Práctico:

$X_1, \dots, X_n$ ← valores.

Estadísticos muestrales

Media muestral: $\boxed{X(n) = \frac{X_1 + \dots + X_n}{n}}$

↳ es un estimador de la $E([X])$ ← variable aleatoria

Varianza muestral: $\boxed{S^2(n) = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X}(n))}$

↳ estima $Var(X)$.

Media muestral. $\frac{X_1 + \dots + X_n}{n} = X(n)$

$E(\bar{X}(n)) = E(X)$

$Var(\bar{X}(n)) = \frac{Var(X)}{n} \to 0$

$n \to \infty$

Ej 2.

$$\int_0^1 g(x)dx \sim \underbrace{E[g(U)]}$$

$$\overline{g(U)}(n) = \frac{g(U_1) + \dots + g(U_n)}{n}$$

$$\sqrt{Var(\overline{g(U)}(n))} = \frac{\sqrt{\boxed{Var(g(U))}}}{\sqrt{n}} \simeq \frac{\sqrt{\bar{S}^2(n)}}{\sqrt{n}} < 0.01$$

d

Ej 5: $\{U_i\}$

$M = n$ si $U_1 \leq U_2 \leq U_3$ y $U_n \leq U_{n-1}$

$$\boxed{P(M > n) = \frac{1}{n!}}$$, Ej $P(M > 2) = \frac{1}{2}$

$$1 - P(M \leq 2) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\underset{\_\_}{1} \quad \underset{\_\_}{2} \quad \underset{\_\_}{3} \quad \ldots \quad \underset{\_\_}{n} \longrightarrow \frac{1}{n!}$$

$$M > n \rightarrow U_1 \leqslant U_2 \leqslant U_3 \leqslant \ldots \leqslant U_n$$

$$P\left(\begin{array}{l}\text{de tener exactamente } n \\ \text{coincidencias en un mazo } n \text{ cartas}\end{array}\right) = \frac{1}{n!}\left(\sum_0^0\right) = \frac{1}{n!}$$

b) $$E(M) = \sum_{n=0}^{\infty} P(M > n) = \sum_{n=0}^{\infty} \frac{1}{n!} = e$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \Rightarrow e^1 = \sum_{n=0}^{\infty} \frac{1}{n!}$$

Serie de Taylor

$$E(M^2) = \sum n^2 \cdot P(M = n) \overset{?}{=}$$

$$E(M^2) \overset{?}{=} \sum_{n=0}^{\infty} P(M^2 > n)$$

$$Var(M) \simeq \underline{0{,}008}$$

$$\int_0^1 g(x)\,dx \approx E(g(U))$$

$$\frac{g(U_1) + \dots + g(U_n)}{n}$$

$$\int_{-\infty}^{\infty} x^2 \exp(-x^2)\,dx = 2\int_0^{\infty} x^2 \exp(-x^2)\,dx$$

$$= \dots = 2 \underbrace{\int_0^1 h(y)\,dy}$$

$$2 \cdot E(h(U)).$$