Procesamiento y Análisis de Imágenes

POINT OPERATIONS

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POINT OPERATIONS

Point operations perform a mapping of the pixel values without changing the size, geometry, or local structure of the image.

Each new pixel value $a' = I'(u, v)$ depends exclusively on the previous value $a = I(u, v)$ at the same position and is thus independent from any other pixel value.

Typical use of point operations:

- modifying image brightness or contrast,
- applying arbitrary intensity transformations ("curves"),
- quantizing (or "posterizing") images,
- global thresholding,
- gamma correction,
- color transformations.

CONTRAST AND BRIGHTNESS

$$f_{contr}(I) = I \cdot \alpha \quad and \quad f_{bright}(I) = I + \beta$$

Original Image

Contrasted image

$\alpha \ = 0.5$

$\alpha = 1.5$

Brightened image

$\beta = 50$

$\beta = -50$

INVERTING IMAGE IN THE RANGE $[0, A_{MAX}]$

$$f_{invert}(a) = a_{max} - a$$

THRESHOLDING SEPARATES THE PIXEL VALUES IN TWO CLASSES

$$f_{threshold}(a) = \begin{cases} a_0 & for\ a < a_{th} \\ a_1 & for\ a \geq a_{th} \end{cases}$$

WITH $0 < a_{th} \leq a_{max}$

Original Image

Inverted image

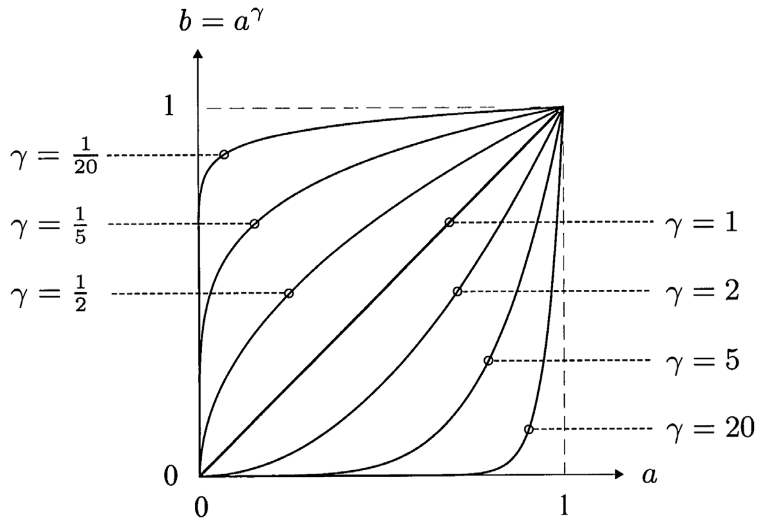
$a_{th} = 127$
$a_0 = 0$
$a_1 = 255$

Thresholded image

Gamma Correction

A pixel value may represent the amount of light falling onto a sensor element in a camera. In the practice, the relationship between a pixel value ant the corresponding physical quantity is usually complex and nonlinear.
For this, it’s necessary to have some kind of “calibrate intensity space” that optimally matches the human perception of the intensity.

Gamma correction is a simple point operation to compensate for the transfer characteristics of different input and output devices and to map them to a unified intensity space.



$$b = f_\gamma(a) = a^\gamma \quad \text{for } a \in \mathbb{R}, \gamma > 0,$$

Gamma Correction

Performing a gamma correction on a pixel value $a \in [0, a_{max}]$ and a gamma value $\gamma > 0$ requires the following the steps:

1. Scale a linearly to $\hat{a} \in [0,1]$
2. Apply the gamma function to $\hat{a}$: $\hat{b} = f_\gamma(\hat{a}) = \hat{a}^\gamma$
3. Scale $\hat{b} \in [0, 1]$ linearly back to $b \in [0,\ a_{max}]$

Formulated in a more compact way

$$b \leftarrow f_{\mathrm{gc}}(a, \gamma) = \left(\frac{a}{a_{\max}}\right)^\gamma \cdot a_{\max}.$$

Point operations involving multiple images

ADD	$ip1 \leftarrow ip1 + ip2$
AVERAGE	$ip1 \leftarrow (ip1 + ip2) \;/\; 2$
DIFFERENCE	$ip1 \leftarrow \lvert ip1 - ip2 \rvert$
DIVIDE	$ip1 \leftarrow ip1 \;/\; ip2$
MAX	$ip1 \leftarrow \max(ip1, ip2)$
MIN	$ip1 \leftarrow \min(ip1, ip2)$
MULTIPLY	$ip1 \leftarrow ip1 \cdot ip2$
SUBTRACT	$ip1 \leftarrow ip1 - ip2$

Alpha blending

Is a simple method for transparently overlaying two images I_{BG} (background image) and I_{FG}(foreground image), transparency es controlled by the value α in the form

$$I'(u,v) = \alpha \cdot I_{BG}(u,v) + (1-\alpha) \cdot I_{FG}(u,v)$$

$$with \leq \alpha \leq 1$$

HISTOGRAM

A histogram(h) of an image(i) represents the frequency distribution function (pdf)of the intensity values that occur in an image.

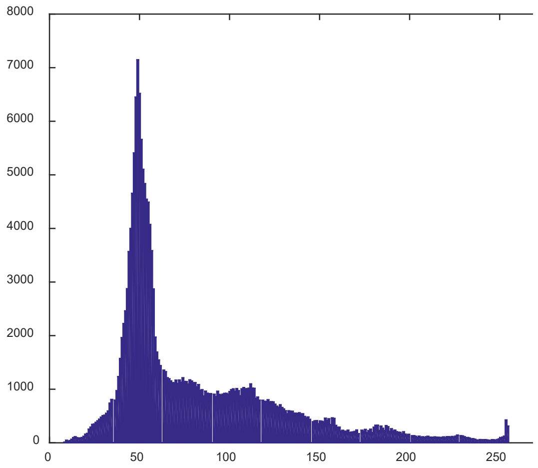
For a grayscale image with intensity values $i(u, v)\epsilon[0, k — l]$ would contain exactly k entries.

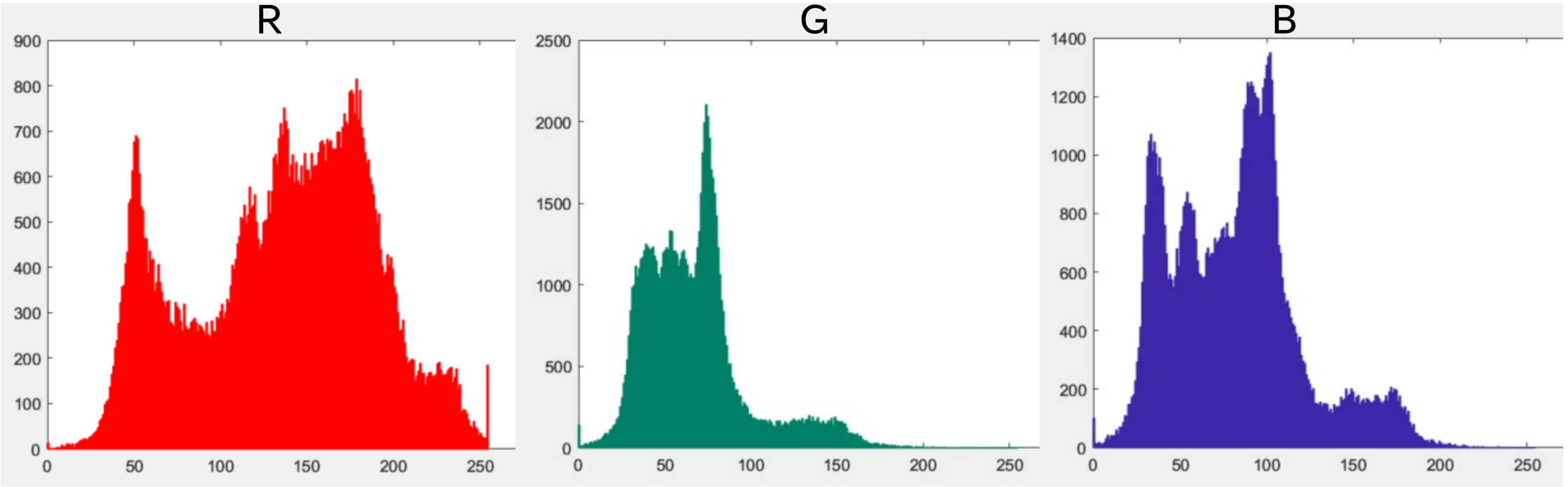
For 8-bit grayscale image, $k = 256$, each individual histogram entry is defined as

