally, and most importantly, the cusp should outperform the logistic curve estimated via nonlinear least squares:

$$\boxed{y_t = \frac{1}{1 + e^{-\alpha_{x,t}/\beta_{x,t}^2}} + \epsilon_t,} \tag{8}$$

for $t = 1, \ldots, T$, $i = 1, \ldots, n$, where y_t, α_i, and β_i have already been defined in equations (1), (3), and (4), and ϵ_t represents zero-mean random disturbances that can be normally distributed, but not necessarily (Seber and Wild 1989). A comparison between the cusp and logistic curve is an optimal indicator of the presence of bifurcation because the logistic function can also model rapid changes in the state variable as a function of the explanatory variables, imitating sudden transitions of the cusp model, but it does not possess degenerate discontinuous points inside the unstable bifurcation region. As the cusp probability density and the logistic functions are not nested models, the likelihood ratio test should not be used in this case (Grasman et al. 2009). Instead, the fit can be compared using the Akaike and Bayesian information criteria (AIC and BIC). The more conservative criterion, BIC, is preferred (Wagenmakers et al. 2005). The lower the information criterion, the better the fit of the estimated model.

Data description and final sample

The datasets were collected from five publicly available online sources. The high, low, opening, and closing daily prices of cryptocurrency are used as background data for the dependent state variable and were downloaded from coinmarketcap.com. Technical explanatory variables representing on-chain activity and trading volumes were retrieved from coinmetrics.io and coinmarketcap.com, respectively. Explanatory variables related to Google Trends were downloaded from trends.google.com, and Wikipedia page visit counts from pageviews.wmcloud.org. Additional financial variables were retrieved from

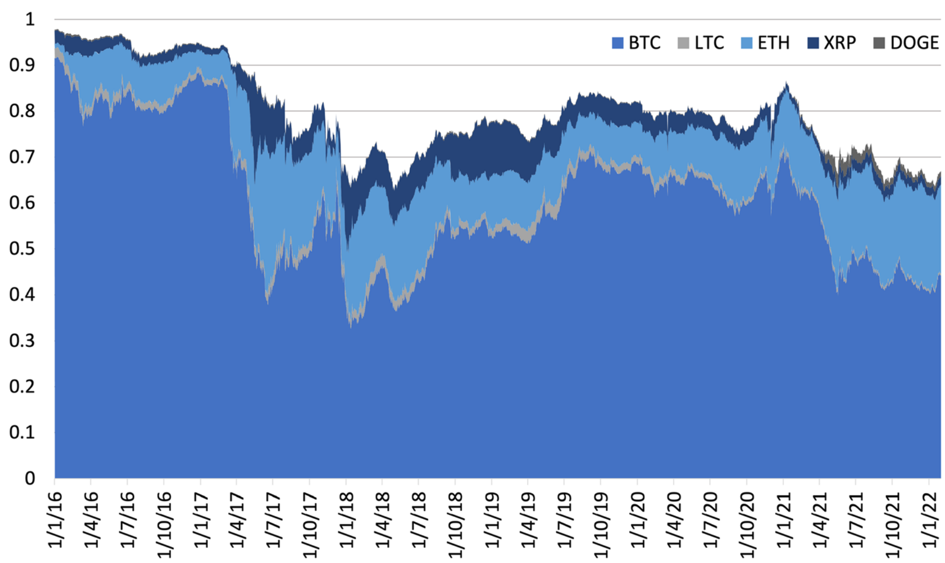

Fig. 2 Combined market capitalization of BTC, LTC, ETH, XRP, and DOGE. *Note:* Depicted in relation to the total cryptocurrency market capitalization between January 1, 2016, and January 31, 2022. Data are retrieved from coinmarketcap.com

Yahoo Finance. All databases were accessed on August 10, 2022. We refer the reader to GitHub repository for downloading the datasets.

The behavior of five well-known cryptoassets was studied. Bitcoin (BTC), Litecoin (LTC), and Ethereum (ETH) are well established within the crypto community and have been at the forefront for a long time. Two more rather specific currencies were also added: XRP by Ripple, which is premined, and Dogecoin (DOGE), which has gained much attention and has recently established a stable position in the TOP 20 according to the market cap. The data span the period from January 1, 2016, to January 31, 2022, which means that more than seven years of rapid development of the crypto-markets is covered, providing us with 2223 daily observations. Figure 2 depicts time-varying combined market capitalization of these five cryptoassets compared to the total cryptocurrency market capitalization. It strongly supports their selection as a representative set of the entire cryptocurrency market, in which Bitcoin retains its dominant power and predictive information potential for other cryptocurrencies over time (Wang and Ngene 2020).

Explanatory variables

Independent variables that are hypothesized to drive the market in the cusp catastrophe model are divided into three groups: technical, information demand-related, and financial variables. For each cryptoasset, data on the following technical variables was collected: total number of active addresses (Addresses, originally downloaded under label AdrActCnt), average daily transaction fee in USD (Fees, originally FeeMeanUSD), mining-related information represented by the hashrate (Hashrate), annualized inflation in percent (Inflation, originally IssContPctAnn), speed of circulation of a given currency inspired by the monetary concept of "velocity of money" (Velocity, originally NVTAdj), and ratio (ExchangeRatio) between the volume of trading on crypto exchanges (Volume) and volume of transactions on the blockchain. The only exception is the premined XRP, for which Hashrate and Inflation data do not exist.

Next, a measure of currency-specific interest of Google search users (GoogleCurrency) and a measure of the overall interest in the cryptocurrency market (GoogleMarket) were used. For each specific cryptoasset, their tickers and full names were combined in Google searches, weighted by their global queries. However, Ripple was used as a full name for XRP, even though it is not precise, as the name is often used in the community. Queries with a low search frequency did not need to pass the minimum search barrier set by Google's algorithm because the reported Google data are based on random sampling. Thus, the sampling was run ten times for each keyword, ensuring a new sample by adding a random alphanumeric sequence and using an average search score. Additional information demand-related variables include Wikipedia page visit counts for individual cryptoassets (Wiki) and the CBOE Volatility Index (VIX). The latter is a well-known measure of stock market volatility expectations based on S&P 500 options. Finally, we also included two financial variables that represent a relatively broad interconnection between the cryptocurrency market and the worldwide economy: log-returns of the S&P 500 index (SP500) and the USD/EUR exchange rate (USDEUR).

For illustrative purposes, Table 3 in the Appendix depicts the descriptive statistics of the original downloaded dataset for BTC. Three statistical tests supplement standard sample moments: for the augmented Dickey–Fuller test (ADF) specified with a constant, with a linear trend, and up to 25 lags (automatically selected according to AIC), H_0 is "unit root presence/covariance nonstationarity," for the Kwiatkowski–Phillips–Schmidt–Shin test (KPSS), H_0 specifies both "level stationarity" and "trend stationarity," and for the Jarque–Bera test (J-B), H_0 is "normality." Generally, the data are nonnormal and, most importantly, some of the time series violate the stationarity assumption.

