

T.P. N° 3

Vectores

2.  $u = \vec{ab}$

a)  $a(2,3)$   $b(1,3)$

$$u = (1-2, 3-3) = (-1, 0) \quad \|u\| = \sqrt{(-1)^2 + (0)^2} = 1$$

b)  $a(0,-1,1)$   $b(1,-2,1)$

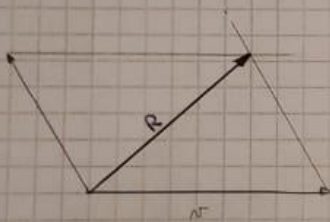
$$u = (1-0, -2+1, 1-1) = (1, -1, 0)$$

$$\|u\| = \sqrt{(1)^2 + (-1)^2 + (0)^2} = \sqrt{2}$$

$$u_v = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$$

3.  $\|u\| = 30$   $\|v\| = 45$   $\alpha = 120^\circ$

$1 \text{ cm} = 10$



$$\|R\| = \sqrt{(30)^2 + (45)^2 + 2 \cdot 30 \cdot 45 \cdot \cos 120^\circ}$$

$$\|R\| = 39,68$$

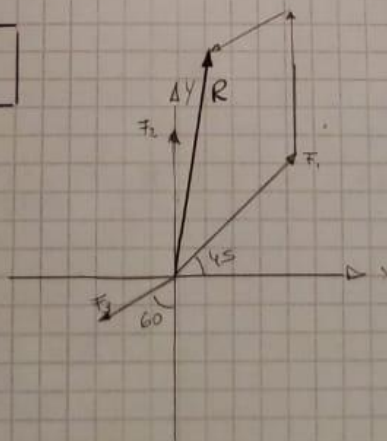
$1 \text{ cm} = 10$

$39 \text{ cm} - x = 39$

4.

$1 \text{ cm} = 10$

$40 \text{ cm} = 40$



NOTA

$$F_{1x} = 30 \cdot \cos 45^\circ = 21,21 \quad F_{1y} = 30 \cdot \sin 45^\circ = 21,21$$

$$F_{2x} = 25 \cdot \cos 90^\circ = 0 \quad F_{2y} = 25 \cdot \sin 90^\circ = 25$$

$$F_{3x} = 15 \cdot \cos 210^\circ = -12,99 \quad F_{3y} = 15 \cdot \sin 210^\circ = -7,5$$

$$R_x = 8,22$$

$$R_y = 38,71$$

$$R = \sqrt{(8,22)^2 + (38,71)^2} = \boxed{39,57}$$

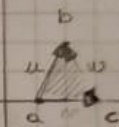
$$\tan \varphi = \frac{R_y}{R_x} = \frac{38,71}{8,22} \Rightarrow \alpha = \boxed{78^\circ 0' 41''}$$

5-  $a(1,0)$   $b(2,2)$   $c(3,0)$

a)  $u = (b-a) = (2-1, 2-0) = (1,2)$

$v = (c-a) = (3-1, 0-0) = (2,0)$

$w = (c-b) = (3-2, 0-2) = (1,-2)$   
 $(-1,2) = (b-c)$



b)  $\|u\| = \sqrt{1^2 + 2^2} = \underline{\underline{\sqrt{5}}}$

$\|v\| = \sqrt{2^2 + 0^2} = \underline{\underline{2}}$

$\|w\| = \sqrt{1^2 + (-2)^2} = \underline{\underline{\sqrt{5}}}$

a)  $\cos \alpha = \frac{u \cdot v}{\|u\| \cdot \|v\|} = \frac{2}{\sqrt{5} \cdot 2} \Rightarrow \underline{\underline{\alpha = 63^\circ 26' 6''}}$

$\cos \beta = \frac{u \cdot w}{\|u\| \cdot \|w\|} = \frac{-1+4}{\sqrt{5} \sqrt{5}} = \frac{3}{5} \quad \underline{\underline{\beta = 53^\circ 7' 48''}}$

$\gamma = 180^\circ - 63^\circ 26' 6'' - 53^\circ 7' 48'' = \underline{\underline{63^\circ 26' 6''}}$

c) isósceles

d) Perímetro =  $l + l + l = \sqrt{5} + 2 + \sqrt{5} = \underline{\underline{6,47}}$

Área =  $\frac{1}{2} \|u\| \|v\| \sin \alpha = \frac{1}{2} \sqrt{5} \cdot 2 \sin 63^\circ 26' 6''$

Área = 2

6.  $u = (-2, 0, 4)$  y  $v = (5, -1, 2)$

$$u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 4 \\ 5 & -1 & 2 \end{vmatrix} = (0\hat{i} + 2\hat{k} + 20\hat{j}) - (0\hat{k} - 4\hat{i} - 4\hat{j})$$

$$2\hat{k} + 20\hat{j} + 4\hat{i} + 4\hat{j} = \underline{\underline{4\hat{i} + 24\hat{j} + 2\hat{k}}}$$

7.  $a = (-1, 1, 0)$   $b = (-1, 0, 4)$  y  $c = (0, 2, 2)$

$$u = (b - a) = (0, -1, 4)$$

$$v = (c - a) = (1, 1, 2)$$

$$\|u\| = \sqrt{0^2 + (-1)^2 + 4^2} = \sqrt{17}$$

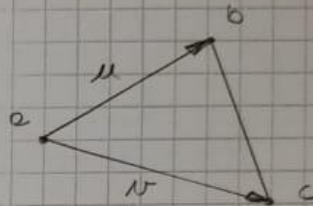
$$\|v\| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$$

$$u \cdot v = 0 - 1 + 8 = 7$$

$$\cos \alpha = \frac{u \cdot v}{\|u\| \|v\|} = \frac{7}{\sqrt{17} \sqrt{6}} = 46^\circ 7' 25''$$

$$\text{Area} = \frac{1}{2} \|u\| \|v\| \sin \alpha = \frac{1}{2} \cdot \sqrt{17} \cdot \sqrt{6} \sin 46^\circ 7' 25''$$

$$\text{Area} = 3,64$$

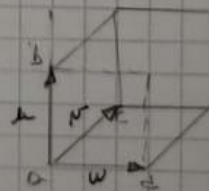


8.  $a = (2, -1, 1)$   $b = (2, 1, 4)$   $c = (4, 3, 1)$   $d = (3, -1, 0)$

$$u = (b - a) = (0, 2, 3)$$

$$v = (c - a) = (2, 4, 0)$$

$$w = (d - a) = (1, 0, -1)$$



NOTA

$$= \begin{vmatrix} 0 & 2 & 3 \\ 2 & 4 & 0 \\ 1 & 0 & -1 \end{vmatrix} = (0+0+0) - (12-0-4) = \boxed{-8}$$

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