$h(i)$ = the number of pixels in an image with the intensity value i

for all $0 < i < k$. more formally stated,

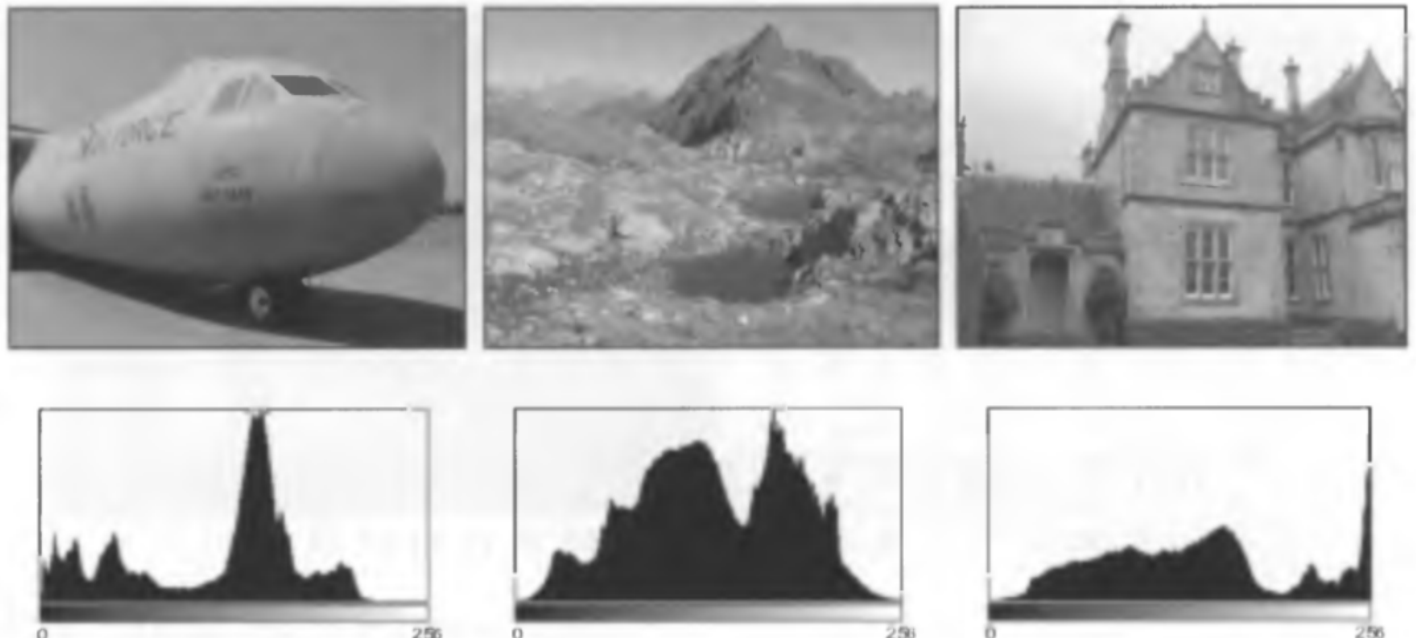
$$h(i) = card\{(u, v)|(u, v) = i\}$$


8000
7000
6000
5000
4000
3000
2000
1000
0
0
50
100
150
200
250


R
G
B
900
800
700
600
500
400
300
200
100
0
50
100
150
200
250
2500
2000
1500
1000
500
1400
1200
1000
800
600
400
200

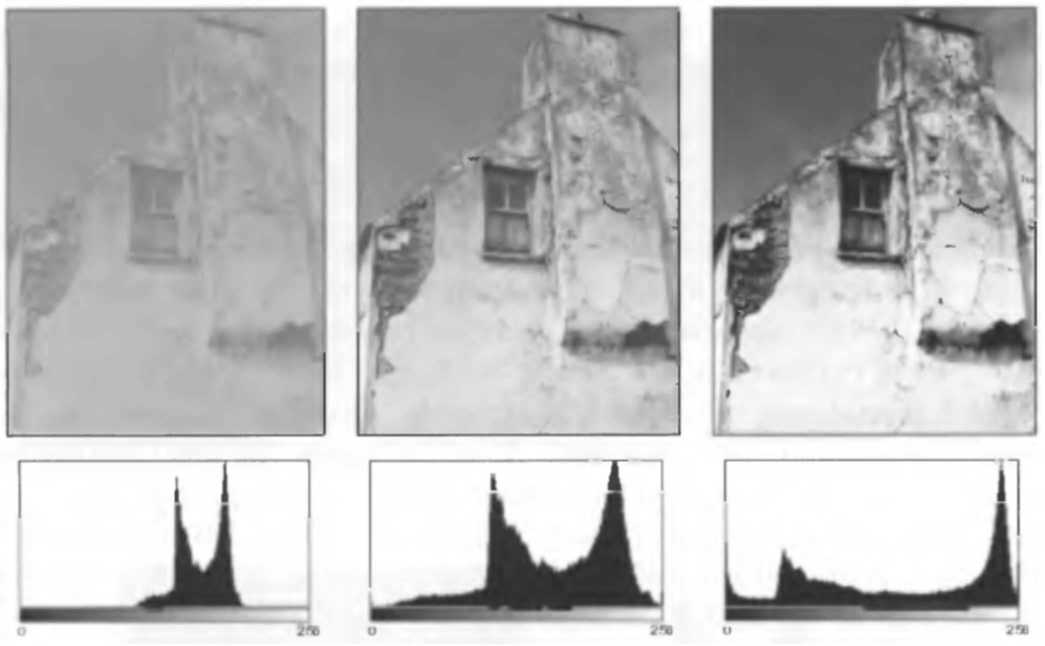
INTERPRETING HISTOGRAM

EXPOSURE IS WHERE A LARGE SPAN OF THE INTENSITY RANGE AT ONE END IS LARGELY UNUSED WHILE THE OTHER END IS CROWDED WITH HIGH-VALUE PEAKS IS REPRESENTATIVE OF AN IMPROPERLY EXPOSED IMAGE.