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Final sample characteristics

Several common data transformations were undertaken to obtain the final sample entered into the estimation routine. First, log-returns for all cryptocurrencies were obtained based on the difference between closing and opening log-prices and for SP500 and USDEUR based on the closing prices. Second, stationarity of all explanatory variables was achieved by first-differencing. The only exceptions are SP500 which was stationary without modification, and ETH Hashrate, which required second-log-differencing. Thus, the researcher was left with a final sample consisting of 2221 daily observations. Third, all explanatory variables were standardized to eliminate numerical impact of the units of measurement and to unify the interpretation of the results. For illustrative purposes, Table 4 in the appendix summarizes descriptive statistics of the final sample for BTC. Due to standardization, the sample mean and standard deviation (SD) are trivially the same for all explanatory variables. The data are still nonnormal, but all currencies' log-returns and explanatory variables achieve stationarity at any reasonable significance level based on the KPSS test; it is further supported by the ADF test which rejects unit root presence at the 1% significance level. Estimated volatility $\widehat{\sigma}_t$ computed according to (6) is nonstationary as it is strongly persistent and exhibits regular clustering properties.

TRANSFORMATIONS

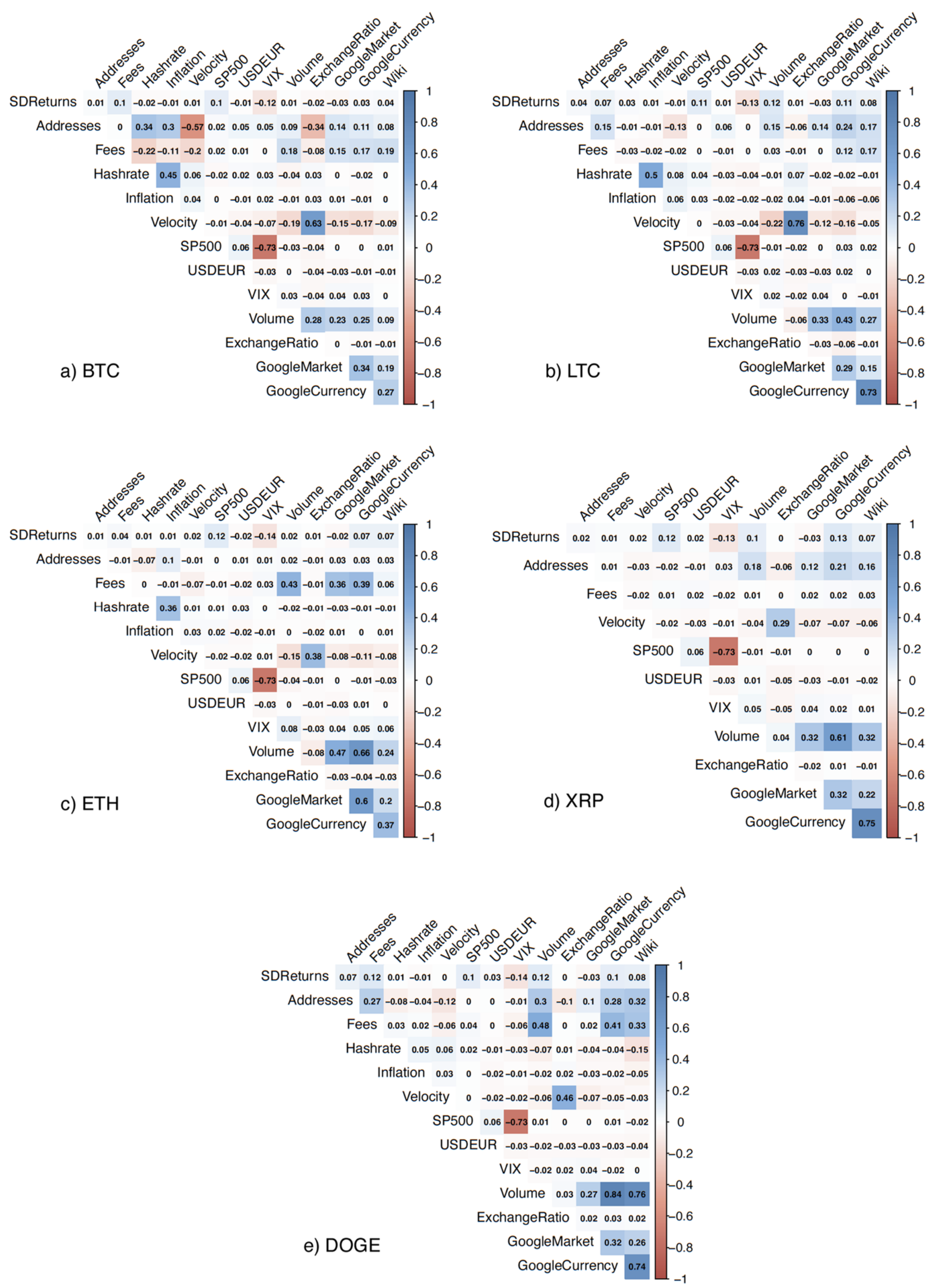

Fig. 3 Correlation matrices of the datasets. *Note:* The color gradually changes from dark blue ($\rho = 1$) through white ($\rho = 0$) to dark red ($\rho = -1$) as the Pearson correlation coefficient ρ decreases; see the specific range of colors on the right side of the figures. "SDReturns" represents $r_t/\widehat{\sigma}_t$. Based on $N = 2221$ observations and rounded to two decimal digits

Finally, Fig. 3 depicts the correlation matrices for all variables that enter the estimation. No serious issues were found concerning possible collinearity, as the higher correlations fall below 0.8 (except for one observation for DOGE), and the strongly correlated variables often act on different sides of the model (e.g., SP500 vs. VIX). However, some interesting connections were observed. The velocity of money and exchange ratio are positively correlated (and rather strongly correlated for BTC and LTC), indicating increased on-chain activity. Google searches for specific assets generally have a weak correlation with overall market searches, supporting their inclusion in the analysis. However, LTC, XRP, and DOGE exhibited strong positive correlation with Wikipedia page visit counts. Finally, fees and off-chain volumes are most correlated with investors' attention toward ETH and DOGE.

Research hypotheses

The model was structured under the assumption that the following variables: Addresses, Fees, Hashrate, Inflation, Velocity, SP500, and USDEUR represent the fundamental component of the pricing mechanism of cryptoassets. Of the variables listed above, the five technical variables are primarily blockchain-based metrics; they either represent technical parameters of the system and are practically exogenous to the system (Inflation) with its security provided by the validators/miners (Hashrate) or on-chain congestion due to its use (Addresses, Fees, Velocity). The two financial variables are included following current literature on interconnectedness between cryptocurrencies and the global economy. Corbet et al. (2020) showed how macroeconomic news announcements influence Bitcoin price. Most importantly, an increase in positive (negative) news surrounding unemployment rates and durable goods decreases (increases) Bitcoin returns. Zhu et al. (2017) found that (macro)economic factors such as consumer price index (CPI), US Dollar Index, Dow Jones Industrial Average (DJIA), or Federal Funds Rate influence Bitcoin price in the long run. Unfortunately, daily observations required for our analysis were unavailable for most of the suggested variables. Still, we included a broader S&P 500 index (SP500) as a potential aggregator of overall macroeconomic development in the US. The inclusion of a USD-related variable (USDEUR) is further supported by Dyhrberg (2016), who detected the presence of regional or country-specific effects, as the USD/EUR exchange rate significantly affects the volatility of Bitcoin returns.

