## MATEMÁTICA DISCRETA

# **GRAFO DIRIGIDO**



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# 1. Consigna

- 1. Especificar un grafo dirigido y no ponderado.
- 2. Realizar y probar la implementación de un grafo dirigido y no ponderado con lista de aristas. Calcular el orden de cada operación.
- 3. Para probar el buen funcionamiento de la clase anterior:
  - (I) Hacer un método que permita cargar datos a un grafo (este método debe permita cargar los valores de los nodos y las aristas).
  - (II) Hacer un método que genere un grafo en forma aleatoria.
  - (III) Hacer un método que muestre por pantalla al grafo (puede ser un listado con los valores de los nodos y otro con las aristas o bien el dibujo del grafo).
- 4. Implementar los tres algoritmos que permiten recorrer un grafo (búsqueda plana, BFS y DFS)
- 5. Modificando adecuadamente los métodos anteriores, escribir y probar algoritmos que:
  - (I) Obtengan la cantidad de vértices fuentes y la cantidad de sumideros.
  - (II) Verificar si es débilmente conexo.
  - (III) Dados dos vértices verificar si existe un camino de longitud 2 entre ambos.
  - (IV) Implementar el algoritmo de Warshall.

# 2. Especificación

### 2.1. DirectedGraph

Description: Represents a directed non-weighted graph.

**Digraph**: Max Order  $\rightarrow$  Digraph

Order: O(1)

**Description:** Creates a new empty digraph with the given max order limit. If no max order is given,

the graph has unlimited max order. **Precondition:** Max Order > 0 **Postcondition:** A Digraph is created.

Classification: Constructor

 $\mathbf{add\_vertex} \colon \mathbf{Digraph} \ \mathbf{X} \ \mathbf{Data} \to \mathbf{void}$ 

Order: O(1)

**Description:** Adds a new vertex to the graph and stores the given data.

**Precondition:** 

■ Digraph must exist.

■ Data must exist.

**Postcondition:** New vertex added to the given graph, storing the given data.

Classification: Modifier

 $\mathbf{remove\_vertex}$ : Digraph X Key  $\rightarrow$  Data

Order: O(n)

Description: Finds the vertex referenced by the given key in the digraph and removes it. Returns

the data stored in the vertex.

Precondition:

■ Digraph must exist.

• Key must reference an existing vertex in the digraph.

Postcondition: Vertex removed, and its data returned

Classification: Modifier

 $\mathbf{key\_of} \colon \mathbf{Digraph} \ \mathbf{X} \ \mathbf{Data} \to \mathbf{Key}$ 

Order: O(n)

**Description:** Finds the key referencing the vertex storing the given data in the graph.

**Precondition:** 

■ Digraph must exist.

• The data must be stored in the digraph.

Postcondition: Key referencing the vertex storing the given data is returned.

 $\mathbf{add} \_\mathbf{edge} \text{: Digraph X From Key X To Key} \to \mathbf{void}$ 

Order: O(n)

**Description:** Adds a new edge connecting the vertices referenced by the given keys. The edge goes from the vertex referenced by From Key to the vertex referenced by To Key.

#### **Precondition:**

■ Digraph must exist.

• From Key and To Key must reference vertices in the digraph.

Postcondition: New edge added to the graph, that goes from the vertex referenced by From Key to

the vertex referenced by To Key.

Classification: Modifier

 $\mathbf{remove\_edge}$ : Digraph X From Key X To Key  $\rightarrow$  void

Order: O(n)

**Description:** Finds the edge connecting the vertices referenced by From Key and To Key and

removes it.

Precondition:

• Digraph must exist.

• From Key and To Key must reference a existing vertices in the digraph.

 An edge going from the vertex referenced by From Key to the vertex referenced by To Key must exist.

Postcondition: Edge removed Classification: Modifier

 $\mathbf{get}_{-}\mathbf{vertex}$ : Digraph X Key  $\to$  Data

Order: O(1)

Description: Finds the vertex referenced by Key in the digraph and returns the data stored in it

**Precondition:** 

■ Digraph must exist.

• Key must reference an existing vertex in the digraph.

Postcondition: Return the data stored in the vertex referenced by the given key.

 $\mathbf{get\_adjacency\_list} \colon \mathrm{Digraph} \ \mathbf{X} \ \mathrm{Key} \to \mathrm{Key} \ \mathrm{List}$ 

Order: O(n)

**Description:** Returns the list of all vertices adjacent to the one referenced by the given key in the

#### **Precondition:**

- Digraph must exist.
- Key must reference an existing vertex in the digraph.