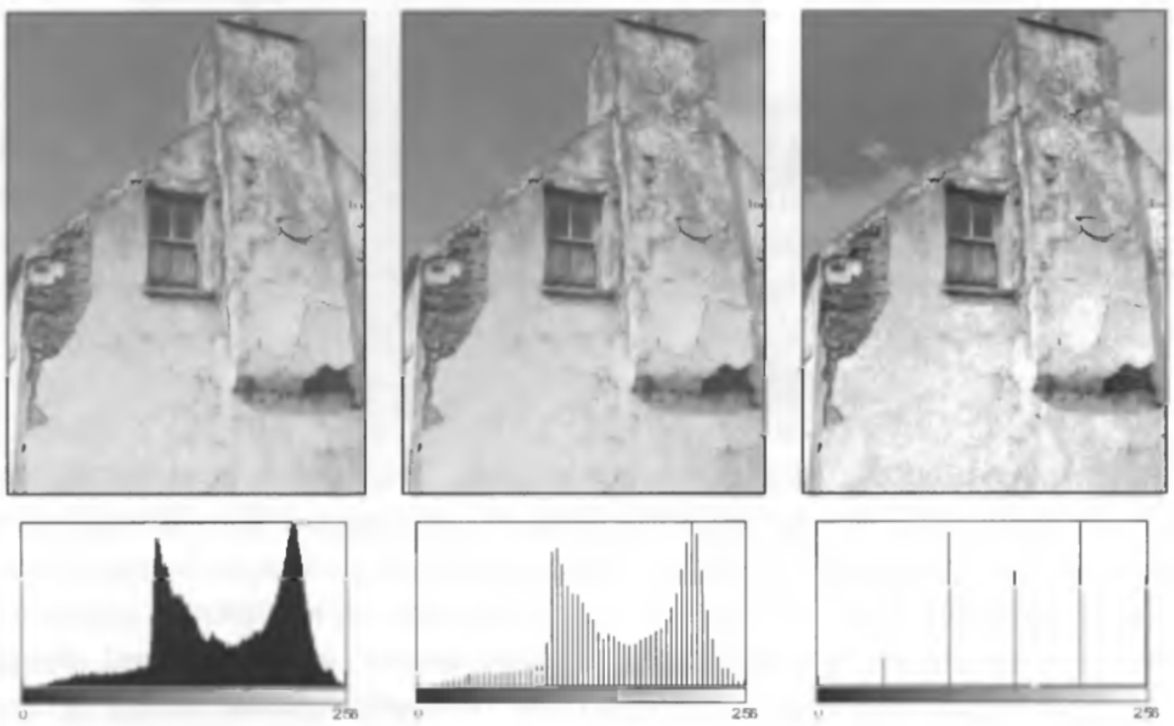
INTERPRETING HISTOGRAM

CONTRAST IS UNDERSTOOD AS A COMBINATION OF THE RANGE OF INTENSITY VALUES EFFECTIVELY USED WITHIN A GIVEN IMAGE AND THE DIFFERENCE BETWEEN THE IMAGE'S MAXIMUM AND MINIMUM PIXEL VALUES.



INTERPRETING HISTOGRAM

DYNAMIC RANGE IS UNDERSTOOD AS THE NUMBER OF DISTINCT PIXEL VALUES IN AN IMAGE.



(a) (b) (c)

CUMULATIVE HISTOGRAM

THE CUMULATIVE HISTOGRAM H(I) IS DEFINED AS

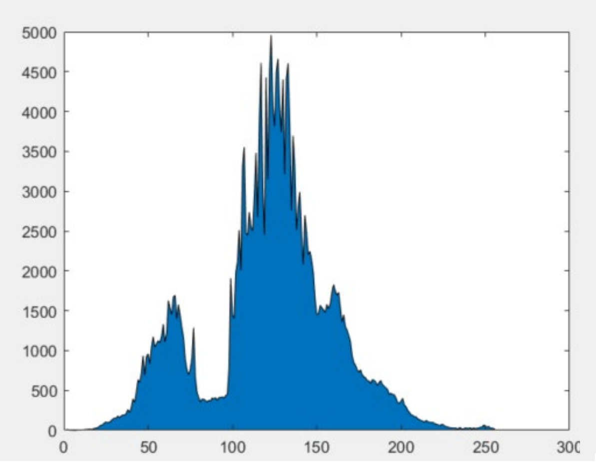
$$H(i) = \sum_{j=0}^{i} h(j) \quad for\ 0 < i < k$$

ALTERNATIVELY, WE CAN DEFINE IT RECURSIVELY

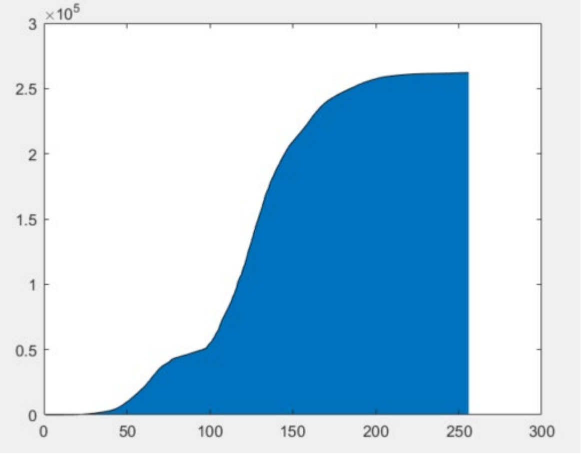
$$H(i) = \begin{cases} h(0) & for\ i = 0 \\ H(i-1) + h(i) & \end{cases}$$