The speculative component is represented by VIX, often called the "fear index." Its inclusion is based on literature suggesting that crypto is perceived as a hedge against stocks and Bitcoin serves as a "safe haven" (Bouri et al. 2017; Selmi et al. 2018; Urquhart and Zhang 2019; Dutta et al. 2020; Shahzad et al. 2019). Volume and ExchangeRatio naturally appear on the speculative side, as most speculative actions in these specific cryptoassets occur off-chain, that is, on centralized exchanges. Moreover, trading volume has been shown to Granger cause extreme negative and positive returns for many cryptocurrencies (Bouri et al. 2019) and contain predictive information for Bitcoin returns (Balcilar et al. 2017). Finally, variables related to Google Trends and Wikipedia views have a rather long tradition of being used as proxies for retail investors' attention (Kristoufek 2013; Garcia et al. 2014).

As the impact of $n = 13$ explanatory variables is studied, this technically translates into a set of linear restrictions of equations (3) and (4), which are as follows: $\alpha_8 = \alpha_9 = \alpha_{10} = \alpha_{11} = \alpha_{12} = \alpha_{13} = 0$ specifies the fundamental side of the market represented by the asymmetry factor $\alpha_{x,t}$, and $\beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = \beta_7 = 0$ specifies the speculative side of the market represented by the bifurcation factor $\beta_{x,t}$. Thus, it is expected that the other reaction coefficients are nonzero, which can be

Table 1 Estimation results

	BTC		*LTC*		*ETH*		*XRP*		*DOGE*	
$\widehat{\omega}_0$	−0.189	**	0.399	***	−0.156	**	0.378	***	−0.891	***
$\widehat{\omega}_1$	0.512	***	0.388	***	0.553	***	0.381	***	0.403	***
$\widehat{\alpha}_0$	−0.332	.	2.259	***	−0.269	*	1.998	***	−4.999	**
$\widehat{\alpha}_{1,Addresses}$	0.078	.								
$\widehat{\alpha}_{2,Fees}$	0.198	***	0.140	**	0.052	.			0.235	***
$\widehat{\alpha}_{3,Hashrate}$							–			
$\widehat{\alpha}_{4,Inflation}$							–			
$\widehat{\alpha}_{5,Velocity}$	0.095	*								
$\widehat{\alpha}_{6,SP500}$	0.147	***	0.197	***	0.190	***	0.285	***		
$\widehat{\alpha}_{7,USDEUR}$										
$\widehat{\beta}_0$	−2.173	***	−4.999	***	−1.699	***	−4.999	***	−4.380	***
$\widehat{\beta}_{8,VIX}$	0.164	*	−0.240	**					0.341	***
$\widehat{\beta}_{9,Volume}$	0.162	**	0.652	***			0.152	.	−0.226	***
$\widehat{\beta}_{10,ExchangeRatio}$					−0.276	***				
$\widehat{\beta}_{11,GoogleMarket}$	0.153	*	−0.322	**			−0.270	*	0.166	***
$\widehat{\beta}_{12,GoogleCurrency}$			0.144	*	0.278	***	0.568	***		
$\widehat{\beta}_{13,Wiki}$	0.142	*								
cusp model										
LL	−3105.510		−3104.456		−3106.319		−3105.382		−3103.173	
AIC	6235.020		6228.912		6228.638		6226.764		6222.346	
BIC	6303.488		6285.970		6274.284		6272.410		6267.988	
Linear model										
LL	−3393.817		−3262.685		−3379.525		−3334.278		−2992.940	
AIC	6807.635		6541.369		6771.050		6680.556		5997.879	
BIC	6864.692		6587.015		6805.285		6714.791		6032.111	
χ^2_2	≤ 0.001		≤ 0.001		≤ 0.001		≤ 0.001		≤ 0.001	
Logistic model										
LL	−3369.692		−3179.475		−3369.088		−3320.334		−2857.437	
AIC	6761.384		6376.950		6752.177		6654.669		5728.873	
BIC	6824.147		6428.302		6792.117		6694.609		5768.810	

***, **, *, and '.' denote significance levels of 0.1%, 1%, 5%, and 10%, respectively

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RESULTS

considered a set of 13 individual research hypotheses for each of the five cryptoassets. We are especially interested in the potential differences between them.

Empirical results

First, we quantitatively evaluate the overall estimation results summarized in Table 1, and then focus on the qualitative interpretation of individual models for each cryptocurrency in detail.

Quantitative evaluation

Following the toolkit in the section on "Estimation methodology and evaluation" section, we observe that for all cryptocurrencies, parameter ω_1 is estimated to be strongly statistically significant and of reasonable magnitudes between 0.381 and 0.553. This

Table 2 Variables remaining in final models

	BTC	LTC	ETH	XRP	DOGE
Addresses	✓				
Fees	✓	✓	✓		✓
Hashrate				–	
Inflation				–	
Velocity	✓				
SP500	✓	✓	✓	✓	
USDEUR					
VIX	✓	✓			✓
Volume	✓	✓		✓	✓
ExchangeRatio			✓		
GoogleMarket	✓	✓		✓	✓
GoogleCurrency		✓	✓	✓	
Wiki	✓				
cusp $\succ$ logistic	✓	✓	✓	✓	

means that equation (7) approximates a well-defined smooth transformation of measured market returns. Additionally, there are multiple statistically strongly significant explanatory variables driving the fundamental and speculative sides of the market, represented by equations (3) and (4), respectively, for all the analyzed cryptoassets. However, for DOGE, only $\alpha_{2,Fees}$ is significant among the variables on the fundamental side, and for XRP (for which Hashrate and Inflation data do not exist as it is premined), only $\alpha_{6,SP500}$ is statistically significant. Moreover, for ETH, the p value for $\alpha_{2,Fees}$ is exactly 0.100364, but we retained it in the model because the difference from the borderline p value of 0.10 is minuscule.

According to the log-likelihoods (LL), the nonlinear cusp catastrophe model exhibits a significantly better fit than the multiple linear regression model for all cryptocurrencies except DOGE. This is indicated either by its considerably higher log-likelihood values or by the likelihood ratio test (χ^2_2), based on which we can confidently reject the null hypothesis that log-likelihoods are at the same level. While the null hypothesis is also rejected for DOGE, this result indicates that the linear model fits better as it exhibits a higher log-likelihood.

More importantly, as per BIC and AIC, in the case of all four cryptocurrencies other than DOGE, the catastrophe model outperforms the logistic regression model. While the latter can also replicate rapid changes in the state variable, it does not possess degenerate discontinuous points. While the difference cannot be rigorously tested statistically, it is most pronounced for BTC, ETH, and XRP. Based on the overall quantitative evaluation of the estimation results, we can partially conclude that for BTC, LTC, ETH, and XRP, the cusp catastrophe model explains the data considerably better than the two natural benchmarks—linear and logistic regressions.

Qualitative interpretation

Table 2 summarizes the statistically significant variables (at least at the 90% confidence level) for the cryptoassets under study. We now focus on these factors separately.

