Sessio 2 del millor curs

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Example 3.

Solve the PDE

$$\boxed{u_x + 2xy^2u_y = 0} \tag{1}$$

The characteristic curves satisfy the ODE $dy/dx = 2xy^2/1 = 2xy^2$. To solve the ODE, we separate variables: $dy/y^2 = 2xdx$; hense $-1/y = x^2 - C$, so that

$$y = (C - x^2)^{-1}.$$

These curves are the characteristics. Again, $u(x, y)$ is a constant on each such curve.m (Check it by writing it out.) So $u(x, y) = f(C)$, where f is an arbitrary function. Theredore, the general solution of is obtained by solving (1) is obtained by solving (2) for C. That is,

$$\boxed{u(x, y) = f\left(x^2 + \frac{1}{y}\right).} \tag{2}$$

Again this is easily checked by differentiation, using the chain rule: $u_x = 2xf'(x^2 + \frac{1}{y})$ and $u_y = -(\frac{1}{y^2}f'(x^2 + \frac{1}{y})$, whence $u_x + 2xy^2u_y = 0$. □

$$\left\{ \frac{x}{1+\frac{x}{1+\frac{x}{\frac{x}{e}}}} \middle| x \in \mathbb{R} \right\}, \emptyset, \{\textstyle\sum_3\}$$
$$\{(x, y) \in \mathbb{R}^2 | \, x \in A\}$$