SO THAT, FOR K LEVELS

$$H(K-1) = \sum_{j=0}^{K-1} h(j) = M \cdot N$$

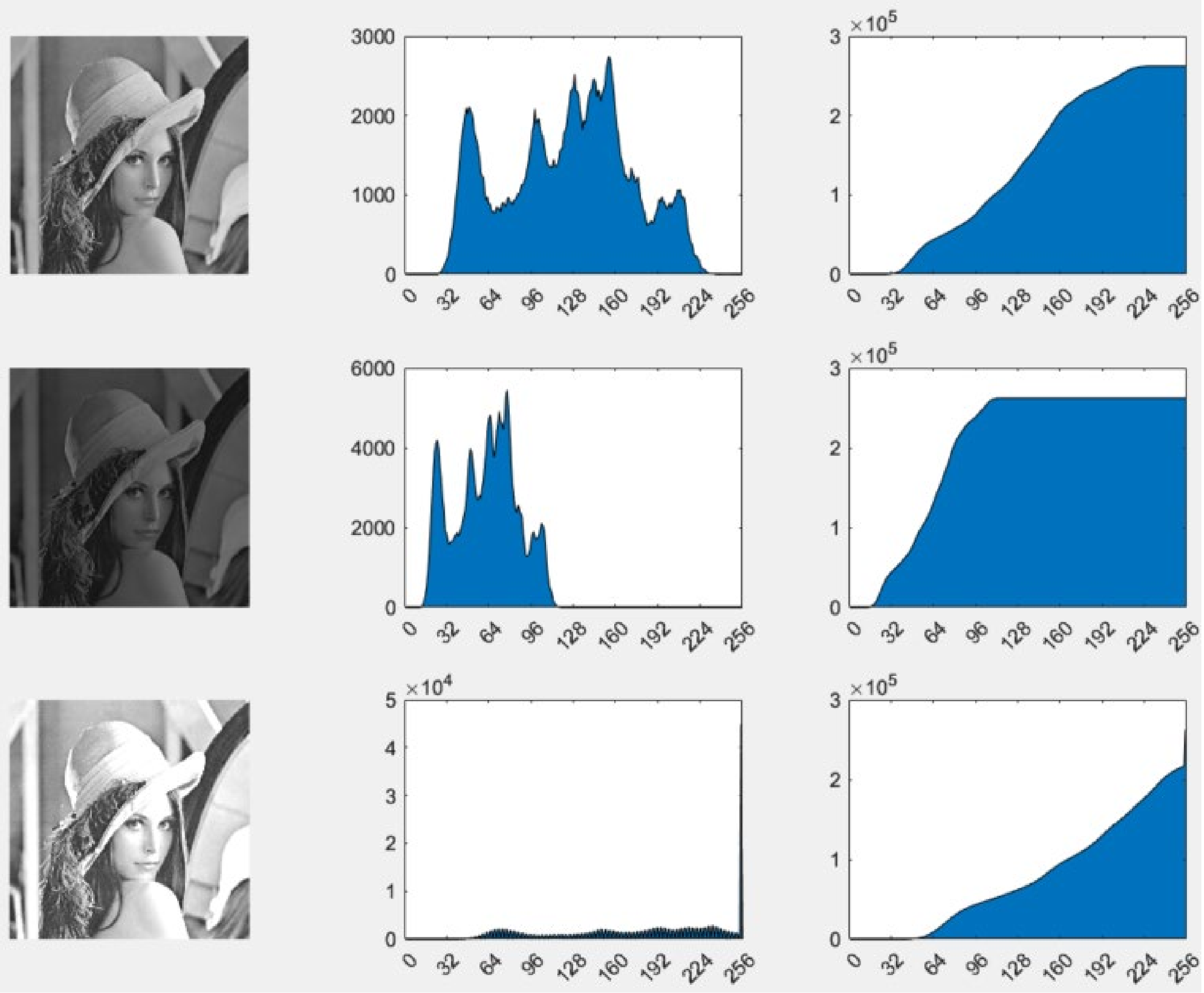


$h(i)$



$\mathrm{H}(i)$

POINT OPERATIONS AND HISTOGRAMS

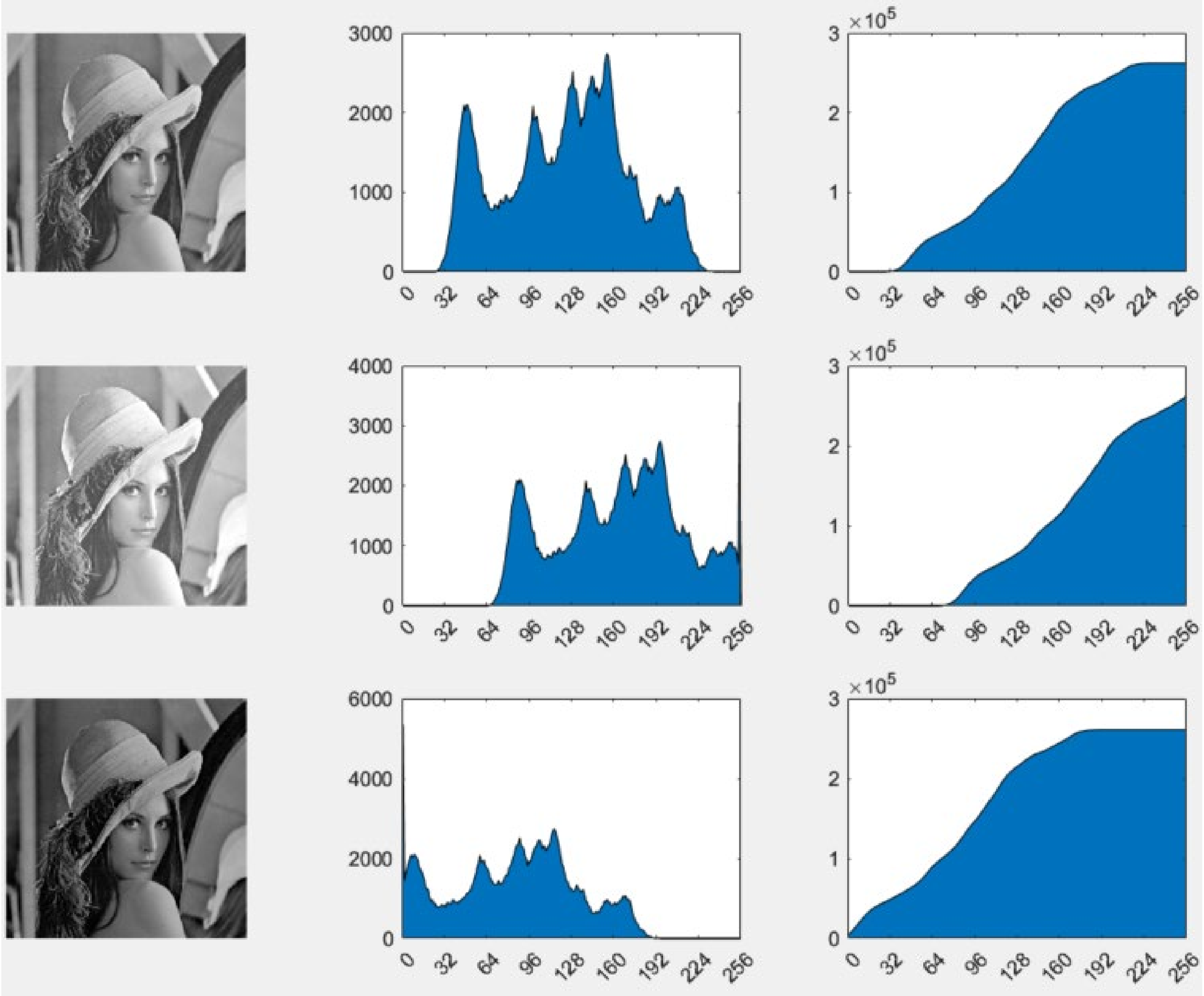


Original Image

$contrast\ \alpha = 0.5$

$contrast\ \alpha = 1.5$

POINT OPERATIONS AND HISTOGRAMS



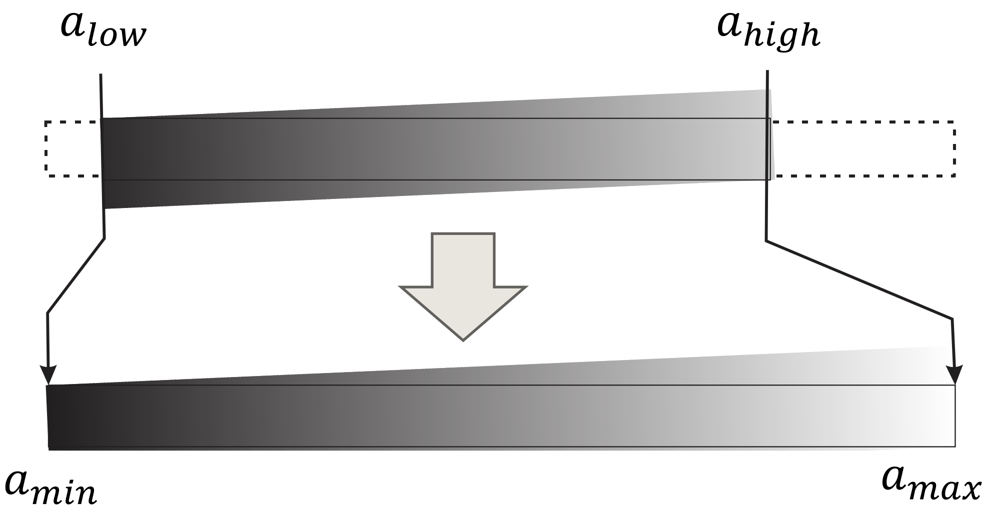
Original Image

Brightness +40

Brightness -40

Automatic contrast adjustment

It´s a point operation whose task is to modify the pixels such that the available range of intensity values is fully covered, mapping the darkness(a_{low}) and brightness (a_{high}) pixels from an image to the maximus (a_{max}) and minimums (a_{min}) intensity values.