Bitcoin

Starting with Bitcoin, both the fundamental and speculative core dynamics are rich in significant variables. On the fundamental side, on-chain transaction fees were found to be the most dominant driver of dynamics. On-chain activity propagating into increasing fees as the network congests, represents a clear fundamental price driver, as it measures network usage. Similarly, as active addresses also provide information about network activity, their significance might be mitigated by the more dominant transaction fee effect, although their correlation is only mild at 0.34, see Fig. 3. Increased on-chain activity is also captured by the velocity variable, which is statistically significant. Putting together the significance of all three variables, there is strong evidence that the increased on-chain activity, that is, transactions between wallets outside of the centralized exchanges, drives Bitcoin price dynamics, which is an evident fundamental factor. The other two fundamental variables are insignificant. Hashrate, sometimes referred to as network security but often considered to be the amount of computational power the miners are willing to contribute to the system for the chance of getting rewards, does not affect the price. This is in agreement with previous studies on the interaction between Bitcoin price and hashrate (Kristoufek 2020; Song and Aste 2020; Marthinsen and Gordon 2022). Inflation, representing newly minted coins, has no tangible effect on price dynamics; it seems to be priced in from a long-term perspective, as new emissions are known and imprinted in the protocol.

On the speculative side, attention-based variables and volume on exchanges are statistically significant, showing that attention—a proxy for retail investors' interest—strongly drives price dynamics. As Wikipedia page views are also statistically significant, they seem to provide additional information about Bitcoin on top of Google searches. This result is unique for Bitcoin as Wikipedia page views are insignificant for the other cryptoassets under study. Interestingly, Google searches are significant for the entire crypto-market and not just for Bitcoin. Again, this is a rather unique observation in the set, indicating the important role played by Bitcoin in the entire crypto-market dynamics. The exchange ratio remains insignificant, which suggests that the possibly increasing volume on centralized exchanges compared with on-chain transfers, does not affect price. Among the real-world variables, SP500 on the fundamental side and VIX on the speculative side are statistically significant. Bitcoin is thus not detached from conventional financial markets. It is rich in drivers on both sides of the spectrum, which interact in a highly nonlinear manner and go through bifurcation phases, as the catastrophe model outperforms both the linear and logistic specifications.

Hashrate, Inflation, and USDEUR are insignificant for all the studied cryptoassets. Insignificance of Hashrate is in line with Kristoufek (2020) pointing towards Bitcoin price driving the hashrate rather than the other way around. This direction seems to have prevailed in recent data and also in other mining-based coins (including all but XRP in our case). Insignificance of Inflation can be attributed to its protocol-imprinted status as minting of new coins is mainly predetermined and thus, likely priced in. Consequently, it does not cause unexpected shocks to the dynamics. Exchange rate probably does not possess enough variability to be an essential factor in explaining the often-turbulent dynamics of cryptoassets.

Litecoin

Turning to Litecoin, one must note that it has been introduced as a fork (and "lite" version) of Bitcoin, building on the Proof-of-Work transaction confirmation, but with a different hashing function (SHA-256 for Bitcoin and Scrypt for Litecoin, where the latter is more memory-centric and the former is more computational power centric), presumably leading to a lower concentration of miners and shorter block times and, therefore, faster transactions. Similar to the case of Bitcoin, the cusp catastrophe model explains price dynamics rather well. On the fundamental side, there are transaction fees and the S &P 500 index. On the speculative side, there are attention-based measures, volume on exchanges, and the VIX index. Thus, the dynamics are driven in a similar manner as that of the largest cryptoasset, albeit with tiny differences (insignificant velocity and Wikipedia searches). However, the interpretation remains the same. In addition, the analysis shows that even smaller cryptoassets are driven by their own fundamental and speculative components rather than being simply driven by the market leader. This is highlighted by the fact that even Litecoin-specific Google searches are significant in addition to general crypto-markets queries.

Ethereum

Ethereum provides a smart-contract protocol that can be used to build decentralized applications via its Ethereum Virtual Machine; thus, it is structurally very different from the first two cryptoassets. Many tokens (mainly of the ERC-20 type) have been deployed in the Ethereum network. When transacting on the network through decentralized applications, the user must pay fees (gas) in ETH, giving it ongoing utility and demand. In fact, fees are the only ETH-specific fundamental driver of price dynamics. On the fundamental side, SP500 has an even stronger effect on the ETH than BTC, similar to LTC. This is in line with the notion that altcoins are more sensitive to standard financial market moves than Bitcoin. On the speculative side, exchange ratio is found to be statistically significant, indicating that the imbalance between on-chain and off-chain activities (centralized exchanges) plays an important role. In addition, Google Trends searches for Ethereum also play an important role in the dynamics, detached from the overall market sentiment.

It is surprising that the fundamental part of the ETH dynamics is relatively weak. In a way, ETH's position comes in waves, where it cyclically becomes the victim of its own success. Often, there are applications that become too popular, and their frequent transactions congest the network, causing such hikes in fees that users are driven away. Alternatively, there are situations when some tokens traded only on decentralized exchanges, such as UniSwap become very popular and attract a massive amount of transactions, again congesting the system. The most profaned example of the latter is Shiba Inu. Such episodes have led to increased interest in either alternative blockchains (Binance Smart Chain, Cardano, Avalanche, Solana) or layer 2 solutions and roll-ups (Polygon, Optimism, Arbitrum) of the Ethereum network.

XRP

Since XRP is premined and does not adhere to the Proof-of-Work consensus, it has neither inflation nor hashrate. It primarily serves as an intermediary mechanism between two currencies in a SWIFT-like environment. Ripple Labs, the company behind XRP, has ongoing issues and conflicts with the SEC regarding the statute of XRP and its presentation to potential investors. This has led to various legal battles, causing large exogenous shocks to the pricing mechanism. This is reflected in the dynamics being dominated by the speculative components of most of the five analyzed cryptoassets. XRP is the only analyzed asset with no significant coin-specific fundamental component. Here, we see that only SP500 drives the fundamental dynamics; its effect on XRP is the strongest compared to its effect on the fundamental sides of the other cryptoassets. The speculative components point toward metrics based on Google Trends, likely reflecting the above-mentioned controversies and ongoing legal battles, the news of which apparently drives pricing. Even though the structure of the significant variables might seem less appealing than in previous cases, they tell a logical story of the coin's history. Importantly, in this case also the cusp model outperformed the linear and logistic models by a margin similar to that of the previous three coins. The complex interactions between the limited set of significant variables still lead to emergence of bifurcation, and consequently, endogenous bubbles and busts.

Dogecoin

Dogecoin is the "father of memecoins" created as a fork of Bitcoin with some changed parameters, most notably its possibly unlimited supply, as opposed to the hard-capped 21 million Bitcoins ever to be minted. DOGE is the only cryptoasset under study for which the cusp catastrophe model does not outperform either the logistic or the linear model specification. The logistic model is dominant, pointing towards a nonlinear response of the returns dynamics to the independent variables in both fundamental and speculative sectors of the market. Thus, DOGE dynamics or better price formation mechanism is less complex than that of the other four assets examined herein. Even then, it is subjected to both sides of the price formation process. While transaction fees drive the fundamental component, similar to the other cryptocurrencies, the speculative side is strongly driven by proxies based on Google Trends, not for the currency-specific, but for the overall interest in the entire cryptocurrency market.

