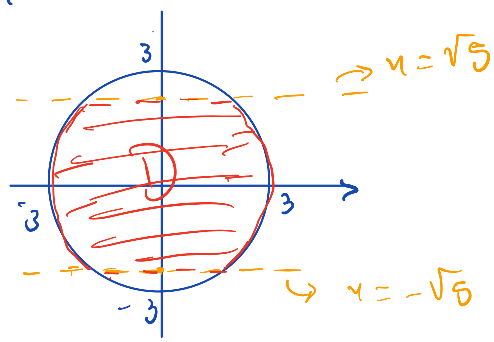
1. $f(x,y) = \sqrt{9-x^2-y^2} + \ln(5-y^2)$

(a) $D = \{(x,y) \in \mathbb{R}^2 : 9-x^2-y^2 \geq 0 \wedge 5-y^2 > 0\} =$

$= \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \leq 9 \wedge (-\sqrt{5} < y < \sqrt{5})\}$



(b) $\text{Int}\, D = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 < 9 \wedge (-\sqrt{5} < y < \sqrt{5})\} \neq D$,

logo D não é aberto.

(c) $\dfrac{\partial f}{\partial x} = \dfrac{\partial}{\partial x}\left((9-x^2-y^2)^{1/2} + \ln(5-y^2) \right) =$

$= \frac{1}{2}(9-x^2-y^2)^{-1/2}(-2x) = -\dfrac{x}{\sqrt{9-x^2-y^2}}$ //

$\dfrac{\partial f}{\partial y} = \dfrac{\partial}{\partial y}\left((9-x^2-y^2)^{1/2} + \ln(5-y^2)\right) = \dfrac{1}{2}(9-x^2-y^2)^{-1/2}(-2y)$

$+ \dfrac{-2y}{5-y^2} = -\dfrac{y}{\sqrt{9-x^2-y^2}} - \dfrac{2y}{5-y^2}$ //

$\dfrac{\partial f}{\partial x}(1,2) = -\dfrac{1}{\sqrt{9-1-4}} = -\dfrac{1}{2}$ //

$\dfrac{\partial f}{\partial y}(1,2) = -\dfrac{2}{2} - \dfrac{4}{5-4} = -5$ //

$\nabla f(1,2) = \left(-\dfrac{1}{2}, -5\right)$ //

(d) $h(x,y) = \ln(5-y^2)$

$CD_h = \{ h(x,y) : (x,y) \in D_h \} =$

$= \{ \ln(5-y^2) : -\sqrt{5} < y < \sqrt{5} \} =$

$=]-\infty, \ln 5]$

$0 < 5 - y^2 \leq 5$

$-\infty < \ln(5-y^2) \leq \ln 5$

2. (a) Verdadeira. É uma curva de nível 5.

$f(x,y) = 5 \Leftrightarrow x^2+y^2+4 = 5 \Leftrightarrow x^2+y^2 = 1$

(b) Falsa.

A superfície de nível 4 é definida por

$x^2+y^2+z^2+1 = 4 \Leftrightarrow x^2+y^2+z^2 = 3.$

(c) Verdadeira.

$|f(x,y)| \leq \sqrt{x^2+y^2} \Leftrightarrow -\sqrt{x^2+y^2} \leq f(x,y) \leq \sqrt{x^2+y^2}$

Como $\lim\limits_{(x,y)\to(0,0)} -\sqrt{x^2+y^2} = \lim\limits_{(x,y)\to(0,0)} \sqrt{x^2+y^2} = 0$,

então $\lim\limits_{(x,y)\to(0,0)} f(x,y) = 0$ e, como tal, tb todos os limites direccionais de f, em $(0,0)$, existem e são iguais a 0.

(d) Verdadeira.

$\lim\limits_{\substack{(x,y)\to(0,0)\\ y=0}} \frac{2y^2}{x^4+y^2} = \lim\limits_{x\to 0} \frac{0}{x^4} = 0$ //

$\lim\limits_{\substack{(x,y)\to(0,0)\\ y=x^2}} \frac{2y^2}{x^4+y^2} = \lim\limits_{x\to 0} \frac{2x^4}{2x^4} = 1$ //

Logo, não existe $\lim\limits_{(x,y)\to(0,0)} \dfrac{2y^2}{x^4+y^2}$.