The mapping function is defined as

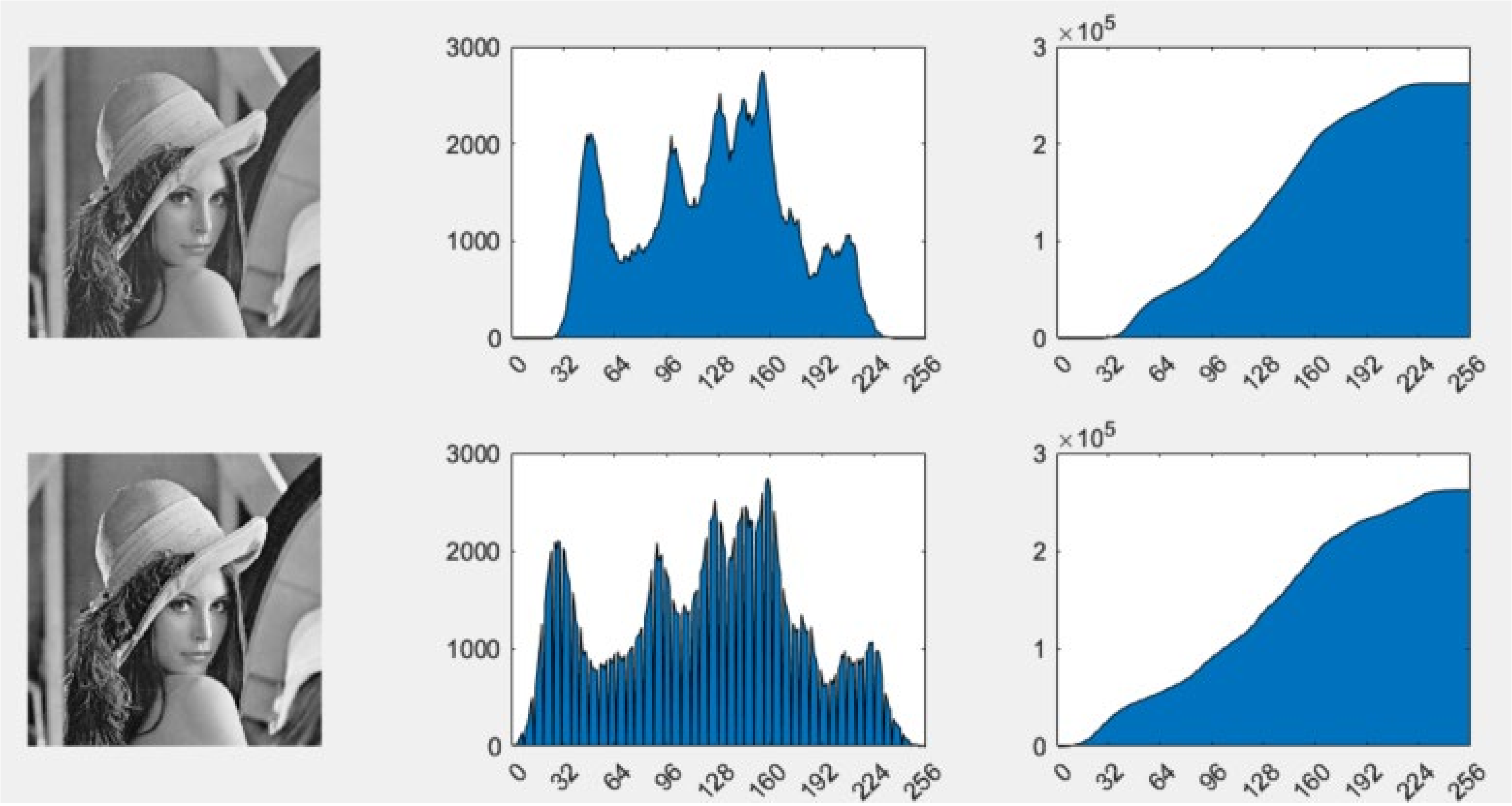
$$f_{ac}(a) = a_{\min} + \left(a - a_{\mathrm{low}}\right) \cdot \frac{a_{\max} - a_{\min}}{a_{\mathrm{high}} - a_{\mathrm{low}}},$$

$$where\ a_{high} \neq a_{low}$$

For an 8-bit image $a_{min} = 0$ and $a_{max} = 255$, simplifying the function to

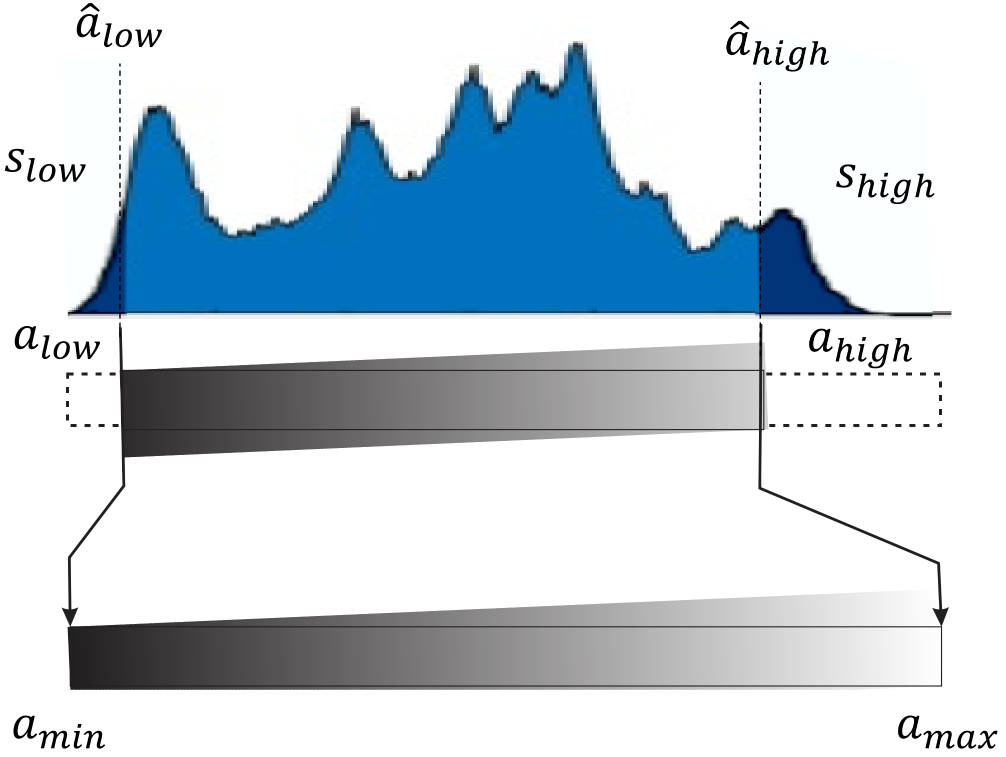
$$f_{ac}(a) = (a - a_{\mathrm{low}}) \cdot \frac{255}{a_{\mathrm{high}} - a_{\mathrm{low}}}.$$

Automatic contrast adjustment



Modified Auto-contrast

The automatic contrast could be strongly influenced by the few pixels in the extremes, which may not be representative the main image content. This can be avoided excluding a percentage of pixels at the upper (s_{high}) y lower (s_{low}) ends of the intensity range.