Postcondition: Return a list of keys referencing all adjacent vertices to the one referenced by the

given key.

Classification: Analyzer

 $\mathbf{edge\_exists}$ : Digraph X From Key X To Key  $\rightarrow$  Boolean

Order: O(n)

Description: Verifies whether an edge from the vertex referenced by From Key to the vertex

referenced by To Key exists.

#### **Precondition:**

■ Digraph must exist.

• From Key and To Key must reference a existing vertices in the digraph.

#### Postcondition:

■ True if the edge exists.

• False if the edge does not exist.

Classification: Analyzer

 $\mathbf{random\_digraph}$ : Vertex Amount X Edge Amount  $\rightarrow$  Digraph

Order: O(n)

Description: Creates a random digraph with the given amount of vertices and edges.

Precondition:

• Vertex Amount > 0

■ Edge Amount > 0

Postcondition: A random digraph with the given amount of vertices and edges is created.

Classification: Constructor.

## 2.2. Graph Related Algorithms

Description: Algorithms related to graph theory

 $plain\_search: Graph \rightarrow Key List$ 

Order: O(n)

**Description:** Given a graph returns a list of keys referencing all its vertices in no particular order.

Precondition: Graph must exist

**Postcondition:** Return a list of keys referencing all vertices in the graph.

Classification: Analyzer

**dfs**: Graph  $\rightarrow$  Key List

Order:  $O(n^2)$ 

**Description:** Given a graph returns a list of keys referencing all its vertices using Depth First Search,

starting from the vertex first inserted in the graph.

Precondition: Graph must exist

Postcondition: Return a list of keys referencing all vertices in the graph using Depth First Search.

Classification: Analyzer

 $\mathbf{bfs} \colon \mathsf{Graph} \to \mathsf{Key} \ \mathsf{List}$ 

Order:  $O(n^2)$ 

Description: Given a graph returns a list of keys referencing all its vertices using Breath First

Search, starting from the vertex first inserted in the graph.

Precondition: Graph must exist

**Postcondition:** Return a list of keys referencing all vertices in the graph using Breath First Search.

Classification: Analyzer

 $number\_of\_sources: Graph \rightarrow Number$ 

Order: O(n)

**Description:** Given a graph returns the number of source vertices.

Precondition: Graph must exist

Postcondition: Return the amount of source vertices in the graph

Classification: Analyzer

 $number\_of\_sinks: Graph \rightarrow Number$ 

Order: O(n)

**Description:** Given a graph returns the number of sink vertices.

**Precondition:** Graph must exist

Postcondition: Return the amount of sink vertices in the graph

 $\mathbf{is\_weakly\_connected} \colon \mathbf{Graph} \to \mathbf{Boolean}$ 

Order:  $O(n^2)$ 

**Description:** Given a graph, checks whether it is weakly connected.

Precondition: Graph must exist

Postcondition:

• True if the graph is weakly connected.

• False if the graph is not weakly connected.

Classification: Analyzer

warshall: Graph  $\rightarrow$  Transitive Closure

Order:  $O(n^3)$ 

**Description:** Given a graph calculates its transitive closure using Warshall's.

Precondition: Graph must exist

Postcondition: The transitive closure must have the capability of calculating whether there is a

path between two vertices in O(1) time.

Classification: Analyzer

save: Graph X Name X View X Format  $\rightarrow$  void

Order: O(n)

**Description:** Given a graph, stores its graphic representation in the given format with the given

name. A view of the file is given if View is True.

#### **Precondition:**

■ Graph must exist

- Name must be a valid file name
- If given, view must be True or False (default False)
- If given, format must be a valid file format for the Graphviz program (default 'pdf').

#### Postcondition:

- A file containing DOT code for Graphviz and an image in the given format are created.
- If View is True, the created image is displayed.

**path\_exists**: Graph x From Key x To Key X Length X Transitive Closure  $\to$  Boolean **Order:**  $O(n^2)$ 

Description: Checks if a path between the vertices referenced by From Key and To Key exist.

- A length or a transitive closure can be given, not both.
- If a length is given, the path can be no longer than the length.
- If a Transitive Closure is given the path is checked in it, achieving O(1) performance.

#### **Precondition:**

- Digraph must exist.
- From Key and To Key must reference a existing vertices in the digraph.
- If given, length > 0.

#### Postcondition:

- True if a path exists.
- False if a path does not exist.