Only Bitcoin and Dogecoin are mainly driven by overall crypto-market attention rather than coin-specific searches. In the case of Bitcoin, it can be attributed to the fact that Bitcoin is often treated and perceived as a proxy for the whole market. However, in the case of Dogecoin, one might speculate that the memecoin will react to the overall market hype rather than the interest in Dogecoin specifically. It might be interesting to compare these with other memecoins, but these are primarily new phenomena and often short-lived with short time series (with the most prominent exception being the Shiba Inu token). Nevertheless, this remains a promising avenue for future inspections. The speculative component turns out to be rich in drivers, as the VIX and Volume are also significant for DOGE. The overall market stress and volume on exchanges form its pricing dynamics.

Interestingly, DOGE is the only analyzed coin not driven by SP500 on the fundamental side. Thus, its long-term dynamics are distinct from conventional financial markets. As is the case for BTC and LTC, VIX plays its role on the speculative side. Therefore, only Bitcoin and its forks react to conventional market stress levels on the fundamental side.

Conclusion

The price formation of cryptoassets is always a controversial topic, primarily because of their unprecedented building blocks and features. However, these features also include very detailed data structures that allow study of interactions within the system, which one can hardly imagine for standard financial assets and instruments. The returns dynamics of five cryptoassets with various specifications (including Bitcoin, Ethereum, Litecoin, XRP, and Dogecoin) were studied. This study focuses on the interplay between the fundamental and speculative parts of the pricing mechanism with possible bifurcation episodes within the cusp catastrophe model. All the cryptoassets studied herein, except for Dogecoin, demonstrate that their price and returns dynamics emerge from complex interactions between fundamental and speculative components, including episodes of price bifurcation characteristics of market collapse events. Bitcoin shows the strongest fundamentals, with four out of seven potential fundamental factors being statistically significant. Interestingly, technical factors connected to the cryptoasset's supply (Inflation and Hashrate) do not play a significant role in the price formation process in any of the analyzed coins. It is mainly on-chain activity and (macro)economic factors (stock market price dynamics and uncertainty) that drive the dynamics. The speculative component is primarily driven by investor attention and (potentially resulting) off-chain activity. Notably, crypto-markes are not detached from conventional financial markets as both sides of the market dynamics are affected by stock market price dynamics represented by S &P 500 index and stock market uncertainty measured by the VIX index.

The results clearly show that the price dynamics of the top cryptoassets are formed by both fundamental and speculative components. This opens up a wide space for the further exploration of related topics. Our results suggest that fundamental factors must be included in pricing models when forecasting and developing trading strategies, which is often not the case. The connection with conventional financial markets should also be taken into account. In risk management within the crypto-market segment, the presented significant price dynamics drivers should be considered as factors possibly driving variance and even correlations among cryptoassets. One can easily imagine, for example, that on-chain activity is a structural factor not necessarily connected to a single asset, as crypto-investors are often interested and invested in various projects on different blockchains. The available width of cryptoasset datasets is a huge advantage in such analyses, but it also poses some challenges as possible factors need to be chosen carefully, and thus, possibly arbitrarily, or via data mining and machine learning techniques. In addition, the separation between fundamental and speculative factors is not clear-cut, as one can often find an explanation for both classifications. This is related to another common issue in crypto-analyses—possible endogeneity, which is complicated to tackle in such an intertwined system. Such limitations do not spare conventional financial markets, but much lower data availability makes this less evident. For cryptoassets, specifically Bitcoin, Kubal and Kristoufek (2022) and Kristoufek (2023) provided an outline of how to treat endogenous crypto-systems, although multi-assets treatment poses additional challenges. However, these are possible issues for interpreting the economics behind it, but not for forecasting and portfolio studies that do not consider the classification. Either way, factors other than those that are autoregressive and speculative, help describe and explain price dynamics and should be considered when constructing relevant models.

From the investors' perspective, complex interactions between the fundamental and speculative components with a tendency towards bifurcations form a tricky playing field. The structural and fundamental characteristics of the analyzed coins suggest that long-term investors should be rewarded for their investments as long as the technology progresses. However, the strength of speculative factors indicates turbulent periods. The least favored type of investor in cryptoassets is the one in between—an investor who is neither a trend chaser benefiting from riding the hype nor a long-term investor holding towards pension. Holding long but not long enough seems to be the worst strategy. This also points to a standard limitation of practically all studies on cryptoassets pricing—regulatory uncertainty. Even though regulations and changes in the rules of the game are an issue for conventional finance and its pricing as well, it is of a different magnitude for cryptoassets.

Alternatively, given the events of 2022, exogenous, unpredictable shocks to the system could cause cascading dynamics to affect (not only) mid-term investors as well. The 2022 drama of FTX, a prominent centralized exchange, shows the complexity of the market and the associated risks, and demonstrates the consequences of an unregulated market lacking proper audits, control mechanisms, and basal risk controls. This brings us to a possible extension of the current cusp model. Allowing for heavy-tailed innovations in the system would represent extreme shocks and might be more appropriate for crypto-markets, specifically for future studies focusing on smaller coins or tokens and their cusp-like dynamics. These would bypass the need for inclusion of systemic risk measures in the tightly interconnected system that the crypto-markets are. Such measures are difficult to construct reliably given crypto-markets specifics, and they still need to be improved in the literature (Kim et al. 2021).

From the big-picture perspective, our results not only provide a basis for forecasting, portfolios, and investment applications but also contribute to the structural discussion about cryptoassets and their classification. Although our study is not the first to examine both fundamental and speculative components of cryptoassets' pricing dynamics within a single model, it is the first to compare and discuss the building blocks of various cryptoassets, each representing a different part of the market, instead of focusing solely on Bitcoin, which is still a standard. In addition, we construct our models with a wide array of variables on both sides of the price formation process, within and outside the crypto-markets. Notably, altcoin pricing dynamics are shown to follow their own paths, partially following Bitcoin as a dominant cryptoasset but with coin-specific drivers of both the fundamental and speculative components. Economically sound models are thus worth exploring, not only for Bitcoin but also for altcoins.

Appendix

See Tables 3 and 4.

Table 3 Descriptive statistics of the BTC original dataset

Statistic	Mean	SD	Min	Max	Skew	Kurt	ADF	KPSS	J-B
AdrActCnt (counts)	758k	202k	317k	1367k	0.44	− 0.55	0.09	≤ 0.01	≤ 0.001
FeeMeanUSD (USD)	3.64	6.81	0.07	60.95	3.90	19.31	≤ 0.01	≤ 0.01	≤ 0.001
HashRate	64.3M	58.1M	0.7M	216.9M	0.49	− 1.12	0.96	≤ 0.01	≤ 0.001
IssContPctAnn (%)	3.80	1.93	0.71	12.02	1.48	2.73	0.04	≤ 0.01	≤ 0.001
NVTAdj	65.86	32.23	15.22	252.90	1.27	2.01	≤ 0.01	≤ 0.01	≤ 0.001
SP (log-returns)	1.0e−5	0.01	− 0.06	0.05	− 0.61	12.90	≤ 0.01	≥ 0.10	≤ 0.001
USDEUR (log-returns)	2.0e−5	1.4e−4	− 1.6e−3	1.7e−3	0.09	19.36	≤ 0.01	≤ 0.01	≤ 0.001
VIX (level)	17.96	8.11	9.14	82.69	2.73	12.09	≤ 0.01	≤ 0.01	≤ 0.001
VolumeExch (USD)	17.7B	20.8B	28.5M	3.5e11	2.97	30.44	0.02	≤ 0.01	≤ 0.001
ExchangeRatio	5.50	6.32	0.14	52.02	2.13	6.02	0.25	≤ 0.01	≤ 0.001
GoogleTrendsCrypto	0.90	1.11	0.01	8.41	2.16	4.95	0.03	≤ 0.01	≤ 0.001
GoogleTrends	0.81	0.80	0.13	11.48	5.44	50.97	≤ 0.01	≤ 0.01	≤ 0.001
Wiki (counts)	17.4k	23.4k	3.8k	344.7k	6.43	58.61	≤ 0.01	≤ 0.01	≤ 0.001

