THE McCULLOCH-PITTS MODEL

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Introduction

The precise, electrochemical way in which the brain's neurons work is beyond the scope of this paper. Indeed, much has still to be discovered by biologists. Nevertheless, one aspect of the function of a neuron is fundamental: the effect of the synapses is variable, and it is this variability that gives the neuron its adaptability. Very early explanations saw the process of synaptic change as one of growth, and proposed that new synaptic connections could be made (grown) between neurons while, at other times, existing connections could be broken. But in 1943 the neurophysiologist Warren McCulloch and the logician Walter Pitts proposed a model in which synaptic changes are continuous and it is their model that is the basis of neural net calculations in contemporary neural computing.

The Model

In this model (which we shall call MCP for McCulloch-Pitts) the adaptability comes from representing the synaptic action by a variable weight which determines the degree to which a neuron should 'take notice' of firing signals that take place at the synapse concerned. The neuron is thought to take firing signals at all its synapses into account by summing their effects, both excitatory and inhibitory, and thereby determining whether it should or should not fire. In the MCP model it is assumed that firing at the axon of a neuron may be represented by the number 1 and no firing at the axon bv the number 0. When it is not known what this number is, the state of the axon of the neuron is given the label X. The effect of a synapse is represented by a weight W, which can (somewhat arbitrarily) take values in the range between -1 and + 1. The effect on a neuron of any particular synapse is then the product: XW.

The negative values of W represent inhibitory synapses. As there are many synapses another label j must be attached to each of these numbers. So X_j and W_j are the input and weight of the jth synapse. To account for all the synapses, MCP merely adds these effects and compares them to some threshold T. If the total sum exceeds this threshold then the neuron fires. What we have just described is the firing rule for an MCP node. This is usually stated in simple mathematical form as follows:

The neuron fires (i.e. its output F = 1) if the following inequality is true:

$$X_1W_1 + X_2W_2 \ldots X_jW_j \ldots X_nW_n > T$$

Some Algebra

The simple way in which the firing rule of the MCP model is stated makes it amenable to some easy algebra from which one may understand what is happening. For example, it is possible to answer the question: 'Given a desired truth table, what values must the weights and threshold be given in order to achieve it?'

To make things simple, but without sacrificing any of the character of this analysis, we take an MCP with only two inputs: X_1 and X_2. A desired truth table is shown below:

X_1:	0	1	1	1
X_2:	0	1	0	1
F:	0	1	1	1

The general form of the firing rule for a 2-input MCP becomes:

$$F = 1 \text{ if } X_1W_1 + X_2W_2 > T$$

The truth table provides values for X_1, X_2, and F that can be inserted into this inequality as follows:

For the right-hand column of the truth table we have X_1 = 1, X_2, = 1 while F = 1 indicates that the threshold must be exceeded, hence:

$$1 * W_1 + 1 * W_2 > T$$

or, more cleanly,

$$W_1 + W_2 > T \qquad (1)$$

Similarly, working from right to left across the columns of the truth tables we get another three inequalities:

$W_1 > T$ (2)
$W_2 > T$ (3)
$0 < T$ (4)

The last of these (4), is obtained from knowing that both X_1 and X_2 are 0 but that the left side of the inequality must be less than T so as to make F = 0. We recall that W_1 and W_2 must be in the range between -1 and +1 (as suggested in the original MCP formulation), and deduce that consequently, T must be in the range between -2 and +2 (to accommodate $W_1 + W_2$ irrespective of the values of W_1 and W_2 within their allowed range).

Now, the desired function, through the above inequalities further constrains these values as follows: (4) restricts T to the range between 0+ and 2 (0+ reads: just over 0); (1) and (2) both require that T be less than 1- (just less than 1), then restricting W_1 and W_2 to the range between T+ and 1. So a suitable set of values for all three unknowns may be obtained by selecting some value for T between 0 and 1-, say 0.5, then selecting W_1 and W_2 in the range 0.5+ to 1, say 0.7.

Several insights may be gleaned from this way of describing the MCP node. First, it is clear that if (2) and (3) are satisfied, this implies that (1) will be automatically satisfied. This leads to a second conclusion. Had the truth table for the right-hand column been: $X_1 = 1$, $X_2 = 1$ and F = 0, with all the other columns being as they are, the function just could not have been achieved at all. In fact, for a 2-input MCP node only 14 of the 16 possible truth tables can be achieved. Things get worse as the number of inputs, n, increases, with the number of achievable functions becoming only a small fraction of the possible ways of constructing a truth table.

The algebra can also throw some light on the need for negative weights. First, let's look at a function that requires negative weights:

X_1:	0	0	1	1
X_2:	0	1	0	1
F:	1	0	1	1

In the usual way, inequalities may be generated from this:

$W_1 + W_2 > T$
$W_1 > T$
$W_2 < T$
$0 > T$

The last of these implies that T is negative and the second last inequality implies that W_2, having to be less than T, must also be negative. (As a matter of interest, T = -0.2, W_2 = -0.3 and W1 = 0.5 would satisfy the inequalities.)

The intuitive notion is that inverting (replacing 0 by 1 and vice versa) the line leading to the negative weight returns the weight to being positive. In terms of the example, this causes the inversion of the second line of the truth table, giving:

X_1:	0	0	1	1
X_2:	1	0	1	0
F:	1	0	1	1

This is now the same as the first example (but written in a different sequence) which requires only positive weights. This confirms the intuition.

Conclusion

Most of what is being discussed in neural computing owes its origins to the work of McCulloch and Pitts and those who have worked with the MCP model. The concept of a variable wieght is appealing as it is easily related to the process of making and breaking connections. Also the idea that the body of the neuron performs a simple sum and threshold operation is easy to grasp.

Bibliography

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