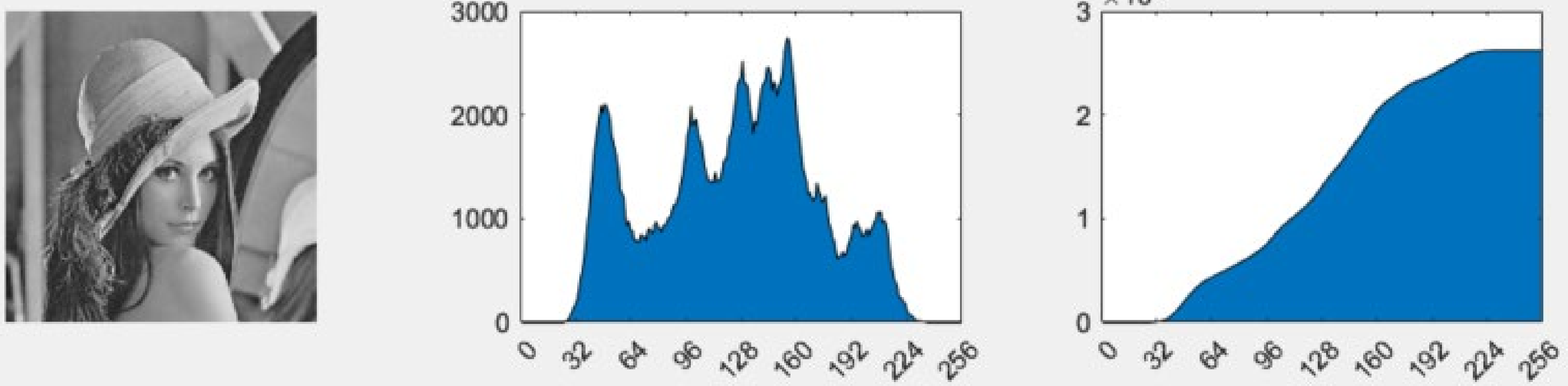


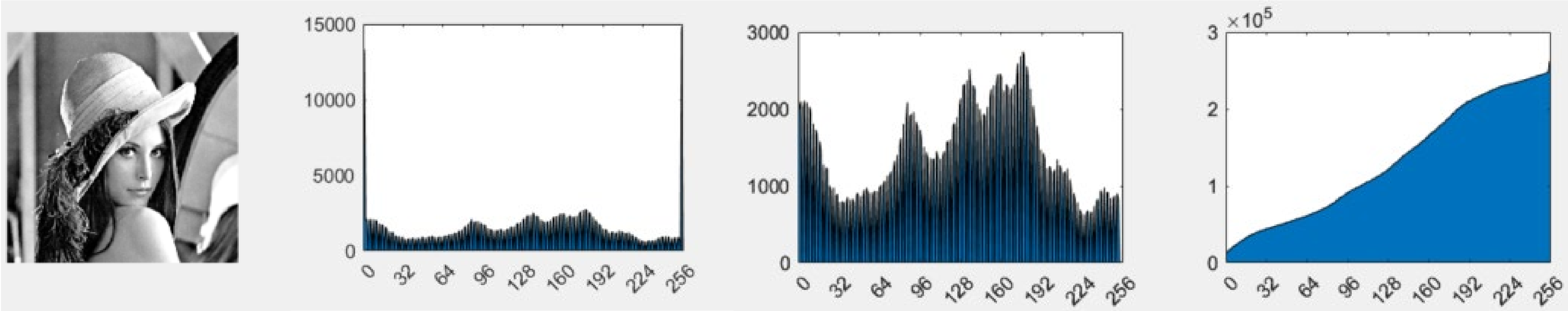
$$\hat{a}_{\text{low}} = \min\{\, i \mid \mathsf{H}(i) \geq M \cdot N \cdot s_{\text{low}} \},$$
$$\hat{a}_{\text{high}} = \max\{\, i \mid \mathsf{H}(i) \leq M \cdot N \cdot (1 - s_{\text{high}}) \},$$

$$f_{\text{mac}}(a) = \begin{cases} a_{\min} & \text{for } a \leq \hat{a}_{\text{low}} \\ a_{\min} + \left(a - \hat{a}_{\text{low}}\right) \cdot \dfrac{a_{\max} - a_{\min}}{\hat{a}_{\text{high}} - \hat{a}_{\text{low}}} & \text{for } \hat{a}_{\text{low}} < a < \hat{a}_{\text{high}} \\ a_{\max} & \text{for } a \geq \hat{a}_{\text{high}}. \end{cases}$$

Modified Auto-contrast

Original image





a) Processed image with s_{low}= $s_{high} = 0.05$

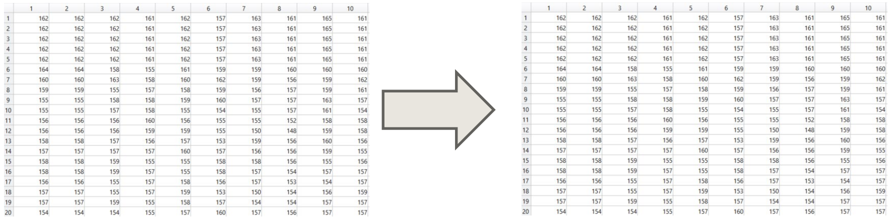
b)Full histogram

c) Histogram cutting a_{min} and a_{max}

d) Cumulative histogram

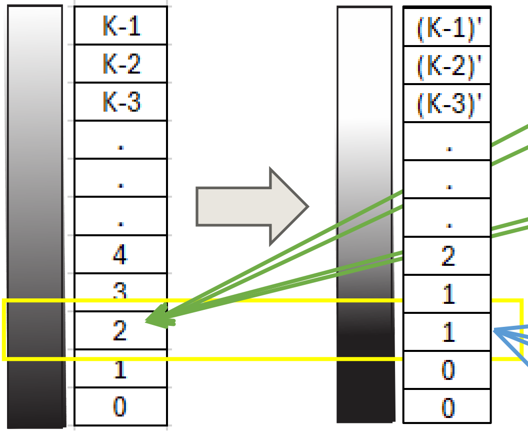
WORKING WITH LOOK UP TABLES

The quantity of operations depends on image size(MxN), but for every pixels with the same intensity values the output is same. For avoid calculate the same operations a lot of times, it´s possible perform the operation using a *look up table*(LUT) from intensity values



$$
\begin{aligned}
&For\ r = 1\ to\ nRows\\
&\quad For\ c = 1\ to\ nCols\\
&\qquad image(r,c) = operation(r,c)\\
&\quad end\\
&end
\end{aligned}
$$

In most of the cases a point operation is more expensive than a replace operation



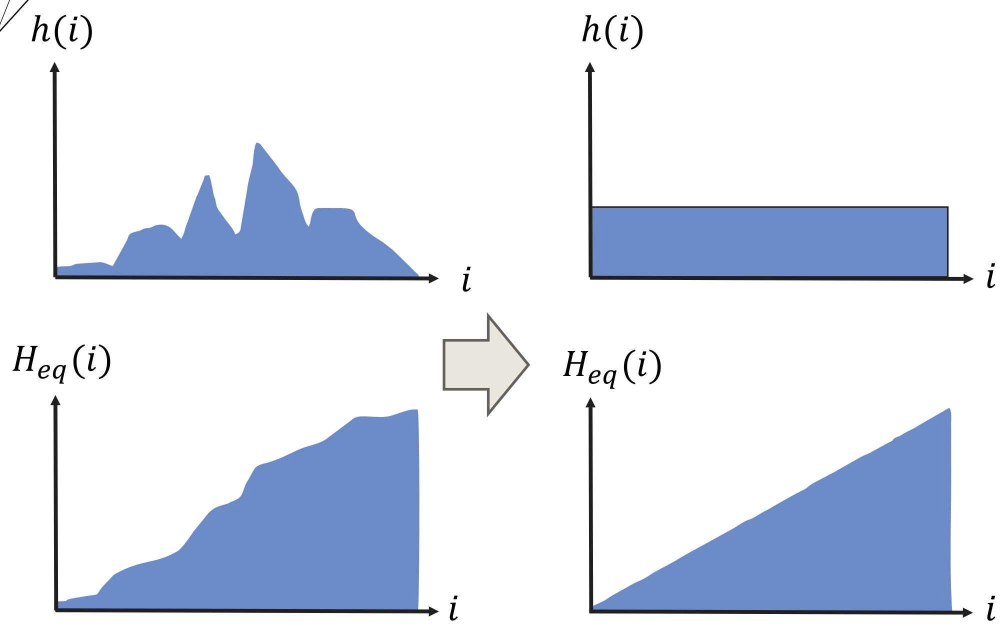
136	160	159	149	92	43
14	163	109	34	2	228
93	49	144	246	93	8
35	1	218	41	149	189
148	2	48	89	130	160
195	27	115	61	250	40
222	227	120	164	231	138
133	213	158	133	2	30
120	2	254	226	71	82
15	248	241	108	255	71

247	127	161	66	73	149
112	167	193	[illegible]	1	248
161	43	131	107	180	185
50	213	66	89	44	76
109	1	116	104	167	153
70	122	229	31	138	216
195	234	46	29	162	173
100	98	124	54	1	4
117	1	193	78	102	195
127	72	88	33	161	26

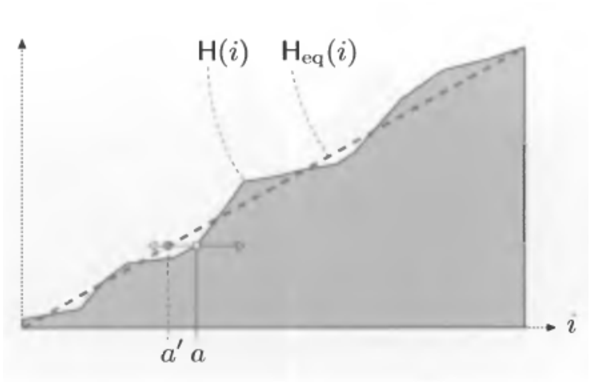
$$
\begin{aligned}
&For\ i = 1\ to\ IntensityLevels\\
&\quad LUT(i) = operation\ for\ intensity\ i\\
&end\\
&For\ r = 1\ to\ nRows\\
&\quad For\ c = 1\ to\ nCols\\
&\qquad newImage(r,c) = LUT(image(r,c))\\
&\quad end\\
&end
\end{aligned}
$$

Histogram equalization

The goal of histogram equalization is to find and apply a point operation such that the histogram of the modified image approximates a uniform distribution.