$N = 2223$ observations, *p* values reported for the statistical tests

Table 4 Descriptive statistics of the BTC final sample

Statistic	Mean	SD	Min	Max	Skew	Kurt	ADF	KPSS	J-B
Returns	0.002	0.04	− 0.47	0.23	− 0.77	11.69	≤ 0.01	≥ 0.10	≤ 0.001
$\widehat{\sigma}$	0.03	0.02	0.00	0.24	2.67	12.04	≤ 0.01	≤ 0.01	≤ 0.001
Addresses	0.00	1.00	− 4.73	3.81	0.28	0.84	≤ 0.01	≥ 0.10	≤ 0.001
Fees	0.00	1.00	− 12.48	10.55	1.29	40.68	≤ 0.01	≥ 0.10	≤ 0.001
Hashrate	0.00	1.00	− 8.06	5.08	0.02	5.54	≤ 0.01	≥ 0.10	≤ 0.001
Inflation	0.00	1.00	− 5.13	5.94	0.21	2.98	≤ 0.01	≥ 0.10	≤ 0.001
Velocity	0.00	1.00	− 5.93	5.32	− 0.64	4.69	≤ 0.01	≥ 0.10	≤ 0.001
SP500	0.00	1.00	− 8.26	7.48	− 0.61	12.89	≤ 0.01	≥ 0.10	≤ 0.001
USDEUR	0.00	1.00	− 8.47	8.54	− 0.11	11.22	≤ 0.01	≥ 0.10	≤ 0.001
VIX	0.00	1.00	− 10.19	14.36	3.34	50.46	≤ 0.01	≥ 0.10	≤ 0.001
Volume	0.00	1.00	− 27.59	26.81	− 0.76	493.36	≤ 0.01	≥ 0.10	≤ 0.001
ExchangeRatio	0.00	1.00	− 10.96	9.10	− 0.62	23.41	≤ 0.01	≥ 0.10	≤ 0.001
GoogleMarket	0.00	1.00	− 12.47	18.26	2.23	78.44	≤ 0.01	≥ 0.10	≤ 0.001
GoogleCurrency	0.00	1.00	− 14.85	28.65	9.37	333.49	≤ 0.01	≥ 0.10	≤ 0.001
Wiki	0.00	1.00	− 16.91	19.77	3.47	146.57	≤ 0.01	≥ 0.10	≤ 0.001

$N = 2221$ observations, *p* values reported for the statistical tests

Abbreviations

ADF	Augmented Dickey–Fuller test
AIC	Akaike information criterion
BIC	Bayesian information criterion
BTC	Bitcoin
CPI	consumer price index
DJIA	Dow Jones Industrial Average
DOGE	Dogecoin
ETH	Ethereum
J-B	Jarque–Bera test
KPSS	Kwiatkowski–Phillips–Schmidt–Shin test
LTC	Litecoin
SD	standard deviation
SP500	Standard & Poor's 500
USD	United States dollar

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CRediT author statement: J.K.: Methodology, Software, Validation, Formal analysis, Investigation, Resources, Data curation, Writing—original draft, Writing—review & editing, Visualization, Supervision; L.K.: Conceptualization, Investigation, Data curation, Writing—original draft, Writing—review & editing, Project administration, Funding acquisition. Both authors read and approved the final manuscript.

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Availability of data and materials
The datasets collected and analyzed during the current study and a sample R code for an illustrative replication of the results are available in the GitHub repository: github.com/jirikukacka/Kukacka_Kristoufek_2023 [created 2022-09-15].

Declarations

Competing interests
The authors declare that they have no competing interests.

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References

Adcock R, Gradojevic N (2019) Non-fundamental, non-parametric Bitcoin forecasting. Physica A 531:121727
Alessandri G, Perinelli E, De Longis E, Theodorou A (2018) Second-order growth mixture modeling in organizational psychology: an application in the study of job performance using the cusp catastrophe model. Nonlinear Dynamics Psychol Life Sci 22(1):53–76
Alexander C, Dakos M (2020) A critical investigation of cryptocurrency data and analysis. Quant Finance 20(2):173–188
Atsalakis GS, Atsalaki IG, Pasiouras F, Zopounidis C (2019) Bitcoin price forecasting with neuro-fuzzy techniques. Eur J Oper Res 276(2):770–780
Balcilar M, Bouri E, Gupta R, Roubaud D (2017) Can volume predict Bitcoin returns and volatility? A quantiles-based approach. Econ Model 64:74–81. https://doi.org/10.1016/j.econmod.2017.03.019
Barunik J, Vosvrda M (2009) Can a stochastic cusp catastrophe model explain stock market crashes? J Econ Dyn Control 33:1824–1836
Barunik J, Kukacka J (2015) Realizing stock market crashes: stochastic cusp catastrophe model of returns under time-varying volatility. Quant Finance 15(6):959–973. https://doi.org/10.1080/14697688.2014.950319
Bedi P, Nashier T (2020) On the investment credentials of bitcoin: a cross-currencyy perspective. Res Int Bus Financ 51(January):101087
Bouri E, Molnár P, Azzi G, Roubaud D, Hagfors LI (2017) On the hedge and safe haven properties of Bitcoin: is it really more than a diversifier? Financ Res Lett 20:192–198. https://doi.org/10.1016/j.frl.2016.09.025
Bouri E, Lau CKM, Lucey B, Roubaud D (2019) Trading volume and the predictability of return and volatility in the cryptocurrency market. Financ Res Lett 29:340–346. https://doi.org/10.1016/j.frl.2018.08.015
Chen D-GD, Lin F, Chen XJ, Tang W, Kitzman H (2014) Cusp catastrophe model: a nonlinear model for health outcomes in nursing research. Nurs Res 63(3):211
Chen D-G, Gao H, Ji C, Chen X (2021) Stochastic cusp catastrophe model and its Bayesian computations. J Appl Stat 48(13–15):2714–2733. https://doi.org/10.1080/02664763.2021.1922993
Chen H, Zhang Y, Liu H, Meng X, Du W (2018) Cause analysis and safety evaluation of aluminum powder explosion on the basis of catastrophe theory. J Loss Prev Process Ind 55:19–24. https://doi.org/10.1016/j.jlp.2018.05.017
Cheung A, Roca E, Su J-J (2015) Crypto-currency bubbles: an application of the Phillips–Shi–Yu (2013) methodology on Mt. Gox bitcoin prices. Appl Econ 47(23):2348–2358
Chow PT, Cheung SO, Yiu TW (2012) A cusp catastrophe model of withdrawal in construction project dispute negotiation. Autom Constr 22:597–604. https://doi.org/10.1016/j.autcon.2011.12.006. Planning Future Cities-Selected papers from the 2010 eCAADe Conference
Clark A (2006) Modeling the net flows of us mutual funds with stochastic catastrophe theory. Eur Phys J B Condens Matter Complex Syst 50(4):659–669
Cobb L, Zacks S (1985) Applications of catastrophe theory for statistical modeling in the biosciences. J Am Stat Assoc 80(392):793–802
Cobb L (1978) Stochastic catastrophe models and multimodal distributions. Behav Sci 23(4):360–374. https://doi.org/10.1002/bs.3830230407