(e) verdadeira

$$\nabla f(x,y) = (\cos x, \cos y)$$

$$\|\nabla f(x,y)\| = \sqrt{\cos^2 x + \cos^2 y} \leq \sqrt{1+1} = \sqrt{2} //$$

$$-\|\nabla f(x,y)\| \leq Df_{\vec{u}}(x,y) \leq \|\nabla f(x,y)\|$$

$$-\sqrt{2} \leq Df_{\vec{u}}(x,y) \leq \sqrt{2} //$$

3. $\lim\limits_{(x,y)\to(0,0)} g(x,y) =$

$$= \lim_{(x,y)\to(0,0)} \underbrace{\left|\frac{x^2}{x^2+y^2}\right|}_{\text{limitada}} \cdot \underbrace{\operatorname{sen} y}_{\to 0} = 0 = g(0,0).$$

Logo g é contínua em $(0,0)$.

4. $$\underbrace{\operatorname{sen}(xyz) - x - 2y - 3z}_{F(x,y,z)} = 0 \qquad P_0 = (2,-1,0)$$

(a) $$\frac{\partial F}{\partial x}(P_0)(x-2) + \frac{\partial F}{\partial y}(P_0)(y+1) + \frac{\partial F}{\partial z}(P_0)\, z = 0$$

Ora, $$\frac{\partial F}{\partial x} = yz\cos(xyz) - 1 \Big|_{P_0} = -1 //$$

$$\frac{\partial F}{\partial y} = xz\cos(xyz) - 2 \Big|_{P_0} = -2$$

$$\frac{\partial F}{\partial z} = xy\cos(xyz) - 3 \Big|_{P_0} = -5$$

A eq. do plano pedido é

$$-(x-2) - 2(y+1) - 5z = 0$$

$$-x + \cancel{2} - 2y - \cancel{2} - 5z = 0 \Leftrightarrow x + 2y + 5z = 0$$

(b) A eq. da recta normal pedida é

$$(x,y,z) = (2,-1,0) + \lambda(-1,-2,-5), \quad \lambda \in \mathbb{R}$$

ou

$$\frac{x-2}{-1} = \frac{y+1}{-2} = \frac{z}{-5}$$

5. $v = x^2 \operatorname{tg}(3y) + y e^{xy}$, com

$x = s + 2t$ e $y = st$.

$$\begin{array}{ccc} & v & \\ x & & y \end{array}$$

$$\frac{\partial v}{\partial s} = \frac{\partial v}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial s}$$

$$= \left(2x \operatorname{tg}(3y) + y^2 e^{xy}\right) \cdot 1 + \left(x^2\, 3 \sec^2(3y) + e^{xy} + y^2 e^{xy}\right) t$$

Qdo $s = 0$ e $t = 1$, vem $x = 2$ e $y = 0$

$$\frac{\partial v}{\partial s}(0,1) = \left(4 \operatorname{tg} 0 + 0^2 e^0\right) + 4 \cdot 3 \frac{1}{\cos^2 0} + e^0 = 13$$

6. $$f(x,y) = \frac{x^3}{3} + \frac{y^2}{2} + 2xy + 5x + y$$

$$\begin{cases} \dfrac{\partial f}{\partial x} = 0 \\[2ex] \dfrac{\partial f}{\partial y} = 0 \end{cases} \quad \begin{cases} x^2 + 2y + 5 = 0 \\[2ex] y + 2x + 1 = 0 \end{cases} \quad \begin{cases} x^2 - 4x - 2 + 5 = 0 \\[2ex] y = -2x - 1 \end{cases}$$

$$\begin{cases} x^2 - 4x + 3 = 0 \\[2ex] \text{—} \end{cases} \quad \begin{cases} x = \dfrac{4 \pm \sqrt{16 - 12}}{2} \\[2ex] \text{—} \end{cases} \quad \begin{cases} x = \dfrac{4 \pm 2}{2} \\[2ex] \text{—} \end{cases} \quad \begin{cases} x = 3 \vee x = 1 \\[2ex] \end{cases}$$

$$P_0 = (3, -7) \qquad P_1 = (1, -3)$$

$$\frac{\partial^2 f}{\partial x^2} = 2x \; ; \quad \frac{\partial^2 f}{\partial y^2} = 1 \; ; \quad \frac{\partial^2 f}{\partial x \partial y} = 2$$

$$\Delta(3,-7) = \begin{vmatrix} 6 & 2 \\ 2 & 1 \end{vmatrix} = 2 > 0 \;\wedge\; \frac{\partial^2 f}{\partial x^2}(P_0) > 0.$$

Logo P_0 é ponto de mínimo

$$\Delta(1,-3) = \begin{vmatrix} 2 & 2 \\ 2 & 1 \end{vmatrix} = -2 < 0.$$

Logo P_1 é ponto de sela.