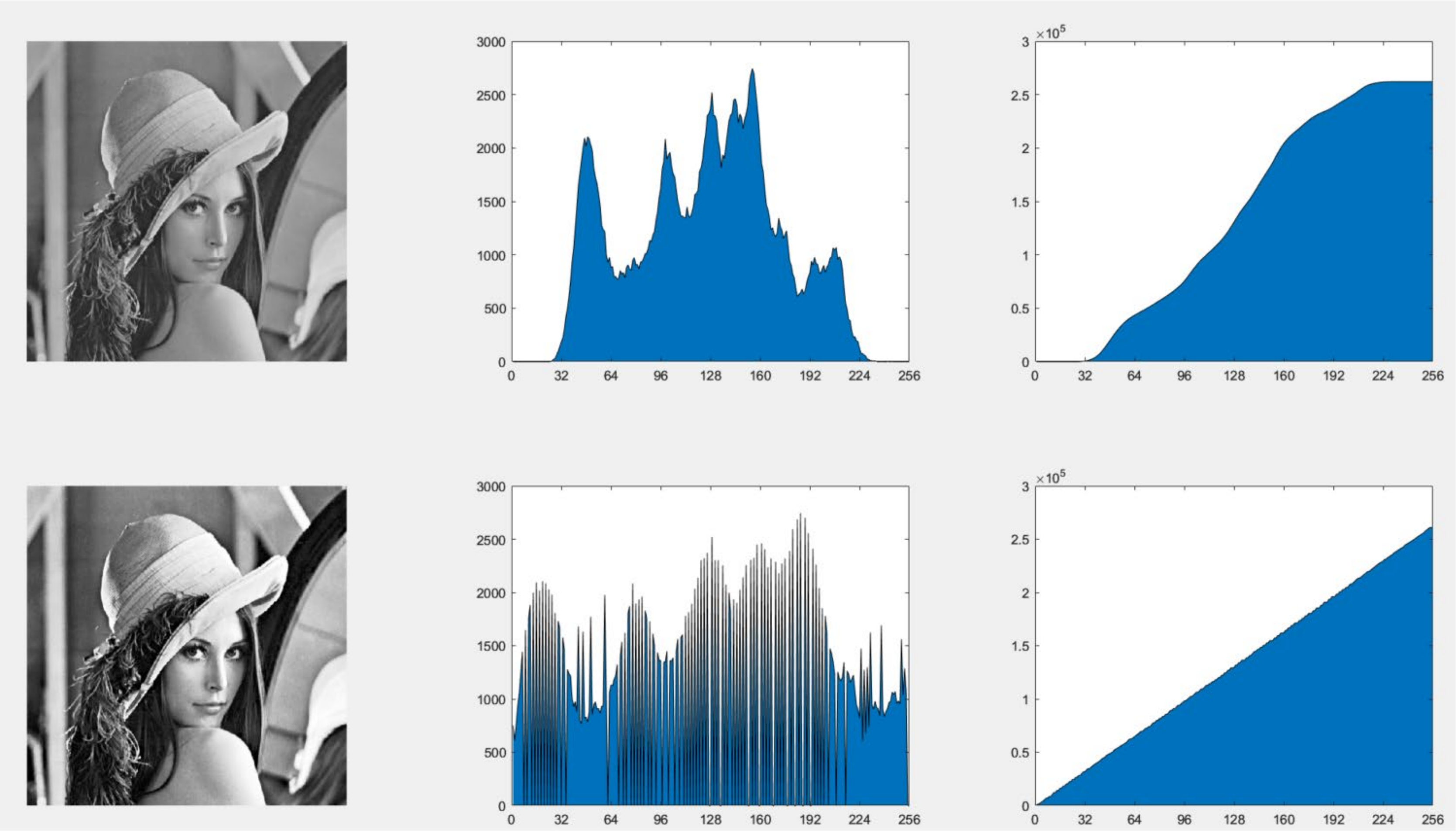


Using the cumulative histogram that has a uniform linear ramp distribution, we looking for a point operation that shift the histogram lines for approximate it.



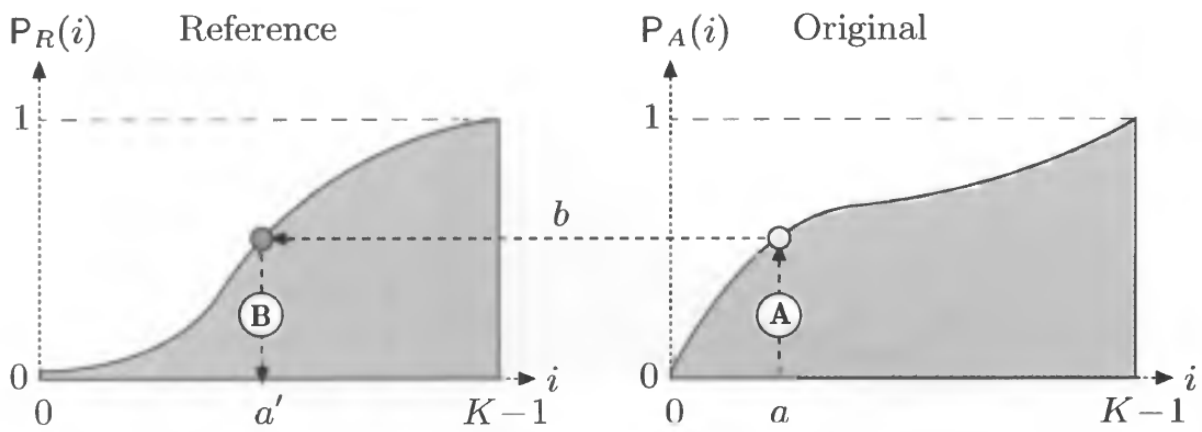
$$f_{eq}(a) = \left\lfloor \mathsf{H}(a) \cdot \frac{K-1}{MN} \right\rfloor ,$$

Histogram equalization



Histogram Specification

The goal of histogram equalization consists of approximate a uniform distribution, for histogram specification histogram the goal is approximate to arbitrary reference distribution. For this, using inverse function from reference distribution the new intensity value a' is computed



Given an image I_A with a distribution P_A looks for matching a reference distribution P_R as closely as possible, converting the original image I_A toa new image $I_{A'}$ by a point operation such that