Cobb L (1981) Parameter estimation for the cusp catastrophe model. Behav Sci 26(1):75–78. https://doi.org/10.1002/bs.3830260107
Cobb L, Watson B (1980) Statistical catastrophe theory: an overview. Math Model 1(4):311–317. https://doi.org/10.1016/0270-0255(80)90041-X
Cobb L, Koppstein P, Chen NH (1983) Estimation and moment recursion relations for multimodal distributions of the exponential family. J Am Stat Assoc 78(381):124–130. https://doi.org/10.1080/01621459.1983.10477940
Corbet S, Lucey B, Yarovaya L (2018) Datestamping the Bitcoin and Ethereum bubbles. Financ Res Lett 26:81–88
Corbet S, Larkin C, Lucey BM, Meegan A, Yarovaya L (2020) The impact of macroeconomic news on Bitcoin returns. Eur J Finance 26(14):1396–1416. https://doi.org/10.1080/1351847X.2020.1737168
Diks C, Wang J (2016) Can a stochastic cusp catastrophe model explain housing market crashes? J Econ Dyn Control 69:68–88. https://doi.org/10.1016/j.jedc.2016.05.008
Dutta A, Das D, Jana RK, Vo XV (2020) Covid-19 and oil market crash: revisiting the safe haven property of gold and Bitcoin. Resour Policy 69:101816. https://doi.org/10.1016/j.resourpol.2020.101816
Dyhrberg AH (2016) Bitcoin, gold and the dollar—a GARCH volatility analysis. Financ Res Lett 16:85–92. https://doi.org/10.1016/j.frl.2015.10.008
Fang F, Chung W, Ventre C, Basios M, Kanthan L, Li L, Wu F (2021) Ascertaining price formation in cryptocurrency markets with machine learning. Eur J Finance. https://doi.org/10.1080/1351847X.2021.1908390
Fang F, Ventre C, Basios M, Kanthan L, Martinez-Rego D, Wu F, Li L (2022) Cryptocurrency trading: a comprehensive survey. Financ Innov 8(1):1–59
Fasoulakis ZC, Avraam TP, Raftoyiannis IG (2015) Dynamic buckling of partially-sway frames with varying stiffness using catastrophe theory. Int J Non-Linear Mech 71:116–126. https://doi.org/10.1016/j.ijnonlinmec.2014.10.002
Fry J (2018) Booms, busts and heavy-tails: the story of Bitcoin and cryptocurrency markets? Econ Lett 171:225–229
Garcia D, Tessone C, Mavrodiev P, Perony N (2014) The digital traces of bubbles: feedback cycles between socio-economic signals in the Bitcoin economy. J R Soc Interface 11:2014623
Garcia D, Schweitzer F (2015) Social signals and algorithmic trading of Bitcoin. R Soc Open Sci 2(9):150288
Garman MB, Klass MJ (1980) On the estimation of security price volatilities from historical data. J Bus 53(1):67–78
Ghorbani MA, Khatibi R, Sivakumar B, Cobb L (2010) Study of discontinuities in hydrological data using catastrophe theory. Hydrol Sci J 55(7):1137–1151. https://doi.org/10.1080/02626667.2010.513477
Gradojevic N, Tsiakas N (2021) Volatility cascades in cryptocurrency trading. J Empir Financ 62(June):252–265
Grasman RPPP, van der Maas HLJ, Wagenmakers E-J (2009) Fitting the cusp catastrophe in R: a cusp package primer. J Stat Softw 32(8):1–27
Guastello SJ, Correro AN, Marra DE (2019) Cusp catastrophe models for cognitive workload and fatigue in teams. Appl Ergon 79:152–168. https://doi.org/10.1016/j.apergo.2018.08.019
Hartelman PAI (1997) Stochastic catastrophe theory
Hayes AS (2019) Bitcoin price and its marginal cost of production: support for a fundamental value. Appl Econ Lett 26(7):554–560
Kim A, Trimborn S, Härdle WK (2021) VCRIX—a volatility index for crypto-currencies. Int Rev Financ Anal 78:101915. https://doi.org/10.1016/j.irfa.2021.101915
Kostomarov DP, Echkina EY, Inovenkov IN (2012) Application of the catastrophe theory in studying the magnetic reconnection process. Math Models Comput Simul 4:135–143. https://doi.org/10.1134/S2070048212020081
Kristoufek L (2013) Bitcoin meets Google Trends and Wikipedia: quantifying the relationship between phenomena of the Internet era. Sci Rep 3(1):1–7
Kristoufek L (2015) What are the main drivers of the Bitcoin price? Evidence from wavelet coherence analysis. PLoS ONE 10(4):1–15. https://doi.org/10.1371/journal.pone.0123923
Kristoufek L (2019) Is the Bitcoin price dynamics economically reasonable? Evidence from fundamental laws. Physica A 536:120873. https://doi.org/10.1016/j.physa.2019.04.109
Kristoufek L (2020) Bitcoin and its mining on the equilibrium path. Energy Econ 85:104588. https://doi.org/10.1016/j.eneco.2019.104588
Kristoufek L (2023) Will Bitcoin ever become less volatile? Financ Res Lett 51:103353. https://doi.org/10.1016/j.frl.2022.103353
Kubal J, Kristoufek L (2022) Exploring the relationship between Bitcoin price and network's rate within endogenous system. Int Rev Financ Anal 84:102375. https://doi.org/10.1016/j.irfa.2022.102375
Kukacka J, Kristoufek L (2020) Do 'complex' financial models really lead to complex dynamics? Agent-based models and multifractality. J Econ Dyn Control 113:103855. https://doi.org/10.1016/j.jedc.2020.103855
Kukacka J, Kristoufek L (2021) Does parameterization affect the complexity of agent-based models? J Econ Behav Organ 192:324–356. https://doi.org/10.1016/j.jebo.2021.10.007
Kyriazis N, Papadamou S, Corbet S (2020) A systematic review of the bubble dynamics of cryptocurrency prices. Res Int Bus Financ 54:101254
Lange R, Oliva TA, McDade SR (2000) An algorithm for estimating multivariate catastrophe models: GEMCAT II. Stud Nonlinear Dyn Econom. https://doi.org/10.2202/1558-3708.1062
Lee AD, Li M, Zheng H (2020) Bitcoin: speculative asset or innovative technology? J Int Finan Markets Inst Money 67:101209
Lux T (2021) Can heterogeneous agent models explain the alleged mispricing of the S &P 500? Quant Finance. https://doi.org/10.1080/14697688.2021.1909744
Lv S, Yang R, Huang C (2017) Contusion and recovery of individual cognition based on catastrophe theory: a computational model. Neurocomputing 220:210–220. https://doi.org/10.1016/j.neucom.2016.05.099 Recent Research in Medical Technology Based on Multimedia and Pattern Recognition
Marthinsen JE, Gordon SR (2022) The price and cost of bitcoin. Q Rev Econ Finance. https://doi.org/10.1016/j.qref.2022.04.003
Mostafa MM (2020) Catastrophe theory predicts international concern for global warming. J Quant Econ 18(3):709–731
Mudassir M, Bennbaia S, Unal D, Hammoudeh M (2020) Time-series forecasting of Bitcoin prices using high-dimensional features: a machine learning approach. Neural Comput Appl 1–15