$$\mathsf{P}_{A'}(i) \approx \mathsf{P}_R(i) \qquad \text{for } 0 \leq i < K.$$

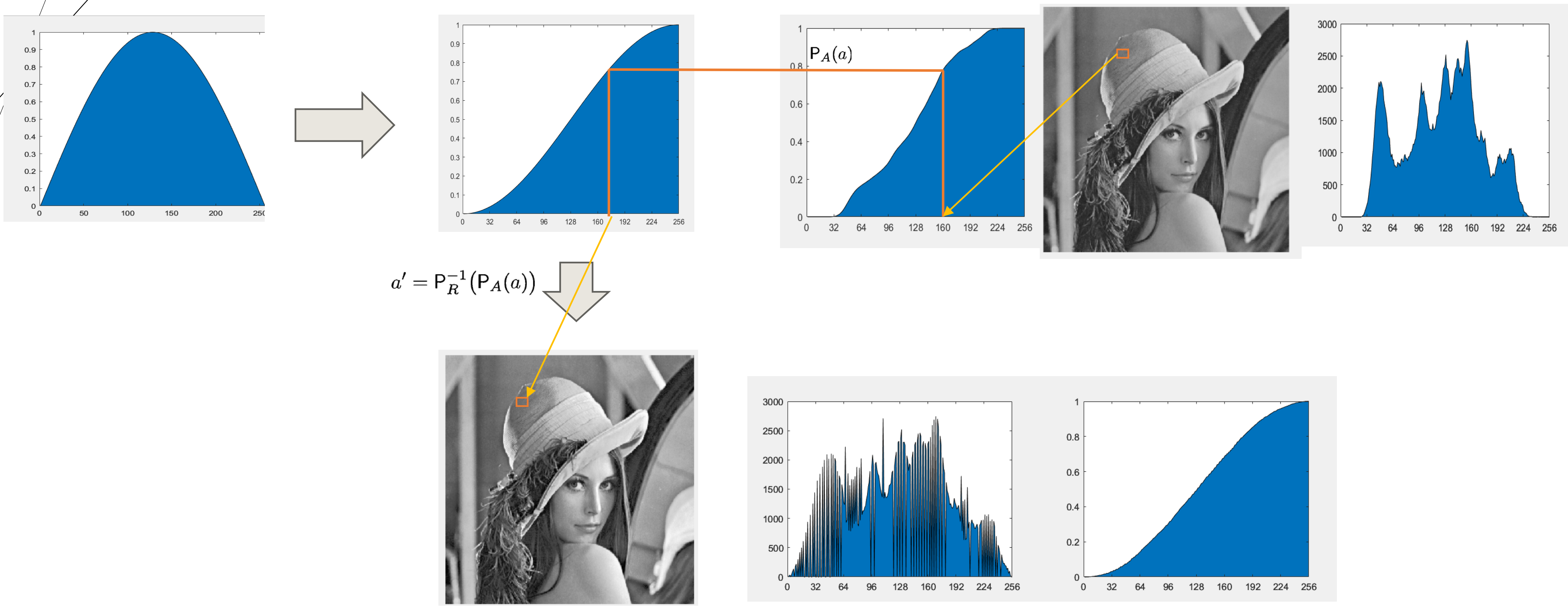
The new pixel value a' is computed by

$$a' = \mathsf{P}_R^{-1}(\mathsf{P}_A(a)),$$

P_R must be invertible

Histogram Specification

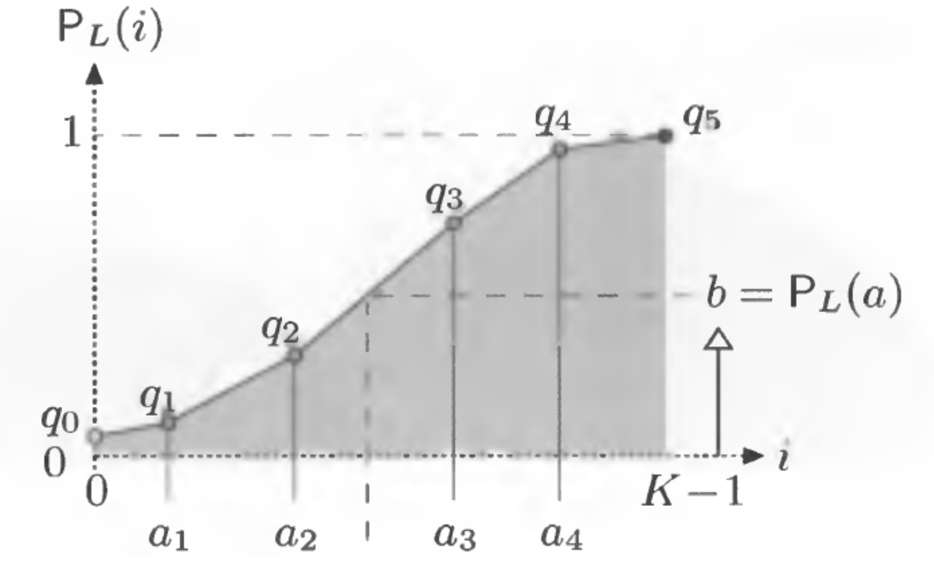
Example: histogram specification to positive sine function



Histogram Specification – Piecewise linear distribution

For a reference distribution specified as a piecewise linear function $P_L(i)$ as a sequence of $N+1$ pairs

$$\mathcal{L} = [\langle a_0, q_0\rangle, \langle a_1, q_1\rangle, \ldots \langle a_k, q_k\rangle, \ldots \langle a_N, q_N\rangle],$$



Where each pair, called controls points, consist of an intensity values a_k and the corresponding function value q_k (with $0 \le a_k < K, a_k < a_{k+1}$ and $0 \le a_k < 1$). The two endpoints $\langle 0, q_0\rangle$ and $\langle k - 1{,}1\rangle$.

The continuous values of $P_L(i)$ are obtained by linear interpolation between the control points as

$$P_L(i) = \begin{cases} q_m + (i - a_m) \cdot \dfrac{(q_{m+1} - q_m)}{(a_{m+1} - a_m)} & \text{for } 0 \le i < K-1 \\ 1 & \text{for } i = K-1, \end{cases}$$

Where $m = \max\{j \in \{0, \ldots N-1\} \mid a_j \le i\}$ is the index of the segment $\langle a_m, q_m\rangle \to \langle a_{m+1}, q_{m+1}\rangle$ which overlaps the position i

Histogram Specification - Piecewise linear distribution

For the inverse distribution function $P_L^{-}(b)$ for $b \in [0,1]$, due to $P_L(i)$ is in general not invertible for values $b < P_L(0)$. We use the “semi-inverse” of the reference distribution as

$$\mathsf{P}_L^{-1}(b) = \begin{cases} 0 & \text{for } 0 \leq b < \mathsf{P}_L(0) \\ a_n + (b - q_n) \cdot \dfrac{(a_{n+1} - a_n)}{(q_{n+1} - q_n)} & \text{for } \mathsf{P}_L(0) \leq b < 1 \\ K - 1 & \text{for } b \geq 1. \end{cases}$$

Where $n = \max\{j \in \{0, \ldots N-1\} \mid q_j \leq b\}$ is the index of the segment $\langle a_n, q_n \rangle \to \langle a_{n+1}, q_{n+1} \rangle$ which overlaps the argument value b

Histogram Specification- Piecewise linear distribution algorithm

PiecewiseLinearHistogram(h_A, $\mathcal{L}_R$)

h_A: histogram of the original image.
$\mathcal{L}_R$: reference distribution function, given as a sequence of $N+1$ control points $\mathcal{L}_R = [\langle a_0, q_0\rangle, \langle a_1, q_1\rangle, \ldots \langle a_N, q_N\rangle]$, with $0 \le a_k < K$ and $0 \le q_k \le 1$.

Let $K \leftarrow \mathrm{Size}(\mathsf{h}_A)$
Let $\mathsf{P}_A \leftarrow \mathrm{Cdf}(\mathsf{h}_A)$ ▷ Cdf is the cumulative distribution in [0,1]
Create a table $f_{\mathrm{hs}}[\,]$ of size K ▷ mapping function f_{hs}
for $a \leftarrow 0 \ldots (K-1)$ **do**
  $b \leftarrow \mathsf{P}_A(a)$
  if $(b \le q_0)$ **then**
    $a' \leftarrow 0$
  else if $(b \ge 1)$ **then**
    $a' \leftarrow K-1$
  else
    $n \leftarrow N-1$
    while $(n \ge 0) \wedge (q_n > b)$ **do** ▷ find line segment in $\mathcal{L}_R$
      $n \leftarrow n-1$
    $a' \leftarrow a_n + (b - q_n) \cdot \dfrac{(a_{n+1} - a_n)}{(q_{n+1} - q_n)}$
  $f_{\mathrm{hs}}[a] \leftarrow a'$
return f_{hs}.

Adjusting to a given histogram (histogram matching)

If we want to adjust one image to the histogram od another image, the reference distribution $P_R(i)$ is not continuous and thus, in general, cannot be inverted, for histogram matching, it’s necessary work with discrete reference distribution.

```
MatchHistograms(h_A, h_R)
    h_A: histogram of the target image
    h_R: reference histogram (of same size as h_A)

    Let K ← Size(h_A)
    Let P_A ← Cdf(h_A)
    Let P_R ← Cdf(h_R)
    Create a table f_hs[ ] of size K
    for a ← 0...(K−1) do
        j ← K−1
        repeat
            f_hs[a] ← j
            j ← j − 1
        while (j ≥ 0) ∧ (P_A(a) ≤ P_R(j))
    return f_hs.
```

The