Oliva TA, Desarbo WS, Day DL, Jedidi K (1987) Gemcat: a general multivariate methodology for estimating catastrophe models. Behav Sci 32(2):121–137. https://doi.org/10.1002/bs.3830320205
Papacharalampous AE, Vlahogianni EI (2014) Modeling microscopic freeway traffic using cusp catastrophe theory. IEEE Intell Transp Syst Mag 6(1):6–16. https://doi.org/10.1109/MITS.2013.2272183
Park PY, Abdel-Aty M (2011) A stochastic catastrophe model using two-fluid model parameters to investigate traffic safety on urban arterials. Accid Anal Prev 43(3):1267–1278. https://doi.org/10.1016/j.aap.2011.01.009
Piyaratne MKDK, Zhao H, Meng Q (2013) APHIDSim: a population dynamics model for wheat aphids based on swallowtail catastrophe theory. Ecol Model 253:9–16. https://doi.org/10.1016/j.ecolmodel.2012.12.032
Poston T, Stewart I (2014) Catastrophe theory and its applications. Courier Corporation
Roopnarine PD (2008) Catastrophe theory. In: Jorgensen E-CSE, Fath B (eds) Encyclopedia of ecology. Academic Press, Oxford, pp 531–536. https://doi.org/10.1016/B978-008045405-4.00146-4
Rosser JBJ (2007) The rise and fall of catastrophe theory applications in economics: was the baby thrown out with the bathwater? J Econ Dyn Control 31:3255–3280
Sebastião H, Godinho P (2021) Forecasting and trading cryptocurrencies with machine learning under changing market conditions. Financ Innov 7(1):1–30
Seber G, Wild C (1989) Nonlinear regression. Wiley
Selmi R, Mensi W, Hammoudeh S, Bouoiyour J (2018) Is Bitcoin a hedge, a safe haven or a diversifier for oil price movements? A comparison with gold. Energy Econ 74:787–801. https://doi.org/10.1016/j.eneco.2018.07.007
Shahzad SJH, Bouri E, Roubaud D, Kristoufek L, Lucey B (2019) Is Bitcoin a better safe-haven investment than gold and commodities? Int Rev Financ Anal 63:322–330. https://doi.org/10.1016/j.irfa.2019.01.002
She J, Zhang H, Kang Y, Feng Y, Zhong Y, Yang B (2020) Cusp catastrophe model for plugging pressure prediction of lost circulation control in fractured reservoirs. J Petrol Sci Eng 186:106705. https://doi.org/10.1016/j.petrol.2019.106705
Song Y-D, Aste T (2020) The cost of Bitcoin mining has never really increased. Front Blockchain. https://doi.org/10.3389/fbloc.2020.565497
Stamovlasis D, Tsaparlis G (2012) Applying catastrophe theory to an information-processing model of problem solving in science education. Sci Educ 96(3):392–410. https://doi.org/10.1002/sce.21002
Stamovlasis D, Vaiopoulou J (2017) The role of dysfunctional myths in a decision-making process under bounded rationality: a complex dynamical systems perspective. Nonlinear Dyn Psychol Life Sci 21(3):267–288
Sutiksno DU, Ahmar AS, Kurniasih N, Susanto E, Leiwakabessy A (2018) Forecasting historical data of Bitcoin using ARIMA and α-Sutte indicator. J Phys Conf Ser 1028:012194
Thom R (1975) Structural stability and morpohogenesis. Benjamin
Urquhart A, Zhang H (2019) Is Bitcoin a hedge or safe haven for currencies? An intraday analysis. Int Rev Financ Anal 63:49–57. https://doi.org/10.1016/j.irfa.2019.02.009
Wagenmakers E-J, Molenaar P, Grasman RP, Hartelman PAI, van der Maas HLJ (2005) Transformation invariant stochastic catastrophe theory. Physica D 211(3):263–276
Wang J, Ngene GM (2020) Does Bitcoin still own the dominant power? An intraday analysis. Int Rev Financ Anal 71:101551. https://doi.org/10.1016/j.irfa.2020.101551
Wang W, Liu S, Zhang S, Chen J (2011) Assessment of a model of pollution disaster in near-shore coastal waters based on catastrophe theory. Ecol Model 222(2):307–312. https://doi.org/10.1016/j.ecolmodel.2010.09.007
Wang Y, Weidmann UA, Wang H (2017) Using catastrophe theory to describe railway system safety and discuss system risk concept. Saf Sci 91:269–285. https://doi.org/10.1016/j.ssci.2016.08.026
Weidlich W, Huebner H (2008) Dynamics of political opinion formation including catastrophe theory. J Econ Behav Organ 67(1):1–26. https://doi.org/10.1016/j.jebo.2007.02.002
Wesselbaum D (2017) Catastrophe theory and the financial crisis. Scott J Political Econ 64(4):376–391. https://doi.org/10.1111/sjpe.12133
Wheatley S, Sornette D, Huber T, Reppen M, Gantner RN (2019) Are Bitcoin bubbles predictable? Combining a generalized Metcalfe's law and the log-periodic power law singularity model. R Soc Open Sci 6(6):180538
White R, Marinakis Y, Islam N, Walsh S (2020) Is Bitcoin a currency, a technology-based product, or something else? Technol Forecast Soc Chang 151:119877
Wu C-H, Lu C-C, Ma Y-F, Lu R-S (2018) A new forecasting framework for bitcoin price with LSTM. In: 2018 IEEE international conference on data mining workshops (ICDMW). IEEE, pp 168–175
Xiaoping Z, Jiahui S, Yuan C (2010) Analysis of crowd jam in public buildings based on cusp-catastrophe theory. Build Environ 45(8):1755–1761. https://doi.org/10.1016/j.buildenv.2010.01.027
Xu M, Chen X, Kou G (2019) A systematic review of blockchain. Financ Innov 5(1):1–14
Zahler RS, Sussmann H (1977) Claims and accomplishments of applied catastrophe theory. Nature 269(10):759–763
Zeeman EC (1974) On the unstable behaviour of stock exchanges. J Math Econ 1:39–49
Zhu Y, Dickinson D, Li J (2017) Analysis on the influence factors of Bitcoin's price based on VEC model. Financ Innov 3(1):1–13

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